Kondo effect in metals and nanostructures

Ramón Aguado ICMM-CSIC

Bibliography:

• P. Coleman, “Introduction to Many Body Physics” (chapters 16 and 17), Cambridge University Press, 2015

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The Abrikosov-Suhl resonance (or “Kondo peak”) is a direct consequence of Fermi liquid behaviour in the strong coupling limit. This physics is similar as the one giving the “coherence peaks” in Mott insulators (Leni’s lecture).
Abrikosov-Suhl resonance

Temperature dependence

\[ t \equiv \frac{T}{T_K} \]

- - - t=43.1
- - - t=19.1
- - - t=3.78
- - - t=0.75
- - - t=0.33
- - - t=0.66
Empty $\rightarrow$ Mixed-valence $\rightarrow$ Kondo (scaling)
This lecture

• Kondo effect in mesoscopic systems: quantum dots and nanotubes, non-equilibrium effects, etc.

• New developments: hybrid systems (competition between superconductivity and Kondo), Shiba states, etc.
Quantum Dots
SINGLE ELECTRON TRANSISTOR

Source \[ C_s, R_s \] \[ C_g \] \[ C_d, R_d \] Drain

\[ V_g \] \[ V_{sd} \]
CHARGING ENERGY

Isolated island: Charge is quantized

\[ Q = Ne \]

Energy

\[ E = \frac{Q^2}{2C} = \frac{e^2}{2C}N^2 \]

Island close to a gate: the gate shifts the potential and induces charge

\[ E = \frac{e^2}{2C}(N - \frac{C_gV_g}{e})^2 \]
CHARGING ENERGY

\[ E = \frac{e^2}{2C} \left( N - \frac{C_g V_g}{e} \right)^2 \]

Electrons can enter the island when:

\[ E(N + 1, V_g) = E(N, V_g) = \frac{e^2}{C} \]
\[ \mu_{\text{dot}}(N + 1, V_g) - \mu_{\text{dot}}(N, V_g) = \frac{e^2}{C} + \Delta \varepsilon \]
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\[ \mu_{\text{dot}}(N + 1, V_g) - \mu_{\text{dot}}(N, V_g) = \frac{e^2}{C} + \Delta \varepsilon \]

\[ \mu_L - \mu_R = eV_{SD} \]

Coulomb Diamonds
Small quantum dots are artificial realizations of the Anderson model!!
Small quantum dots are artificial realizations of the Anderson model!!

The question is can we go from Coulomb Blockade to Kondo at low enough temperatures?
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Enhancement of conductance as a consequence of the Kondo resonance: note that current flows at fixed occupancy (spin flips).
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The question is can we go from Coulomb Blockade to Kondo at low enough temperatures?

Enhancement of conductance as a consequence of the Kondo resonance: note that current flows at fixed occupancy (spin flips)
Kondo effect in a single-electron transistor

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A Tunable Kondo Effect in Quantum Dots

Sara M. Cronenwett, Tjerk H. Oosterkamp, Leo P. Kouwenhoven

A tunable Kondo effect has been realized in small quantum dots. A dot can be switched from a Kondo system to a non-Kondo system as the number of electrons on the dot is changed from odd to even. The Kondo temperature can be tuned by means of a gate voltage as a single-particle energy state nears the Fermi energy. Measurements of the temperature and magnetic field dependence of a Coulomb-blockaded dot show good agreement with predictions of both equilibrium and nonequilibrium Kondo effects.
Differential conductance

Kondo resonance

$\Delta I/\Delta V$
Direct measurement of the Kondo resonance!!

Resonance in the middle of the Coulomb Blockade valley: very sharp indicating long lifetime (many-body Kondo) as opposed to single particle
Quantitative Tests of Kondo Theory

\[ T_K = \frac{\sqrt{\Gamma U}}{2} \exp\left(\frac{\pi \varepsilon_0 (\varepsilon_0 + U)}{\Gamma U}\right) \]

G-G, Kastner et al., PRL 81, 5225 (1998)
Kondo resonance splits with magnetic field with twice the Zeeman splitting. Why???
At finite bias voltage there are two Fermi surfaces, each with its own Kondo resonance.
At finite bias voltage there are two Fermi surfaces, each with its own Kondo resonance.
Low-Temperature Transport Through a Quantum Dot: The Anderson Model Out of Equilibrium

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(Received 7 December 1992)

The infinite-\(U\) Anderson model is applied to nonequilibrium transport through a quantum dot containing two spin levels weakly coupled to two leads. At low temperatures, the Kondo peak in the equilibrium density of states is split upon the application of a voltage bias. The split peaks, one at the chemical potential of each lead, are suppressed by nonequilibrium dissipation. In a magnetic field, the Kondo peaks shift away from the chemical potentials by the Zeeman energy, leading to an observable peak in the differential conductance when the nonequilibrium bias equals the Zeeman energy.

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NCA+Keldysh
Disclaimer

• Beyond the superficial description of “splitting by voltage and magnetic field” there are a lot of theoretical developments (not discussed here).

• With relevant questions such as: how voltage=non-equilibrium effects induces dephasing?, how the logarithmic divergences are cut-off? etc.

- Symmetric unitary limit of $2e^2/h$ conductance has been achieved in a SET
- Conductance saturates at low $T$

Tunneling into a Single Magnetic Atom: Spectroscopic Evidence of the Kondo Resonance

V. Madhavan, W. Chen, T. Jamneala, M. F. Crommie, N. S. Wingreen

The Kondo effect arises from the quantum mechanical interplay between the electrons of a host metal and a magnetic impurity and is predicted to result in local charge and spin variations around the magnetic impurity. A cryogenic scanning tunneling microscope was used to spatially resolve the electronic properties of individual magnetic atoms displaying the Kondo effect. Spectroscopic measurements performed on individual cobalt atoms on the surface of gold show an energetically narrow feature that is identified as the Kondo resonance—the predicted response of a Kondo impurity. Unexpected structure in the Kondo resonance is shown to arise from quantum mechanical interference between the d orbital and conduction electron channels for an electron tunneling into a magnetic atom in a metallic host.

www.sciencemag.org • SCIENCE • VOL. 280 • 24 APRIL 1998

SAME YEAR OF THE QUANTUM DOT EXPERIMENTS!!
FANO LINESHAPES: interference of two paths

\[ \tilde{E}_0^2 + \tilde{E}_0^2 + 1 = \tilde{E}_0 \]

\[ \tilde{E}_0 \approx \frac{(\tilde{E} + q)^2}{\tilde{E}_0^2 + 1} \]

\[ \tilde{E} = \frac{E - E_0}{\Gamma} \]
Double dots: two-impurity

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Splitting of Kondo peak as coupling between dots increases.

NCA+Keldysh

• Non-monotonic conductance: development of coherence
• Similar to heavy fermion physics in mesoscopics!!

When the right dot contains an even number of electrons the left dot shows Kondo effect.

Tunable Nonlocal Spin Control in a Coupled–Quantum Dot System

N. J. Craig,1 J. M. Taylor,1 E. A. Lester,1 C. M. Marcus,1* M. P. Hanson,2 A. C. Gossard2

The effective interaction between magnetic impurities in metals that can lead to various magnetic ground states often competes with a tendency for electrons near impurities to screen the local moment (known as the Kondo effect). The simplest system exhibiting the richness of this competition, the two-impurity Kondo system, was realized experimentally in the form of two quantum dots coupled through an open conducting region. We demonstrate nonlocal spin control by suppressing and splitting Kondo resonances in one quantum dot by changing the electron number and coupling of the other dot. The results suggest an approach to nonlocal spin control that may be relevant to quantum information processing.
When the right dot contains an odd number of electrons Kondo effect is suppressed.

Tunable Nonlocal Spin Control in a Coupled–Quantum Dot System

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Tunable coupling
• Two magnetic atoms, one attached to the tip of a STM and one adsorbed on a metal surface, each constituting a Kondo system.
• Cobalt dimers clamped between an STM tip and a gold surface.
• Control of the tip-sample distance with sub-picometer resolution allows to tune the interaction between the two cobalt atoms with unprecedented precision.

Quantum point contacts
Quantum point contacts
Clear shoulder as one increases temperature!!
The differential conductance at finite bias shows clear zero-bias anomalies at low temperatures. Kondo physics in an open system?

Kondo features

The anomaly at “high temperatures” is originated from a magnetic moment (similar to Coulomb blockade, hence the conductance reduction) which is quenched at low temp. by Kondo.
0.7 anomaly

- Theory predicts that such moments can form near pinch-off (Rejec and Meir, Nature 442, 900-903, 2006) but not consensus.
- Importantly, theory also predict that long contacts can host two impurities.
Two-impurity Kondo physics in length-tunable QPCs recently demonstrated

**Odd and even Kondo effects from emergent localization in quantum point contacts**


Odd and even Kondo effects from emergent localization in quantum point contacts

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Splitting and Non-monotonic conductance: development of coherence!!
Nanotubes
Image: Jesper Nygard, Niels Bohr, Copenhagen
Quantum dot behaviour

\[ G_{\text{peak}} \sim \frac{1}{T} \]

- 0.3 K
- 1.7 K
- 4.2 K

- 300 K
- 205 K
- 0.34 K
Quantum dot behaviour

Transport spectroscopy of a tube quantum dot

$B = 0 \text{T}$

$T = 100 \text{ mK}$
Short nanotubes, quantized levels

Valley degeneracy?
Short nanotubes, quantized levels

Yes: Each level is fourfold degenerate (spin+valley).
Visible in Coulomb Blockade!!!
At low temperatures both spin and valley quantum fluctuations

\[ |+, \uparrow \rangle \rightarrow |+, \downarrow \rangle \]
At low temperatures both spin and valley quantum fluctuations.

\[ |-, \uparrow \rangle \xrightarrow{\text{process}} |+, \uparrow \rangle \]
SU(4) Kondo effect?

Experiments from Delft (Pablo Jarillo-Herrero et al)
SU(4) Kondo effect?

Difficult to tell at this level linear conductance is the same for SU(2) and SU(4)!!
Remember Langreth's sum rule for the phase shift

\[ G = N \frac{e^2}{h} \sin^2 \delta = N \frac{e^2}{h} \sin^2 (\pi \langle n \rangle) \]

**SU(N=2)**

\[ \langle n \rangle = \frac{1}{2} \]

\[ G = 2 \frac{e^2}{h} \sin^2 \left( \frac{\pi}{2} \right) = \frac{2e^2}{h} \]

**SU(N=4)**

\[ \langle n \rangle = \frac{1}{4} \]

\[ G = 4 \frac{e^2}{h} \sin^2 \left( \frac{\pi}{4} \right) = \frac{2e^2}{h} \]
SU(4) Kondo effect?

Moreover, it is not obvious whether the valley is conserved during tunneling (e.g. bad contacts). Valley mixing can lead to SU(2) spin Kondo.
SCALING EQUATIONS HAVE TWO FIXED POINTS: SU(2) AND SU(4)

How to detect SU(4) Kondo effect?

Semiclassically, the two orbital states correspond to electrons circling the tube either clockwise or anticlockwise. Such motion has a corresponding orbital moment

$$\mu_{orb} = \frac{eD\nu F}{4}$$

Determination of electron orbital magnetic moments in carbon nanotubes

E. D. Minot*, Yuval Yaish*, Vera Sazonova & Paul L. McEuen

Laboratory of Atomic and Solid-State Physics, Cornell University, Ithaca, New York 14853, USA

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How to detect SU(4) Kondo effect?

An axial magnetic field will couple to this orbital moment!

\[ \Delta E = \pm \mu_{orb} B || = \frac{e D v_F B ||}{4} \]

Determination of electron orbital magnetic moments in carbon nanotubes

E. D. Minot*, Yuval Yaish*, Vera Sazonova & Paul L. McEuen

Laboratory of Atomic and Solid-State Physics, Cornell University, Ithaca, New York 14853, USA
Spin Kondo splits as $\Delta_{\text{Zeeman}}$.

Orbital Kondo splits as $2\Delta_{\text{orbital}}$. 

$B_{||} \neq 0$
Pablo Jarillo-Herrero, Jing Kong, Herre S.J. van der Zant, Cees Dekker, Leo P. Kouwenhoven, Silvano De Franceschi, Nature 434, 484-488 (2005)
Also **pure orbital** Kondo effect is possible at high magnetic fields (spins are fully polarized).
Recently, this physics has been observed in Silicon transistors with single dopants (two-valley degeneracy of Silicon gives exotic Kondo).
Kondo + superconductivity
Kondo + superconductivity

What happens when a magnetic impurity tries to live inside a superconductor?

It turns out that even a classical spin may lower the energy of the system and create bound states inside the superconducting gap.
Yu-Shiba-Rusinov states (YSR)

**Yu (1965)**

BOUND STATE IN SUPERCONDUCTORS WITH PARAMAGNETIC IMPURITIES

Yu Luh (Yu Lu)

Received July 10, 1963
Acta Physica Sinica 21, 75-91 (January, 1965)

A generalized wave function exists around the gap. An analysis of wave function.

Furthermore, the impurity and the observable quantities should be considered.

**Shiba (1968)**

Progress of Theoretical Physics, Vol. 40, No. 3, September 1968

Classical Spins in Superconductors

Hiroyuki SHIBA*3

Department

SUPERCONDUCTIVITY NEAR A PARAMAGNETIC IMPURITY

A. I. Rusinov
P. N. Lebedev Physics Institute, USSR Academy of Sciences
Submitted 28 November 1968
ZhETF Pis. Red. 9, No. 2, 146-149 (20 January 1969)

It is shown in [1] that introduction of a small amount (<1%) of paramagnetic impurities in a superconductor exerts a strong influence on its properties. In particular, the energy gap of the spectrum of such an alloy no longer coincides with the magnitude of the ordering

\[ \omega = \pm \omega_B = \pm \Delta_0 \frac{1 - ((J/2) S \pi \rho)^2}{1 + ((J/2) S \pi \rho)^2} \]

In [1] was carried out in the Born approximation with respect to the scattering of the electrons by the magnetic impurity

**Rusinov (1969)**

...
\[ \omega = \pm \omega_B \equiv \pm \Delta^0 \frac{1 - \left( \frac{J}{2} S \pi \rho \right)^2}{1 + \left( \frac{J}{2} S \pi \rho \right)^2} \]
A spinful impurity in a superconducting host strongly affects superconductivity (spin scattering has pair breaking character).

Well known since the 60's (Yu-Shiba-Rusinov): a classical spin may lower the energy of the system and create sub-gap bound states.

The ground state parity changes as sub-gap levels cross zero energy.

$$\Delta$$

$$0$$

$$-\Delta$$

$$\alpha = \pi \rho_0 JS$$

$$\epsilon_0 = \pm \Delta \frac{1 - \alpha^2}{1 + \alpha^2}$$
For large exchange coupling the spin of the ground states changes. This happens when the energy of the excitation crosses zero energy.

For a review see “Impurity-induced states in conventional and unconventional superconductors”, Balatsky et al, Reviews of Modern Physics, 78, 373 (2006)
THEORETICAL DESCRIPTION:
THE SUPERCONDUCTING ANDERSON MODEL

\[ \mathcal{H}_D = \sum_\sigma \epsilon_{d\sigma} d^\dagger_\sigma d_\sigma + U n_{d\uparrow} n_{d\downarrow} \]

\[ \mathcal{H}_T = \sum_{\alpha,k,\sigma} \left( V_{\alpha k\sigma} c^\dagger_{\alpha k\sigma} d_\sigma + h.c. \right) \]

\[ \mathcal{H}_S = \sum_{k,\sigma} (\epsilon_{S k} - \mu_S) c^\dagger_{S k\sigma} c_{S k\sigma} + \sum_k \left( \Delta c^\dagger_{S k\uparrow} c^\dagger_{S k\downarrow} + h.c. \right) \]
THEORETICAL DESCRIPTION: 
THE SUPERCONDUCTING ANDERSON MODEL

All relevant experimental quantities can be calculated from Green’s functions written in the basis of Nambu spinors

\[ \Psi_\sigma = \begin{pmatrix} d_\sigma \\ d_\sigma^\dagger \end{pmatrix} \]

\[ \hat{G}_\sigma^r(t, t') \equiv -i\theta(t - t') \langle [\Psi_\sigma(t), \Psi_\sigma^\dagger(t')]_+ \rangle \quad \text{etc.,...} \]

\[ \hat{G}_\sigma^r(\omega)^{-1} = \hat{G}_\sigma^r(0)(\omega)^{-1} - \hat{\Sigma}_\sigma(\omega) \]
THEORETICAL DESCRIPTION: THE SUPERCONDUCTING ANDERSON MODEL

Self-energy coming from the coupling to the superconducting reservoir

\[ \hat{\Sigma}(\omega) = \Gamma_S \begin{pmatrix} \frac{\omega}{\sqrt{\Delta^2 - \omega^2}} & \frac{\Delta}{\sqrt{\Delta^2 - \omega^2}} \\ \frac{\Delta}{\sqrt{\Delta^2 - \omega^2}} & \frac{\omega}{\sqrt{\Delta^2 - \omega^2}} \end{pmatrix} \]

\[ \Delta \rightarrow \infty \quad \text{Large gap limit: the self-energy is static} \]

\[ \hat{\Sigma}(\omega) = \Gamma_S \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]
\[ \Delta \rightarrow \infty \quad \text{Large gap limit: the self-energy is static} \]

\[ \hat{\Sigma}(\omega) = \Gamma_S \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

The superconducting reservoir can be “integrated out”. We can write an impurity Hamiltonian with pairing interaction.

\[ \mathcal{H}_D = \sum_\sigma \epsilon_0 d_\sigma^{\dagger} d_\sigma - \Gamma_S (d_\uparrow^{\dagger} d_\uparrow^{\dagger} + d_\uparrow d_\uparrow) + U n_\uparrow n_\downarrow \]
Large gap limit: the self-energy is static

\[ \Delta \rightarrow \infty \]

\[ \mathcal{H}_D = \sum_\sigma \epsilon_0 d^\dagger_\sigma d_\sigma - \Gamma_S (d^\dagger_\uparrow d^\dagger_\uparrow + d_\uparrow d_\uparrow) + U n_\uparrow n_\downarrow \]

This problem can be exactly diagonalized:
four Bogoliubov De Gennes eigenstates

Doublet sector

\[ | \uparrow \rangle \]
\[ | \downarrow \rangle \]
\[ \Delta \rightarrow \infty \] Large gap limit: the self-energy is static

\[ \mathcal{H}_D = \sum_{\sigma} \epsilon_0 d_{\sigma}^\dagger d_{\sigma} - \Gamma_S (d_{\uparrow}^\dagger d_{\uparrow}^\dagger + d_{\uparrow} d_{\uparrow}) + U n_{\uparrow} n_{\downarrow} \]

This problem can be exactly diagonalized:

four Bogoliubov De Gennes eigenstates

BCS Singlet sector

\[ |-\rangle = u |\uparrow\downarrow\rangle + v |0\rangle \]
\[ |+\rangle = -v |\uparrow\downarrow\rangle + u |0\rangle \]

\[ \xi \equiv \epsilon_0 + \frac{U}{2} \]
\[ E_{\pm} = \frac{U}{2} + \xi \pm \sqrt{\xi^2 + \frac{\Gamma_S^2}{S}} \]
Anderson model with normal reservoir

\[ \frac{-\varepsilon_d}{U} \]

- Magnetic
  - \[ \langle n_{d\uparrow} \rangle \neq \langle n_{d\downarrow} \rangle \]
- Non-magnetic
  - \[ \langle n_{d\uparrow} \rangle = \langle n_{d\downarrow} \rangle \]
Anderson model with BCS reservoir

Hubbard and BCS pairing compete in fixing fermion parity of the ground state. Strong U wants a doublet singlet state whereas strong coupling to the superconductor wants BCS singlet ground state.
- Physical Picture: odd-occupied QD acts as a *spinful* quantum impurity coupled to the SC.

- The ground state fermion parity changes from even to odd as sub-gap levels cross zero energy (quantum phase transition). Accordingly, the spin state changes from singlet to doublet.

- These levels are the quantum version of Yu-Shiba-Rusinov (YSR) states (subgap states in a superconductor created by pair-breaking from a classical spin impurity).
For the Anderson model, these sub-gap states are poles of the Green’s function

\[ \hat{G}_\sigma^r(t, t') \equiv -i \theta(t - t') \langle [\Psi_\sigma(t), \Psi_\sigma^\dagger(t')] \rangle_+ \]

\[ \hat{G}_\sigma^r(\omega)^{-1} = \hat{G}_\sigma^r(0)(\omega)^{-1} - \Sigma_\sigma(\omega) \]

Note that in the presence of finite Hubbard U this Anderson impurity problem is highly non-trivial: competition between superconductivity, quantum fluctuations, Coulomb Blockade and Kondo physics.
Starting in the doublet phase, Kondo singlets will compete versus BCS singlets as quantum fluctuations become important!!!
Starting in the doublet phase, Kondo singlets will compete versus BCS singlets as quantum fluctuations become important!!

\[ \Delta \ll T_K \]

Quantum phase transition: ground state changes from doublet to singlet when

\[ \Delta \sim T_K \]
Phase diagram

T. Meng, S. Florens and P. Simon PRB 79, 224521 (2009), etc...
Shiba versus Kondo, doublets versus singlets, and all this...still an open problem.

\[
S = 0
\]

\[
S = \frac{1}{2}
\]

Precise boundaries?
Shiba versus Kondo, doublets versus singlets, and all this...still an open problem

\[ S = 0 \]

As the gap closes, Kondo effect becomes dominant
Shiba versus Kondo, doublets versus singlets, and all this...still an open problem

As the gap closes, Kondo effect becomes dominant

\[ S = 0 \]

\[ S = \frac{1}{2} \]
Shiba versus Kondo, doublets versus singlets, and all this...still an open problem

As the gap closes, Kondo effect becomes dominant

$\Gamma$

BCS

Spin fluctuations, Kondo

Charge fluctuations

$S = 0$

$S = \frac{1}{2}$

$S = 1$

$S = 2$

BCS

Precise boundaries?

$V_g$
Shiba versus Kondo, doublets versus singlets, and all this...

Hybrid systems composed of superconductors and some kind of quantum dot (nanotubes, nanowires, etc) are ideal testbed for all these physical effects!!!
Extremely clean Coulomb diamonds coexisting with induced superconducting gap (carbon nanotube with Niobium electrodes)

Pronounce subgap structure in every fourth diamond

Origin of subgap structure in odd diamonds unclear at that time

Answer: Structure is a direct measurement of YRS states inside the gap (magnetic moment regime, doublet phase)

**S-dot-N** (Local density of states with YSR) (Kondo disregarded!)

Similar physics in STM experiments
Similar physics in STM experiments
Starting from the doublet phase with YRS states inside the gap it is possible to increase the role of Kondo fluctuations by closing the gap with a magnetic field.
Starting from the doublet phase with YRS states inside the gap it is possible to increase the role of Kondo fluctuations by closing the gap with a magnetic field.

Coexistence of Kondo and induced BCS pairing!!
This competition of different physical regimes and effects is very relevant for current experimental efforts towards detecting Majorana fermions in hybrid semiconductor/superconductor systems (Next Talk).