Emergence of Quantum Phases in Novel Materials

Fermi liquid theory

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Outline

❑Why does band theory work? Concepts: Adiabaticity and quasiparticles

- Definition of adiabaticity and quasiparticles
- Quasiparticle weight
- Electronic distribution function
- Quasiparticle decay and spectral function

❑ Energy as a functional of the number of quasiparticles

- Renormalized mass
- Interaction parameters

❑ Measurable quantities. Fermi liquid behavior

- Specific heat
- Mass in ARPES
- Spin susceptibility
- Resistivity

❑Instabilities of the Fermi liquid

Some references

- Introduction to many body physics. Piers Coleman. Cambridge University Press.

- Theory of Quantum Liquids. Phillipe Nozieres, David Pines. Advanced Books Classics.

- A guide to Feynmann Diagrams in the Many Body problem. E.D. Mattuck. Dover Books on Physics

- Metal-insulator transitions. M. Imada, A. Fujimori, Y. Tokura. Rev. Mod. Phys. 70, 1039 (1998)

Why does an independent electron model work at all?

Band theory:

Basis of our understanding of solids

-Successful description

- Metals and insulators

- Dependence on temperature of measurable quantities (Cv, χ, ..)

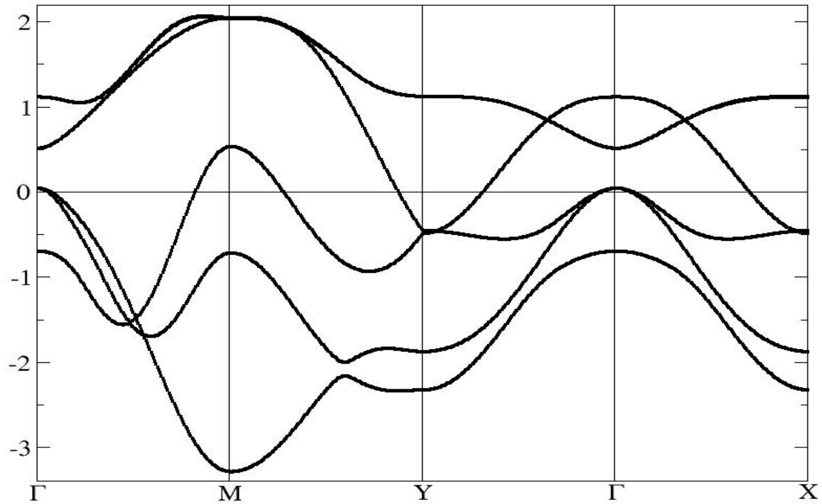


Fig: Calderón et al, PRB, 80, 094531 (2009)

Interactions:

Electrons interact between them (and also with the lattice)

Interaction energies are not small when compared with kinetic energy

Interactions have not been included in the band picture beyond a simple mean field

Why does band theory work?

Fermi liquid theory

Does it always work?

NO (Mott physics, Luttinger liquids …)

What does it mean that the “independent” electron model works?

Band theory:

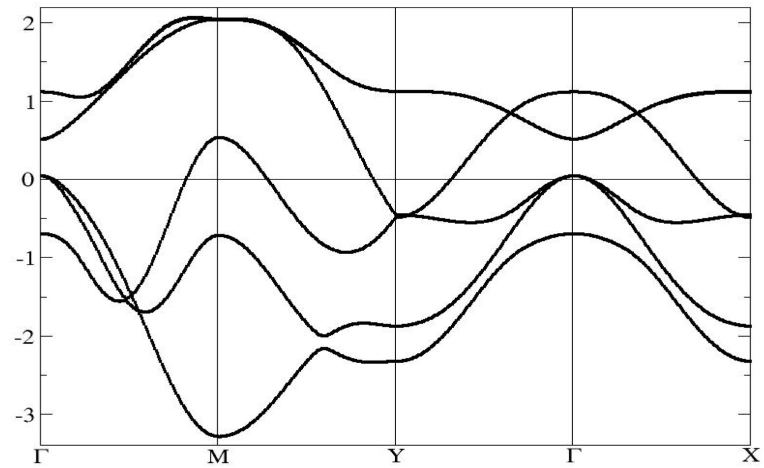
Energy levels filled following Fermi-Dirac distribution. Fermi surface.

Let’s add electron with momentum k to ground state.

The new electron occupies an eigenstate of the system with momentum k.

Energy levels are not modified, just filled. **Rigid Band shift**

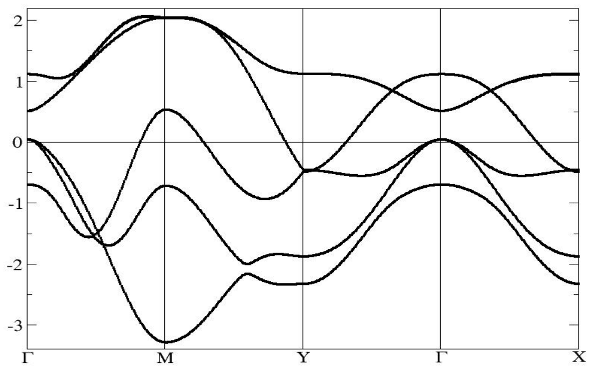
Description in terms of **single particle states**



What does it mean that the "independent" electron model works?

Independent electron model (Band theory):

Rigid band shift. Single particle states



Interacting system:

Add electron with momentum k to ground state with N electrons

The new electron interacts with other electrons in the system

Energy levels are modified, not just filled. **Non rigid band shift expected**

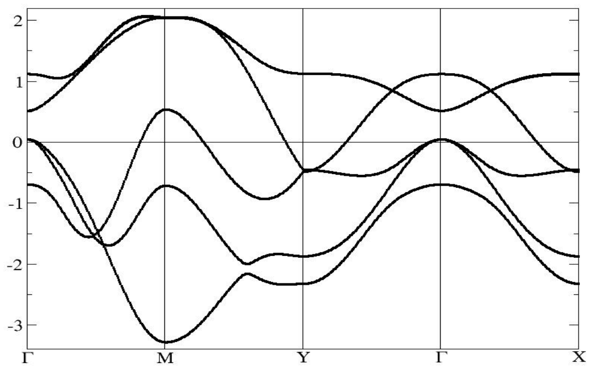
The electron does not fill an eigenstate of the state with N electrons.

Description in terms of **single particle states "does not seem possible".**

"Electronic state expected to decay in excitations of the system".

What does it mean that the “independent” electron model works?

Band theory: Rigid band shift. Single particle states



Interacting system:

Description in terms of **single particle states “does not seem possible”.**

“Electronic state expected to decay in excitations of the system”.

- Pauli principle restricts the phase space for decay of excitations
- The behavior of the system controlled by the low energy excitations. Concept of Quasiparticle

Fermi liquid theory

Fermi liquid theory: general idea

- It justifies the success of band theory

- Focus not in ground state but in **low energy excitations.** Introduces the concept of **quasiparticles**. Valid only at low energies and temperatures.

- Theory written in terms of **parameters**. It goes beyond particular models

- **Phenomenological** theory, but it can be justified with perturbation theory

- **Perturbative** theory, but not restricted to weak interactions.

- Proposed for 3-He: isotropic, no charge, short range interactions, in the continuum limit but it can be generalized to describe electrons in a metal

- **Sometimes it fails**. Correlated electron systems. Non-Fermi liquid behavior

Adiabaticity

- Interacting electronic Hamiltonian $H=H_0+V$ with ground state $|\Psi^*_g>$

we do not know how to solve it

we know how to solve it

- Non interacting electronic Hamiltonian H_0 with ground state $|\Psi_g>$

- Imagine we start at $t=-\infty$ in $|\Psi_g>$ and switch on interactions slowly on time

$$H(t)=H_0+\lambda(t)V \qquad \lambda(t)=e^{-\delta|t|}$$

δ arbitrarily small

Interaction completely switched on at t=0

- Follow the evolution of the ground state from $|\Psi_g>$ to $|\Psi^*_g>$

$$|\Psi_g^*(t)>=V^{ev}|\Psi_g>$$

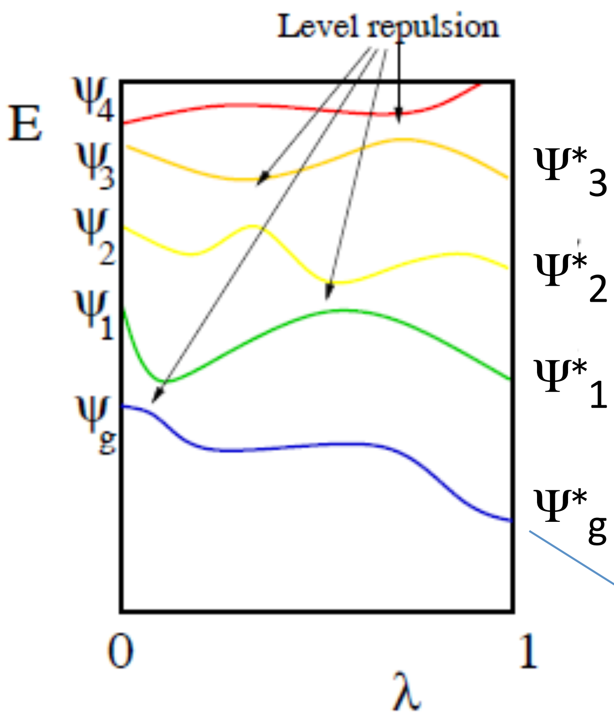
See Coleman's book

Adiabaticity

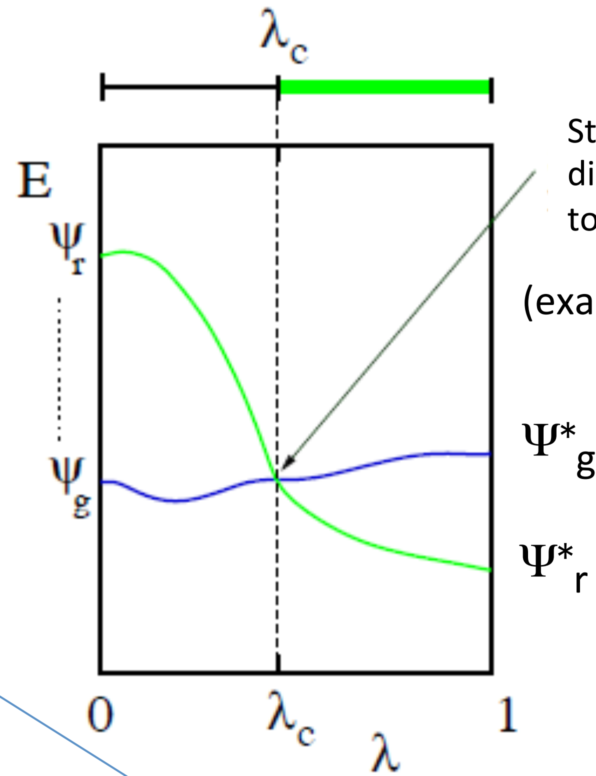
See Coleman's book

$$H = H_0 + \lambda\ V$$

Adiabaticity:



Phase transition:



States which cross have different symmetry or topology …

(example ferromagnetism)

No level crossing :

the ground state of the interacting system can be reached using perturbation theory

Direct connection between the ground state of the interacting & non-interacting hamiltonians

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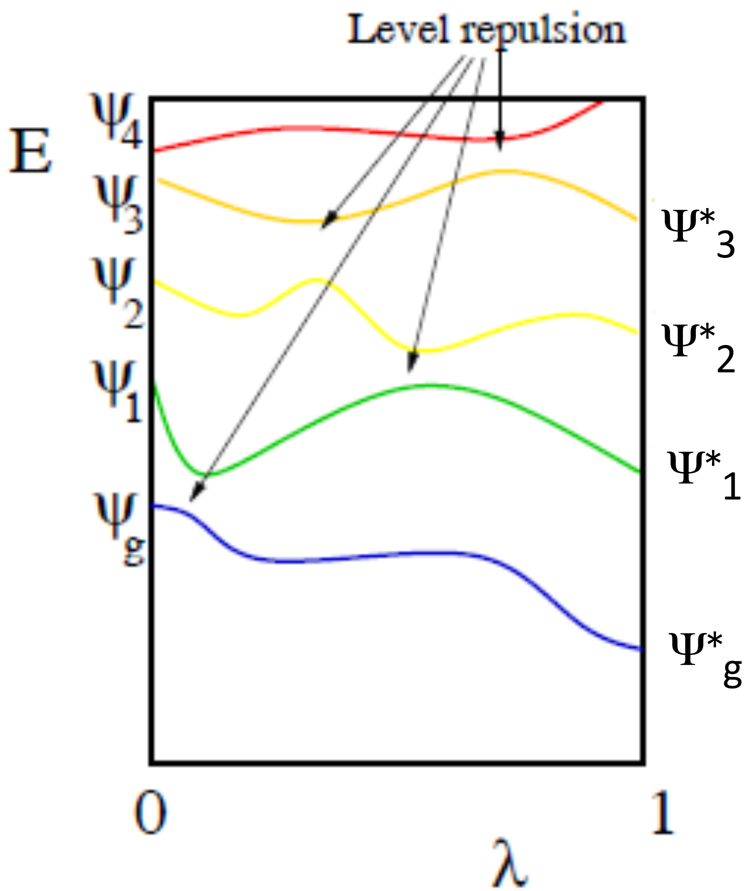
Adiabaticity



No level crossing :

the excited state of the interacting system

can be reached using perturbation theory

Adiabaticity is a requirement for Fermi liquid to work

No phase transition

Excited state.

$C^{\dagger}_{k\sigma}|\Psi*_g>$

Add an electron with momentum K

to **interacting** ground state

See Coleman's book

Adiabaticity and quasiparticles

- Assume the ground states of the interacting $|\Psi^*_g>$ and non-interacting $|\Psi_g>$ N-particle system are connected by adiabaticity

- Consider the non-interacting state of H_0 with N+1 particles (fermions).

$$|k\sigma> = C^+_{k\sigma}\,|\Psi_g>$$

Eigenstate
(Clean system)

The new particle added to an excited state with momentum **k** and spin σ.

(Electron)

Ground state **non-interacting** system with N particles

See Coleman's book

Adiabaticity and quasiparticles

- Assume the ground states of the interacting $|\Psi^*_g>$ and non-interacting $|\Psi_g>$ N-particle system are connected by adiabaticity

- Consider the non-interacting state of H_0 with N+1 particles (fermions).

$$|k\sigma>=C^{+}_{k\sigma}\,|\Psi_g>$$

Eigenstate
(Clean system)

The new particle added
to an excited state with
momentum **k** and spin σ.

Ground state
non-interacting
system with N particles

- Switch on interactions and look at interacting state $H= H_0+ V$ with N+1 particles

$$|k\sigma^*>=V^{ev}|k\sigma>$$

See Coleman's book

Adiabaticity and quasiparticles

Same charge,
spin and momentum

$$|k\sigma^*\rangle = V^{ev}|k\sigma\rangle$$

Interactions conserve
charge, momentum and spin
(Assumption: no spin-orbit coupling)

See Coleman's book

Adiabaticity and quasiparticles

Same charge,
spin and momentum

$$|k\sigma^*\rangle = V^{ev}|k\sigma\rangle$$

Interactions conserve
charge, momentum and spin

$$|k\sigma\rangle = C^{+}_{k\sigma}\,|\Psi_g\rangle$$

Particle: Elementary excitation
of the non-interacting system
Charge e, spin 1/2, momentum k

$$|k\sigma^*\rangle = a^{+}_{k\sigma}\,|\Psi^*_g\rangle$$

Quasiparticle $\longrightarrow$ Elementary excitation of the interacting system with momentum k, spin 1/2, and charge e

See Coleman's book

Adiabaticity and quasiparticles

Same charge,
spin and momentum

Interactions conserve
charge, momentum and spin

$$|k\sigma^*\rangle = V^{ev}|k\sigma\rangle$$

$$|k\sigma\rangle = C^{+}_{k\sigma}\,|\Psi_g\rangle$$

Particle: Elementary excitation of the non-interacting system
Charge e, spin 1/2, momentum k

$$|k\sigma^*\rangle = a^{+}_{k\sigma}\,|\Psi^*_g\rangle$$

Quasiparticle → Elementary excitation of the interacting system with momentum k, spin 1/2, and charge

Different operatos for particle (electron) & quasiparticle

$$a^{\dagger}_{k\sigma} = V^{ev} c^{\dagger}_{k\sigma} V^{ev\dagger}$$

See Coleman's book

Adiabaticity and quasiparticles

- Fermi liquid theory requires adiabaticity
  - Perturbative effect of the interactions
  - No phase transition with respect to the non-interacting system
- Quasiparticle: elementary excitation of the interacting system.

Charge e, spin σ, momentum k ensured by adiabaticity and interactions which conserve charge, spin and momentum

$$|k\sigma*\rangle = a^{+}_{k\sigma}|\Psi*_g\rangle = V^{ev}(C^{+}_{k\sigma}|\Psi_g\rangle)$$

Definition of quasiparticle $a^{\dagger}_{k\sigma} \neq c^{\dagger}_{k\sigma}$

- In an experiment:

electron $c^{+}_{k\sigma}|\Psi*_g\rangle$ STM

$c_{k\sigma}|\Psi*_g\rangle$ ARPES

Adiabaticity and quasiparticles

$c^{\dagger}_{k\sigma}|\Psi^*_g>$

The elementary excitations of the interacting system are the quasiparticles $\mathbf{a}^{\dagger}_{k\sigma}$ not the electrons.

- Write $c^{\dagger}_{k\sigma}$ in terms of the elementary excitations of the interacting system $a^{\dagger}_{k\sigma}$

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} \underbrace{A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)}_{} a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

Generic expression which conserves charge, momentum & spin

If different from zero the electron decays into Particle-hole quasiparticle excitations

higher order decay processes

See Coleman's book

Adiabaticity and quasiparticles

$C^{\dagger}_{k\sigma}|\Psi^*_g>$

- Write the electron operator $c^{\dagger}_{k\sigma}$ in terms of the excitations of the interacting system $a^{\dagger}_{k\sigma}$

Same charge, momentum & spin

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma) a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

A single particle description of $C^{\dagger}_{k\sigma}|\Psi^*_g>$ makes sense if Z_k is finite

higher order decay processes

$$Z_k = |<\Psi^*_g|a_{k\sigma}c^{\dagger}_{k\sigma}|\Psi^*_g>|^2 > 0$$

Quasiparticle weight

Overlap between the state reached adding an electron to the interacting system and the state which results from adding an elementary excitation to the interacting system

See Coleman's book

Adiabaticity and quasiparticles

$c^{\dagger}_{k\sigma} | \Psi *_g >$

- Write the electron operator $c^{\dagger}_{k\sigma}$ in terms of the excitations of the interacting system $a^{\dagger}_{k\sigma}$

Same charge, momentum & spin

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}} a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma) a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

higher order decay processes

$$0 \leq Z_k \leq 1$$

Quasiparticle weight

Z_k Measures the strength of the correlations & & the validity of the Fermi liquid description

- $Z_k=1$ non interacting system
- $Z_k=0$ Fermi liquid theory not applicable
- $Z_k>0$ ensures one to one correspondence between electron and quasiparticle.

See Coleman's book

Adiabaticity and quasiparticles

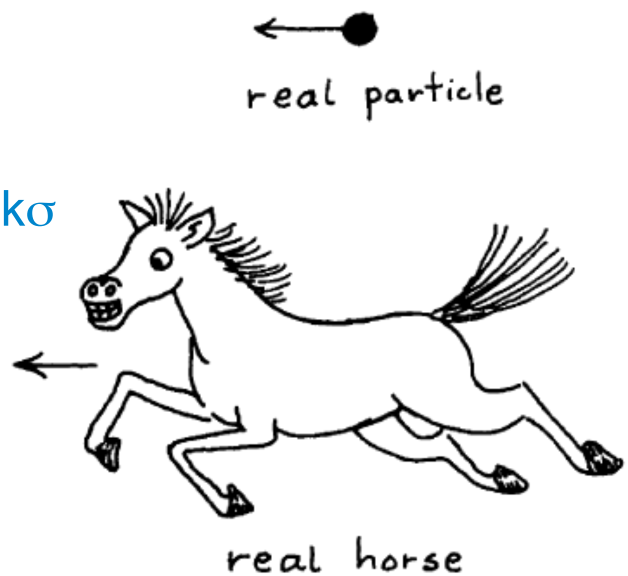
Interacting system, **if Z_k is finite**

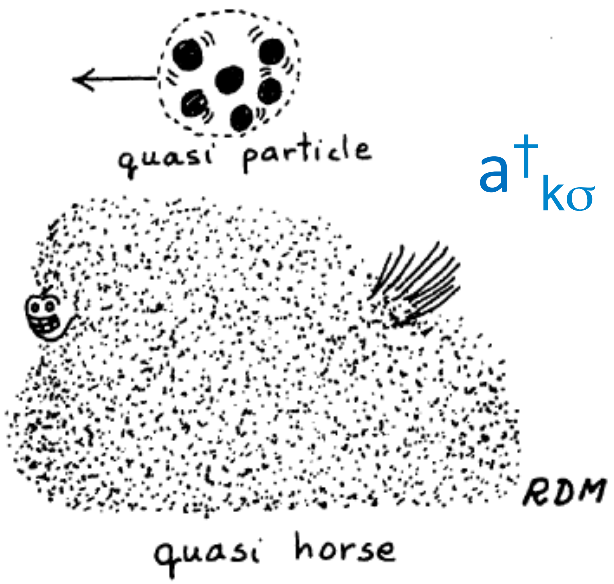
Elementary excitations:

quasiparticles with charge e and spin ½ and momentum k

-The quasiparticles are not electrons but there is a one-to-one correspondence with an el

$C^{\dagger}_{k\sigma}$





$a^{\dagger}_{k\sigma}$

Mattuck's book

Z_k gives the part of the excitation $C^{\dagger}_{k\sigma}|\Psi^{*}_{g}\rangle$ which is in the quasiparticle state

(how much of the quasi-horse is really the horse)

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Adiabaticity and quasiparticles

- Fermi liquid theory requires adiabaticity
  - Perturbative effect of the interactions
  - No phase transition with respect to the non-interacting system
- Quasiparticle: elementary excitation of the interacting system.

Charge e, spin σ, momentum k ensured by adiabaticity and interactions which conserve charge, spin and momentum

$$|k\sigma^*\rangle = a^+_{k\sigma}\,|\Psi^*_g\rangle = V^{ev}\left(C^+_{k\sigma}\,|\Psi_g\rangle\right)$$

- Quasiparticle weight: Overlap between the elementary excitations of the interacting and non-interacting systems. $0 \leq Z_k \leq 1$ strength of correlations

$$c^\dagger{}_{\mathbf{k}\sigma} = \sqrt{Z_\mathbf{k}}\,a^\dagger{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\,a^\dagger{}_{\mathbf{k}_4\sigma_4}a^\dagger{}_{\mathbf{k}_3\sigma_3}a_{\mathbf{k}_2\sigma_2} + \ldots$$

$Z_k>0$ required for the applicability of Fermi liquid theory

Adiabaticity and quasiparticles. Fermi surface

$|\Psi_g> \longrightarrow |\Psi^*_g>$ Adiabatically connected

$c^\dagger_{k\sigma}|\Psi^*_g> \longrightarrow a^\dagger_{k\sigma}$ Quasiparticle

$c_{k\sigma}|\Psi^*_g> \longrightarrow a_{k\sigma}$ Quasihole

Quasiparticle, Quasihole → Fermionic excitations

↓

Fermi-Dirac distribution

Fermi surface

Electronic distribution. Jump at Fermi Surface

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

$$n_k = <\Psi^*{}_g | C^{\dagger}{}_{k\sigma} C_{k\sigma} | \Psi^*{}_g> = Z_k <\Psi^*{}_g | a^{\dagger}{}_{k\sigma} a_{k\sigma} | \Psi^*{}_g> + \text{continuum}$$

Distribution of real electrons

Fermi Dirac statistics

At T=0 the distribution of **quasiparticles** has jump of height 1 at the Fermi surface $\Theta(-\varepsilon_k)$

Continuum:
Amount of the Electronic excitation which is **not in the quasiparticle state**

also called incoherentpart

Electronic distribution. Jump at Fermi Surface

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)a^{\dagger}{}_{\mathbf{k}_4\sigma_4}a^{\dagger}{}_{\mathbf{k}_3\sigma_3}a_{\mathbf{k}_2\sigma_2} + \ldots$$

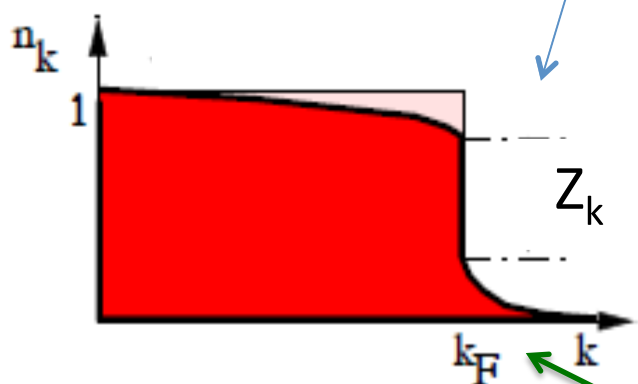
$$n_k = \langle \Psi^*{}_g | C^{\dagger}{}_{k\sigma} C_{k\sigma} | \Psi^*{}_g \rangle = Z_k \langle \Psi^*{}_g | a^{\dagger}{}_{k\sigma} a_{k\sigma} | \Psi^*{}_g \rangle + \text{continuum}$$

Fermi Dirac statistics

Continuum: Amount of the Electronic excitation which is **not in the quasiparticle state**

At T=0 the distribution of **electrons** has a jump of height $\mathbf{Z_k}$ (not 1) at the Fermi level

$$Z_k \; \Theta(-\varepsilon_k)$$



Leakage of electrons away from the Fermi surface (stronger correlations larger leakage)

Adiabaticity and quasiparticles. Fermi surface and decay

$|\Psi_g> \longrightarrow |\Psi^*_g>$ Adiabatically connected

Non-interacting system

$|k\sigma>=C^+_{k\sigma}|\Psi_g>$ **Eigenstate**

Interacting system

$|k\sigma^*>=a^+_{k\sigma}|\Psi^*_g>$ **Not an Eigenstate** $\longrightarrow$ **Finite lifetime** $\tau=\Gamma^{-1}$ (width level)

The quasiparticles also decay

Adiabaticity and quasiparticles. Fermi surface and decay

$|\Psi_g> \longrightarrow |\Psi^*_g>$ Adiabatically connected

Non-interacting system

$|k\sigma>=C^+_{k\sigma}|\Psi_g>$ **Eigenstate**

Interacting system

$|k\sigma^*>=a^+_{k\sigma}|\Psi^*_g>$ **Not an Eigenstate** $\longrightarrow$ **Finite lifetime** $\tau=\Gamma^{-1}$ (width level)

Stability of the quasiparticle requires $\Gamma << \varepsilon$ Decay rate of the quasiparticle Much smaller than its energy

Quasiparticle decay

A quasiparticle with $k>k_F$ cannot decay into an **occupied** state below the FS with $k<k_F$

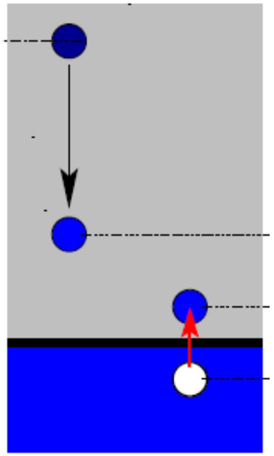
Decay of quasiparticles conserves momentum, charge & spin

$a^\dagger_{k\sigma}$

2 particles and 1 hole → $a^\dagger_{k1\sigma}a^\dagger_{k2\sigma}a_{k3\sigma}$

3 particles and 2 holes → $a^\dagger_{k1\sigma}a^\dagger_{k2\sigma}a^\dagger_{k4}a_{k3\sigma}a_{k3\sigma}$

…



Decay of one quasiparticle
in a 3-body process

Pauli principle restricts
the decay processes allowed

Figs. Coleman's book

Quasiparticle decay

A quasiparticle with $k>k_F$ cannot decay into an **occupied** state below the FS with $k<k_F$

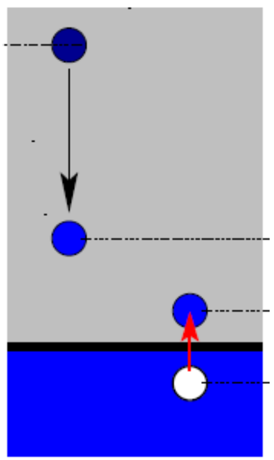
Decay of quasiparticles conserves momentum, charge & spin

$a^\dagger_{k\sigma}$

2 particles and 1 hole $\rightarrow$ $a^\dagger_{k1\sigma}a^\dagger_{k2\sigma}a_{k3\sigma}$

3 particles and 2 holes $\rightarrow$ $a^\dagger_{k1\sigma}a^\dagger_{k2\sigma}a^\dagger_{k4}a_{k3\sigma}a_{k3\sigma}$

...



$$\tau^{-1}=\frac{\varepsilon^{*2}+\pi^2T^2}{E^*_F}$$

$\varepsilon^*=E^*-E^*_F$

Energy from Fermi Surface

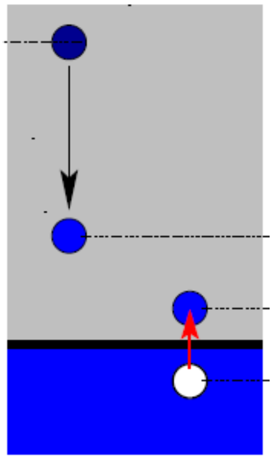
Estimate in 3D based on phase space considerations (Pauli principle) & 3 body decay

Decay of one quasiparticle
3-body process

Quasiparticles well defined at low energies & temperatures

Infinite lifetime at the Fermi surface at zero temperature

Quasiparticle decay



Decay of one quasiparticle
3-body process

$$\frac{\Gamma}{\varepsilon} \sim \varepsilon$$ Estimate in 3D

$$\frac{\Gamma}{\varepsilon} \sim \varepsilon \, \mathrm{Ln} \, \varepsilon$$ Estimate in 2D

$$\frac{\Gamma}{\varepsilon} \sim \mathrm{const}$$ Estimate in 1D

$\varepsilon^*=E^*-E^*_F$

Energy from
Fermi Surface

Quasiparticles well defined at low energies & temperatures

Infinite lifetime at the Fermi surface at zero temperature

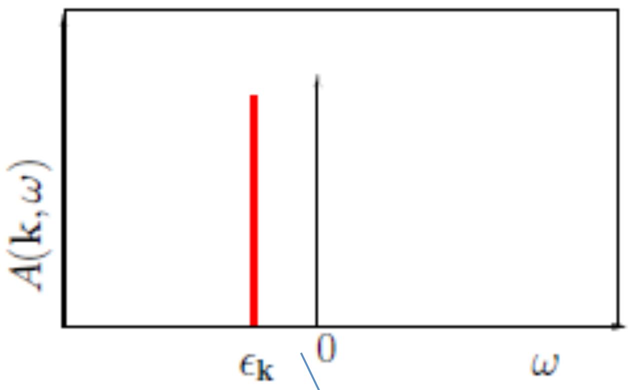
in 3D and 2D but not 1D. Fermi liquid theory fails in 1D

Giuliani PRB 26, 4428 (1982)

Spectral function A(k,ω)

Distribution of excitations created when a particle is added or removed from the system

- Non interacting system

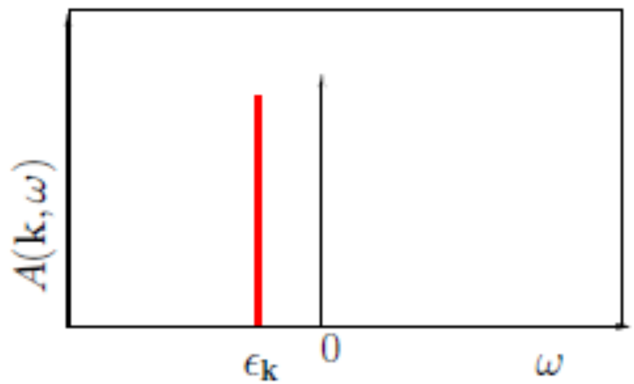


Eigenstate, well defined energy for a given momentum. Infinite lifetime

Spectral function $A(k,\omega)$

Distribution of excitations created when **an electron** is added or removed from the system

- ❑ Non interacting system

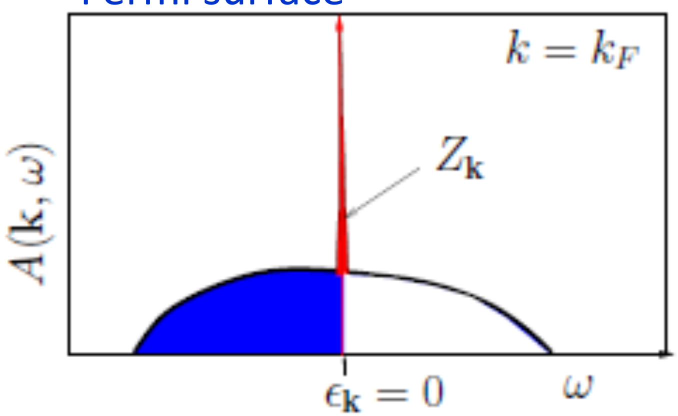


Quasiparticle peak:
Part of the electronic excitation which is in The quasiparticle state
Coherent part

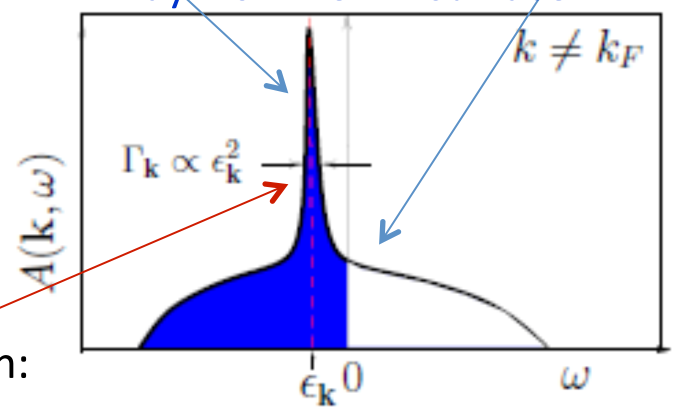
Continuum:
Amount of the Electronic excitation which is **not in the quasiparticle state**
Incoherent part

- ❑ Interacting system

Fermi surface



Away from Fermi surface



Peak width:
Decay of
the quasiparticle

Figs. Coleman's book

Spectral function A(k,ω)

Distribution of excitations created when
an electron is added or removed from the system

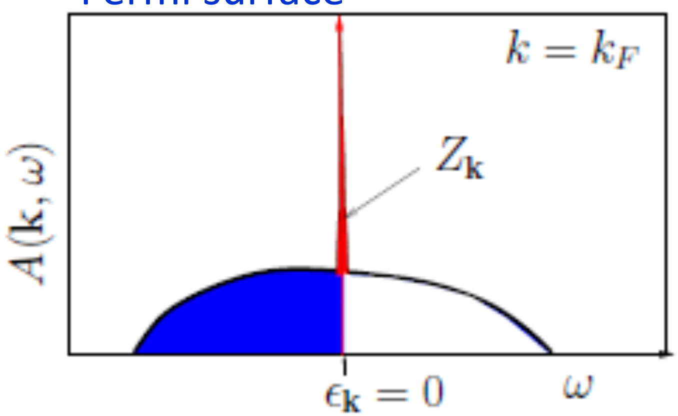
$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\,a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

- Interacting system

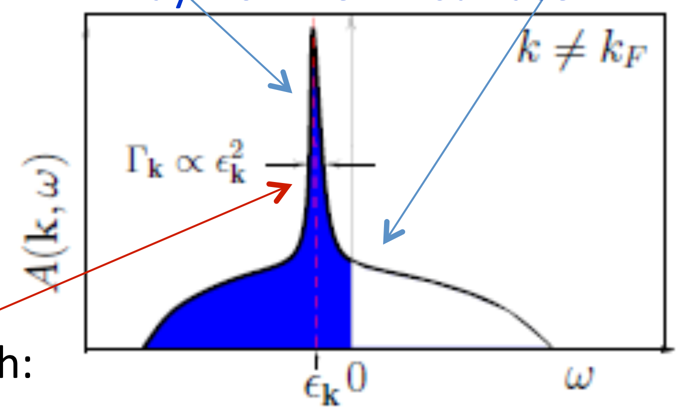
Quasiparticle peak:
Part of the electronic
excitation which is in
The quasiparticle state
Coherent part

Continuum:
Amount of the
Electronic excitation
which is **not in the quasiparticle state**
Incoherent part

Fermi surface



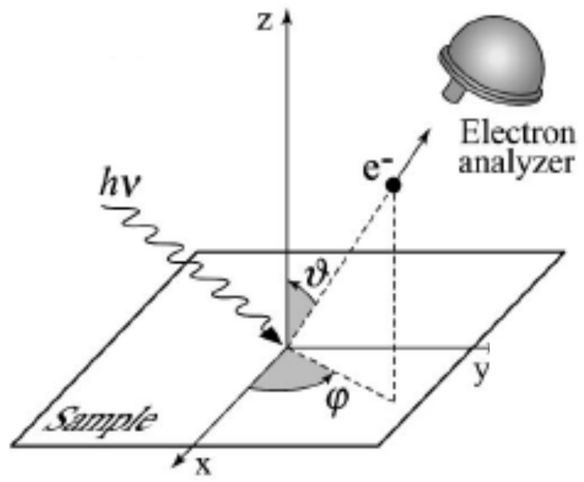
Away from Fermi surface

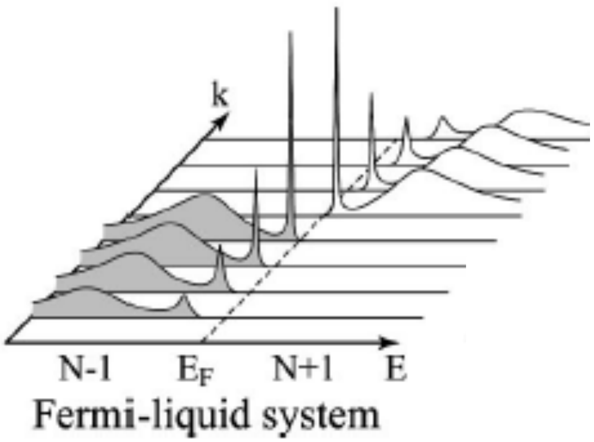


Peak width:
Decay of
the quasiparticle

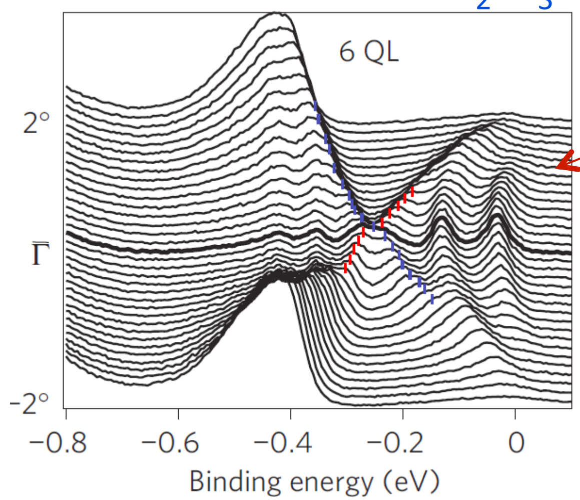
Figs. Coleman's book

Spectral function A(k,ω) and ARPES





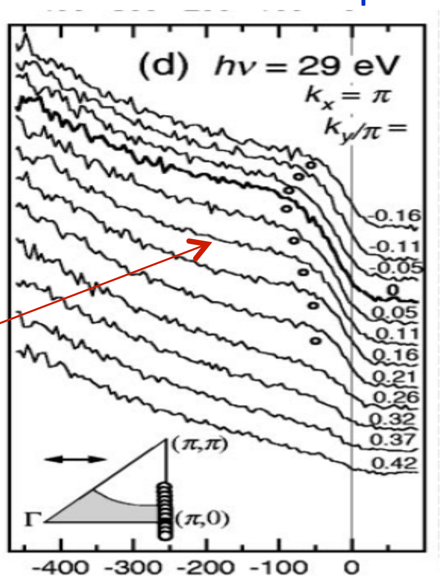
Bi_2Se_3



Well defined Quasiparticle peaks

Badly defined quasiparticle peak

Cuprate



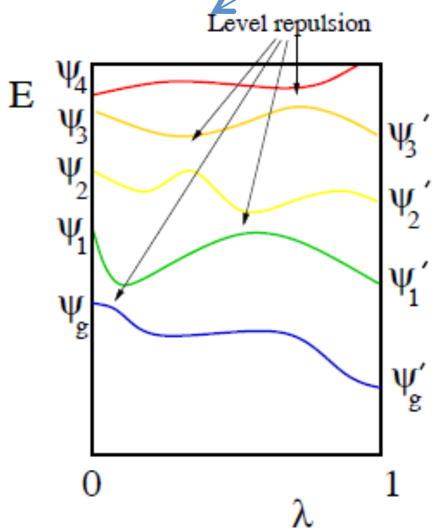
Possibility to map the bands

Figs: Damascelli et al, RMP 2003
Zhang et al, Nat Phys. 2010

Summary: Fermi liquid. Concepts

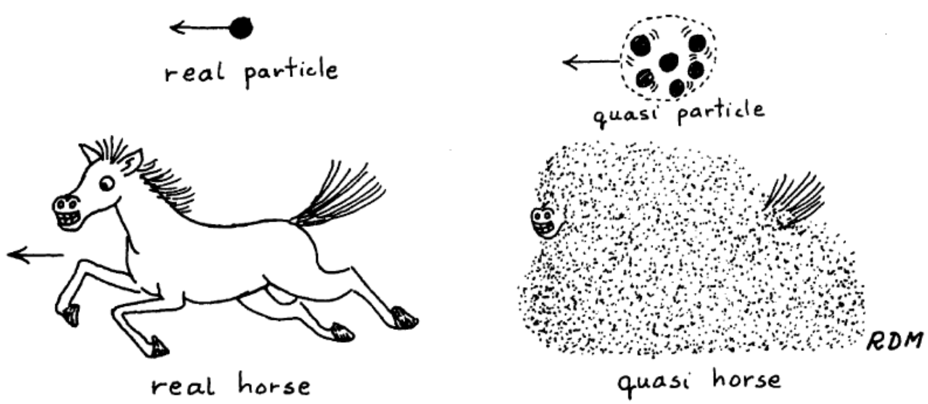
- ❑ Why does band theory work?

- ❑ Fermi liquid: Adiabaticity & Quasiparticles



Interactions connect two ground states with the same symmetry & topology with no phase transition

One to one correspondence between electrons and quasiparticles, respectively the elementary excitations of the non-interacting and interacting systems : same spin, charge and momentum



Summary: Fermi liquid. Concepts

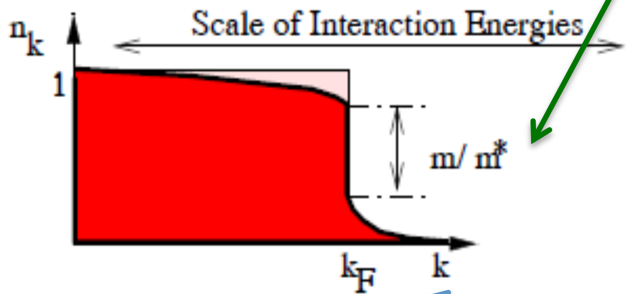
- The electron decays into quasiparticle excitations

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

Quasiparticle weight $0 < Z < 1$ Strength of correlations

Z, inverse of mass enhancement (simple models)

- Quasiparticles are fermions. Fermi surface



Leakage, even at T=0

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Summary: Fermi liquid. Concepts

- Quasiparticles are not eigenstates. Decay. Well defined at low energies and temperatures in 3D, but not in 1D

3D: $$\tau^{-1} = \frac{(\varepsilon^*)^2 + \pi^2 T^2}{E^*_F}$$

Do not confuse the decay of the quasiparticle (width of the quasiparticle level)
with the decay of the electron into quasiparticles

- The electron decays into quasiparticle excitations

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

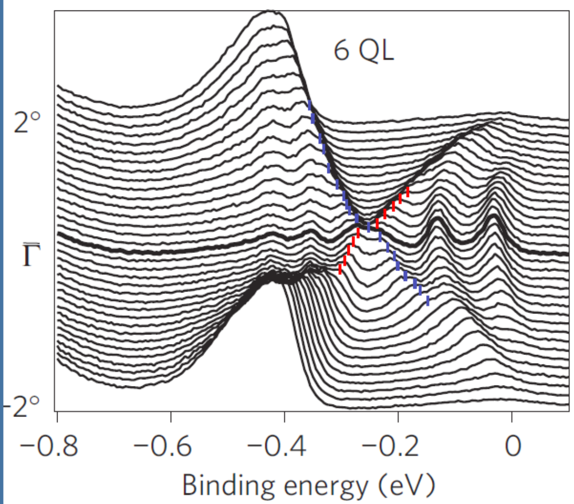
Quasiparticle weight $0 < Z < 1$ Strength of correlations

Summary: Fermi liquid. Concepts

- Quasiparticles are not eigenstates. Decay. Well defined at low energies and temperatures in 3D, but not in 1D

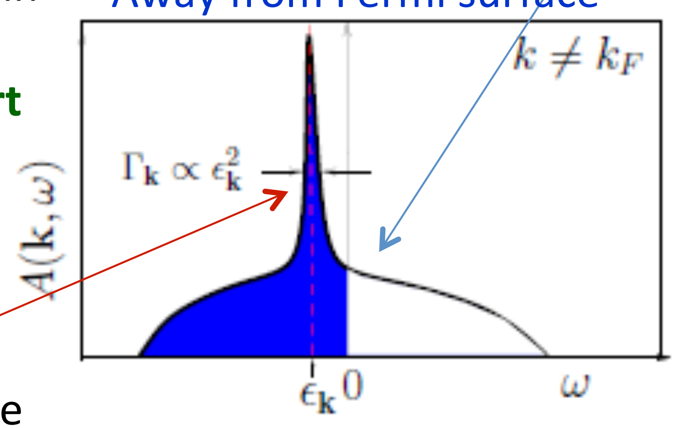
3D: $$\tau^{-1} = \frac{(\varepsilon^*)^2 + \pi^2 T^2}{E^*_F}$$

Continuum:
Amount of the Electronic excitation which is **not in the quasiparticle state**
Incoherent part



Quasiparticle peak:
Part of the electronic excitation which is in the quasiparticle State **Coherent part**

Away from Fermi surface



Peak width:
Decay of
the quasiparticle

Fermi liquid theory

- ❑ Assume adiabaticity: the interacting ground state is perturbatively connected

- ❑Quasiparticles: elementary excitations of the interacting system have a one to one correspondence with elementary excitations of the non-interacting one. Implies quasiparticle weight Z_k finite.

- ❑ Quasiparticles in a metal: fermionic excitations with spin ½ and charge e. Well defined Fermi surface

- ❑ Quasiparticle decay rate in 3D: $\tau^{-1} = \dfrac{(\varepsilon^*)^2+\pi^2T^2}{E^*_F}$

- ❑ Description of the system in terms not of the ground state of the system but in terms of the low energy excitations → deviation from equilibrium δn_p.

- ❑ Energy functional $F[\delta n_p]$ in terms of parameters which can be measured experimentally.

Description in terms of quasiparticles

- Consider non-interacting ground state

$$H = H_0 - \mu N = \sum_{\sigma} (E_{\mathbf{k}} - \mu) c^{\dagger}{}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} \qquad |\psi_g\rangle = \prod_{|\mathbf{k}|<k_F,\ \sigma} c^{\dagger}{}_{\mathbf{k}\sigma} |0\rangle$$

- Define the transformation

$$d^{\dagger}{}_{\mathbf{k}\sigma} = \begin{cases} c^{\dagger}{}_{\mathbf{k}\sigma} & (k > k_F) \quad \text{particle} \\ \mathrm{sgn}(\sigma) c_{-\mathbf{k}-\sigma} & (k > k_F) \quad \text{hole} \end{cases}$$

- Rewrite the hamiltonian

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} |(E_{\mathbf{k}} - \mu)| d^{\dagger}{}_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + F_g$$

Deviation from equilibrium occupation

Excitations

Fermionic excitations
Charge e
Spin σ

Fermi liquid theory: description in terms of quasiparticles

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} |(E_{\mathbf{k}} - \mu)\, d^{\dagger}_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + F_g$$

Excitations (equilibrium occupation of states included in F_g)

$\delta n_{k\sigma} = d^{\dagger}_{k\sigma} d_{k\sigma}$ → Density of excitations

$$\boxed{H_0 - \mu N = F_g + \Sigma_{k\sigma} \varepsilon_{k\sigma} \delta n_{k\sigma} = F\,[\delta n_{k\sigma}]}$$

Excitation energy measured with respect to the ground state

Energy written as a functional of the density of excitations

Fermi liquid theory: description in terms of quasiparticles

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} |(E_{\mathbf{k}} - \mu)\, d^{\dagger}_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + F_g$$

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Excitation energy measured with respect to the ground state

Energy written as a functional of the density of excitations

- Landau's idea: Energy of the interacting system is a functional of the density of quasiparticles $F\,[\delta n_{k\sigma}]$. Expansion around equilibrium. Small $\delta n_{k\sigma}$

$$\delta n_{k\sigma} = a^{\dagger}_{k\sigma} a_{k\sigma}$$

Fermi liquid theory: description in terms of quasiparticles

- Landau's idea: Energy of the interacting system is a functional of the density of quasiparticles $F\,[\delta n_{k\sigma}]$. Expansion around equilibrium. Small $\delta n_{k\sigma}$

$$\boxed{F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)}$$

First order in the expansion

second order in the expansion

$$\varepsilon^*_{k\sigma} = \left.\frac{\partial F}{\partial(\delta n_{k\sigma})}\right|_{\delta n_{k\sigma}=0}$$

$$f_{k\sigma k'\sigma'} = \left.\frac{\partial^2 F}{\partial(\delta n_{k\sigma})\partial(\delta n_{k\sigma})}\right|_{\delta n_{k\sigma}=0}$$

Fermi liquid theory: description in terms of quasiparticles

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The effect of interactions parametrized

Fermi liquid theory: Parameters of the model

- To first order:

Energy of a quasiparticle
in the absence of other quasiparticles

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma}$$

Fermi liquid theory: Parameters of the model

- To first order:

Energy of a quasiparticle
in the absence of other quasiparticles

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma}$$

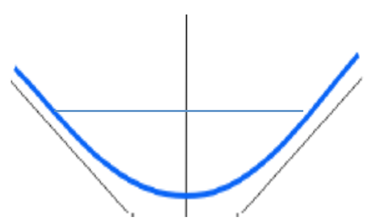
In a non-interacting state (continuum)

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon_{k\sigma}\delta n_{k\sigma}$$

$$\varepsilon_{k\sigma} = k^2/2m - \mu = (k^2 - k_F^2)/2m$$

close to k_F we can linearize the
spectrum (first order expansion in $k-k_F$)

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$



k_F

Fermi liquid theory: Parameters of the model

- To first order:

Energy of a quasiparticle
in the absence of other quasiparticles

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma} \varepsilon^*_{k\sigma} \delta n_{k\sigma}$$

In a non-interacting state (continuum)

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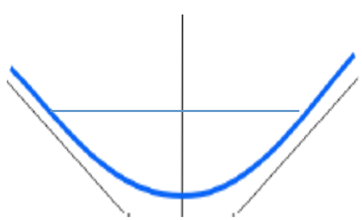

k_F

By analogy, in the interacting system, we linearize the spectrum close to k_F and **define m***

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

Renormalized mass or quasiparticle mass

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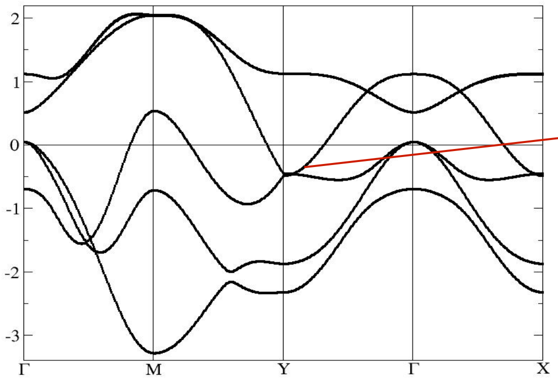
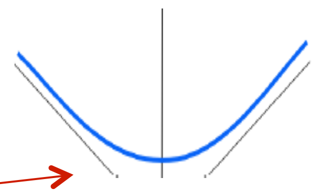
Fermi liquid theory: Parameters of the model

- Non-interacting system:





Band mass different to free electron mass
Modification due to ionic potential

$$m^{-1} = |\partial^2\varepsilon/\partial k^2|$$

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$

- Interacting system:

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

Quasiparticle mass → Different from m due to electronic interactions

Fermi liquid theory: description in terms of quasiparticles

- Landau's idea: Energy of the interacting system is a functional of the density of quasiparticles $F\,[\delta n_{k\sigma}]$. Expansion around equilibrium. Small $\delta n_{k\sigma}$

$$\boxed{F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)}$$

First order in the expansion

second order in the expansion

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

Band mass

$$Z = \frac{m}{m^*}$$

If Z vanishes (no quasiparticle weight) the expansion does not converge
Fermi liquid theory is not applicable

(simple models)

Fermi liquid theory: Interactions between quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

Energy of a quasiparticle in the absence of other quasiparticles

Residual interaction between quasiparticles

Quasiparticles are not eigenstates

Decay of quasiparticle states

Fermi liquid theory: description in terms of quasiparticles

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} |(E_{\mathbf{k}} - \mu)\ d^{\dagger}_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + F_g$$

Excitations (equilibrium occupation of states included in F_g)

$\delta n_{k\sigma} = d^{\dagger}_{k\sigma} d_{k\sigma}$ → Density of excitations

$$\boxed{H_0 - \mu N = F_g + \Sigma_{k\sigma}\varepsilon_{k\sigma}\delta n_{k\sigma} = F\,[\delta n_{k\sigma}]}$$

Excitation energy measured with respect to the ground state

Energy written as a functional of the density of excitations

Non interacting case → Absence of second order term $\Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'} \delta n_{k\sigma} \delta n_{k'\sigma'}$

Fermi liquid theory: Interactions between quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

Energy of a quasiparticle in the absence of other quasiparticles

Interaction between quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}(\varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k'\sigma'})\delta n_{k\sigma}$$

The energy of a quasiparticle Is modified by the presence of other quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon'_{k\sigma}\delta n_{k\sigma}$$

$$\varepsilon'_{k\sigma} = \varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k'\sigma'}$$

Non-rigid band shift

Fermi liquid theory: Interaction parameters

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

Remember assumption of isotropic system

Focus on the Fermi surface $|k|=|k'|=K_F$

Time-reversal invariance (no magnetic field) $\quad f_{k\sigma,k'\sigma'}=f_{-k-\sigma,-k'-\sigma'}$

Fermi surface invariant under reflection $k \rightarrow -k$ $\quad f_{k\sigma,k'\sigma'}=f_{k-\sigma,k'-\sigma'}$

If spin is conserved in general the dependence on spin enters only via their relative orientation

$$f_{k\sigma,k'\sigma'}=f_{kk'}^{s}+f_{kk'}^{a}\sigma.\sigma'$$

Spin symmetric

Spin antisymmetric (exchange)

Pines & Nozieres and Coleman's books

Fermi liquid theory: Interaction parameters

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

$f_{kk'}^{s}$ $\qquad$ $f_{kk'}^{a}$

Focus on the Fermi surface $|k|=|k'|=K_{F.}$ Isotropic system

$f_{kk'}^{s,a}$: dependence only on the angle ξ between k and k' $\rightarrow$ Expansion in Legendre Polynomials

$$f_{kk'}^{s,a} = \Sigma_{l=0}^{\infty} \mathbf{f_l^{s,a}} P_l(\cos\xi)$$

Fermi liquid theory: Interaction parameters

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

$f_{kk'}^{s}$ $\quad$ $f_{kk'}^{a}$

Focus on the Fermi surface $|k|=|k'|=K_F$. Isotropic system

$f_{kk'}^{s,a}$: dependence only on the angle ξ between k and k' → Expansion in Legendre Polynomials

$$f_{kk'}^{s,a} = \Sigma_{l=0}^{\infty} \mathbf{f_l^{s,a}} P_l(\cos\xi)$$

Dimensionless parameters $\quad N^*(E^*_F)\, f_l^{s,a} = \mathbf{F_l^{s,a}}$

Density of quasiparticle states at the Fermi level In the interacting system

Interaction parameters

$F_0^s, F_0^a, F_1^s, F_1^a \ldots$

Can be extracted from experiment and model calculations

Fermi liquid theory: Interaction and parameters. Summary

Expansion of the free energy in terms of the quasiparticle density

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma} \varepsilon^*_{k\sigma} \delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'} \delta n_{k\sigma} \delta n_{k'\sigma'} + O(\delta n^3)$$

Energy of a quasiparticle
In the absence of other
quasiparticles

Residual interactions
between
the quasiparticles

Linearized dispersion around K_F

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k - k_F)$$

Renormalized mass or
quasiparticle mass

$$f_{kk'}^{s,a} = \Sigma_{l=0}^{\infty} f_l^{s,a} P_l(\cos \xi)$$

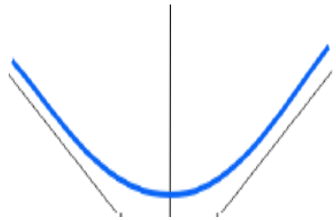
$$N^*(E^*_F)\, f_l^{s,a} = F_l^{s,a}$$

Small number of parameters l=0,1 …

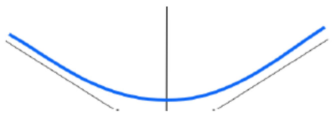
Fermi liquid theory: Measurable quantities

Bandwidth renormalization

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$

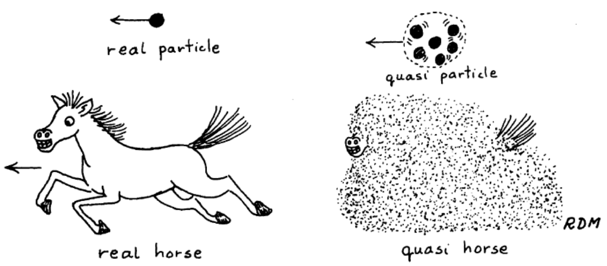


$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$



m*>m always

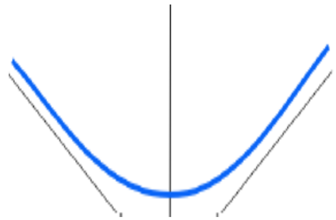
The backflow of the surrounding fluid enhances the mass



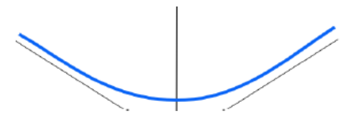
Fermi liquid theory: Measurable quantities

Bandwidth renormalization

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$



$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$



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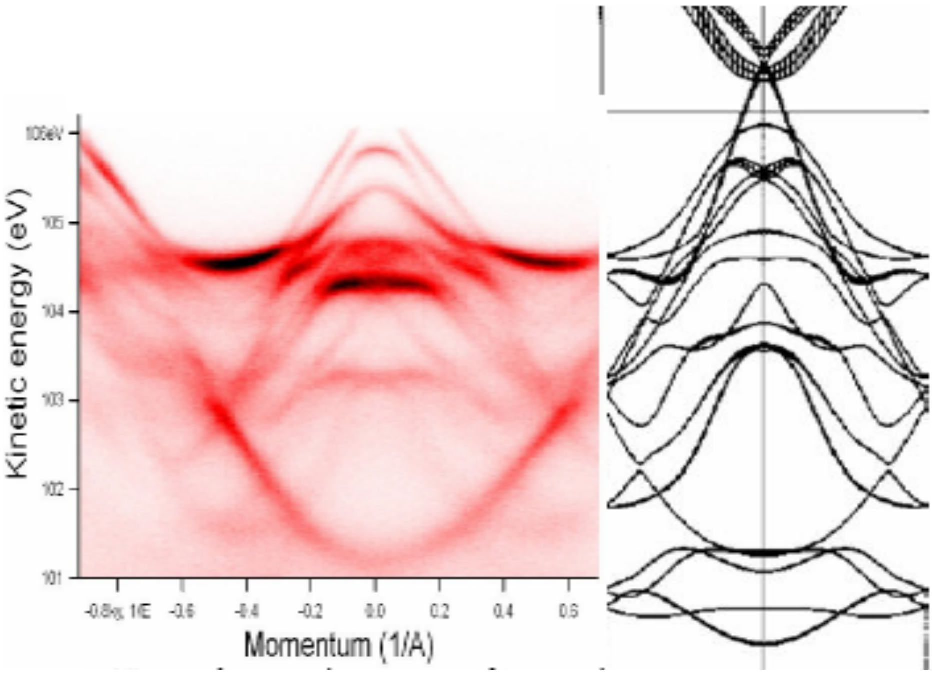


Fig. : Evtushinksy lectures

Bands can be visualized via angle resolved photoemmission ARPES and the effective mass obtained comparing the experimental bands with the LDA prediction (also quantum oscillations)

How large is m^*/m?

Experimental

LDA

Fermi liquid theory: Measurable quantities

Non-interacting Fermi gas: $K_B T << \mu$

From Fermi-Dirac statistics $$n(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/K_B T}+1}$$

- **Specific heat** $$C_v = \left.\frac{\partial F}{\partial T}\right|_{\mu}$$

$$C_v = \gamma\, T$$ **Linear in temperature** $$\gamma = (\pi/3) K^2_B N(E_F) \propto m$$

- **Spin susceptibility** $$\chi_s = \frac{\partial M}{\partial H}$$

$$\chi_s = \mu_B{}^2 N(E_F) \propto m$$ **independent of T**

Fermi liquid theory: Measurable quantities

Non-interacting Fermi gas: $K_B T << \mu$ From Fermi-Dirac statistics

- **Specific heat** $C_v=\gamma\, T$ **Linear in temperature** $\gamma=(\pi/3)K^2_B N(E_F) \propto m$

- **Spin susceptibility** $\chi_s= \mu_B{}^2 N(E_F) \propto m$ **independent of T**

Interacting Fermi liquid: $\delta n_{k\sigma}=n_{k\sigma}(T,\mu)-n_{k\sigma}(0,\mu)$

$$n_{k\sigma} = \frac{1}{e^{-(\varepsilon_{k\sigma}'-\mu)/K_BT}+1}$$

Fermi-Dirac distribution for "interacting" quasiparticle energies

Quasiparticle weight Z_k not present

$$\varepsilon'_{k\sigma}= \varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k'\sigma'}$$

See Pines & Nozieres' and Coleman's books for details on the derivation

Fermi liquid theory: Measurable quantities

Specific heat $C_v = \left.\frac{\partial F}{\partial T}\right|_{\mu}$

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

$F(T)$ $\qquad \propto T^2 \qquad \propto T^4$

Dominates at low T

See Pines & Nozieres' and Coleman's books for details on the derivation

Fermi liquid theory: Measurable quantities

Specific heat $C_v = \left.\dfrac{\partial F}{\partial T}\right|_{\mu}$

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

$F(T)$ $\qquad \propto T^2 \qquad \propto T^4$

$C^*_v = \gamma^* T \qquad \gamma^* = (\pi/3) K^2_B N^*(E^*_F) \propto m^*$

Same temperature dependence as in Fermi gas but with a renormalized proportionality constant γ^*

Fermi liquid theory: Measurable quantities

Specific heat $C_v = \left.\frac{\partial F}{\partial T}\right|_{\mu}$

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

$F(T)$ $\propto T^2$ $\propto T^4$

$C^*_v = \gamma^* T$ $\gamma^* = (\pi/3)K^2_B N^*(E^*_F) \propto m^*$

Same temperature dependence as in Fermi gas but with a renormalized proportionality constant γ^*

Check experimental dependence and measure γ
Can be also compared with Model calculations

Interacting Non-Interacting

$$C^*_v = \frac{m^*}{m} C_v$$

Comparison with LDA like calculations

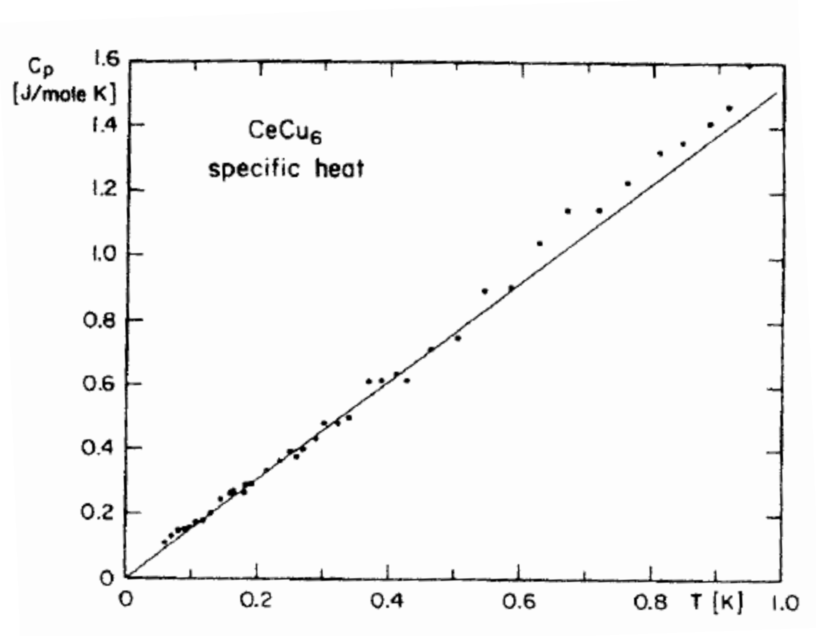
Fermi liquid theory: Measurable quantities

Specific heat

$C_v = \gamma^* T \qquad \gamma^* = (\pi/3) K_B^2 N^*(E_F^*) \propto m^*$

$\gamma^*_{Au,Ag} = 0.67$ mJ $mol^{-1} K^{-1}$ (vs 0.63 mJ $mol^{-1} K^{-1}$ in absence of interaction)

$\gamma^*_{CeCu6} = 1{,}5$ J $mol^{-1} K^{-1}$ → **Heavy fermion**



Fermi liquid theory: Measurable quantities

Specific heat

$C_v = \gamma^* T$ $\quad\quad$ $\gamma^* = (\pi/3) K^2_B N^*(E^*_F) \propto m^*$

(note phonon contribution can dominate $C_v \propto T^3$)

$\gamma^*_{Au,Ag} = 0.67$ mJ mol^{-1} K^{-1} (vs 0.63 mJ mol^{-1} K^{-1} in absence of interaction)

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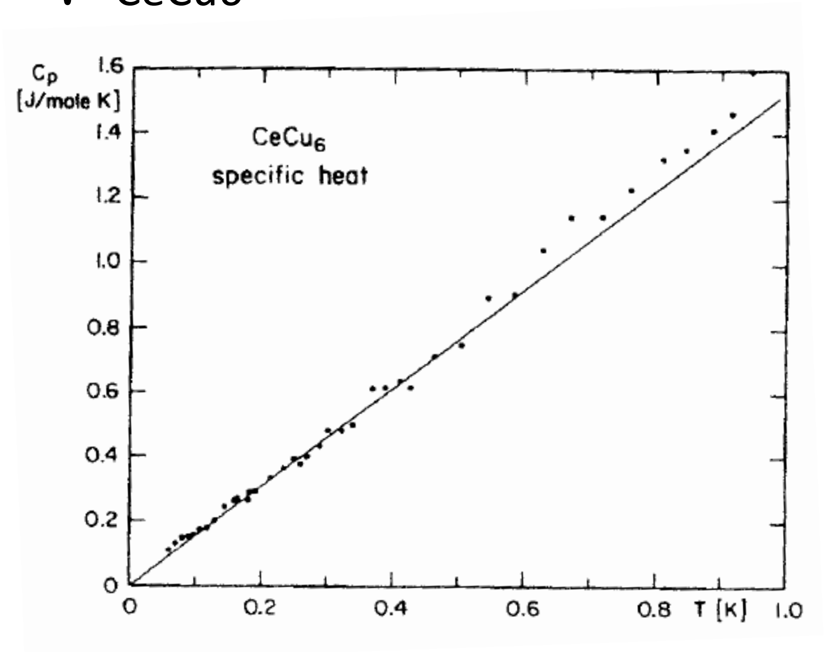

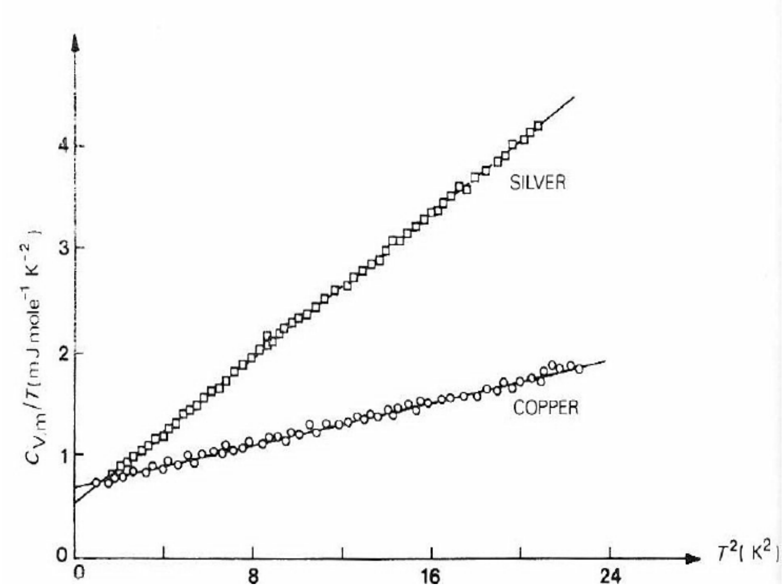

Fermi liquid theory: Measurable quantities

Mass renormalization

Looking at the quasiparticle current

$$m^*=m\,(1+F_1^s)=\frac{m}{1-N(E_F)f^1_s}$$

Spin independent
But directional response

Coleman's book

Fermi liquid theory: Measurable quantities

$$\chi_s = \frac{\partial M}{\partial H}$$

- **Spin susceptibility**

$$\chi^*_s = \frac{\mu_B{}^2 N^*(E_F)}{1 + F_0{}^a} = \frac{(m^*/m)}{1 + F_0{}^a}\,\chi_s$$

independent of T as in Fermi gas

interacting

Susceptibility
Spin dependent
Isotropic response

Non-interacting

Fermi liquid theory: Measurable quantities

○ Spin susceptibility

$$\chi^*_s = \frac{\mu_B^2 N^*(E_F)}{1 + F_0^a} = \frac{(m^*/m)}{1 + F_0^a}\chi_s$$

independent of T
as in Fermi gas

interacting

Susceptibility
Spin dependent
Isotropic response

Non-interacting

Wilson ratio or Stoner enhancement factor

experiment

$$W = \frac{(\gamma^*/\chi^*_s)}{(\gamma/\chi_s)} = \frac{1}{1 + F_0^a}$$

$C_v = \gamma^* T \qquad \gamma^* \propto m^*$

Constant

LDA

Deviation from unity,
signature of electronic correlations

A way to obtain F_0^a

Fermi liquid theory: Measurable quantities. Resistivity

Band theory: classification into metals and insulators

Metal ρ increases with T

Not changed by interactions in a Fermi liquid

Insulator ρ decreases with T

Fermi liquid theory: Measurable quantities. Resistivity

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Drude conductivity $\sigma = ne^2\tau/m^*$ $\qquad \tau^{-1} = \dfrac{\pi^2 T^2}{E^*_F} \propto m^* T^2$

Fermi liquid theory: Measurable quantities. Resistivity

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$$\boxed{\rho = \rho_0 + AT^2 \qquad A \propto (m^*)^2}$$

High resistivity due to interactions

disorder

Quadratic dependence on T

Constant strongly dependent on interactions

Phonon resistivity $\propto T^5$ can dominate in simple metals

Fermi liquid theory: Measurable quantities

$C^*_v = \gamma^* T \qquad \gamma^* \propto m^*$

$\rho = \rho_0 + AT^2 \qquad A \propto (m^*)^2$

$$\frac{A}{(\gamma^*)^2} \sim \text{constant}$$

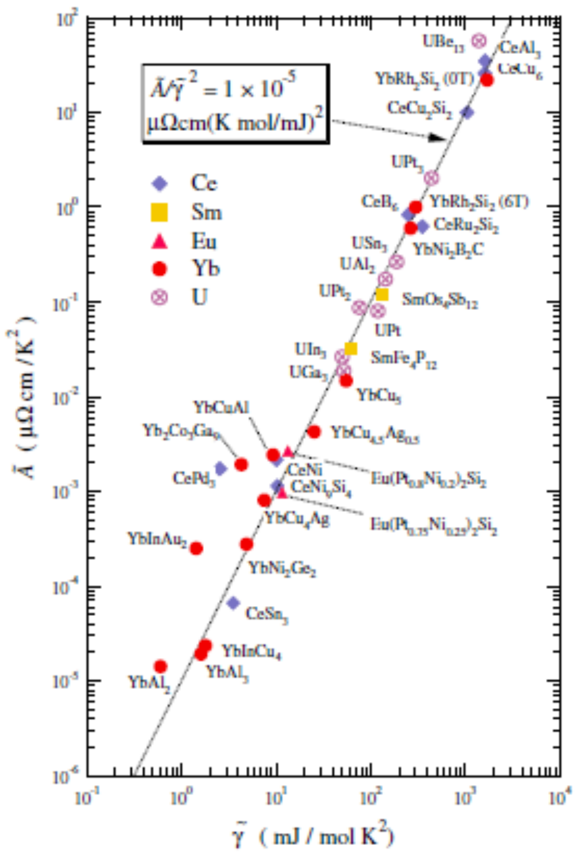
Kadowaki –Woods ratio

Fermi liquid theory: Measurable quantities

$$C^*_v=\gamma^* T \qquad \gamma^* \propto m^*$$

$$\rho=\rho_0 + AT^2 \qquad A \propto (m^*)^2$$

$$\frac{A}{(\gamma^*)^2} \tilde{} \text{ constant}$$



A and γ corrected by degeneracy

Kadowaki –Woods ratio

Fig: Tsuji et al, PRL 94, 057201 (2005)

Fermi liquid behavior

- **Bands observed in photoemission**

Reduction in bandwidth as compared to LDA estimate of m*/m

- **Specific heat linear in temperature.**

$C^*_v = \gamma^* T$

Enhancement of γ estimate of m*/m

Careful in materials with multiple Fermi pockets

- **Temperature independent spin susceptibility**

Enhancement: mass renormalization + Stoner enhancement

- **Resistivity quadratic in temperature**

Interactions enhance resistivity as $(m^*)^2$

$\rho = \rho_0 + AT^2$

Correlated electrons: frequently non-Fermi liquid behavior

Fermi liquid instabilities. Phase transitions

$$m^* = m\,(1+F_1^s) = \frac{m}{1-N(E_F)f^1_s}$$

It could diverge for large enough interactions (Localization. Metal-insulator transition)

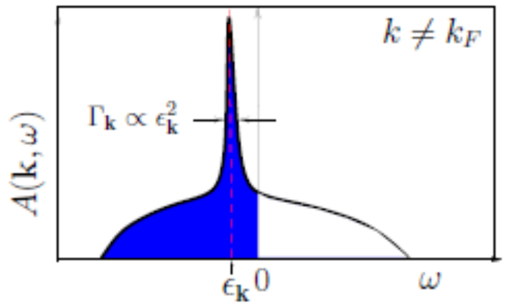
$$\chi^*_s = \frac{\mu_B^2 N^*(E_F)}{1+F_0^a}$$

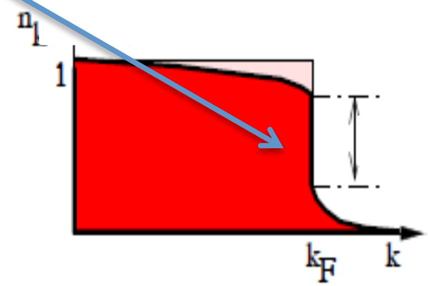
It diverges if $F^0_a=-1$

Fermi liquid theory: Summary

- ❑ Adiabaticity and well defined quasiparticles assumed ($Z_k > 0$). Quasiparticles spin ½ and charge e and momentum k. Fermi surface.

- ❑ In the 3D continuum quasiparticle decay ε^{*2}. Quasiparticles well defined at low energies





Z_k

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

$$N^*(E^*_F)\, f_l^{s,a} = F_l^{s,a}$$

- ❑ Fermi liquid behavior $\rho^* = \rho^*_0 + AT^2$ $C^*_v = \gamma^* T$ χ^*_s Bands in ARPES

- ❑ In simple models $Z = (m/m^*)$ Measurable experimentally!

- ❑ The Fermi liquid can be unstable

Strength of correlations

- ❑ And more: transport equation, collective modes, …