Emergence of quantum phases in novel materials

SUPERCONDUCTIVITY I

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BIBLIOGRAPHY

Collection of reviews

- Conventional SPC "Superconductivity" Edited by Parks. 1968.
- Conventional and unconventional SPC "Superconductivity" 2008 (Fe SPC not included)

"Introduction to Many-body physics" Piers Coleman

"Introduction to superconductivity" Tinkham

SPC history:

"Superconductivity: a very short introduction" S. Blundell

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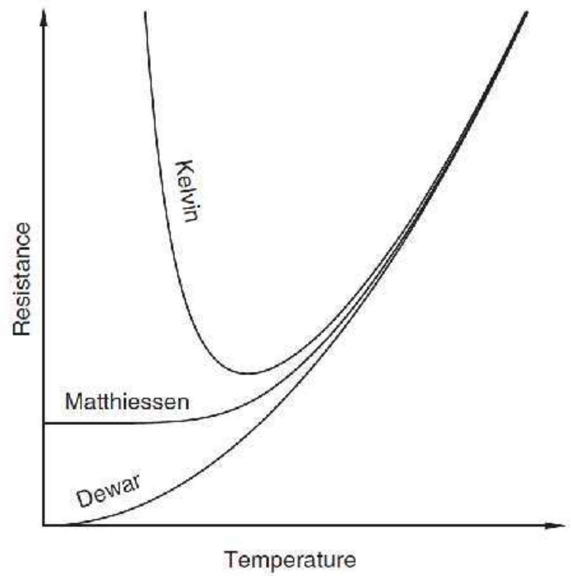
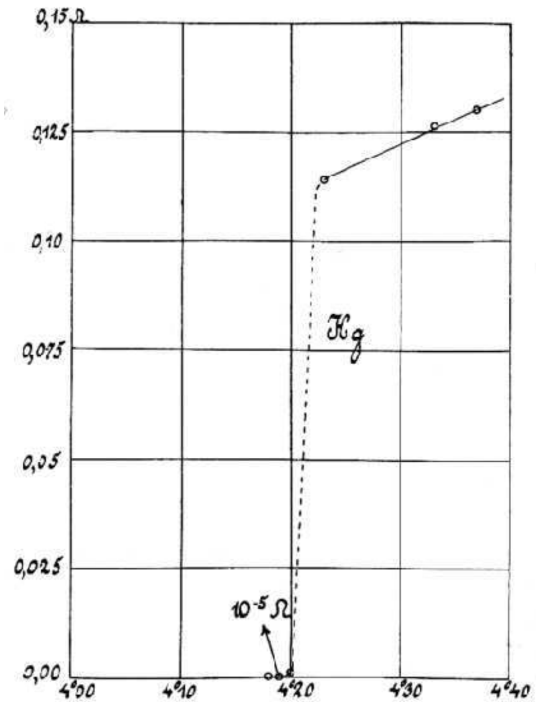
THE DISCOVERY (1911)

Resistivity in metals at low T at the turn of the 20th century





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Heike Kamerlingh Onnes liquefied He: 1st person to measure low T resistivity

S. Blundell "Superconductivity: a very short introduction"

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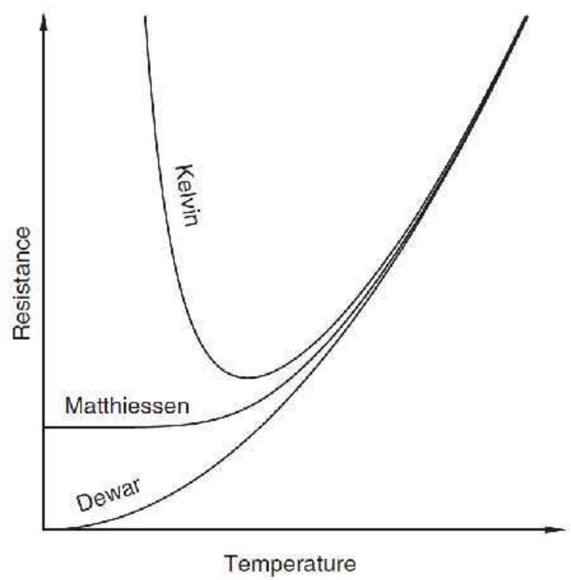

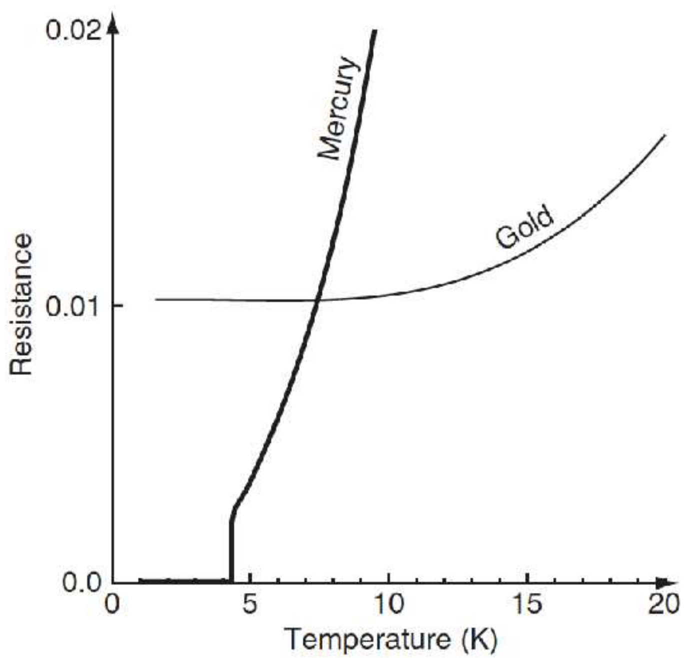

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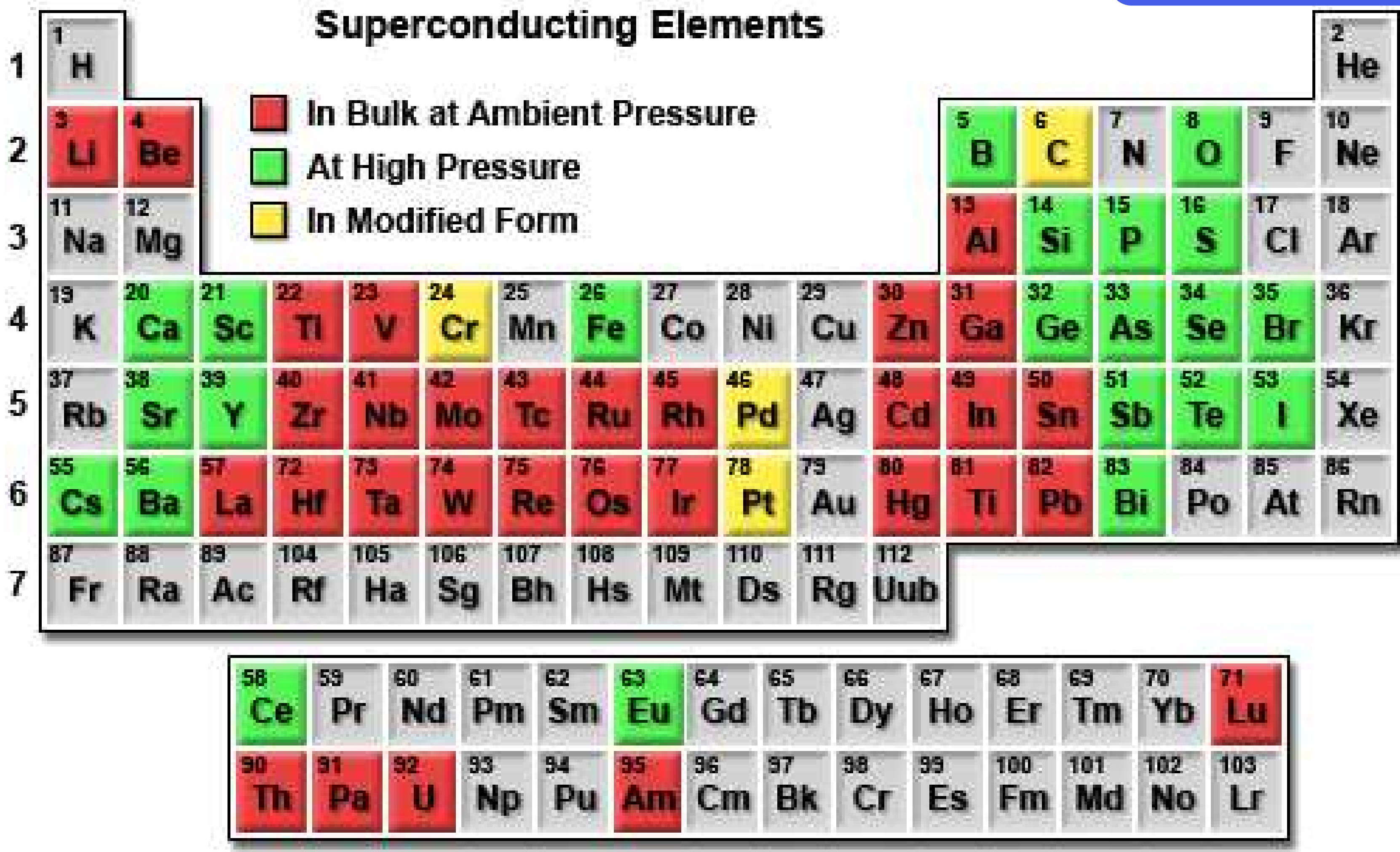

http://www.physics.buffalo.edu/

And many compounds!

https://www.webelements.com/periodicity/superconductivity_temp/

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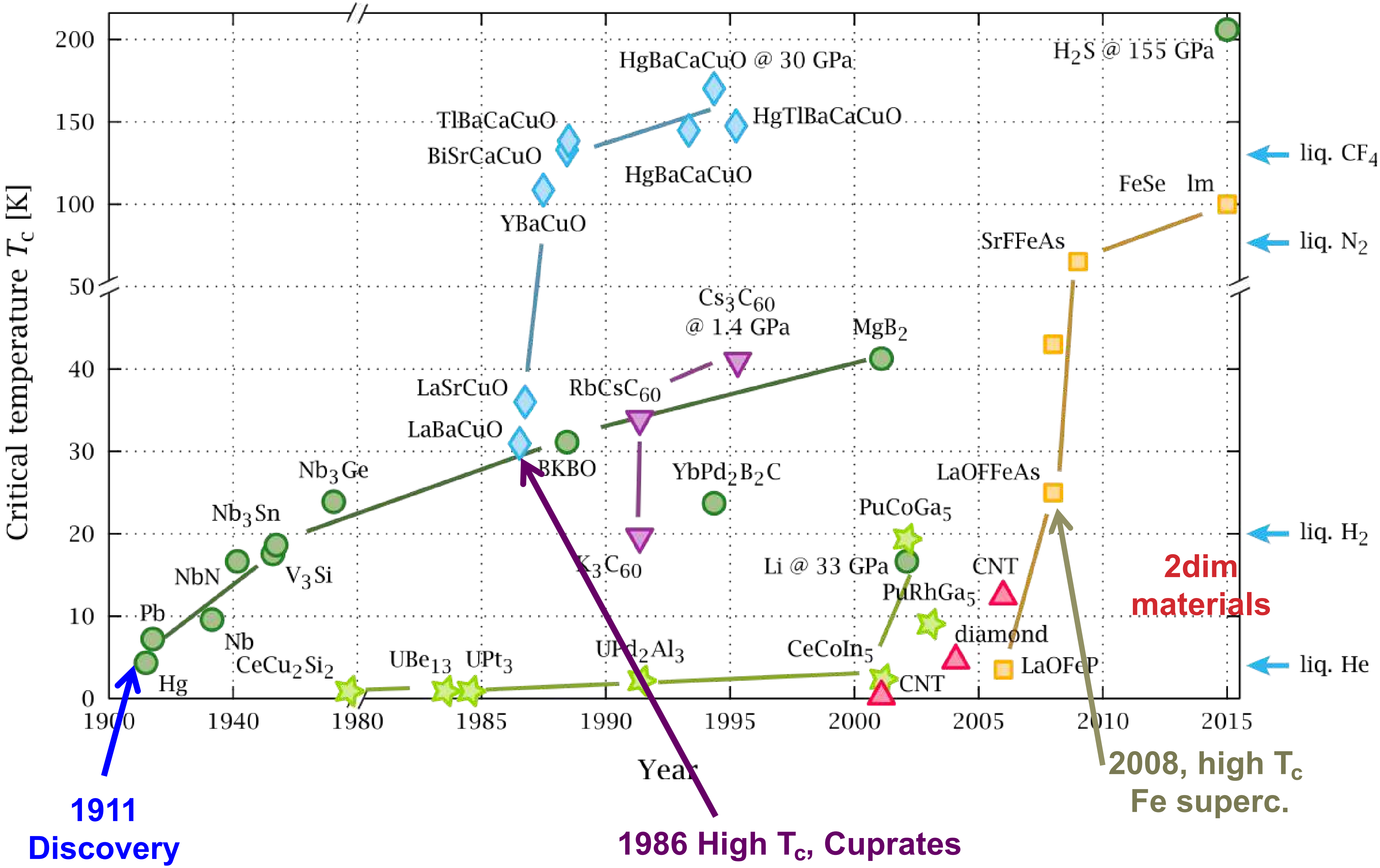

wikipedia

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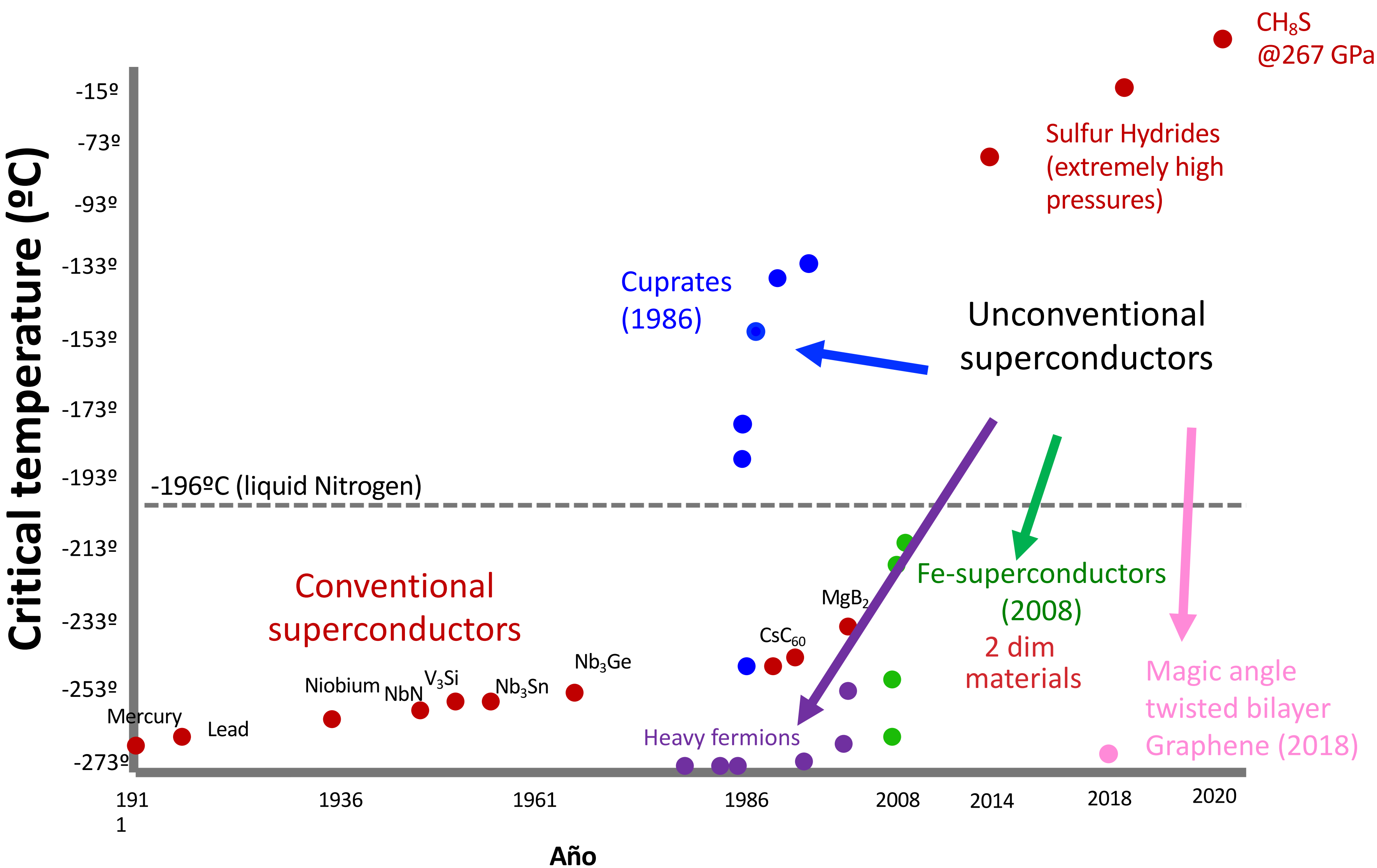
Critical temperature (ºC)
-15º
-73º
-93º
-133º
-153º
-173º
-193º
-213º
-233º
-253º
-273º
-196ºC (liquid Nitrogen)
Conventional superconductors
Mercury
Lead
Niobium
NbN
V_3Si
Nb_3Sn
Nb_3Ge
CsC_{60}
MgB_2
Cuprates (1986)
Unconventional superconductors
Heavy fermions
Fe-superconductors (2008)
2 dim materials
Magic angle twisted bilayer Graphene (2018)
Sulfur Hydrides (extremely high pressures)
CH_8S @267 GPa
1911
1936
1961
1986
2008
2014
2018
2020
Año

OUTLINE

- **Superconductivity I**
  - Properties (zero resistivity, Meissner effect)
  - Understanding (pairing, BCS, Ginzburg-Landau)
  - Electron-phonon interaction (conventional superconductivity)
- **Superconductivity II (unconventional superconductivity)**
  - What are the new issues.
  - What are (some of) the proposals.

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A BIT OF (THE EARLY) HISTORY

1911: Discovery

1933: Meissner effect

1935: London Equations

1937: Superfluidity in ^{4}He

1950: Isotope effect

Electron-electron interaction mediated by phonons

Ginzburg-Landau

1953: Pippard's coherence length

1955: Idea of gap (Bardeen)

Bardeen-Pines effective interaction

1956: Cooper pairs

Abrikosov extension of Ginzburg-Landau (vortices)

1957: BCS

1959: Gor'kov derived Ginzburg-Landau theory from BCS

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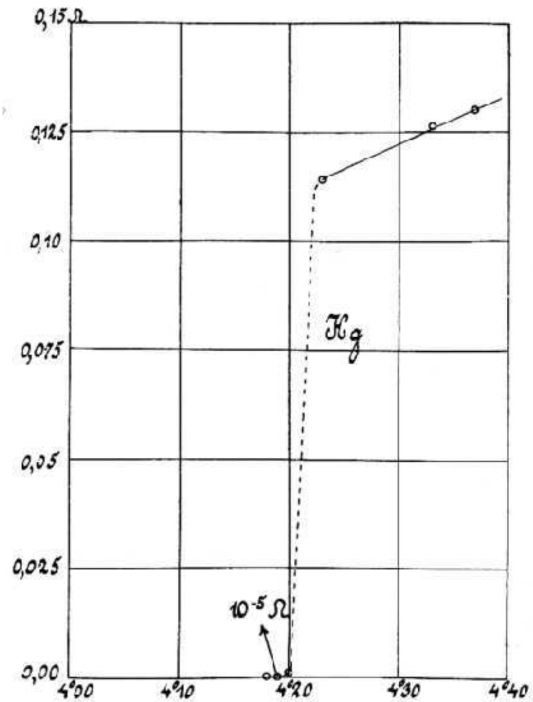
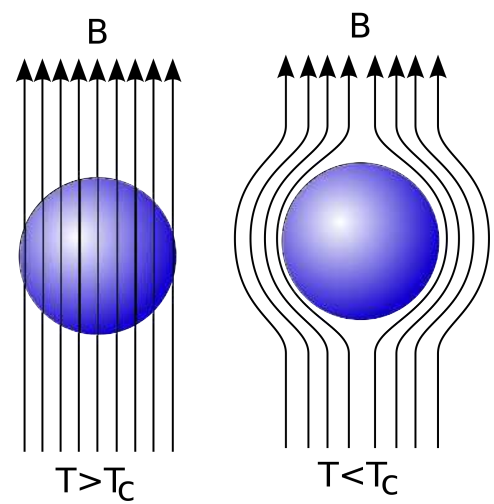
SUPERCONDUCTORS ARE MORE THAN PERFECT CONDUCTORS



http://commons.wikimedia.org/

New lead: The Meissner-Ochsenfeld effect (1933): SUPERCONDUCTORS ARE PERFECT DIAMAGNETS



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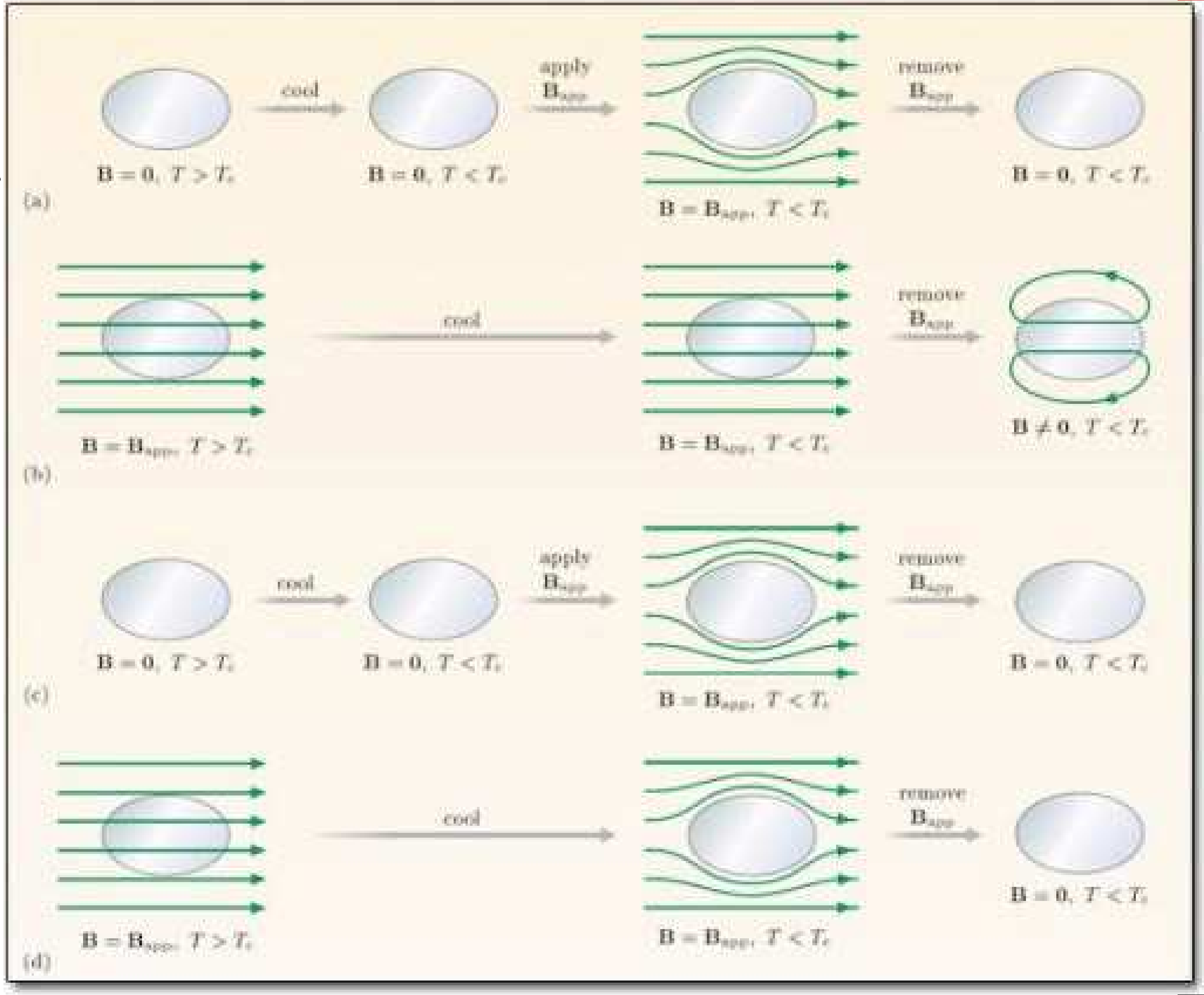
Perfect conductor

Superconductor



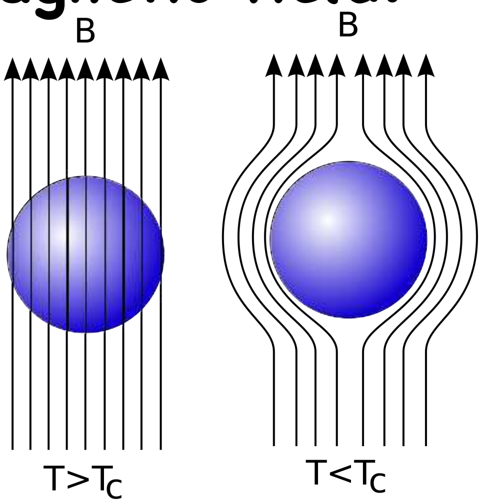
www.open.edu: Superconductivity

THE MEISSNER-OCHSENFELD EFFECT (1933)

Superconductors **expel** magnetic fields

www.icmm.csic.es/superconductividad
Note: this picture corresponds to the mixed state (see later)

Superconductors produce supercurrents that completely screen out the magnetic field.



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LONDON EQUATIONS (1935)

(phenomenological approach)

Ohm's law

(a relation between current and electric field)

↓ London equations

A new relation
between electric current and magnetic field
(inspiration: Meissner effect)

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LONDON EQUATIONS (1935)

(phenomenological approach)

Superconductivity is a macroscopic quantum phenomenon

Long range order of the momentum vector:

if Δp is very small, then Δx is very large (uncertainty principle)

A *condensation* in momentum space (= zero net momentum) rather than in real space

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LONDON EQUATIONS (1935)

Without applied magnetic field, the ground state has zero net momentum (condensation in momentum space). If <p>=0 also under an applied field (the wave-function is "rigid"):

$$\vec{p} = m\vec{v} + e\vec{A}/c$$

<p>=0

$$\vec{J}_s = n_s e \langle \vec{v}_s \rangle = \frac{-n_s e^2 \vec{A}}{mc} = \frac{-\vec{A}}{\Lambda c}$$

$$\Lambda = \frac{m}{n_s e^2}$$

n_s is the density of superconducting electrons

Tinkham's book, Coleman's book

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LONDON EQUATIONS

$$\vec{J}_s = \frac{-\vec{A}}{\Lambda c}$$

Time derivative

Curl

PERFECT CONDUCTIVITY: an electric field accelerates the electrons (in Ohm's law, an electric field sustains their velocity against resistance).

$$\vec{E} = \frac{\partial}{\partial t}\left(\Lambda \vec{J}_s\right)$$

SUPERCURRENT SCREENS MAGNETIC FIELD

$$\vec{h} = -c\nabla \times \left(\Lambda \vec{J}_s\right)$$

$$\Lambda = \frac{m}{n_s e^2}$$

nkham's book, Coleman's book

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LONDON EQUATIONS

nkham's book, Coleman's book

Superconducting electrons do not suffer any scattering.
From Drude model for electrical conductivity

$$m\frac{d\vec{v}}{dt} = e\vec{E} - \frac{m\vec{v}}{\tau}$$

Accelerative supercurrent

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LONDON EQUATIONS

nkham's book, Coleman's book

SUPERCURRENT SCREENS MAGNETIC FIELD

$$\vec{h} = -c\nabla \times \left(\Lambda \vec{J}_s\right)$$

+MAXWELL EQUATION

$$\nabla \times \vec{h} = 4\pi \vec{J}_s / c$$

$$\nabla^2 \vec{h} = \frac{\vec{h}}{\lambda^2} \longrightarrow \vec{h}(r) = \vec{h}(0) e^{-r/\lambda}$$

$$\Lambda = \frac{m}{n_s e^2}$$

$$\lambda^2 = mc^2 / 4\pi n_s e^2$$

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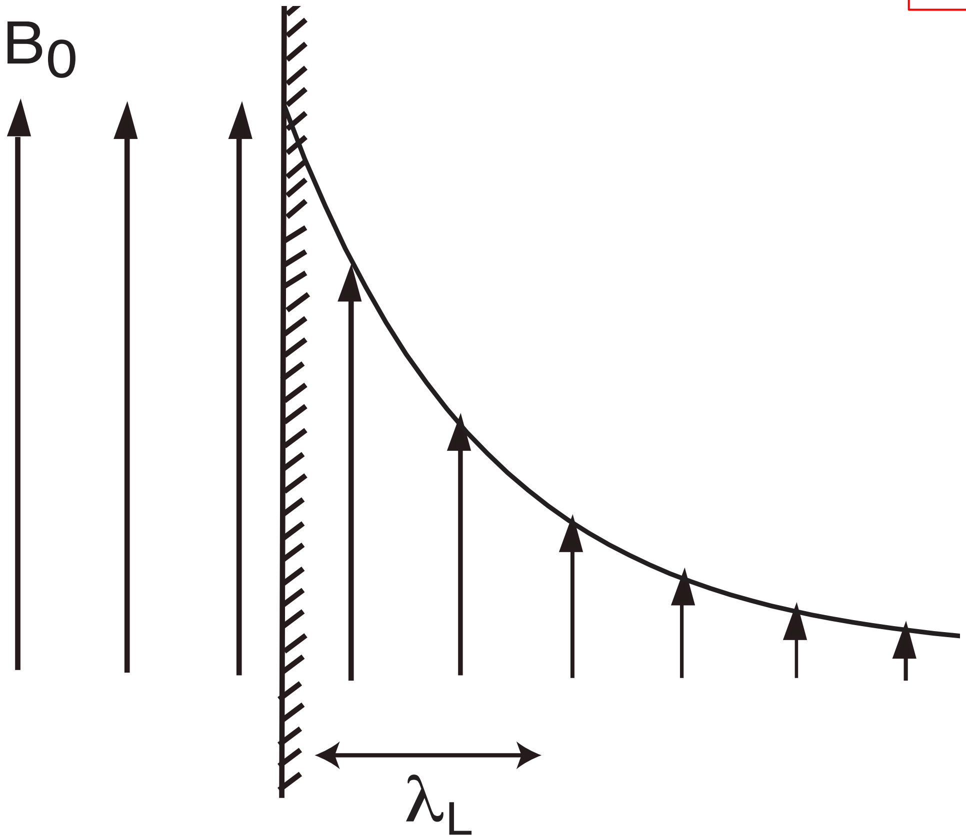
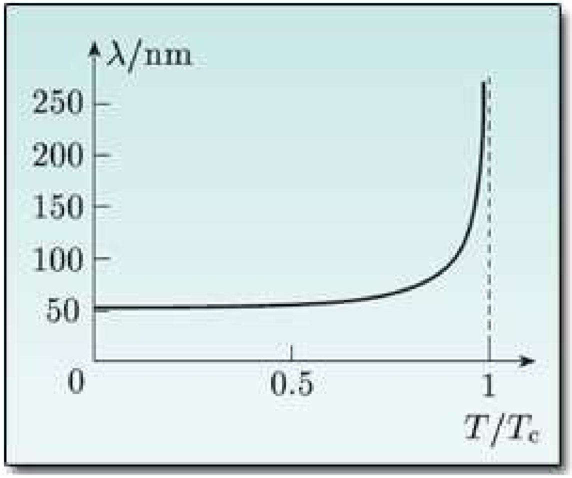
LONDON EQUATIONS

$$\vec{h}(r) = \vec{h}(0)e^{-r/\lambda}$$



LONDON PENETRATION LENGTH

$$\lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}$$



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PIPPARD'S COHERENCE LENGTH

From the uncertainty principle:

Only e^- within k_BT_c of E_F participate in superconductivity

$$\Delta p \approx \frac{k_B T_c}{v_F} \qquad \Delta x \gtrsim \frac{\hbar}{\Delta p} \approx \frac{\hbar v_F}{k_B T_c} \sim \xi_0$$

From a non-local generalization of the London equations

$$\xi_0 = 0.15 \frac{\hbar v_F}{k_B T_c}$$

ξ_0: wave-function size

2 characteristic length scales: penetration and coherence lengths

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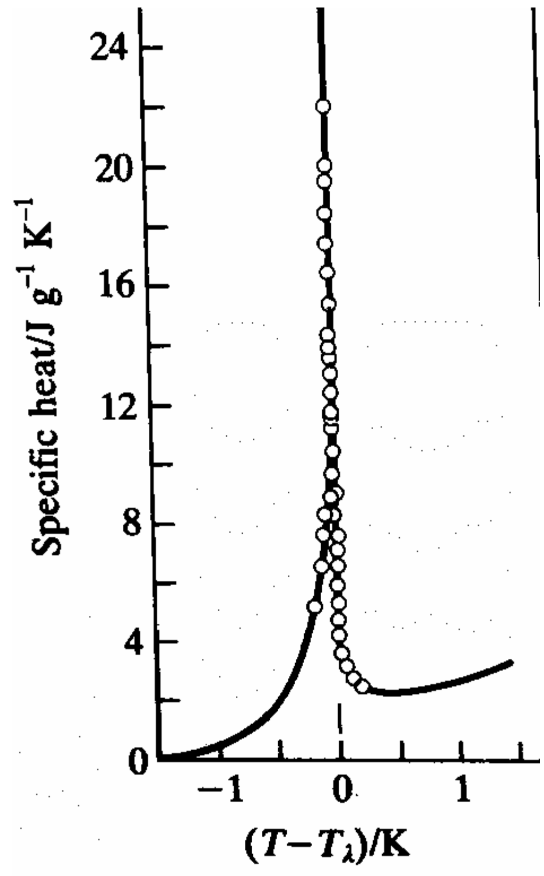
^{4}HE SUPERFLUIDITY (1937)

^{4}He liquifies at 4.2K

(He I: normal quantum liquid)

At 2.17K, transition to superfluid (He II). λ transition.

Viscosity=0.



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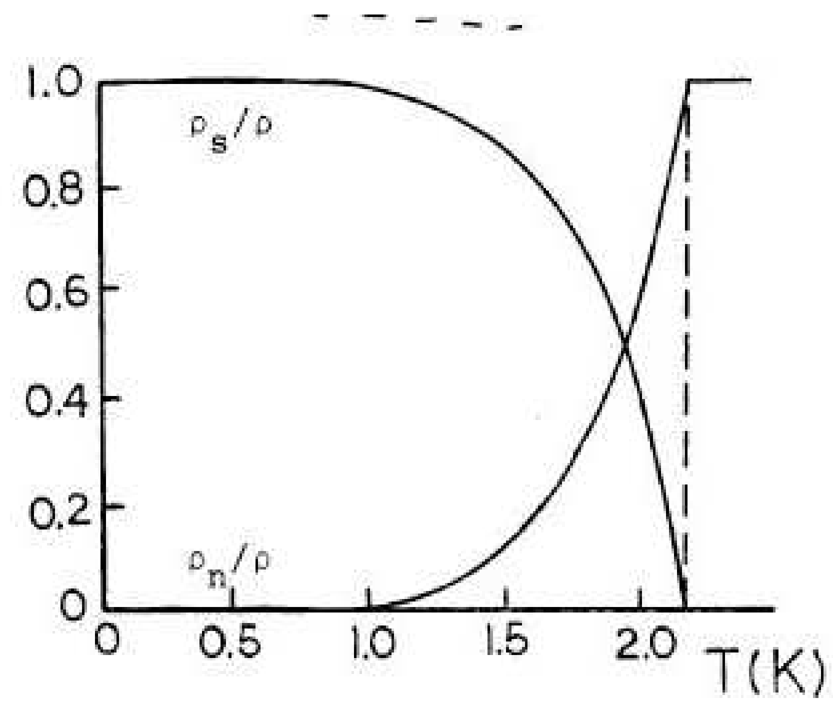
^{4}He SUPERFLUIDITY (1937)

2 fluid model

- normal fluid
- superfluid

Relation to Bose Einstein condensation:

In BEC: no interactions. In superfluid there are interactions (superfluid density is reduced)



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SUPERFLUIDITY & SUPERCONDUCTIVITY

A superconductor is a charged superfluid.

Note: What is condensing in the superconductor??
Electrons are fermions, not bosons!!
Before 1957, a microscopic theory was still missing.

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LANDAU TRANSITION THEORY

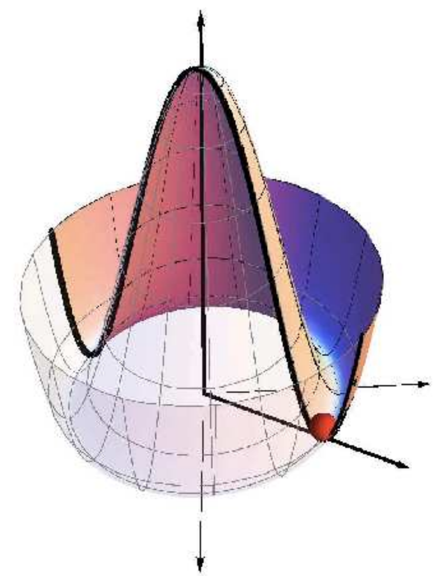
leman's book

Macroscopic theory in terms of order parameters. Valid close to the critical point. The order parameter description is valid for lengths larger than the coherence length.

$$f[\psi] = r(\psi^*\psi) + \frac{u}{2}(\psi^*\psi)^2,$$

$$\psi \equiv \psi_1 + i\psi_2 \equiv |\psi| e^{i\phi}.$$

For $T<T_c$ $\qquad \psi = \sqrt{\frac{|r|}{u}}e^{i\phi}$



A well-defined phase breaks the U(1) continuous symmetry

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GINZBURG-LANDAU (allows for inhomogeneous order parameters)

Coleman's book

New gradient terms. For a superfluid (no charge):

$$f_{GL}[\psi, \nabla\psi] = \frac{\hbar^2}{2m}|\nabla\psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4$$

London's phenomenological wavefunction is the complex order parameter.

$$\psi(x) = |\psi(x)|e^{i\phi(x)}$$

$n_s=|\psi|^2$

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The rigidity of the order parameter is in the gradient term

$$\nabla\psi = (\nabla|\psi| + i\nabla\phi|\psi|)e^{i\phi}$$

$$f_{GL} = \overbrace{\frac{\hbar^2}{2m}|\psi|^2(\nabla\phi)^2}^{\textbf{KE: phase rigidity}} + \overbrace{\left[\frac{\hbar^2}{2m}(\nabla|\psi|)^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4\right]}^{\text{amplitude flucts}}$$

Phase stiffness

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GINZBURG-LANDAU (allows for inhomogeneous order parameters)

New gradient terms. For a superfluid:

$$f_{GL}[\psi, \nabla\psi] = \frac{\hbar^2}{2m}|\nabla\psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4$$

New length scale related to the gradient term: **correlation length** (length scale of the amplitude fluctuations). At T=0, it is the Pippard's coherence length.

$$\xi(T) = \xi(0)\left|1 - \frac{T}{T_c}\right|^{-1/2} ; \xi(0) = \sqrt{\frac{\hbar^2}{2maT_c}}$$

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GINZBURG-LANDAU

Phase stiffness

$$f_{GL} = \frac{\rho_\phi}{2}(\nabla\phi)^2 + \text{constant}$$

Amplitude fluctuations are confined to scales shorter than ξ. For longer lengths those terms are constant.

Calculate the current

$$\vec{J} = -i\frac{\hbar}{2m}\left(\hat{\psi}^\dagger\vec{\nabla}\hat{\psi} - \vec{\nabla}\hat{\psi}^\dagger\hat{\psi}\right)$$

Assuming

$$\psi(x) = \sqrt{n_s(x)}e^{i\phi(x)}$$

$$\mathbf{J}_s = n_s\frac{\hbar}{m}\nabla\phi$$

Superfluid velocity

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GINZBURG LANDAU (now with charge)

$$F[\psi,\mathbf{A}] = \int d^d x \Big[\overbrace{\frac{\hbar^2}{2M}\left|(\nabla - \frac{ie^*}{\hbar}\mathbf{A})\psi\right|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4}^{f_\psi} + \underbrace{\frac{(\nabla\times\mathbf{A})^2}{2\mu_0}}_{f_{EM}} \Big]$$

The free energy is gauge invariant (any change in the phase of the order parameter can be compensated by a gauge transformation)

$$\psi(x) \to \psi(x)e^{i\alpha(x)}, \qquad \mathbf{A} \to \mathbf{A} + \frac{\hbar}{e^*}\nabla\alpha$$

The electromagnetic part of the Free energy has an associated length: the London penetration length.

(Note e*. e*=2e only after BCS theory)

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GINZBURG LANDAU (now with charge)

$$\psi(x) \rightarrow \psi(x)e^{i\alpha(x)}, \qquad \mathbf{A} \rightarrow \mathbf{A} + \frac{\hbar}{e^*}\nabla\alpha$$

By absorbing the phase of the order parameter, the phase stiffness appears as a mass term (gapped excitation).

ANDERSON-HIGGS MECHANISM

(in a neutral superfluid you have Goldstone (massless) modes)

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GINZBURG LANDAU (now with charge)

$$F[\psi,\mathbf{A}] = \int d^d x \Big[\overbrace{\frac{\hbar^2}{2M}\left|(\nabla - \frac{ie^*}{\hbar}\mathbf{A})\psi\right|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4}^{f_\psi} + \underbrace{\frac{(\nabla\times\mathbf{A})^2}{2\mu_0}}_{f_{EM}} \Big]$$

Now

$$\mathbf{J}(x) = \frac{e^*\hbar}{M}|\psi|^2\nabla\phi - \frac{e^{*2}}{M}|\psi|^2\mathbf{A}$$

$$= e^* n_s \overbrace{\frac{\hbar}{M}\left(\vec{\nabla}\phi - \frac{e^*}{\hbar}\mathbf{A}\right)}^{\mathbf{v}_s} = e^* n_s \mathbf{v}_s$$

An external vector potential produces superflow

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D.C. JOSEPHSON EFFECT (1962)

A zero voltage supercurrent should flow between two superconducting electrodes connected by a "weak link".

$$I_s = I_c \sin \Delta\varphi$$

Critical current at the junction

Difference in the phase of the Ginzburg-Landau wave-functions

A.C. JOSEPHSON EFFECT

An applied voltage leads to an oscillatory current

B. Josephson in Park's compilation

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GINZBURG LANDAU (now with charge)

$$F[\psi,\mathbf{A}] = \int d^d x \Big[\overbrace{\frac{\hbar^2}{2M}\left|(\nabla - \frac{ie^*}{\hbar}\mathbf{A})\psi\right|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4}^{f_\psi} + \underbrace{\frac{(\nabla\times\mathbf{A})^2}{2\mu_0}}_{f_{EM}} \Big]$$

The length scale for A is the London penetration length λ.

The ratio between the London penetration length and the coherence length is the **Ginzburg Landau parameter**.

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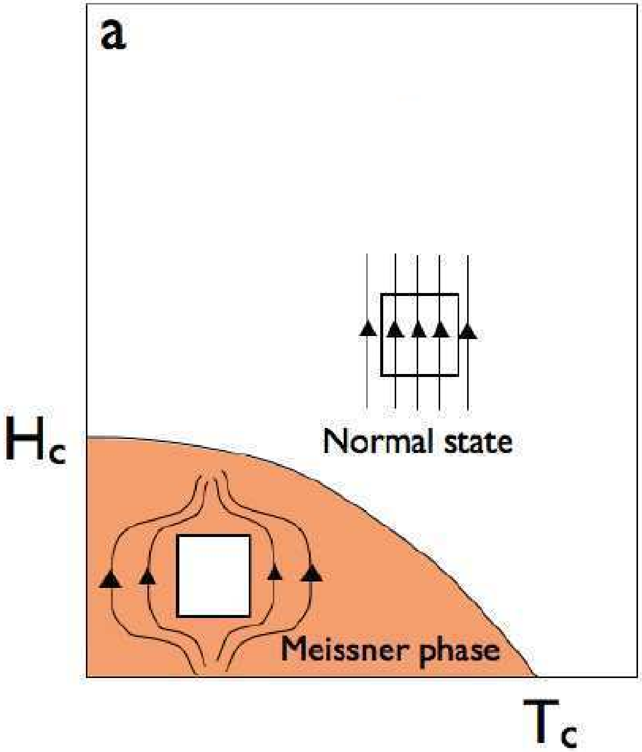
CRITICAL FIELD H_C

There is a maximum value of magnetic field H_c that can be applied before superconductivity is destroyed. There is also a critical current I_c.

First superconductors discovered had a low H_c. This is bad news if you want to build magnets with superconducting cables.

Some superconductors (alloys) didn't exclude the magnetic field completely and had much larger critical fields (1930's)



Coleman's book

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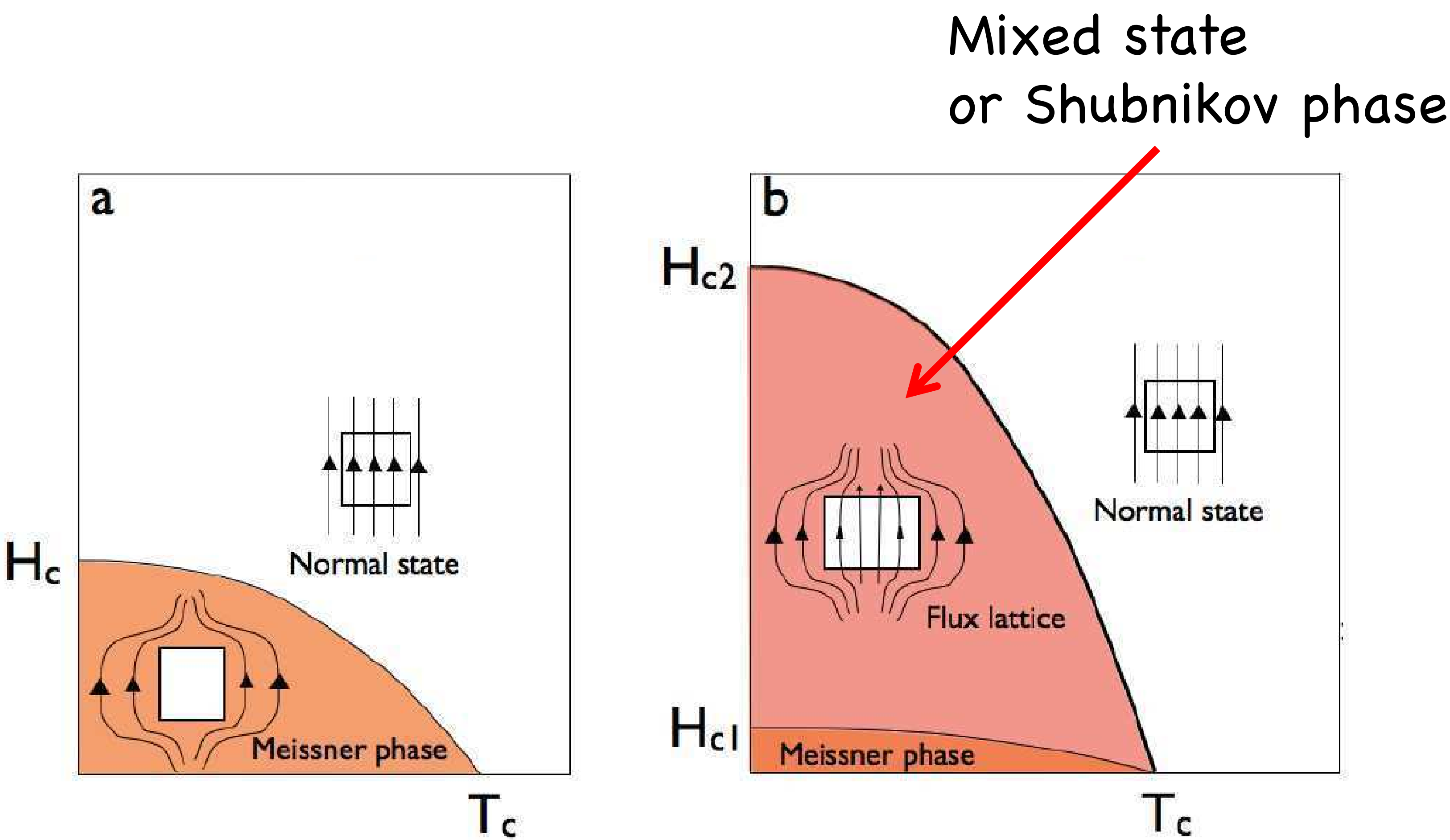
CRITICAL FIELD H_C



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TYPE I/TYPE II SUPERCONDUCTORS

The first superconductors were all Type I (λ<<ξ). Abrikosov explored the opposite possibility.

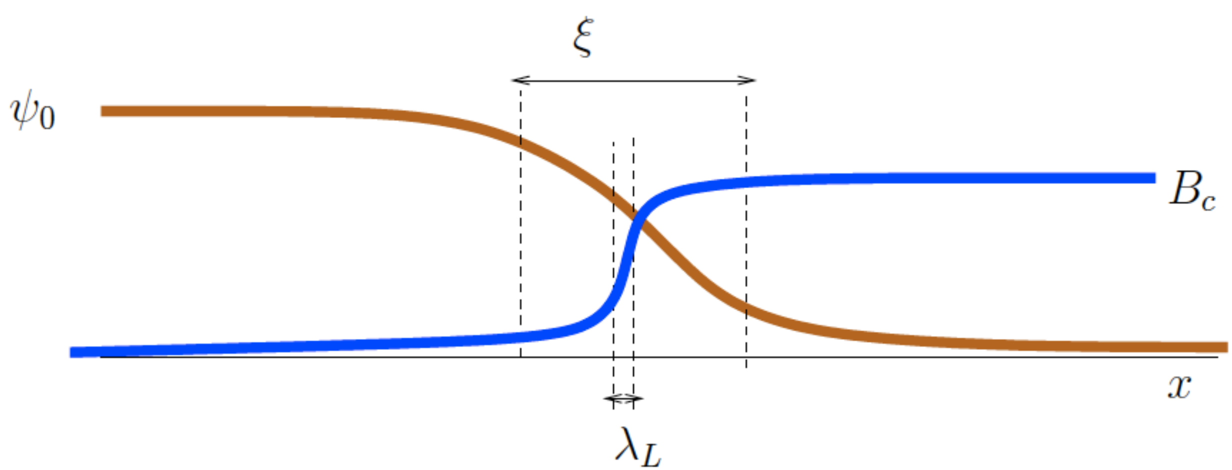
London penetration length

$$\lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}$$

The correlation length

$$\xi(T) = \xi(0)\left|1 - \frac{T}{T_c}\right|^{-1/2} ; \xi(0) = \sqrt{\frac{\hbar^2}{2maT_c}}$$

Coleman's book



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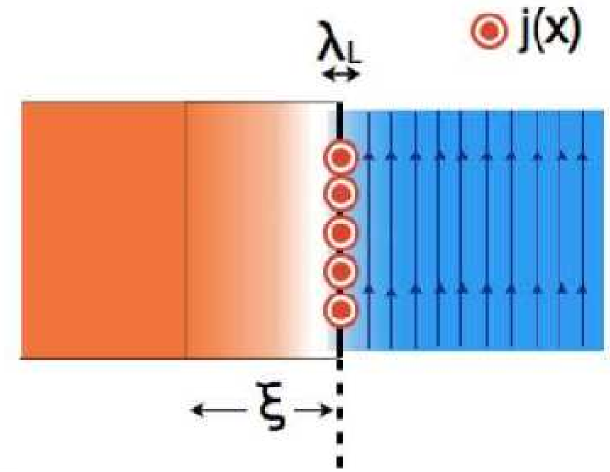
London and Ginzburg-Landau considered λ<ξ.
The supercurrent that screens the applied magnetic field occurs in a thin layer.



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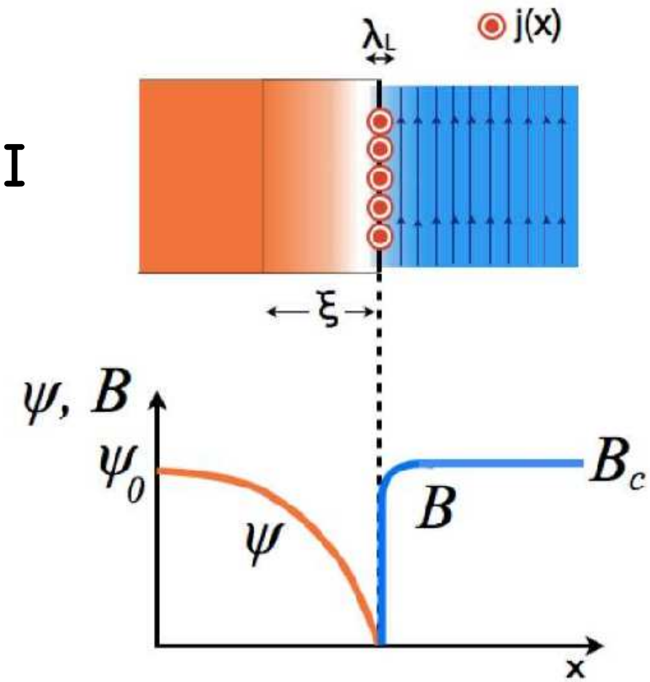
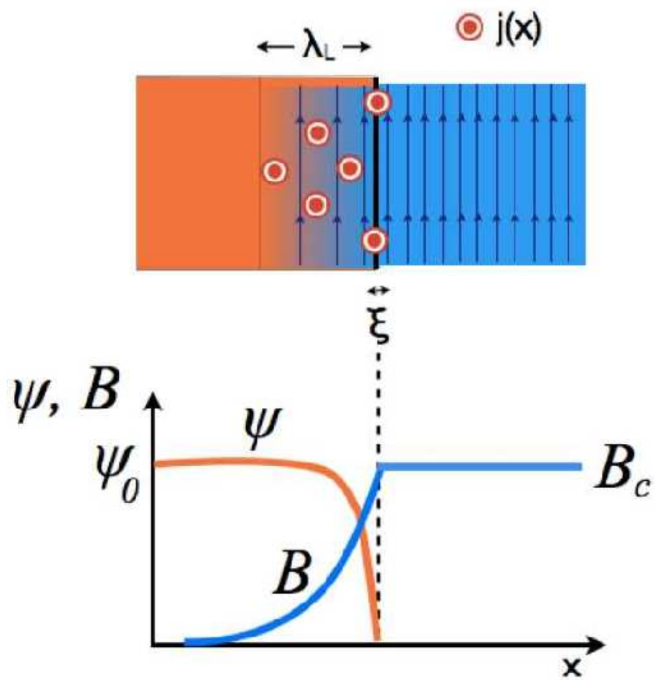
TYPE I/TYPE II SUPERCONDUCTORS

Type I





Type II

Coleman's book

Modifying the wave-function costs a lot of energy. Not much energy gain by allowing H in. N/S interfaces are costly. At H_c there is a first order transition into the normal state.

Modifying the wave-function doesn't cost too much. You gain energy by allowing the magnetic field in. N/S interfaces are favoured**.** Two critical fields**. Vortices**.

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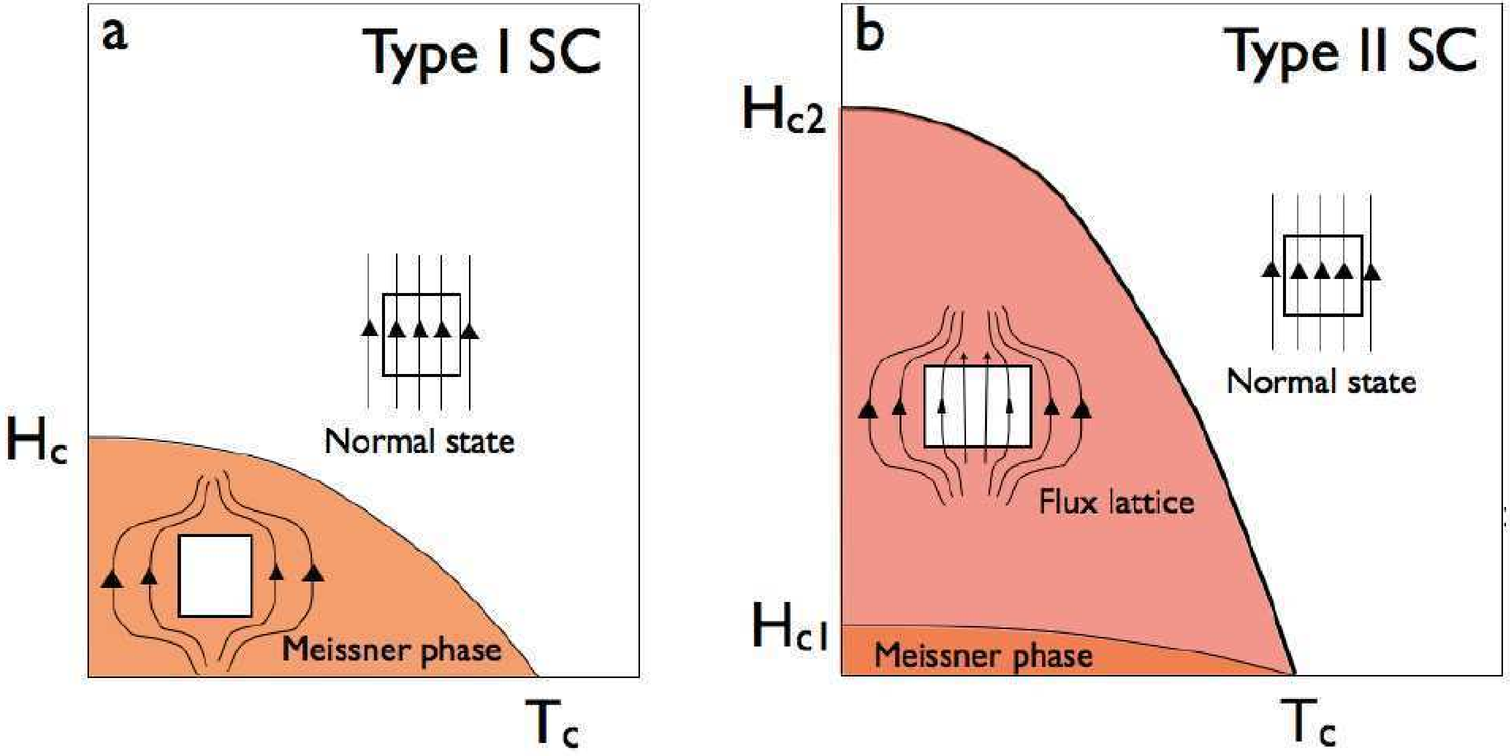
CRITICAL FIELD H_C

$$\frac{\lambda_L}{\xi} < \frac{1}{\sqrt{2}} \qquad\qquad \frac{\lambda_L}{\xi} > \frac{1}{\sqrt{2}}$$



Coleman's book

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SOME EXAMPLES

http://www.qm.phy.cam.ac.uk/teaching/

Compound	T_c [K]	ξ [nm]	λ [nm]	$\kappa = \lambda/\xi$	
Al	1.18	1600	50	0.03	type-I
Pb	7.19	83	39	0.47	type-I
Nb	9.25	40	44	1.1	border
Nb_3Sn	18.2	3.6	124	34	type-II
$YBa_2Cu_3O_{7-\delta}$	90	(1.5)	(130)	(87)	type-II

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CRITICAL FIELDS IN TYPE II

$$\frac{\lambda_L}{\xi} > \frac{1}{\sqrt{2}}$$

$$B_{c2} = \frac{\phi_0}{2\pi\xi^2}, \quad B_{c1} = \frac{\phi_0}{4\pi\lambda^2} \quad \text{and} \quad B_{c1}B_{c2} = B_c^2$$

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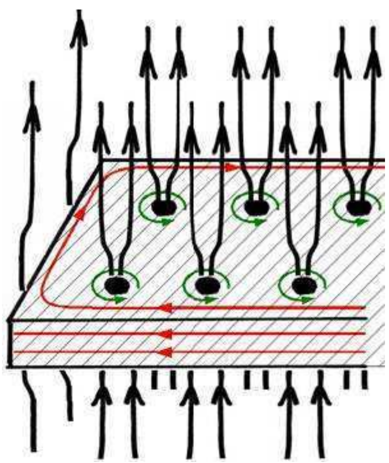
VORTICES

In type II superconductors, part of the field can go through the superconductor. A supercurrent circulates around the magnetic flux to screen it: Vortices.

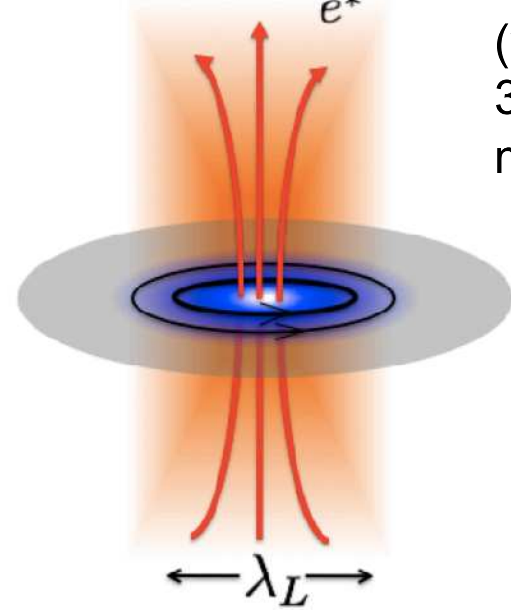


http://www.supraconductivite.fr/

Superconductivity is suppressed at the vortex core

The flux is quantized

$$\Phi_0 = \frac{h}{e^*}$$



(vortex in 3dim XY model)

Coleman's book

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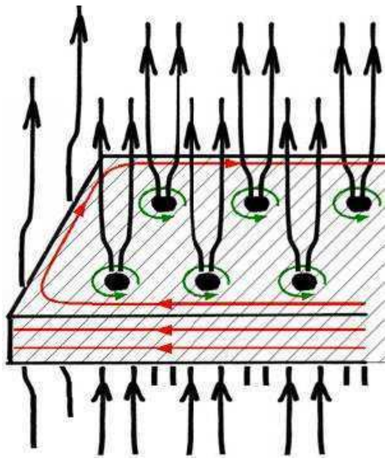
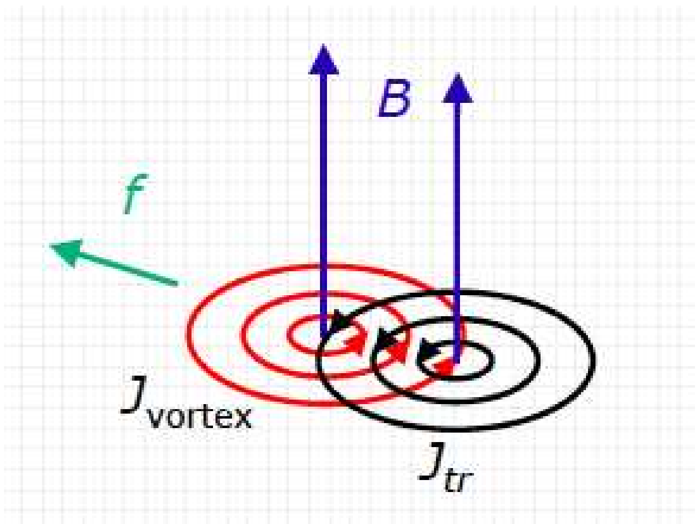
VORTICES

In type II superconductors, part of the field can go through the superconductor. A supercurrent circulates around the magnetic flux to screen it: Vortices.



http://www.supraconductivite.fr/



Coleman's book

Vortices repel each other (they have the same topological charge)

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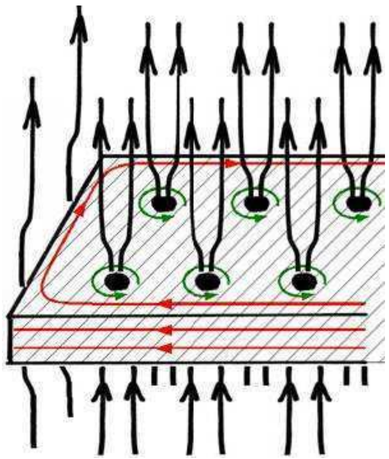
VORTICES

In type II superconductors, part of the field can go through the superconductor. A supercurrent circulates around the magnetic flux to screen it: Vortices.



http://www.supraconductivite.fr/

Pinning!!

www.icmm.csic.es/superconductividad

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PINNING

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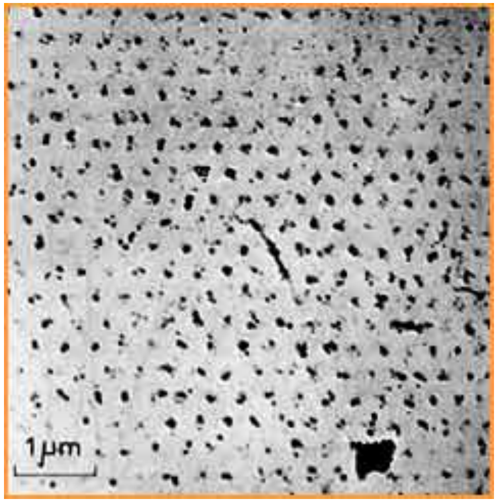
calderon@icmm.csic.es

VORTICES

In type II superconductors, part of the field can go through the superconductor. A supercurrent circulates around the magnetic flux to screen it: Vortices.
From London eqs: the magnetic flux is quantized. Vortices order in a regular array.

First observation of an array of vortices



Physics Letters 24A, 526 (1967)

Supercurrent exert forces on vortices leading to dissipation. Impurities pin the vortices so they cannot move.

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TOWARDS A MICROSCOPIC MODEL

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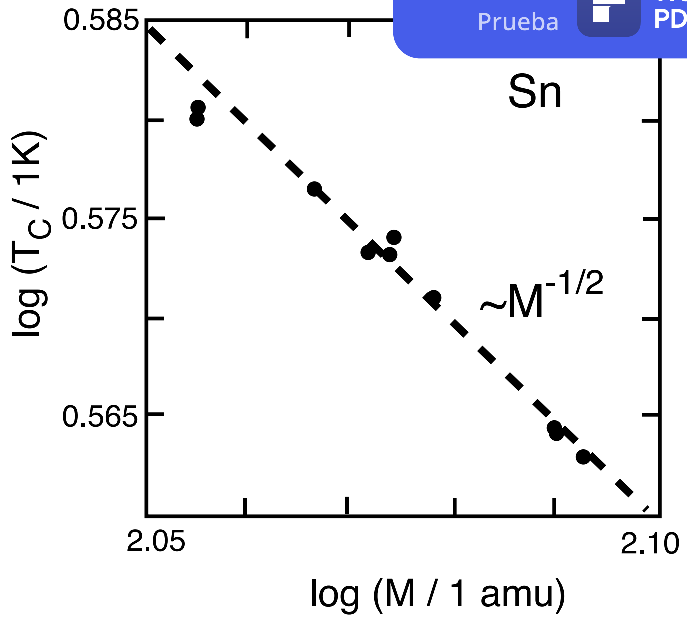
ISOTOPE EFFECT

(C. Reynolds et al, E. Maxwell)



Evidence that superconductivity was related to electron-phonon interaction

$$T_c \propto \omega_D \approx \frac{1}{\sqrt{M}}$$

Independently, Fröhlich suggested that SPC occurs due to electron-lattice interaction. Also Bardeen, independently, as soon as he knew about the isotope effect.

$$V_{eff}(\mathbf{k},\mathbf{k}') = -g^2_{\mathbf{k}-\mathbf{k}'} \frac{2\omega_{\mathbf{k}-\mathbf{k}'}}{\omega^2_{\mathbf{k}-\mathbf{k}'} - (\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'})^2}$$

Formulation of the electron-phonon interaction. Leads to attractive energy.

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THE IDEA OF A GAP

Bardeen proposed that a gap in the spectrum would lead to the rigidity of the wave-function. Some experiments were showing a sign of a gap (activated specific heat).

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EFFECTIVE INTERACTION

Include electron-phonon interaction (must be important due to isotope effect). Phonon of frequency ω_q.

$$V_q + |M_q|^2 \left(\frac{1}{\hbar\omega - \hbar\omega_q} + \frac{1}{-\hbar\omega - \hbar\omega_q} \right) = V_q + \frac{2|M_q|^2}{\hbar} \frac{\omega_q}{\omega^2 - \omega_q^2}$$

Change in E for the e-

M_q: electron-phonon matrix element. If electron-phonon interaction is too weak (good metals), no superconductivity arises

Overscreening: The effective electron-electron interaction can be negative (attractive)

The dynamics of the phonons is much slower than the electrons'

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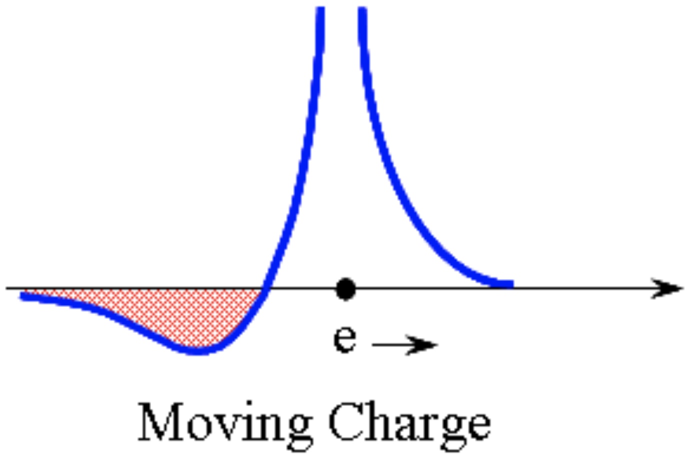
EFFECTIVE INTERACTION

Include electron-phonon interaction (must be important due to isotope effect). Phonon of frequency ω_q.

$$V_q + |M_q|^2 \left(\frac{1}{\hbar\omega - \hbar\omega_q} + \frac{1}{-\hbar\omega - \hbar\omega_q} \right) = V_q + \frac{2|M_q|^2}{\hbar} \frac{\omega_q}{\omega^2 - \omega_q^2}$$

Attraction if $\omega < \omega_q$



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BCS theory (1957)

Bardeen-Cooper-Schrieffer

Interaction-pairs-wavefunction

Nobel prize 1972

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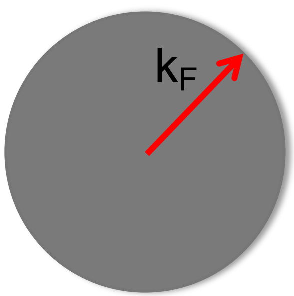
COOPER PAIRS (1956)

Weak attractions can bind pairs of electrons.

Requires the existence of a Fermi sea (doesn't work for free particles)



Add two electrons on top of the Fermi sea

$$|\Psi\rangle = \Lambda^{\dagger}|FS\rangle$$

Assume zero momentum and singlet pairing (expected lowest energy)

With pair creation operator

$$\Lambda^{\dagger} = \sum_{\mathbf{k}} \phi_{\mathbf{k}} c^{\dagger}{}_{\mathbf{k}\downarrow} c^{\dagger}{}_{-\mathbf{k}\uparrow}$$

pair wavefunction

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COOPER PAIRS (1956)

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c^{\dagger}{}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \hat{V}$$

$$H|\Psi\rangle = \sum_{|\mathbf{k}|>k_F} 2\epsilon_{\mathbf{k}} \phi_{\mathbf{k}} |\mathbf{k}_P\rangle + \sum_{|\mathbf{k}|,\ |\mathbf{k}'|>k_F} |\mathbf{k}_P\rangle\langle \mathbf{k}_P|\hat{V}|\mathbf{k}'_P\rangle \phi_{\mathbf{k}'} .$$

$$E\phi_{\mathbf{k}} = 2\epsilon_{\mathbf{k}}\phi_{\mathbf{k}} + \sum_{|\mathbf{k}'|>k_F} \langle \mathbf{k}_P|\hat{V}|\mathbf{k}'_P\rangle \phi_{\mathbf{k}'}$$

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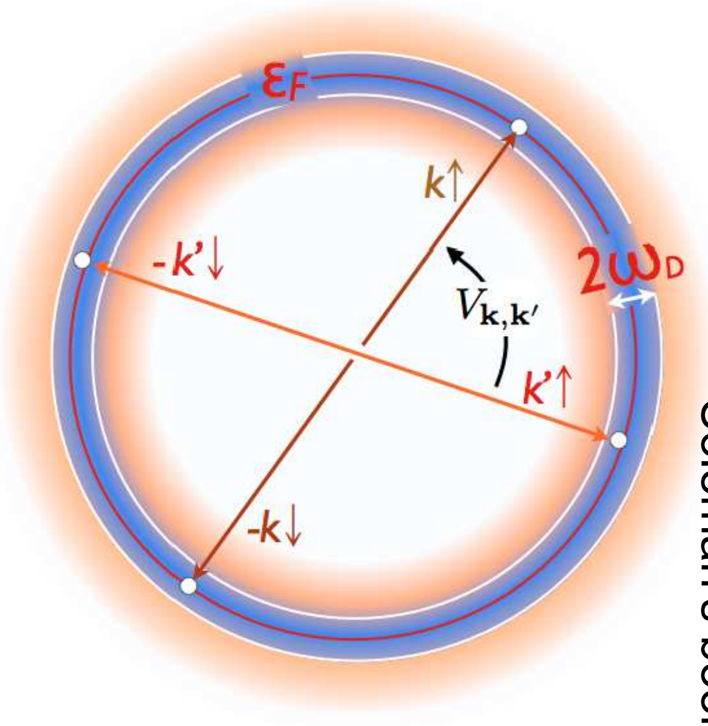
COOPER PAIRS (1956)

Simplified model interaction

$$V_{\mathbf{k},\mathbf{k}'} = \begin{cases} -g_0/V & (\,|\epsilon_{\mathbf{k}}|,\ |\epsilon_{\mathbf{k}'}| < \omega_D) \\ 0 & \text{otherwise} \end{cases}$$



Coleman's book

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COOPER PAIRS (1956)

$$(E - 2\epsilon_{\mathbf{k}})\phi_{\mathbf{k}} = -\frac{g_0}{V} \sum_{0<\epsilon_{\mathbf{k}'}<\omega_D} \phi_{\mathbf{k}'},$$

$$\phi_{\mathbf{k}} = -\frac{g_0/V}{E - 2\epsilon_{\mathbf{k}}} \sum_{0<\epsilon_{\mathbf{k}'}<\omega_D} \phi_{\mathbf{k}'},$$

....

$$E = -2\omega_D e^{-\frac{2}{g_0 N(0)}}.$$

Coleman's book

Fermi sea is unstable to the formation of Cooper pairs

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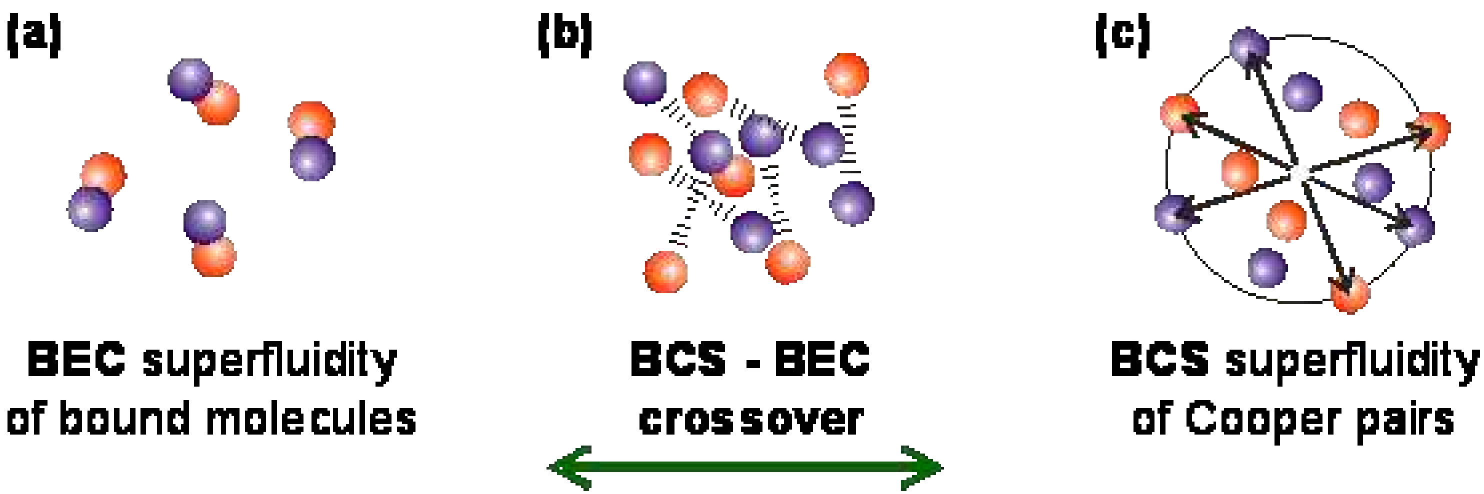
COOPER PAIRS (1956)

The degeneracy of the Fermi sea N(0) implies pairs form even for a very weak pairing interaction

$$E = -2\omega_D e^{-\frac{2}{g_0 N(0)}}.$$

The Cooper pair has a small smearing in k space → a large extension (coherence length) in real space (10^4 Å for Al).

Pairs overlap.



http://www.qm.phy.cam.ac.uk/teaching/

http://users.physics.harvard.edu/~greiner/fermicondensates.html

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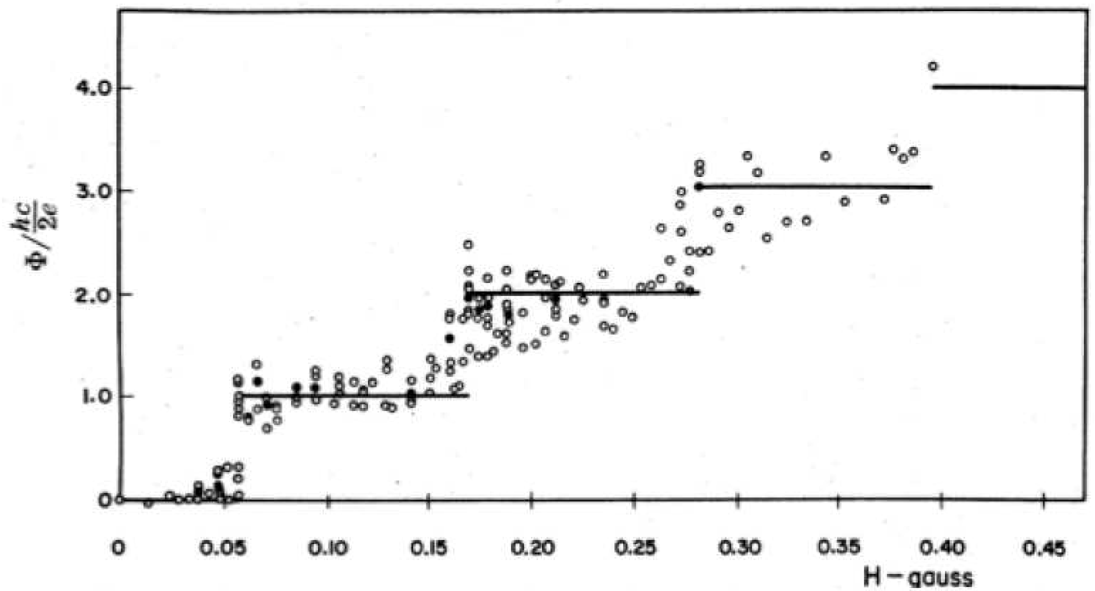
OBSERVATION OF COOPER PAIRS

The vortex magnetic flux is quantized (London, 1950)

$$\Phi_0 = \frac{h}{e^*} \equiv \frac{h}{2e}.$$

Magnetic flux in a cylinder provides a proof for the charge of the Cooper pair.



Phys. Rev. Lett. 7, 43–46 (1961)

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SCHRIEFFER: WAVE-FUNCTION

Coherent superposition of Cooper pair operators

$$|\psi_{BCS}\rangle = \exp[\Lambda^{\dagger}]|0\rangle, \quad \Lambda^{\dagger} = \sum_{\mathbf{k}} \phi_{\mathbf{k}} c^{\dagger}{}_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow}$$

$$|\psi_{BCS}\rangle = \prod_{\mathbf{k}} \exp[\phi_{\mathbf{k}} c^{\dagger}{}_{\mathbf{k}\uparrow} c^{\dagger}{}_{-\mathbf{k}\downarrow}]|0\rangle = \prod_{\mathbf{k}} (1 + \phi_{\mathbf{k}} c^{\dagger}{}_{\mathbf{k}\uparrow} c^{\dagger}{}_{-\mathbf{k}\downarrow})|0\rangle.$$

And the BCS Hamiltonian

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

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GAP EQUATION

Start from pairing Hamiltonian

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

Mean field approximation

Tinkham's book

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$$

$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - \sum_k \left(\Delta_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \Delta^*_k c_{-k\downarrow} c_{k\uparrow} \right)$$

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GAP EQUATION

Diagonalize (Bogoliubov transformation)

$$\alpha^\dagger_{k\uparrow} = c^\dagger_{k\uparrow} u_k + c_{-k\downarrow} v_k$$

$$\alpha_{-k\downarrow} = c_{-k\downarrow} u^*_k - c^\dagger_{k\uparrow} v^*_k$$

$$H = \sum_{k\sigma} E_k \alpha^\dagger_{k\sigma} \alpha_{k\sigma}$$

$$E_{\mathbf{k}} = \sqrt{\epsilon^2_{\mathbf{k}} + |\Delta_{\mathbf{k}}|^2}$$

Tinkham's book

The self consistent eq for the gap is:

$$\Delta_{\mathbf{k}} = -\frac{1}{2} \sum_{\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon^2_{\mathbf{k}'} + |\Delta_{\mathbf{k}'}|^2}} V_{\mathbf{k},\mathbf{k}'}$$

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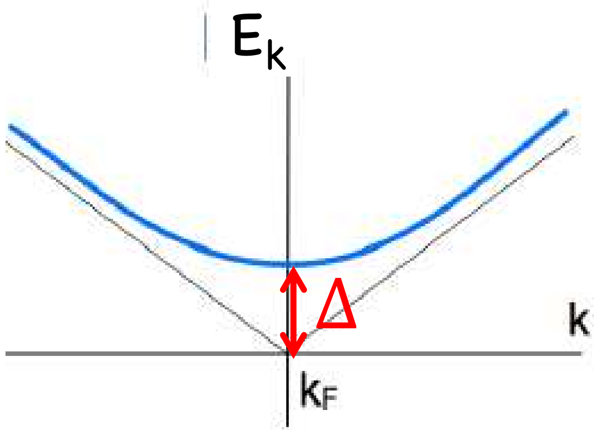
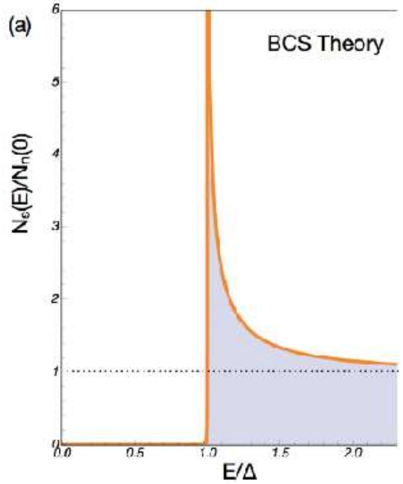
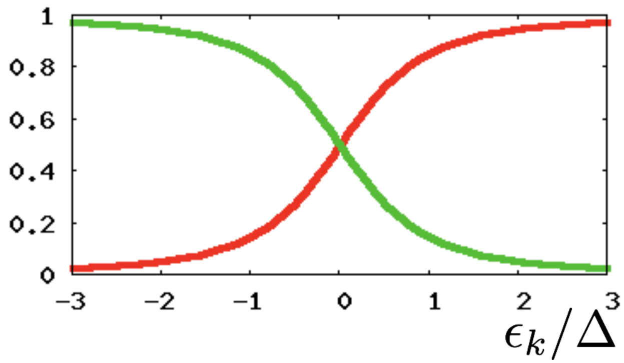
BCS THEORY

$$E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}|^2}$$ Excitation energy of quasiparticles





$$\alpha^{\dagger}_{k\uparrow} = c^{\dagger}_{k\uparrow} u_k + c_{-k\downarrow} v_k$$



http://www.qm.phy.cam.ac.uk/teaching/

Coleman's book

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BCS THEORY

Explicit equation for Δ (T=0)

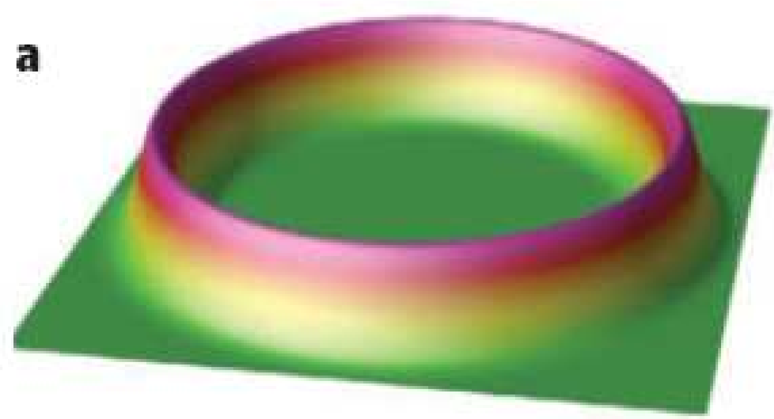
$$\Delta_{\mathbf{k}} = -\frac{1}{2}\sum_{\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}} V_{\mathbf{k},\mathbf{k}'}$$

Within BCS $V_{kk'} = -V$ within $\hbar\omega_D$ of E_F

Then $\Delta_{kk'}=\Delta$ (isotropic)

$$\Delta \approx 2\hbar\omega_D e^{-1/N(0)V}$$

Isotropic pairing function: s-wave



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BCS THEORY

For finite T

$$\Delta_{\mathbf{k}} = -\frac{1}{2}\sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon^2_{\mathbf{k}'} + |\Delta_{\mathbf{k}'}|^2}} \tanh\left(\frac{\beta}{2}\sqrt{\epsilon^2_{\mathbf{k}'} + |\Delta_{\mathbf{k}'}|^2}\right)$$

$$k_B T_c = 1.13\hbar\omega_D \exp[-1/VN(E_F)]$$

$$\Delta \approx 2\hbar\omega_D e^{-1/N(0)V}$$

$$\frac{\Delta(0)}{k_B T_c} = 2/1.33 = 1.764$$

Tinkham's book

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BCS THEORY

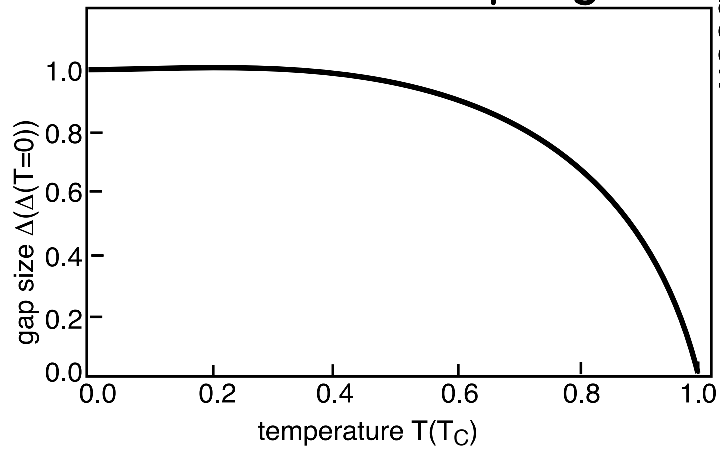
For finite T

$$\Delta_{\mathbf{k}} = -\frac{1}{2}\sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}} \tanh\left(\frac{\beta}{2}\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}\right)$$

For weak coupling superconductors

$$\frac{\Delta(T)}{\Delta(0)} \sim 1.76\left(1 - \frac{T}{T_c}\right)^{1/2}$$

Universal Δ(T) curve in the weak coupling limit



Tinkham's book

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BCS THEORY

Condensation energy=
energy of superconducting state - energy of normal state

$$\Delta W = -\frac{1}{2} N(0)\Delta^2$$

The magnetic critical field which destroys superconductivity

$$\frac{1}{2} N(0)\Delta^2 = \frac{H_c^2}{8\pi}$$

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MIGDAL-ELIASHBERG
(INCLUSION OF RETARDATION EFFECTS)

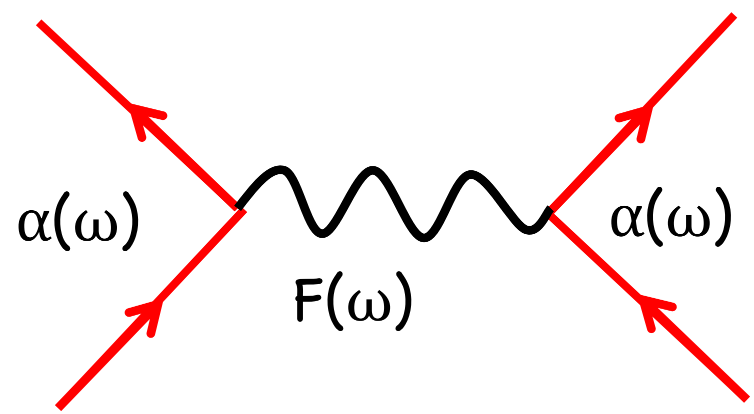
BCS theory included retardation effects in a minimal way by introducing an energy cut-off: the Debye frequency.

Eliashberg treated the electron-phonon interaction including correctly the retardation effects.

$$T_c = \mathcal{F}(\alpha^2 F(\omega), \mu^*)$$

Electron-phonon spectral function

Coulomb pseudopotential



Migdal (1957): vertex corrections $O(m/M)^{1/2}$ can be dropped (equivalent to adiabaticity condition)

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MIGDAL-ELIASHBERG

BCS theory included retardation effects in a minimal way by introducing an energy cut-off: the Debye frequency.

Eliashberg treated the electron-phonon interaction including correctly the retardation effects.

$$T_c = \mathcal{F}(\alpha^2 F(\omega), \mu^*) \qquad \Delta = G(\alpha^2 F(\omega), \mu^*)$$

$$\frac{2\Delta}{k_B T_c} = 3.53\left[1 + 12.5\left(\frac{T_c}{\omega_{ln}}\right)^2 \log\left(\frac{\omega_{ln}}{2T_c}\right)\right]$$

Coupling strength

PRB 29, 184 (1984)

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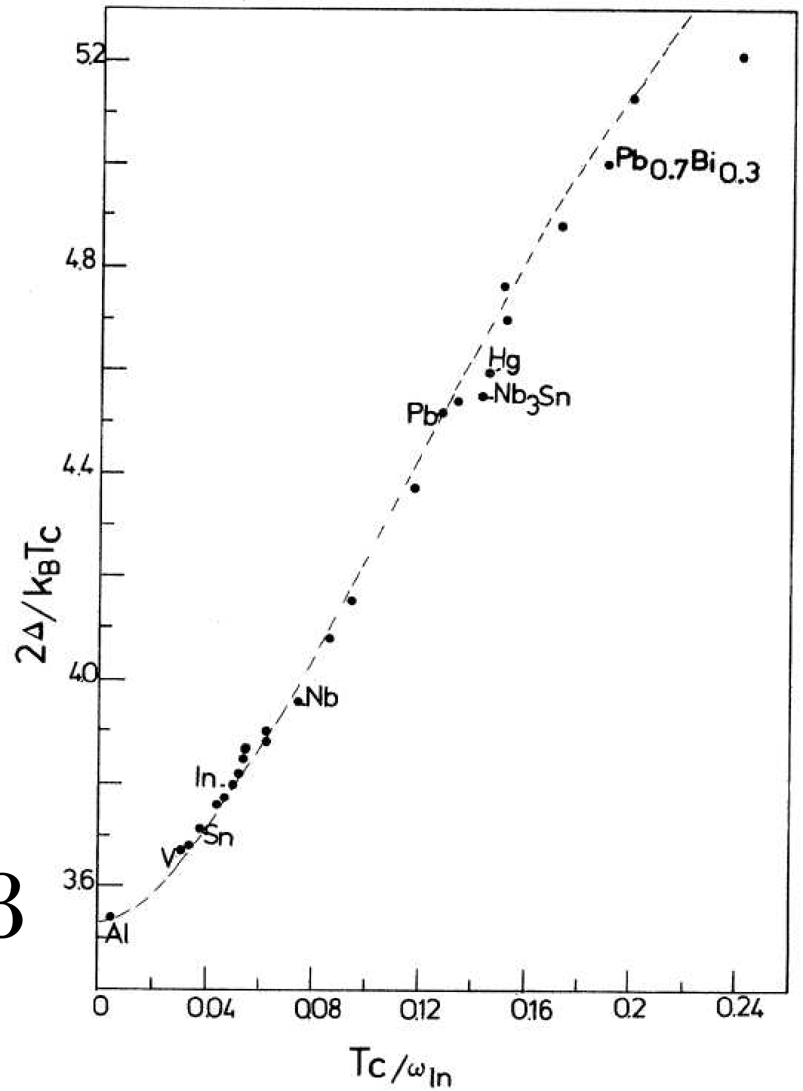
MIGDAL-ELIASHBERG

$$\frac{2\Delta(0)}{k_B T_c} = 3.53$$



PRB 29, 184 (1984)

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MAX T_C FOR PHONON MEDIATED SUPERCONDUCTIVITY

McMillan PR 167, 331

$$T_c \propto \langle \omega_{ph} \rangle \exp[-1/(\lambda - \mu^*)]$$

Input: phonon spectrum

With the electron phonon coupling constant

$$\lambda = 2\int_0^\infty \frac{\alpha^2 F(\omega)}{\omega}$$

TABLE VIII. The predicted maximum superconducting transition temperature for four classes of materials found from the observed T_c and λ and Fig. 11, together with the observed

Pure metals

Alloys

Metal	T_c (°K)	λ	T_c^{max} (°K)	Observed maximum T_c	Material
Pb	7.2	1.3	9.2	8.8	Pb–Bi
Nb	9.2	0.82	22	10.8	Zr–Nb
V_3Si	17	0.82	40		
Nb_3Sn			28	20	Nb_3Al–Nb_3Ge

Note:

MgB_2 Tc=40K

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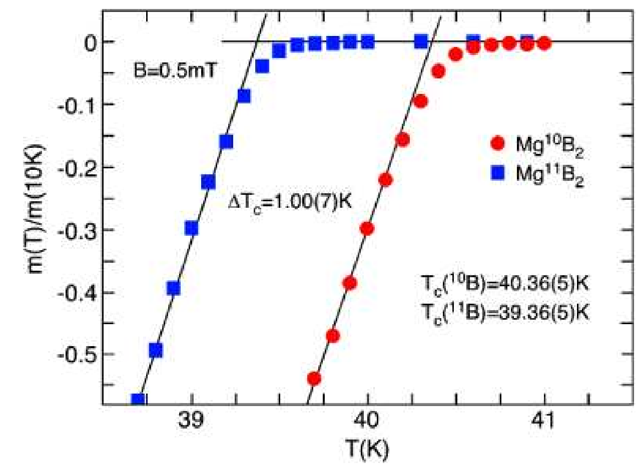
Canfield and Crabtree, P

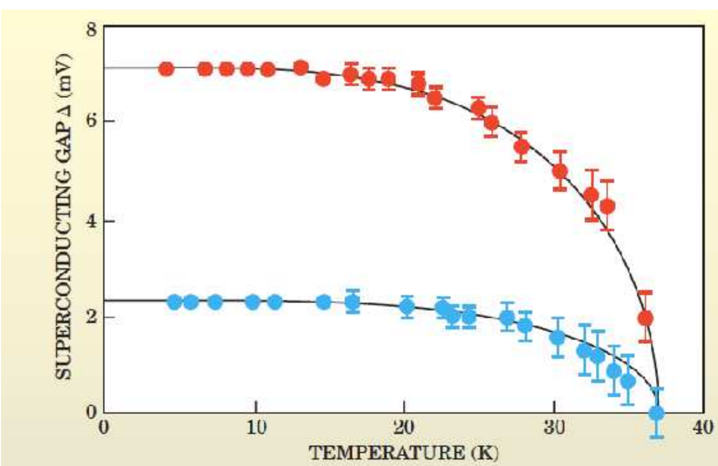
MgB_2 (2001)

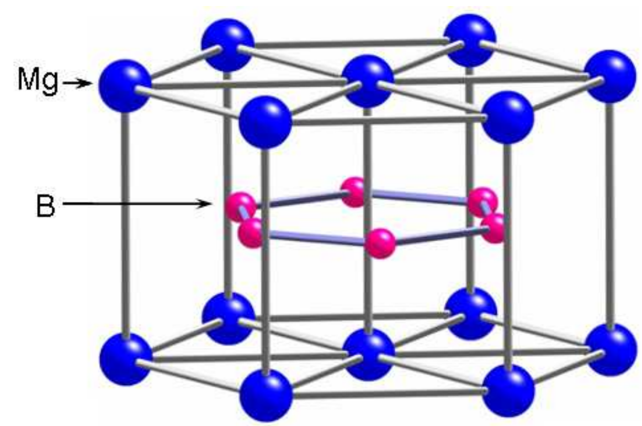
T_c=39 K. **MULTIBAND SUPERCONDUCTOR**

Conventional SPC (electron-phonon interaction). 2 differentiated bands. One very strong phonon (in the B plane) coupled to the conduction electrons. 2 energy gaps with different electron-phonon strengths.

JPCM 24 233201







DOI: 10.5772/17117

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RECORD T_C FOR CONVENTIONAL SUPERCONDUCTIVITY

H_2S T_c=190K!! (at 200 GPa)
arXiv:1412.0460

T_c=203 K Nature 525, 73–76 (2015)

LaH_{10} T_C=250 K at 170 GPa

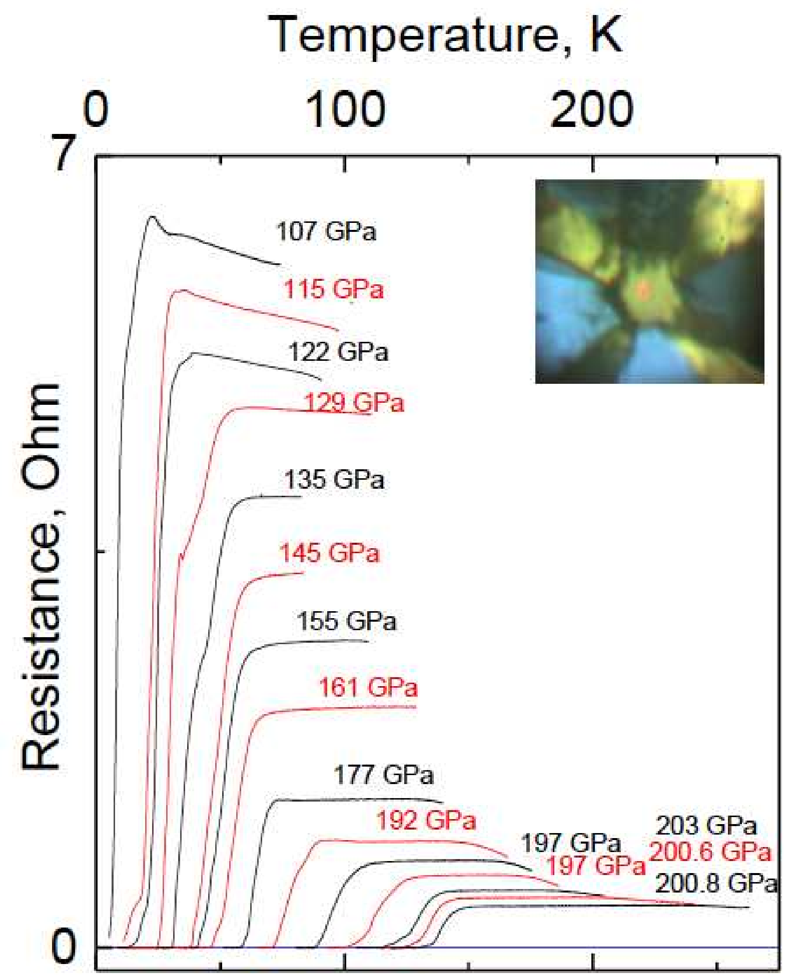
Nature 569, 528–531 (2019)

H_2S dissociates in these conditions leading to H_3S, which was predicted to be a superconductor at high pressures.

Hydrogen atoms are light → high phonon frequencies expected. Large Tc predicted Ashcroft PRL 21, 1748 (1968); 21, 1748 (2004)

Possible role of anharmonic phonons.

Errea et al PRL 114, 157004



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DIRTY SUPERCONDUCTORS (SUPERCONDUCTING ALLOYS)

ANDERSON THEOREM

Non-magnetic impurities in an isotropic (s-wave) superconductor do not affect superconductivity: same T_c, Δ and density of states. Time-reversal symmetry is not broken.

Valid as long as the disorder potential does not localize the conduction electrons.

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DIRTY SUPERCONDUCTORS (SUPERCONDUCTING ALLOYS)

Magnetic impurities (which break time-reversal symmetry) depress T_c and modify the BCS density of states. (Anderson-Gor'kov).

Pair breaking interactions: increase the energy of one member of the pair and decrease the energy of the other (by affecting spin or momentum).

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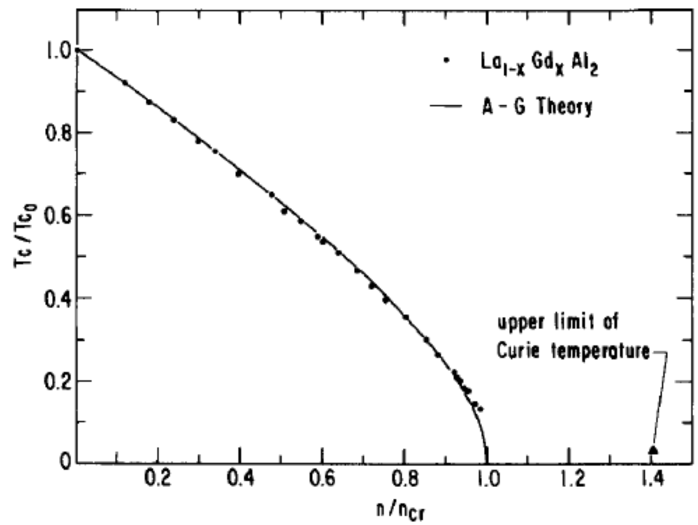
DIRTY SUPERCONDUCTORS (SUPERCONDUCTING ALLOYS)

Magnetic impurities (which break time-reversal symmetry) depress T_c and modify the BCS density of states. (Anderson-Gor'kov).

Paramagnetic impurities can suppress superconductivity

$$\ln T_c/T_{c_0} = \Psi\left(\tfrac{1}{2}\right) - \Psi\left(\tfrac{1}{2} + 0.140 n T_{c_0}/n_{cr} T_c\right)$$

$n_{cr} = 0.88\ T_{c0}$



Maple, Phys. Lett. A 26, 513 (1968)

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DIRTY SUPERCONDUCTORS (SUPERCONDUCTING ALLOYS)

Magnetic impurities (which break time-reversal symmetry) depress T_c and modify the BCS density of states. (Anderson-Gor'kov).

n_{cr}=0.88 T_{c0} but the gap disappears for n'=0.91 n_{cr}.
GAPLESS SUPERCONDUCTIVITY Phillips, PRL 10, 96 (1963)

Pair breaking parameter $\alpha = \dfrac{1}{\tau_s}$

2-fluid model

Finite lifetime → energy broadening → states in the gap.

↓

The gap goes to zero faster than T_c

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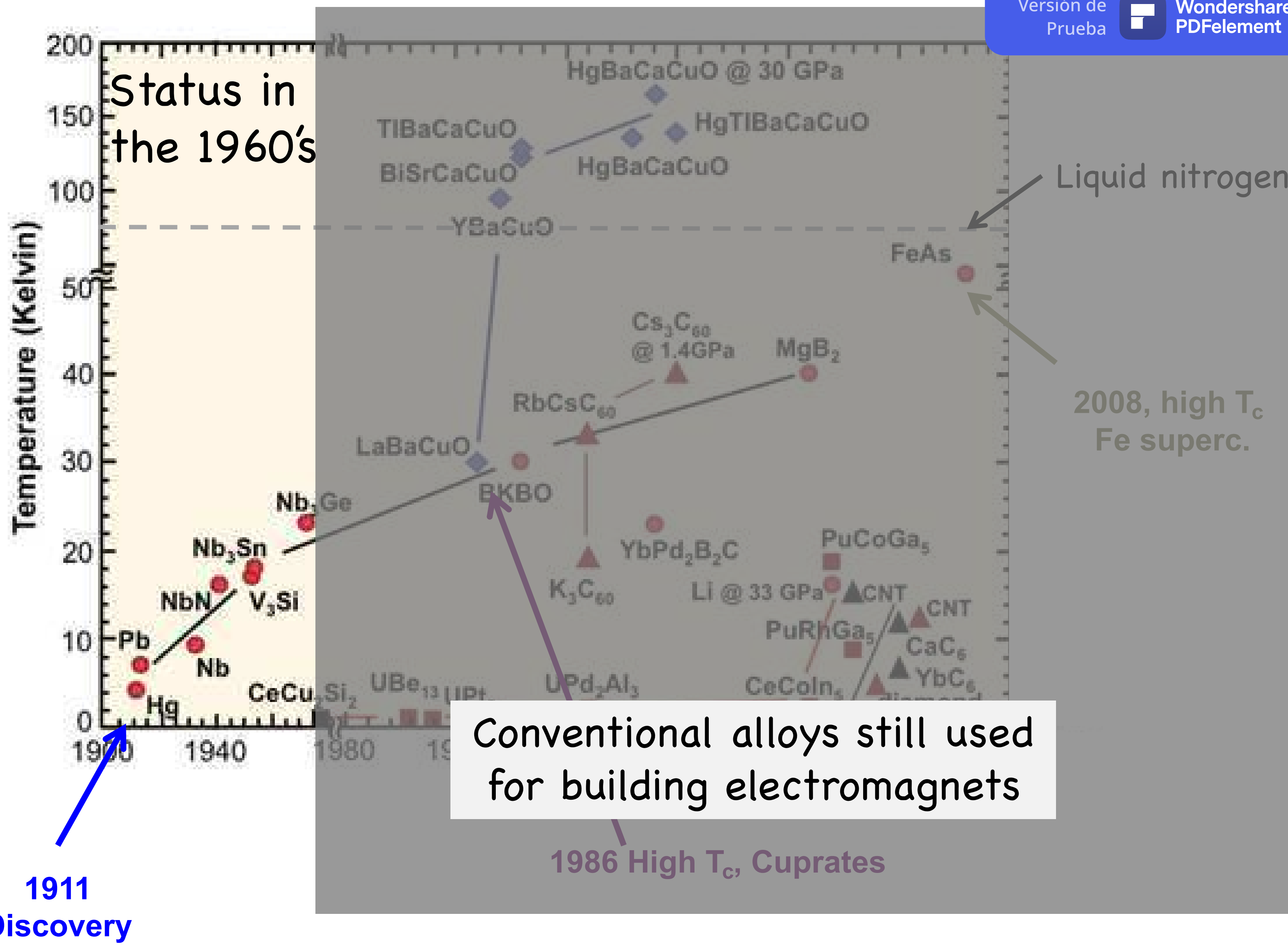

http://www.ccas-web.org/

Mazin, Nature

MATTHIAS'S RULES

- High symmetry (cubic best) is good
- High density of electronic states is good.
- Stay away from oxygen
- Stay away from magnetism (PM impurities destroy superconductivity)
- Stay away from insulators
- Stay away from theorists

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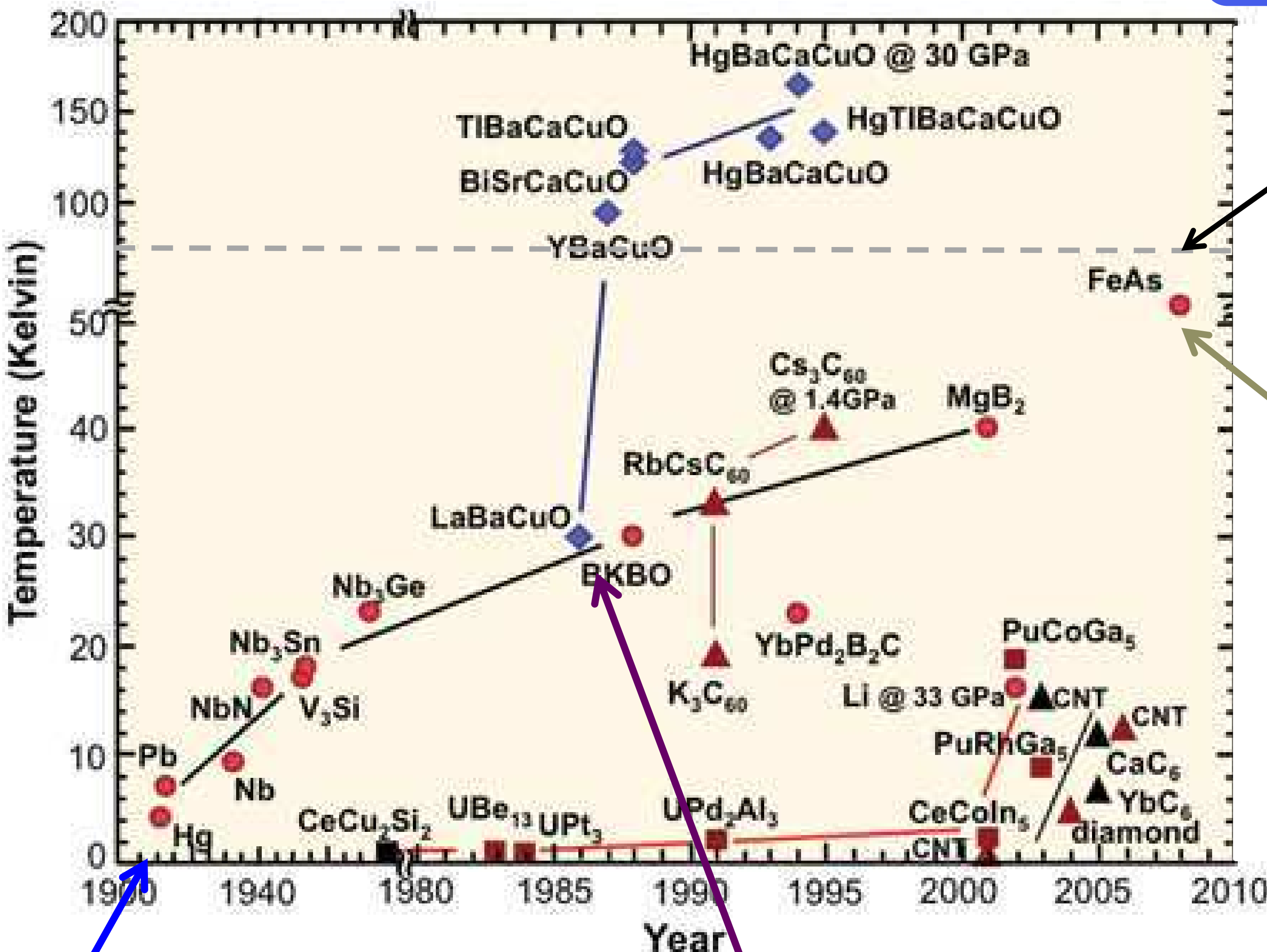
HOWEVER, THERE WERE MANY SURPRISES AHEAD…



Liquid nitrogen

2008, high T_c Fe superc.

1986 High T_c, Cuprates

1911 Discovery

http://www.ccas-web.org/