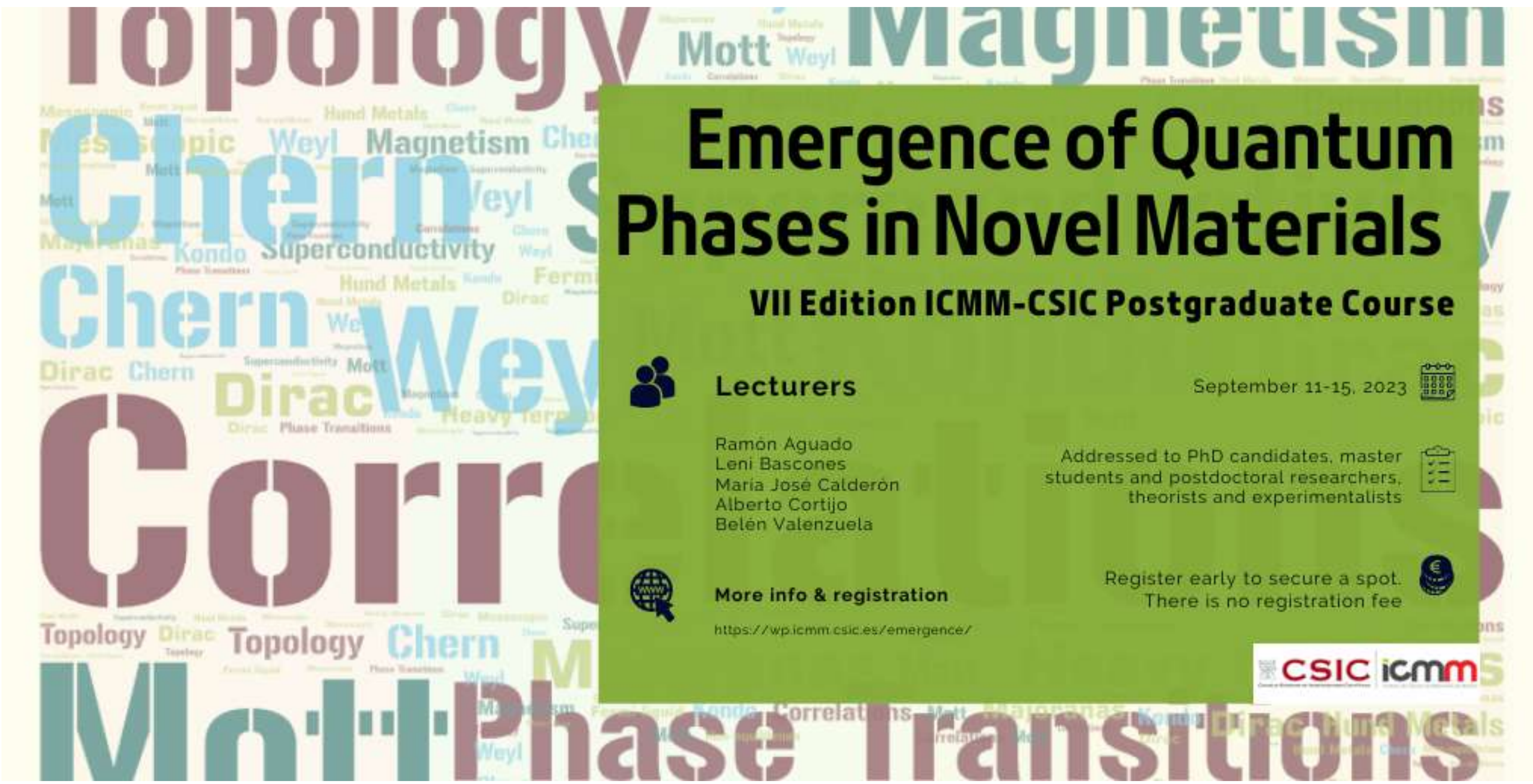
DIRAC MATERIALS IN CONDENSED MATTER

PART III: Topological Semimetals

Alberto Cortijo



OUTLINE

1. Weyl semimetals.

Definition. Bandstructure, types

2. Berry phase physics.

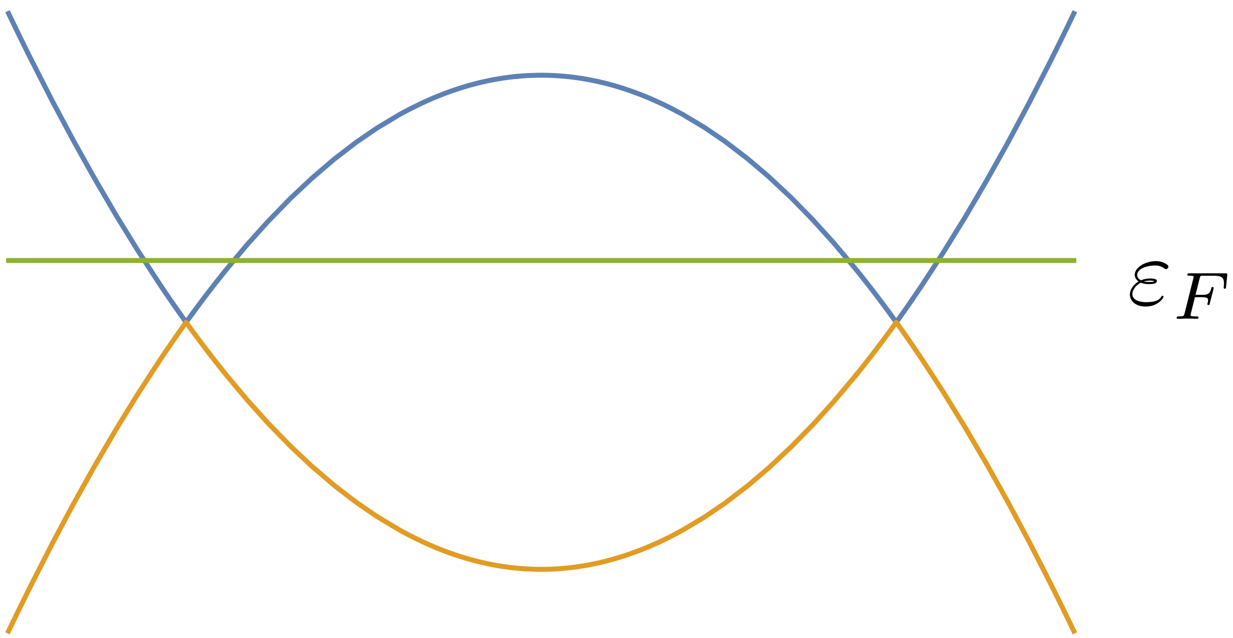
From band structure to Berry connection. Nielsen Ninomiya theorem. QAHE & Fermi arcs.

3. Transport properties in Weyl semimetals.

Semiclassical approach (why and when). Chiral anomaly. The monopole paradigm and Anomaly related transport. Beyond the monopole paradigm.

4. Conclussions

I. Weyl semimetals.



Herrig, PR 52, 365 (1937)

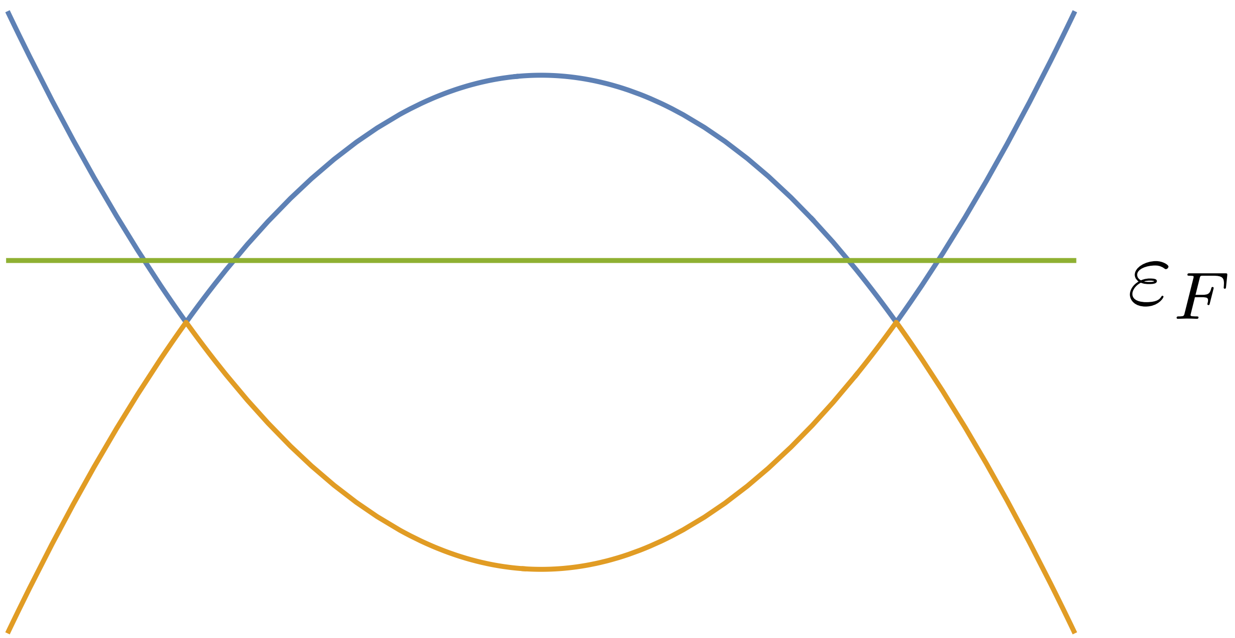
Conduction and valence bands touch each other at a discrete set of points in the BZ

$$H = sv\boldsymbol{\sigma} \cdot (\boldsymbol{k} - s\boldsymbol{b})$$

Wigner Von Neumann theorem (avoided crossing)

$$H = \begin{pmatrix} a_1 + a_2 & b_1 - ib_2 \\ b_1 + ib_2 & a_1 - a_2 \end{pmatrix}$$

I. Weyl semimetals.



Herrig, PR 52, 365 (1937)

In 2d: $\boldsymbol{k} \in \mathbb{R}^2$

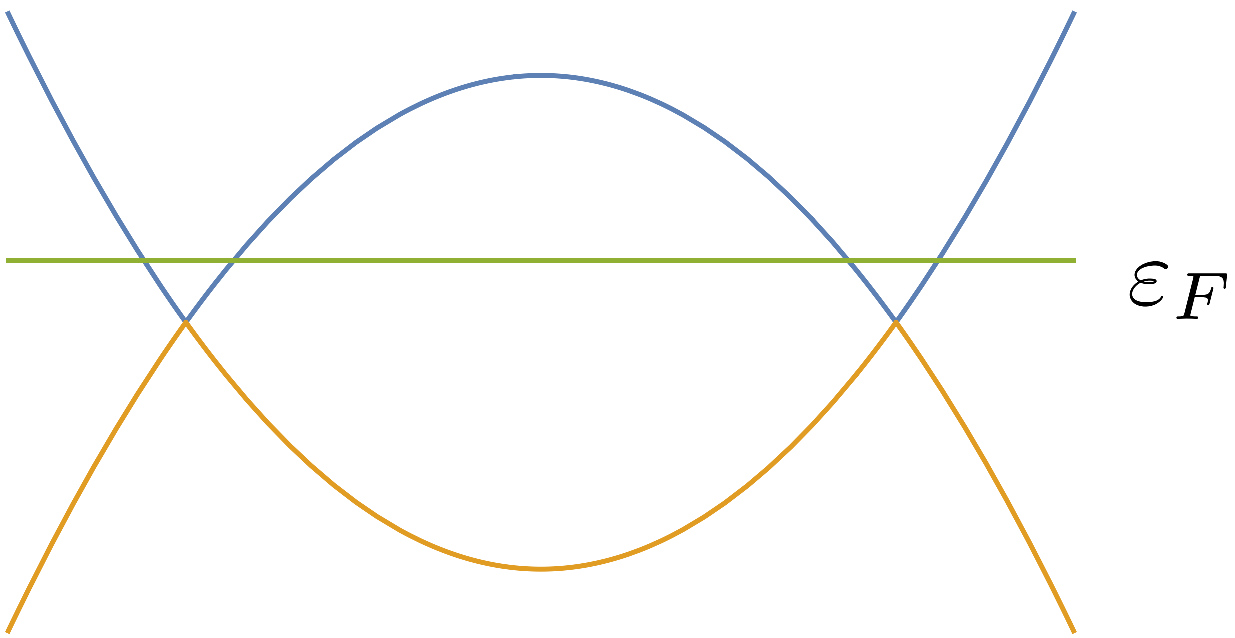
Wigner Von Neumann theorem
(avoided crossing)

$$H = \begin{pmatrix} a_1 + a_2 & b_1 - ib_2 \\ b_1 + ib_2 & a_1 - a_2 \end{pmatrix}$$

$$E_\pm = a_1 \pm \sqrt{a_1^2 + b_1^2 + b_2^2}$$

an extra parameter is needed to tune: important role of symmetries

I. Weyl semimetals.



Herrig, PR 52, 365 (1937)

Wigner Von Neumann theorem
(avoided crossing)

Band crossing are not symmetry-enforced but accidental

$$H = \sigma_1 b_1 + \sigma_2 b_2 + \sigma_3 a_3$$

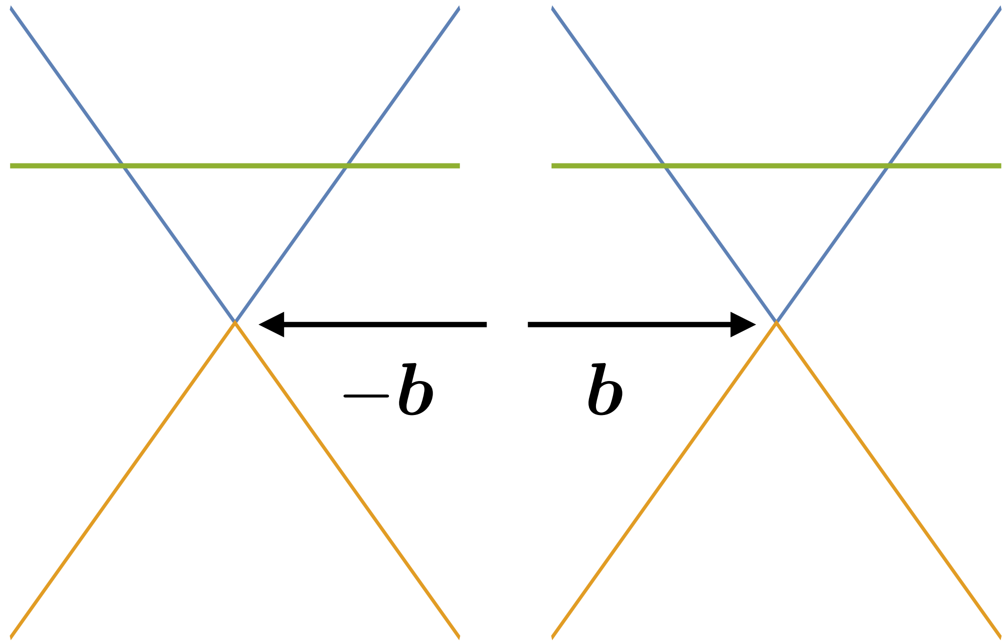
$$H = v_{ij} \sigma_i k_j$$

In 3d: $\boldsymbol{k} \in \mathbb{R}^3$

$$\chi = sign(\det(v_{ij}))$$

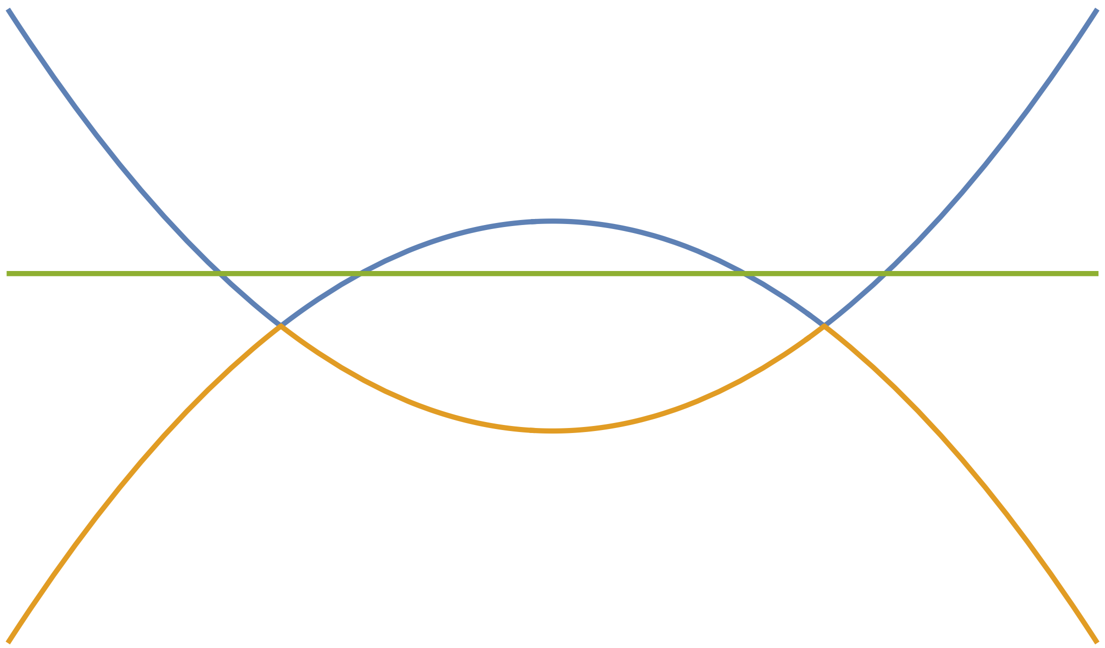
chirality

1. Weyl semimetals.



$$H = sv\boldsymbol{\sigma} \cdot (\boldsymbol{k} - s\boldsymbol{b})$$

I. Weyl semimetals.



Herrig, PR 52, 365 (1937)

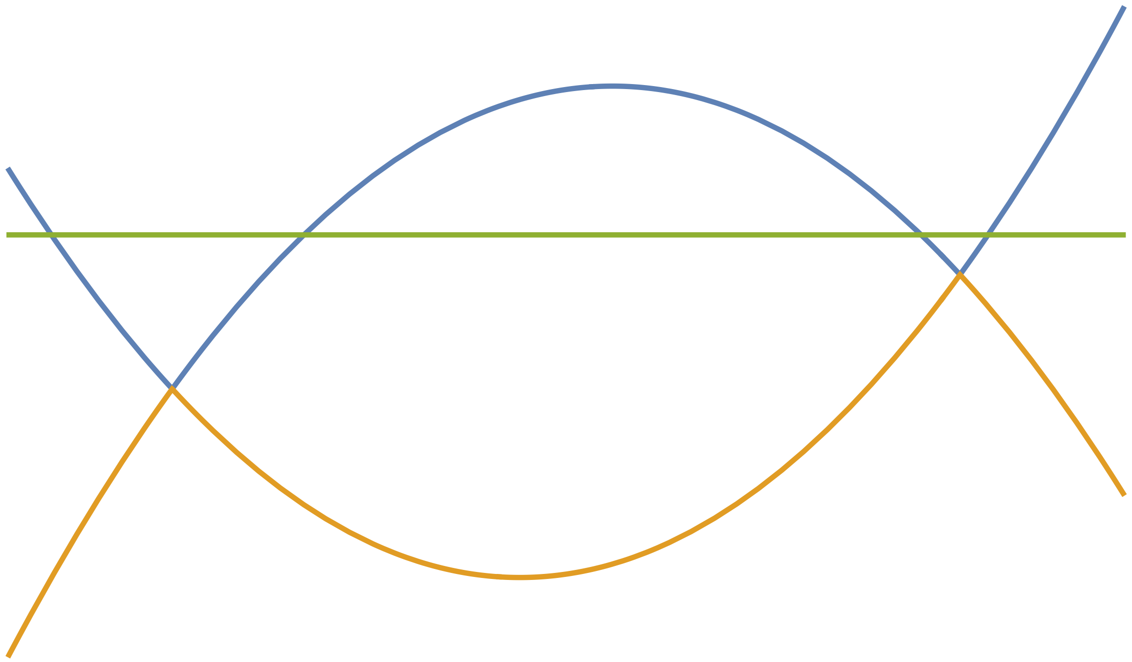
$$H = f_0(\boldsymbol{k})\sigma_0 + \sum_i \sigma_i f_i(\boldsymbol{k})$$

$$f_0 = 0$$

I. Weyl semimetals.

$$H = f_0(\boldsymbol{k})\sigma_0 + \sum_i \sigma_i f_i(\boldsymbol{k})$$

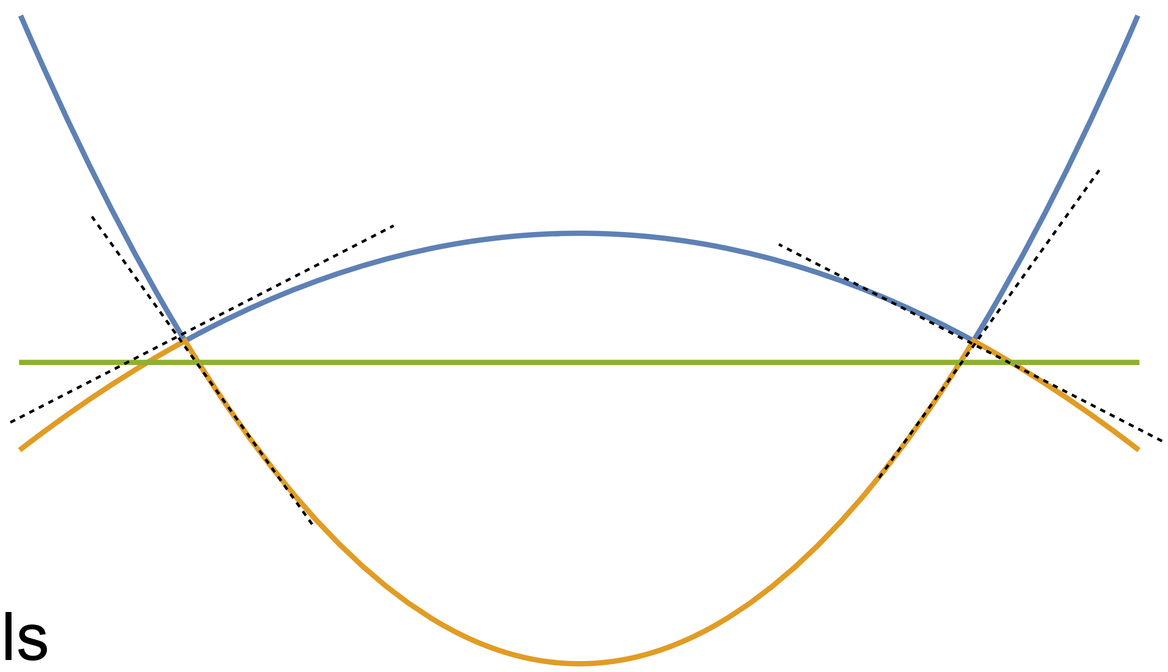
Weyl points at different energies



f_0 Odd function

Bradlyn et al, Science 353 (2016)

f_0 Even function



Tilted Weyl semimetals

Soluyanov et al, Nature 527, 495 (2015)

I. Weyl semimetals.

$$H = f_0(\boldsymbol{k})\sigma_0 + \sum_i \sigma_i f_i(\boldsymbol{k})$$

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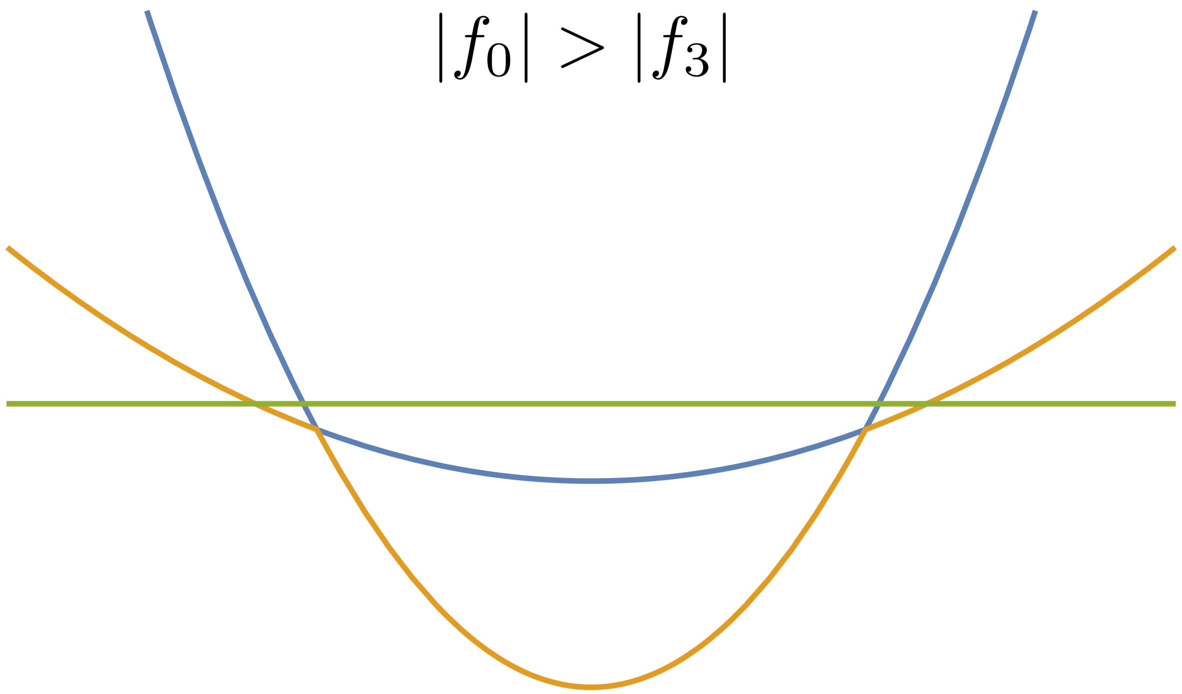
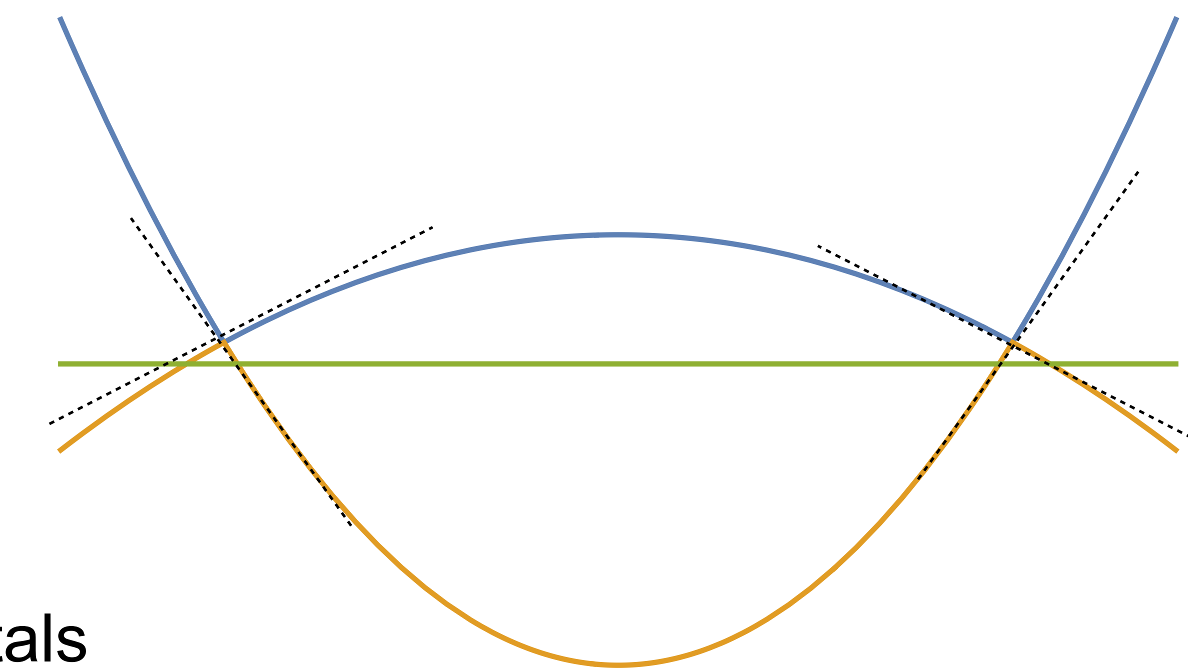


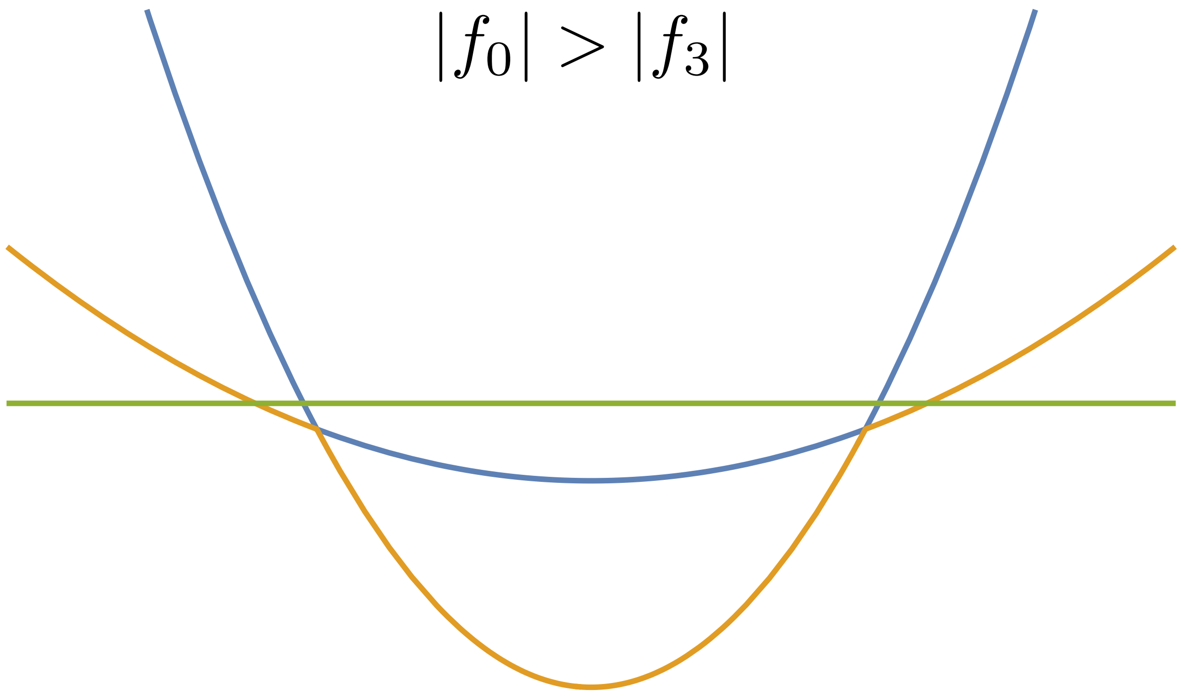
Type II Weyl semimetals

Soluyanov et al, Nature 527, 495 (2015)



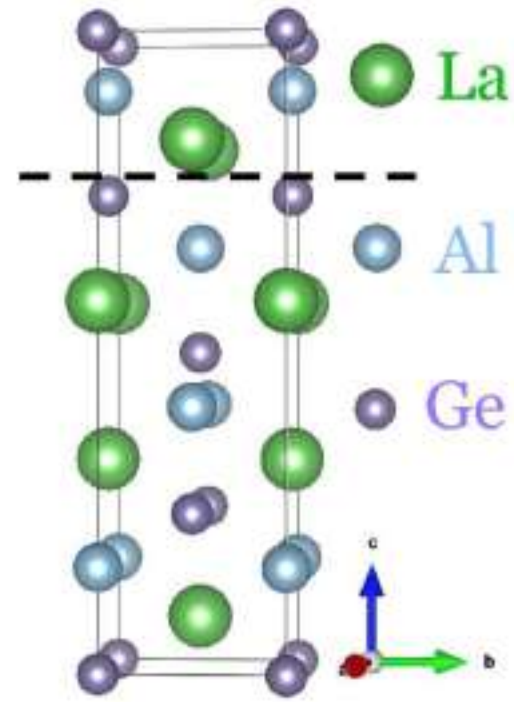
Tilted Weyl semimetals

1. Weyl semimetals.



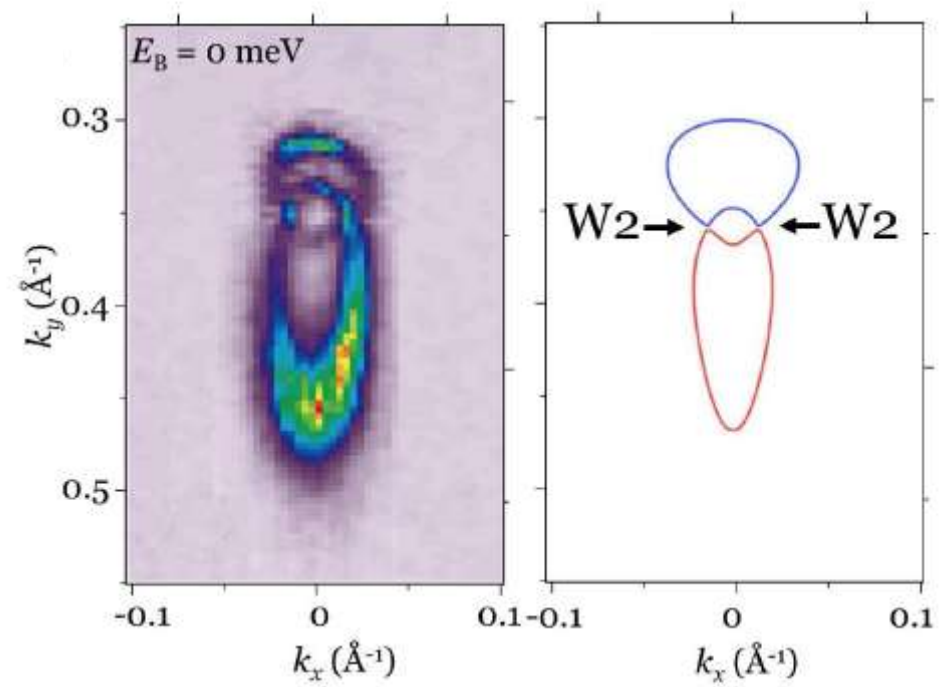
Type II Weyl semimetals

Soluyanov et al, Nature 527, 495 (2015)

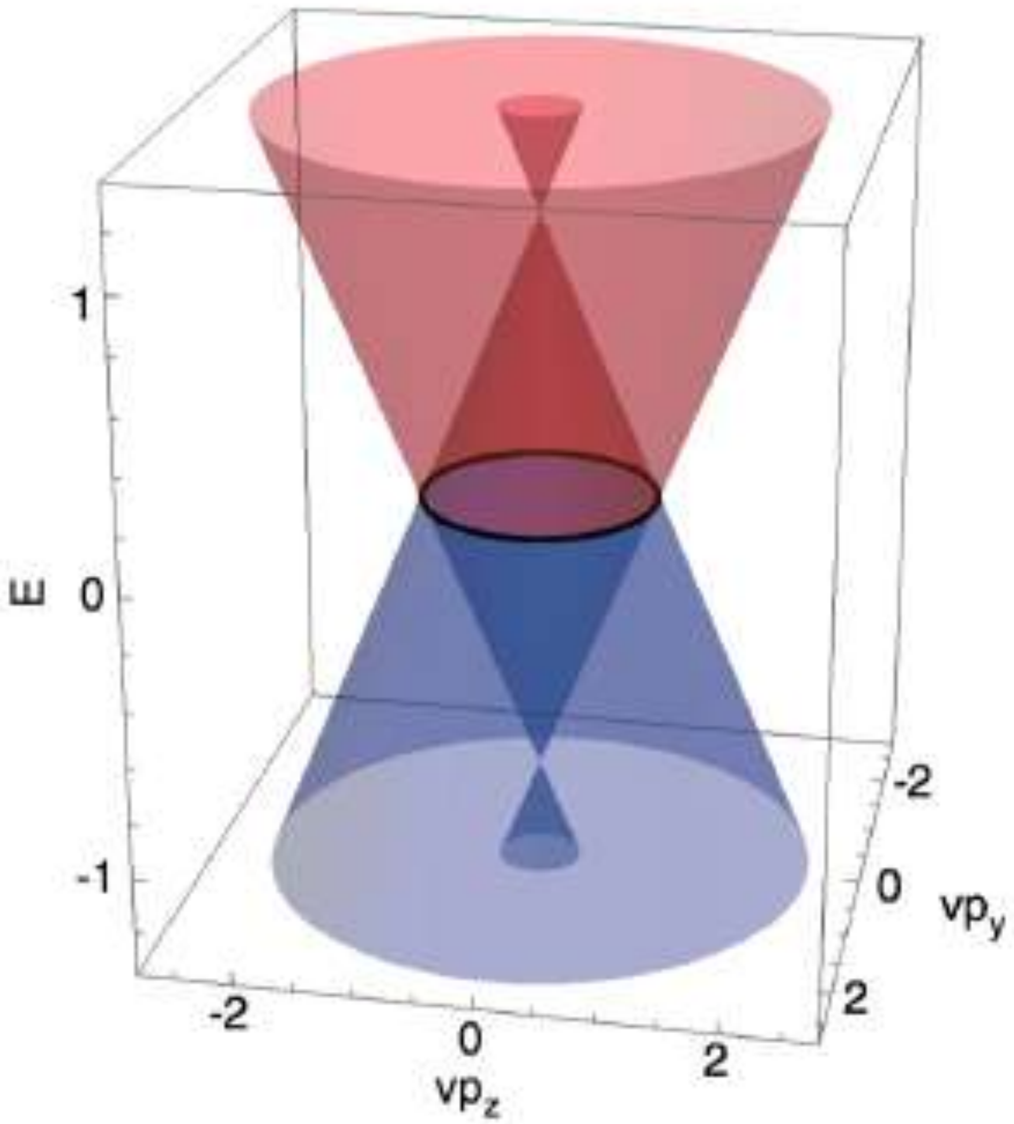


$LaAlGe$

SY Xu et al. arXiv:1603.0731



1. Weyl semimetals.



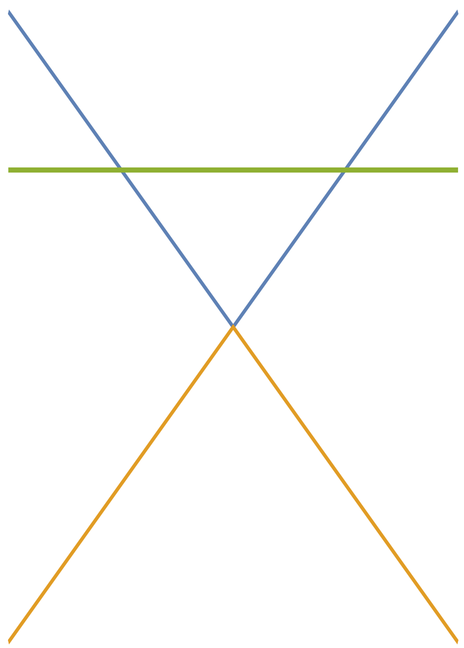
$$H = f_0(\boldsymbol{k})\sigma_0 + \sum_i \sigma_i f_i(\boldsymbol{k})$$

Tune one f to zero

Severe constraints from symmetries

Nodal line semimetals

2. Berry phase physics.

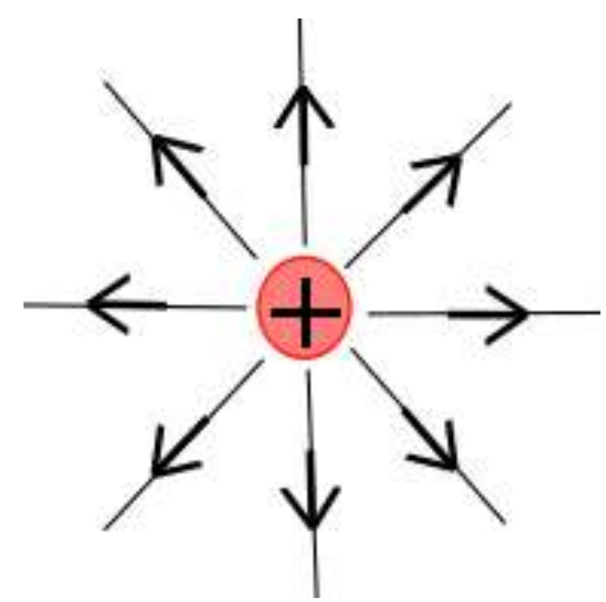


$$H = sv\boldsymbol{\sigma}\cdot(\boldsymbol{k} - s\boldsymbol{b})$$

$$\chi = sign(\det(v_{ij})) = s$$

$$\boldsymbol{\Omega} = -s\frac{1}{2}\frac{\boldsymbol{k}}{k^3}$$

monopole form

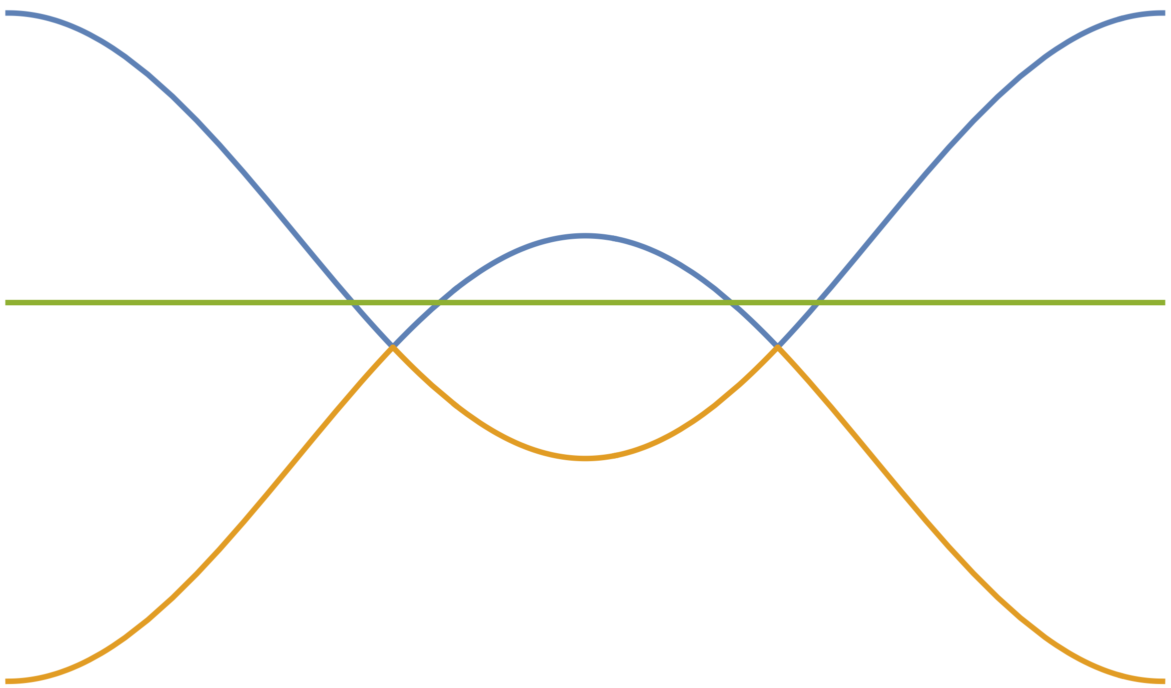


$$\boldsymbol{\mathcal{A}}(\boldsymbol{k}) = i\left\langle u_{\boldsymbol{k}}|\partial_{\boldsymbol{k}} u_{\boldsymbol{k}}\right\rangle$$

$$\boldsymbol{\Omega}(\boldsymbol{k}) = \partial_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}(\boldsymbol{k})$$

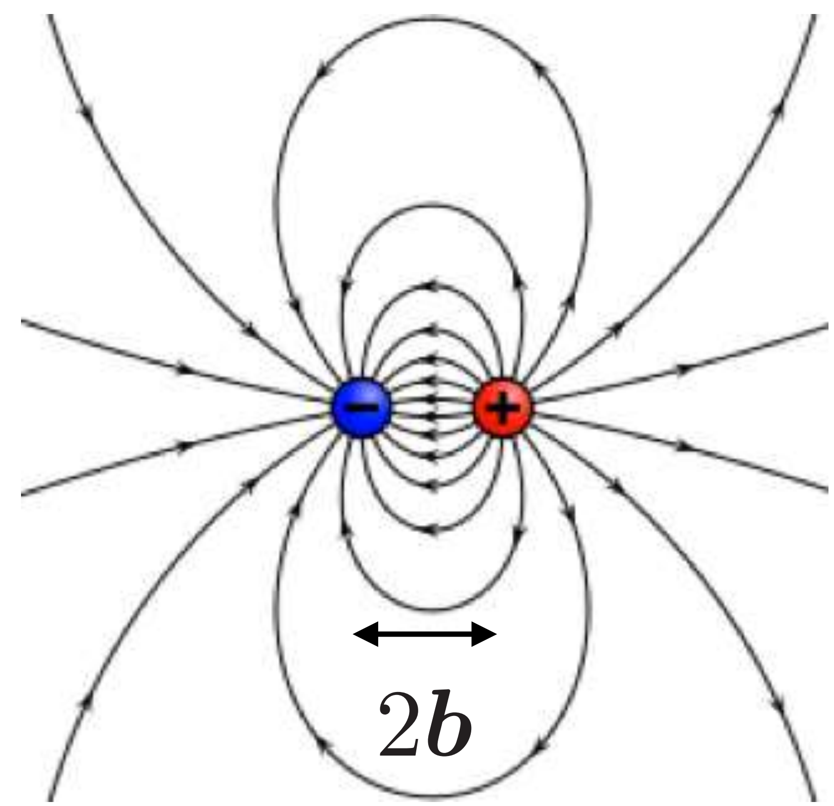
Nielsen, Ninomiya, NucPhysB 185, 20 (1981)

2. Berry phase physics.



at least a source and drain of “force” lines

But, again, everything must be periodic



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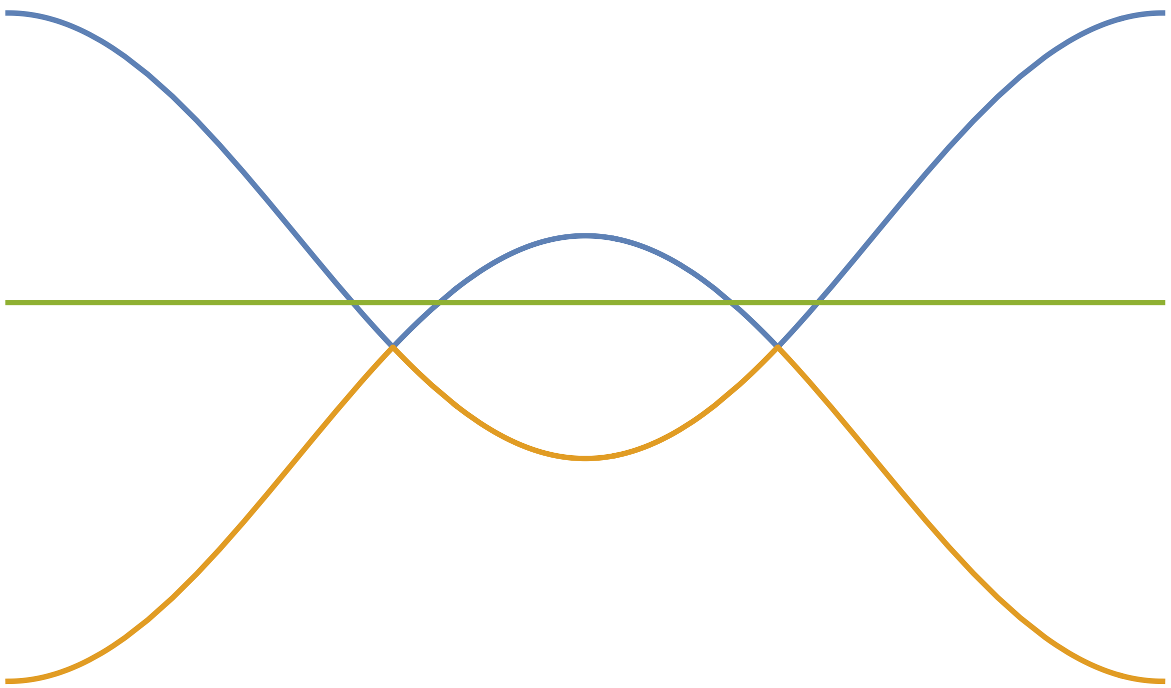
2. Berry phase physics.



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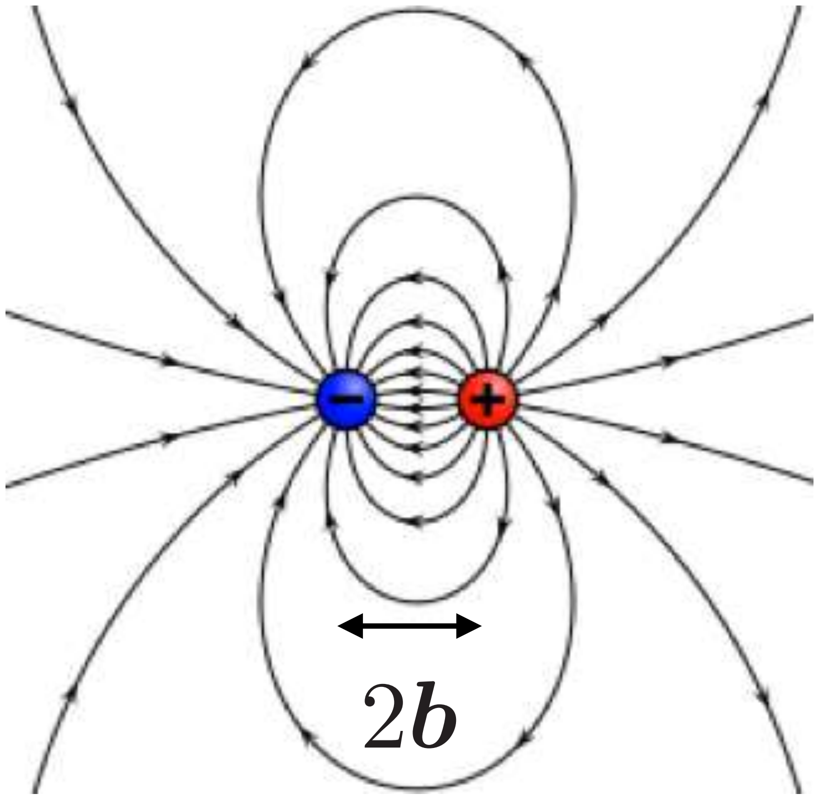


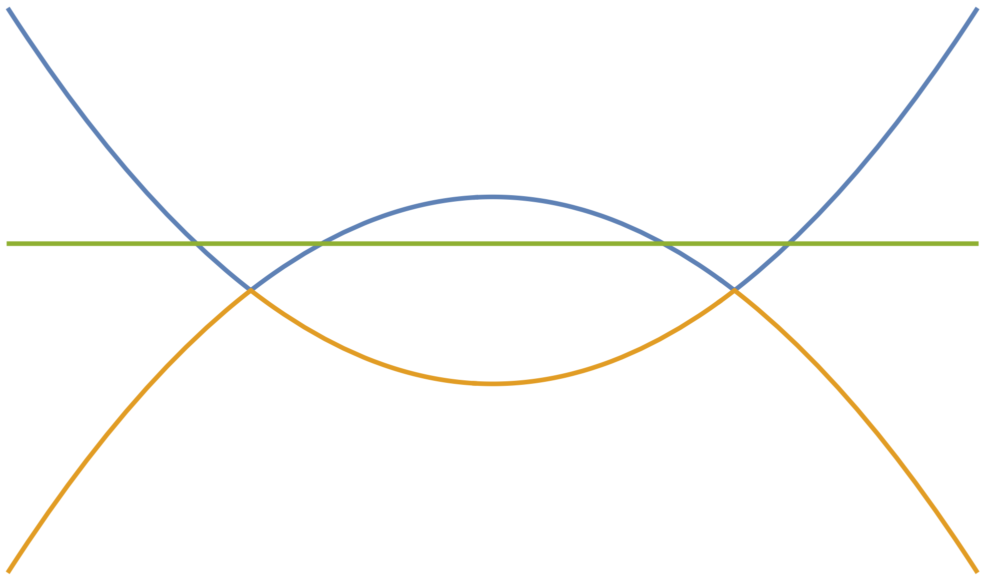
Nielsen-Ninomiya theorem:

In a lattice, band crossings with opposite chirality always come in pairs

Nielsen, Ninomiya, NucPhysB 185, 20 (1981)

2. Berry phase physics.

Fermi arcs:



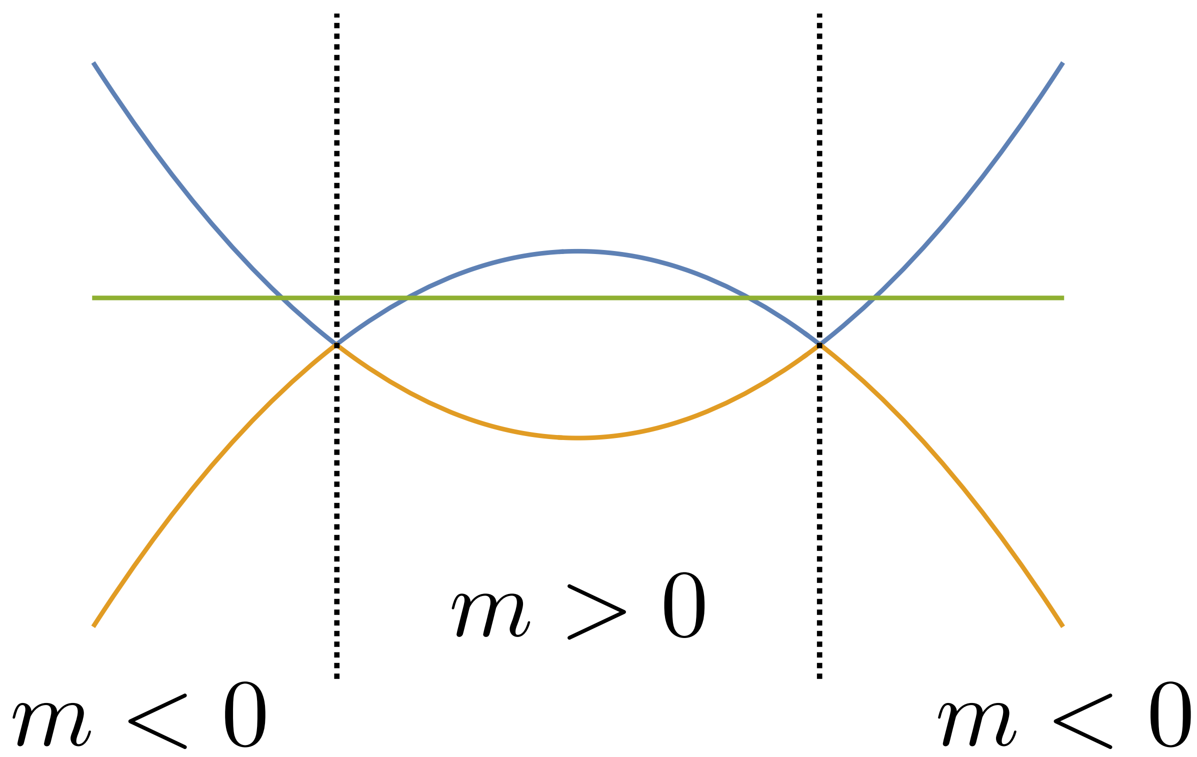
$$H(\boldsymbol{k}) = \sigma_1 v k_1 + \sigma_2 v k_2 + \sigma_3 m(k_3)$$

$$m(k_3) = m_0 - \beta k_3^2$$

Weyl nodes @ $\boldsymbol{b}_\pm = (\mathbf{0}, \pm\sqrt{m_0/\beta})$

2. Berry phase physics.

Fermi arcs:



$$H(\boldsymbol{k}) = \boldsymbol{\sigma} \cdot \boldsymbol{k}_\perp + \sigma_3 m(\boldsymbol{k})$$

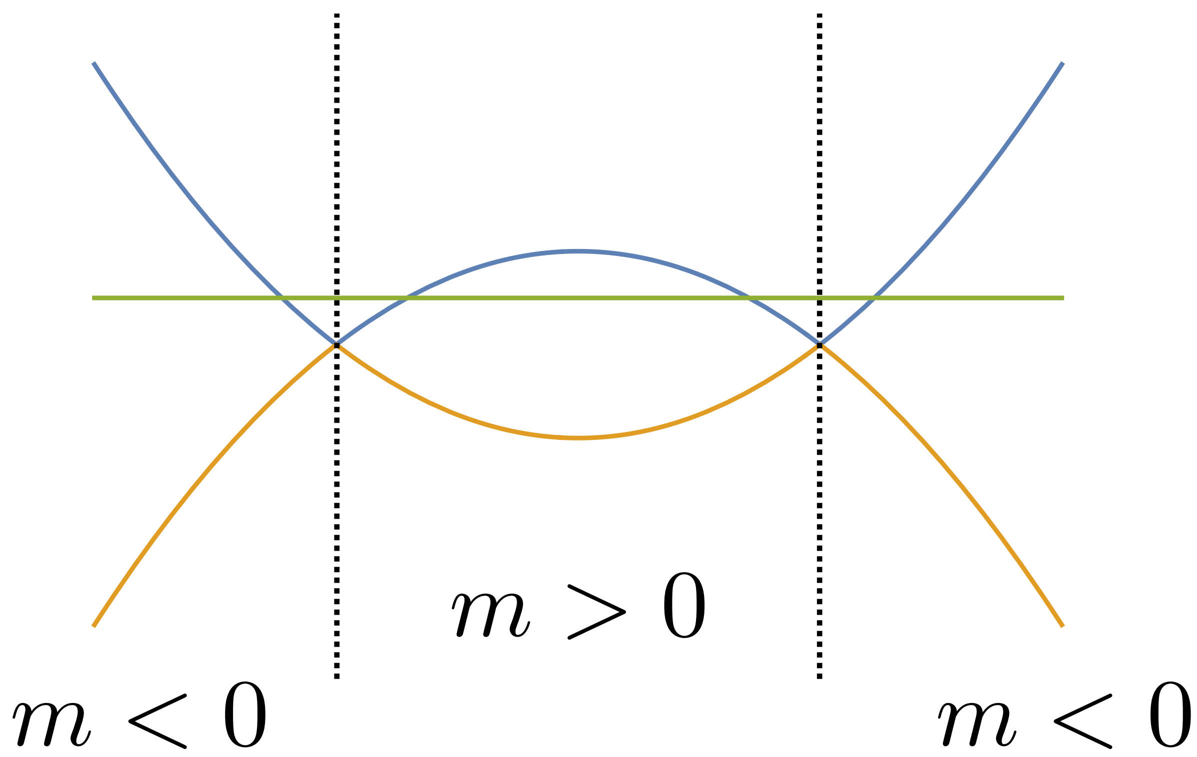
$$m = m_0 - \beta k_3^2 - \beta |\boldsymbol{k}_\perp|^2$$

$$\sigma_{12}^{3D} = \int \frac{dk_3}{2\pi} \sigma_{12}^{2D}$$

$$\sigma_{12}^{2D} = \frac{1}{2}(sign(m_0 - \beta k_3^2) + sign(\beta))$$

2. Berry phase physics.

Fermi arcs:



$$H(\boldsymbol{k}) = \boldsymbol{\sigma} \cdot \boldsymbol{k}_\perp + \sigma_3 m(\boldsymbol{k})$$

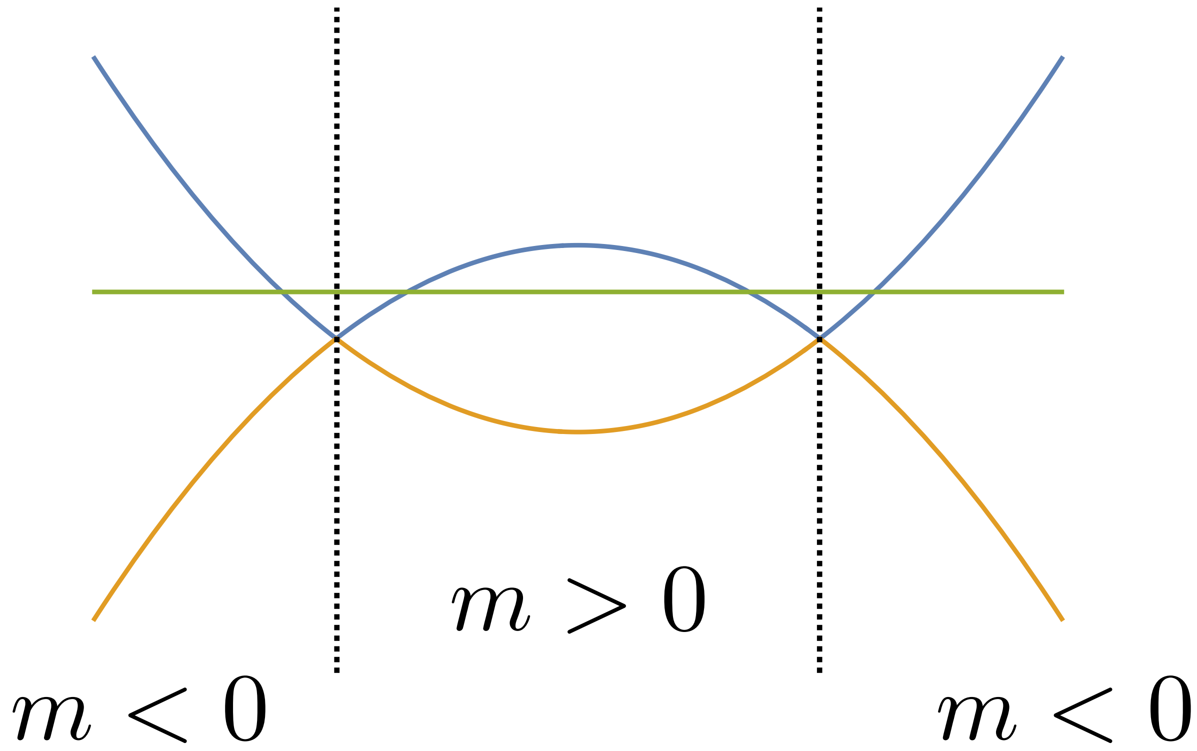
$$m = m_0 - \beta k_3^2 - \beta |\boldsymbol{k}_\perp|^2$$

$$\sigma_{12}^{3D} = \int \frac{dk_3}{2\pi} \sigma_{12}^{2D}$$

$$\sigma_{12}^{3D} = \frac{e^2}{4\pi^2} \int_{-b_3}^{b_3} dk_3 = \frac{e^2}{2\pi^2} b_3$$

2. Berry phase physics.

Fermi arcs:



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$$m = m_0 - \beta k_3^2 - \beta |\boldsymbol{k}_\perp|^2$$

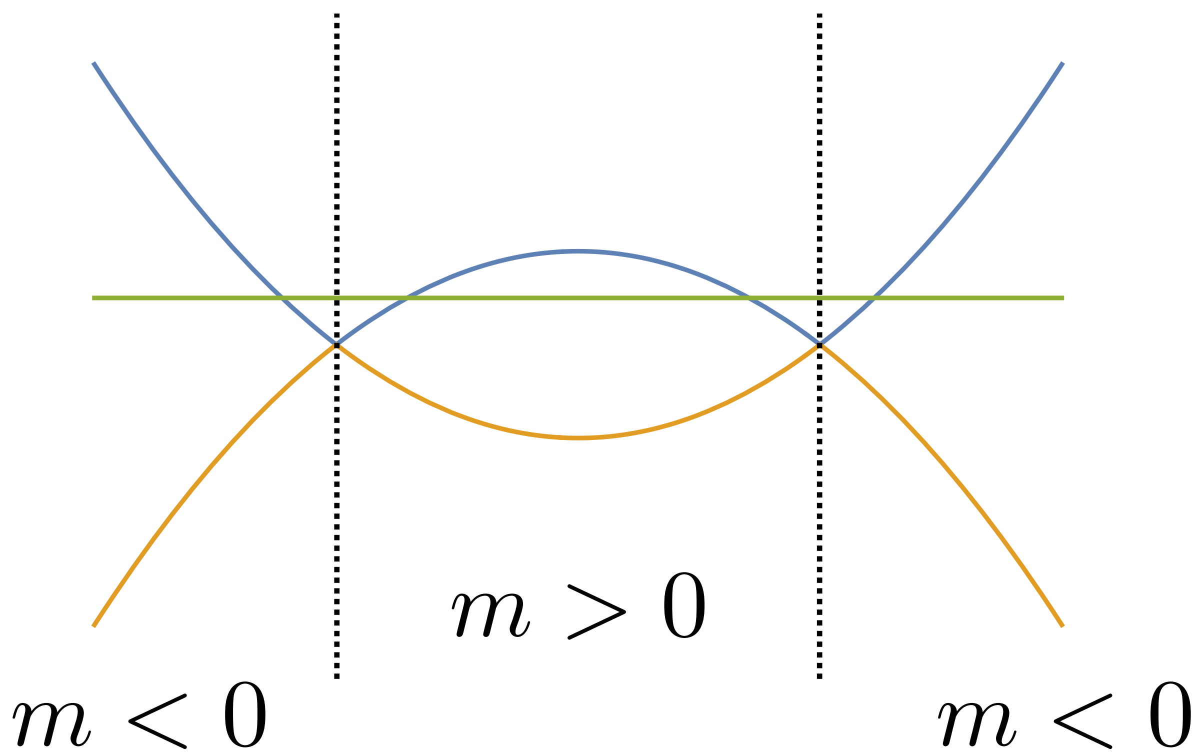
$$\sigma_{12}^{3D} = \int \frac{dk_3}{2\pi} \sigma_{12}^{2D}$$

$$\boldsymbol{J} = \frac{e^2}{2\pi^2} \boldsymbol{b} \times \boldsymbol{E}$$

anomalous quantum Hall effect

2. Berry phase physics.

Fermi arcs:



$$H(\boldsymbol{k}) = \boldsymbol{\sigma} \cdot \boldsymbol{k}_{\perp} + \sigma_3 m(\boldsymbol{k})$$

$$m = m_0 - \beta k_3^2 - \beta |\boldsymbol{k}_{\perp}|^2$$

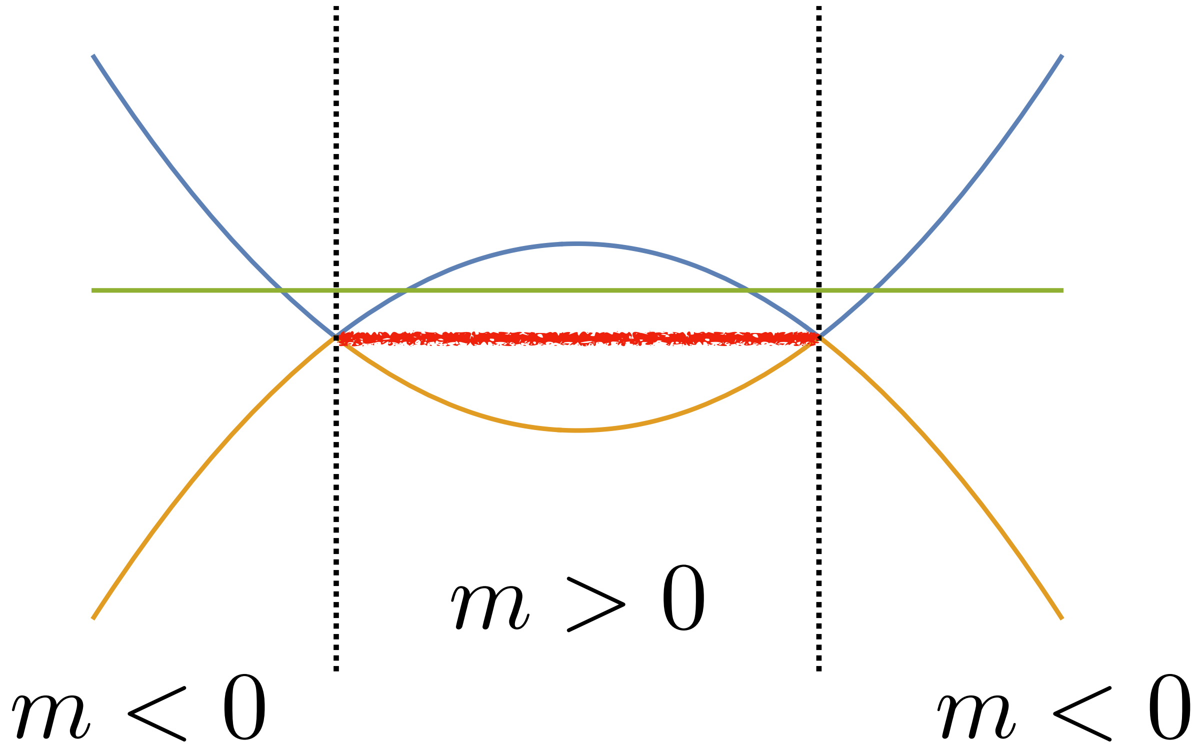
$$\sigma_{12}^{3D} = \int \frac{dk_3}{2\pi} \sigma_{12}^{2D}$$

$$W = \boldsymbol{J} \cdot \boldsymbol{E} = \frac{e^2}{2\pi^2} \boldsymbol{b} \times \boldsymbol{E} \cdot \boldsymbol{E} = 0$$

Ground state Hall-like responses are non-dissipative and thus allowed in equilibrium (!!)

2. Berry phase physics.

Fermi arcs:



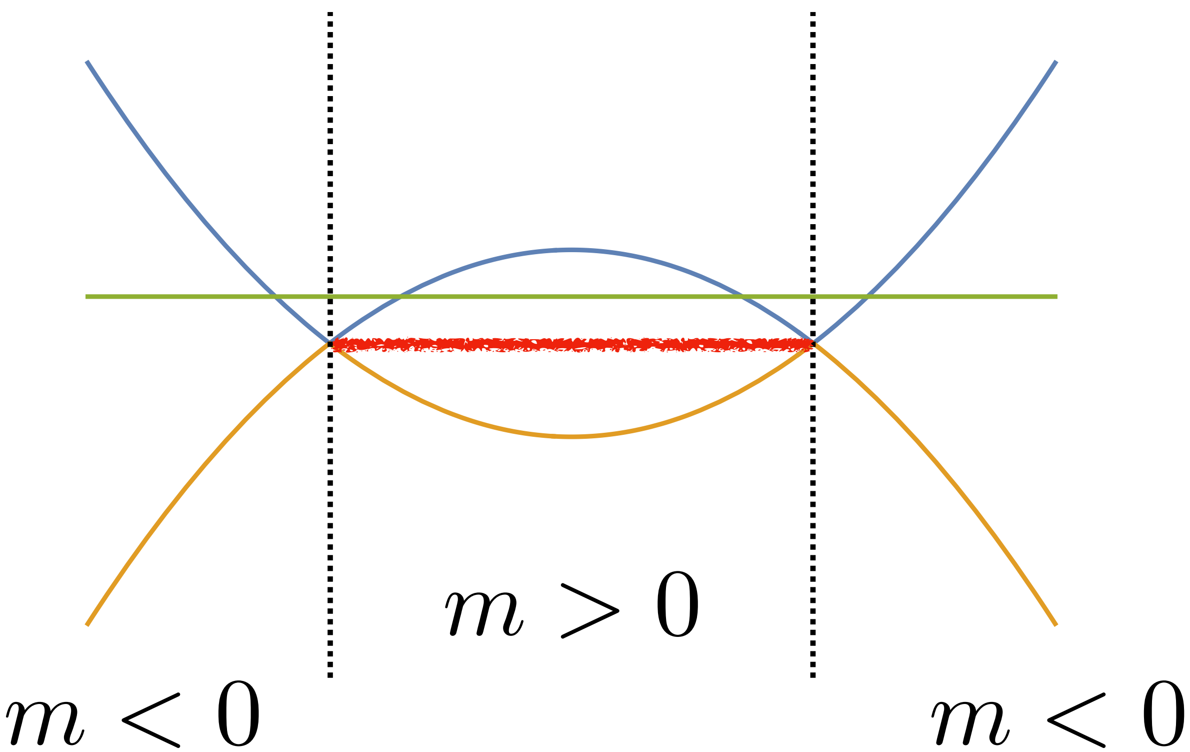
$$\boldsymbol{J} = \frac{e^2}{2\pi^2}\boldsymbol{b} \times \boldsymbol{E}$$

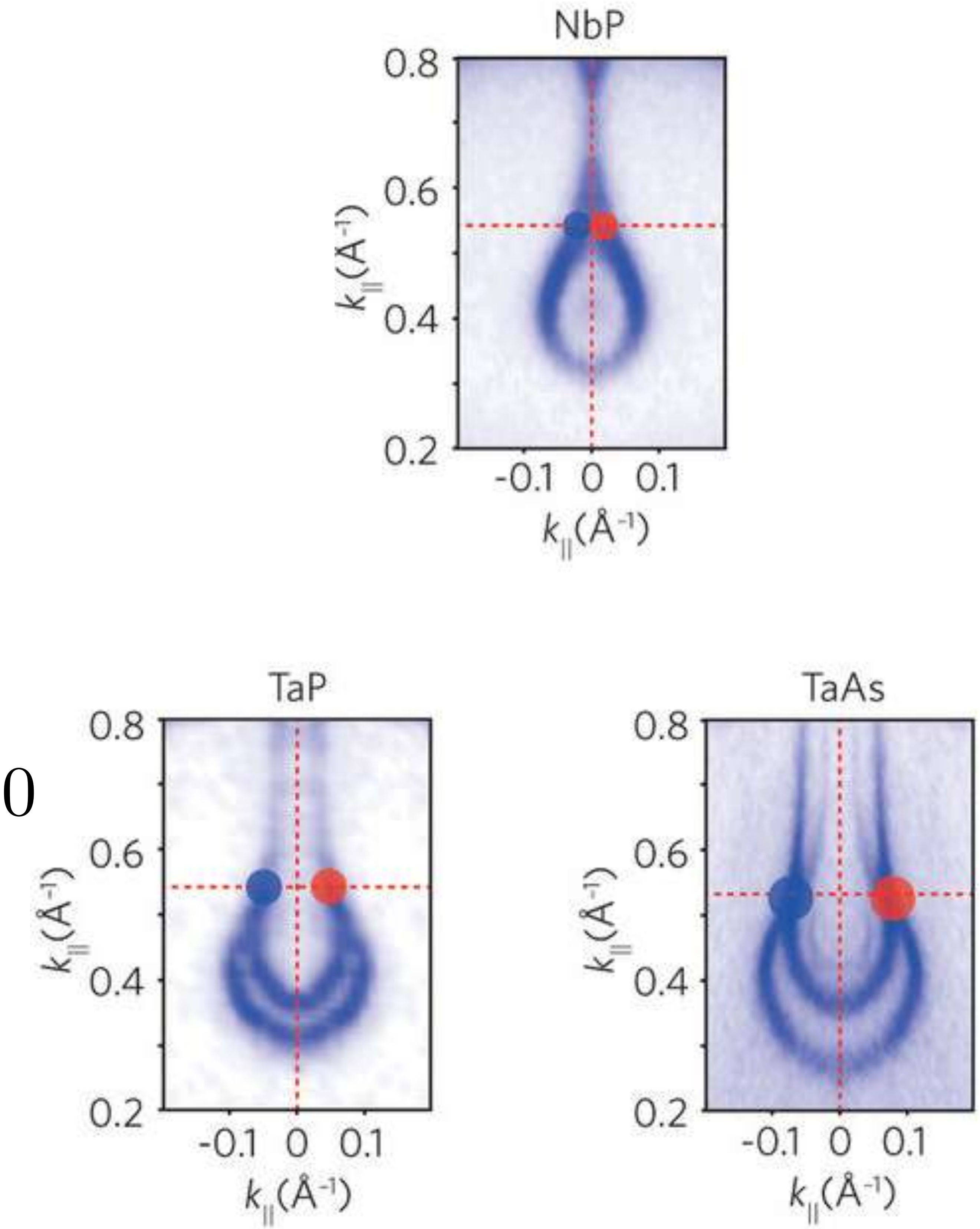
$$\sigma_{12}^{3D} = \frac{1}{2\pi}\int_{-b_3}^{b_3} dk_3 \sigma_{12}^{2D}$$

We have as many 1D chiral states as values of k_3 contributing to σ_{12}^{3D}

2. Berry phase physics.

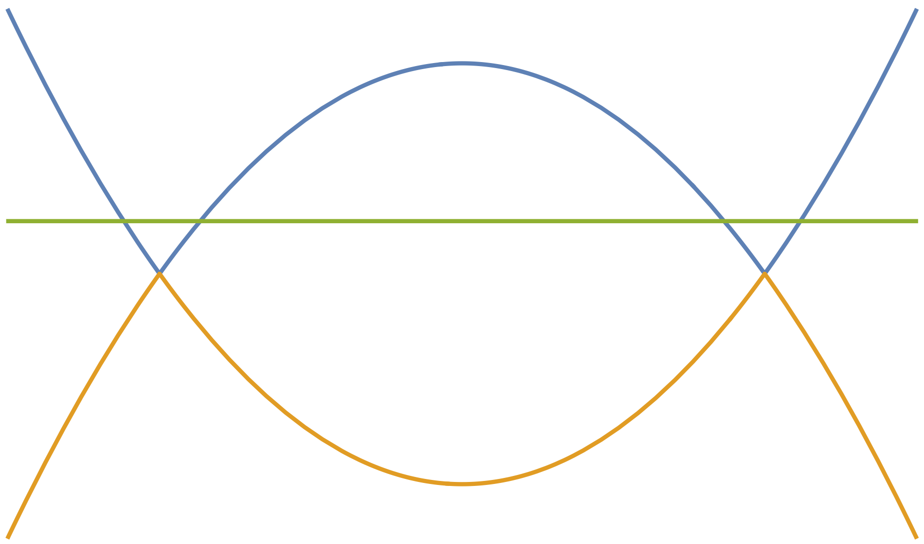
Fermi arcs:





ZK Liu et al. Nat. Mat. 16, 27 (2016)

3. Transport properties in Weyl semimetals.



ε_F

A Weyl semimetal is a topologically nontrivial metallic system

$$\rho(\varepsilon) \sim \varepsilon^2$$

Two main phenomenological effects related to gapless (massless) chiral fermions

CHIRAL ANOMALY

CHIRAL MAGNETIC EFFECT

3. Disgression: Quantum anomaly

Failure of a symmetry at the classical level to remain a symmetry when quantum effects are considered

$$[H_0, U] = 0$$

symmetry

susceptibility

$$\mathcal{O} = \chi_0 V$$

observable "current"

driving force

breaks the symmetry

$$H_0 \to H = H_0 + mH_1$$

$$\mathcal{O} = \chi[m]V$$

$$\chi[m \to 0] \neq \chi_0$$

$$[H, U] = m[H_1, U] \neq 0$$

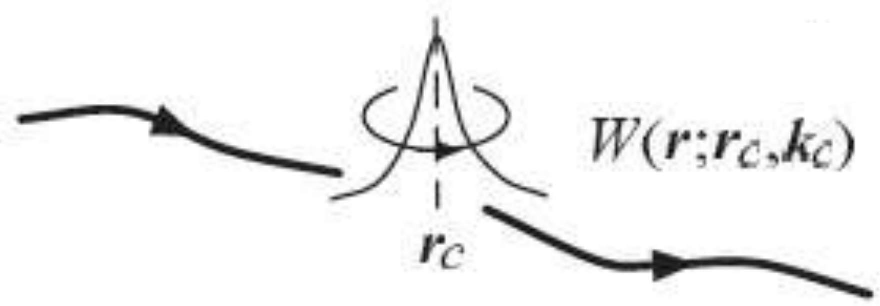
"anomalous"

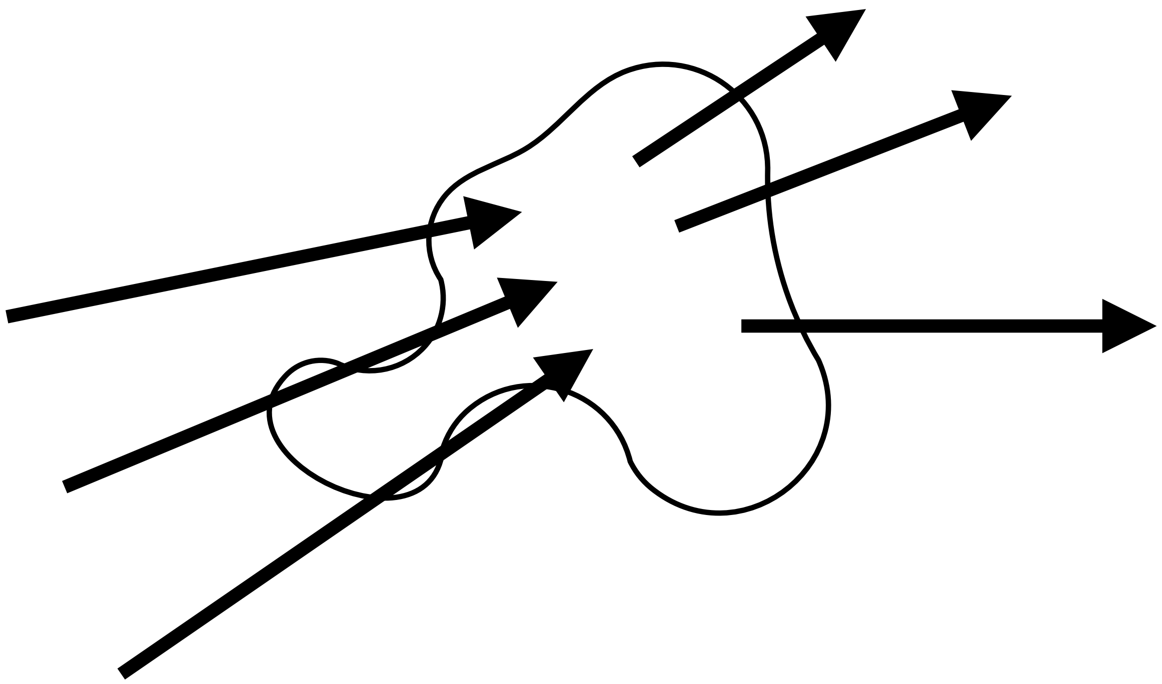
3. Transport properties: Chiral kinetic theory

$$\dot{f}(\mathbf{k},\mathbf{r}) + \dot{\mathbf{k}}\cdot\nabla_{\mathbf{k}} f(\mathbf{k},\mathbf{r}) + \dot{\mathbf{r}}\cdot\nabla_{\mathbf{r}} f(\mathbf{k},\mathbf{r}) = I_c[f]$$

forces diffusion scattering

$$dN = f(t,\boldsymbol{r},\boldsymbol{k})d\boldsymbol{r}d\boldsymbol{k}$$





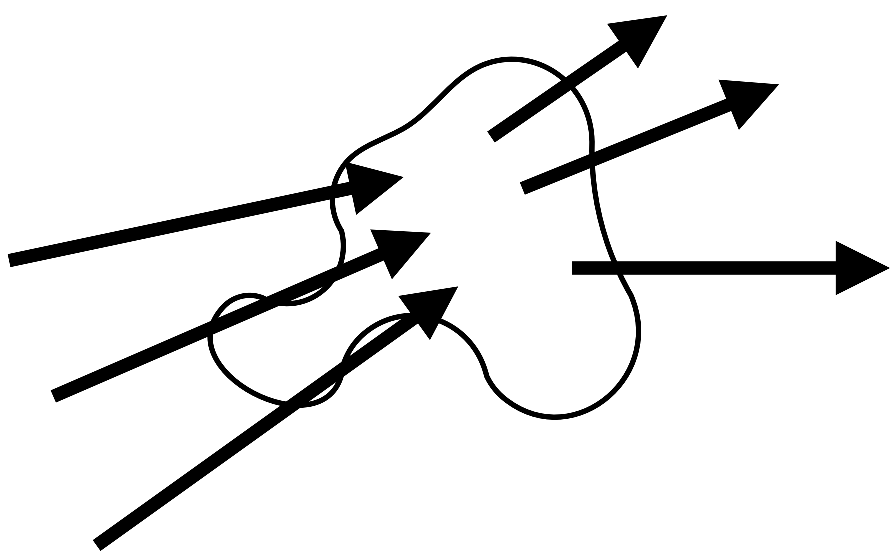
$$l \gg l_F$$

in essence, “geometrical optics” is applicable

3. Transport properties: Chiral kinetic theory

$$\dot{f}(\mathbf{k},\mathbf{r}) + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} f(\mathbf{k},\mathbf{r}) + \dot{\mathbf{r}} \cdot \nabla_{\mathbf{r}} f(\mathbf{k},\mathbf{r}) = I_c[f]$$

forces diffusion scattering



$$I_c[f] = \int (d\boldsymbol{k}') W_{\boldsymbol{k}\boldsymbol{k}'} (f(\boldsymbol{k}',\boldsymbol{r}) - f(\boldsymbol{k},\boldsymbol{r}))$$

Elastic (impurity) scattering

3. Transport properties: Chiral kinetic theory

semiclassical dynamics:

Time dependent variational principle

Sundaram, Niu, PR 53, 7010 (1996)

M Marder “Condensed Matter Physics”

$$\mathcal{L} = \langle\psi|\, i\hbar\partial_t - H \,|\psi\rangle$$

$$|\psi\rangle = \sum_{\boldsymbol{k}} |W| e^{i\boldsymbol{k}_c \boldsymbol{r}_c - \chi(\boldsymbol{r}_c) - \chi'(\boldsymbol{k}_c)} \, |u_{\boldsymbol{k}}\rangle$$

$$\boldsymbol{k}_c = \langle\psi| - i\hbar\partial_{\boldsymbol{r}} - e\boldsymbol{A} \,|\psi\rangle \;\Rightarrow \chi(\boldsymbol{r}_c) = \int_{\boldsymbol{r}_c} \boldsymbol{A}(\boldsymbol{r}')\cdot d\boldsymbol{r}'$$

$$0 = \langle\psi|\, \boldsymbol{r} - \boldsymbol{r}_c \,|\psi\rangle \Rightarrow \chi'(\boldsymbol{k}_c) = i(\boldsymbol{k} - \boldsymbol{k}_c)\cdot \boldsymbol{\mathcal{A}}(\boldsymbol{k}_c)$$

3. Transport properties: Chiral kinetic theory

semiclassical dynamics:

1. Time dependent variational principle

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$$|\psi\rangle = \sum_{\boldsymbol{k}} |W| e^{i\boldsymbol{k}_c\boldsymbol{r}_c - \chi(\boldsymbol{r}_c) - \chi'(\boldsymbol{k}_c)}\,|u_{\boldsymbol{k}}\rangle$$

$$\mathcal{L} = -e\dot{\boldsymbol{r}}\cdot\boldsymbol{A}(\boldsymbol{r}) + \hbar\boldsymbol{k}\cdot\dot{\boldsymbol{r}} + \hbar\dot{\boldsymbol{k}}\cdot\boldsymbol{\mathcal{A}}(\boldsymbol{k}) - \varepsilon^0(\boldsymbol{k}) - e\boldsymbol{m}(\boldsymbol{k})\cdot\boldsymbol{B}$$

$$\frac{\partial\mathcal{L}}{\partial\boldsymbol{q}} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\boldsymbol{q}}} \qquad \boldsymbol{q} = (\boldsymbol{r},\boldsymbol{k})$$

3. Transport properties: Chiral kinetic theory

semiclassical dynamics:

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M Marder "Condensed Matter Physics"

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$$\mathcal{L} = -e\dot{\boldsymbol{r}}\cdot\boldsymbol{A}(\boldsymbol{r}) + \hbar\boldsymbol{k}\cdot\dot{\boldsymbol{r}} + \hbar\dot{\boldsymbol{k}}\cdot\boldsymbol{\mathcal{A}}(\boldsymbol{k}) - \varepsilon^0(\boldsymbol{k}) - e\boldsymbol{m}(\boldsymbol{k})\cdot\boldsymbol{B}$$

$$\dot{\boldsymbol{k}} = e\boldsymbol{E} + e\dot{\boldsymbol{r}}\times\boldsymbol{B} \qquad \dot{\boldsymbol{r}} = \boldsymbol{v}_{\boldsymbol{k}} + \dot{\boldsymbol{k}}\times\boldsymbol{\Omega}_{\boldsymbol{k}}$$

3. Transport properties: Chiral kinetic theory

Equations of motion:

$$D = 1 + e\boldsymbol{\Omega} \cdot \boldsymbol{B}$$

$$\dot{\boldsymbol{k}} = e\boldsymbol{E} + e\dot{\boldsymbol{r}} \times \boldsymbol{B}$$

$$\dot{\boldsymbol{r}} = \boldsymbol{v_k} + \dot{\boldsymbol{k}} \times \boldsymbol{\Omega_k}$$ Berry phase

$$\boldsymbol{v_k} = \frac{\partial \varepsilon_{\boldsymbol{k}}}{\partial \boldsymbol{k}} - e\frac{\partial \boldsymbol{m_k}}{\partial \boldsymbol{k}} \cdot \boldsymbol{B}$$

$$D\dot{\boldsymbol{r}} = \boldsymbol{v_k} + e\boldsymbol{E} \times \boldsymbol{\Omega_k} + e(\boldsymbol{\Omega_k} \cdot \boldsymbol{v_k})\boldsymbol{B}$$

$$D\dot{\boldsymbol{k}} = e\boldsymbol{E} + e\boldsymbol{v_k} \times \boldsymbol{B} + e^2(\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega_k}$$

3. Transport properties: Chiral anomaly

$$\dot{\rho}_s = \int d\boldsymbol{k} \dot{f}_s$$

$$\int d\boldsymbol{k} \dot{f}_s + \int d\boldsymbol{k} \dot{\boldsymbol{k}} \partial_{\boldsymbol{k}} f_s = \int d\boldsymbol{k} \int d\boldsymbol{k}' W_{\boldsymbol{k}\boldsymbol{k}'} (f'_{s'} - f_s)$$

3. Transport properties: Chiral anomaly

$$\dot{\rho}_s = \int d\boldsymbol{k} \dot{f}_s$$

$$\int d\boldsymbol{k} \dot{f}_s = \int d\boldsymbol{k} f_s \partial_{\boldsymbol{k}} \cdot \dot{\boldsymbol{k}} + \int d\boldsymbol{k}' f'_{s'} \int d\boldsymbol{k} W_{\boldsymbol{k}\boldsymbol{k}'} - \int d\boldsymbol{k} f_s \int d\boldsymbol{k}' W_{\boldsymbol{k}\boldsymbol{k}'}$$

$$\dot{\boldsymbol{k}} = e\boldsymbol{E} + e\boldsymbol{v}_{\boldsymbol{k}} \times \boldsymbol{B} + e^2(\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega}_{\boldsymbol{k}}$$

3. Transport properties: Chiral anomaly

$$\dot{\rho}_s = \int d\boldsymbol{k} \dot{f}_s$$

$$\int d\boldsymbol{k} \dot{f}_s = \int d\boldsymbol{k} f_s \partial_{\boldsymbol{k}} \cdot \dot{\boldsymbol{k}} + \int d\boldsymbol{k}' f'_{s'} \int d\boldsymbol{k} W_{\boldsymbol{k}\boldsymbol{k}'} - \int d\boldsymbol{k} f_s \int d\boldsymbol{k}' W_{\boldsymbol{k}\boldsymbol{k}'}$$

$$\partial_{\boldsymbol{k}} \cdot \dot{\boldsymbol{k}} = e^2 (\boldsymbol{E} \cdot \boldsymbol{B}) \partial_{\boldsymbol{k}} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}}$$

$$\partial_{\boldsymbol{k}} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}} = s\delta(\boldsymbol{k})$$

For (linear) Weyl fermions

3. Transport properties: Chiral anomaly

$$\dot{\rho}_s = \int d\boldsymbol{k} \dot{f}_s$$

$$\dot{\rho}_s = s\frac{e^2}{8\pi^3}(\boldsymbol{E}\cdot\boldsymbol{B})f_s(0) - \frac{1}{2\tau_s}\left(\int d\boldsymbol{k} f_s - \int d\boldsymbol{k}' f_{s'}\right)$$

$$\partial_{\boldsymbol{k}} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}} = s\delta(\boldsymbol{k})$$

For (linear) Weyl fermions

$$\frac{1}{2\tau_s} = \int d\boldsymbol{k} W_{\boldsymbol{k}\boldsymbol{k}'}$$

Intervalley lifetime

3. Transport properties: Chiral anomaly

$$\dot{\rho}_s = \int d\boldsymbol{k} \dot{f}_s$$

$$\dot{\rho}_s = s\frac{e^2}{8\pi^3}(\boldsymbol{E}\cdot\boldsymbol{B})f_s(0) - \frac{1}{2\tau_s}(\rho_s - \rho_{s'})$$

$$\partial_{\boldsymbol{k}} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}} = s\delta(\boldsymbol{k})$$

For (linear) Weyl fermions

$$\frac{1}{2\tau_s} = \int d\boldsymbol{k} W_{\boldsymbol{k}\boldsymbol{k}'}$$

Intervalley lifetime

3. Transport properties: Chiral anomaly

$$\dot{\rho}_s = s\frac{e^2}{8\pi^3}(\boldsymbol{E}\cdot\boldsymbol{B})f_s(0) - \frac{1}{2\tau_s}(\rho_s - \rho_{s'})$$

$$\dot{\rho} = \dot{\rho}_+ + \dot{\rho}_- = 0$$ by gauge invariance

$$(f_+(0) = -f_-(0))$$

$$\dot{\rho}_5 \equiv \dot{\rho}_+ - \dot{\rho}_- = 2\frac{e^2}{8\pi^3}(\boldsymbol{E}\cdot\boldsymbol{B})f_+(0) - \frac{1}{\tau_s}(\rho_+ - \rho_-)$$

3. Transport properties: Chiral anomaly

$$\dot{\rho}_s = s\frac{e^2}{8\pi^3}(\boldsymbol{E}\cdot\boldsymbol{B})f_s(0) - \frac{1}{2\tau_s}(\rho_s - \rho_{s'})$$

$$\dot{\rho} = \dot{\rho}_+ + \dot{\rho}_- = 0$$

by gauge invariance

$$(f_+(0) = -f_-(0))$$

$$\dot{\rho}_5 = \frac{e^2}{4\pi^3}(\boldsymbol{E}\cdot\boldsymbol{B})f_+(0) - \frac{1}{\tau_s}\rho_5$$

Chiral charge is NOT conserved
(if we impose gauge invariance)

intervalley scattering
also modifies the non conservation
of chiral charge

3. Transport properties: Chiral magnetic effect

$$\dot{\boldsymbol{x}}_s = \boldsymbol{v}_{\boldsymbol{k}} + e\boldsymbol{E} \times \boldsymbol{\Omega}_s + e(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})\boldsymbol{B}$$

$$\boldsymbol{J}_s = e \int d\boldsymbol{k} f_s \dot{\boldsymbol{x}}_s = e^2 \boldsymbol{B} \int d\boldsymbol{k} f_s (\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})$$

For (linear) Weyl fermions $\boldsymbol{\Omega}_{\boldsymbol{s}} \cdot \boldsymbol{v}_{\boldsymbol{k}} = s\frac{v}{2k^2}$

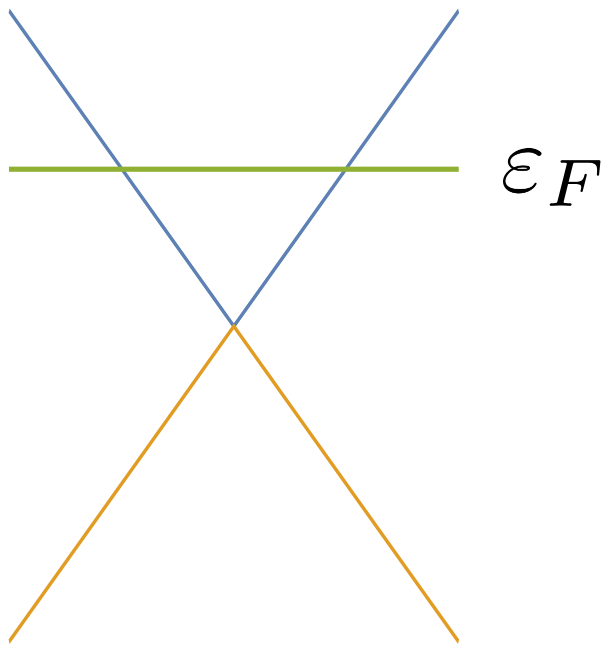
$$\boldsymbol{J}_s = e \int d\boldsymbol{k} f_s \dot{\boldsymbol{x}}_s = s\boldsymbol{B}\frac{ve^2}{4\pi^2} \int_0^{k_F} dk f_s$$

3. Transport properties: Chiral magnetic effect

$$\boldsymbol{J}_s = e \int d\boldsymbol{k} f_s \dot{\boldsymbol{x}}_s = s\boldsymbol{B}\frac{e^2}{4\pi^2}\int d\varepsilon f_s(\varepsilon)$$

$\varepsilon = \pm vk$ (linear Weyl fermions)

Full distribution function



$$\boldsymbol{J}_+^{(I)} = \boldsymbol{B}\frac{e^2}{4\pi^2}\varepsilon_F$$

Equilibrium current!!

(another) Bloch theorem:
no currents in equilibrium
(Ground state)

3. Transport properties: Negative magnetoresistance

Out of equilibrium properties: apply an electric field

$$D_s \dot{\boldsymbol{x}} = \boldsymbol{v_k} + e\boldsymbol{E} \times \boldsymbol{\Omega}_s + e(\boldsymbol{\Omega}_s \cdot \boldsymbol{v_k})\boldsymbol{B}$$

CME Precursor

$$D_s \dot{\boldsymbol{k}} = e\boldsymbol{E} + e\boldsymbol{v_k} \times \boldsymbol{B} + e^2(\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega}_s$$

Chiral anomaly precursor

$$D_s \dot{\boldsymbol{k}} \cdot \partial_{\boldsymbol{k}} f_s = I_c[f_s, f_{s'}]$$

Boltzmann equation

$$f_s \sim f_0(\varepsilon_{\boldsymbol{k}}) + f_{1s} \qquad f_{1s} \sim \mathcal{O}(\boldsymbol{E})$$

3. Transport properties: Negative magnetoresistance

Out of equilibrium properties: apply an electric field

$$D_s \dot{\boldsymbol{x}} \sim e(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})\boldsymbol{B}$$

CME precursor

$$D_s \dot{\boldsymbol{k}} \cdot \boldsymbol{v}_{\boldsymbol{k}} \sim e^2 \boldsymbol{E} \cdot \boldsymbol{B}(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})$$

Chiral anomaly precursor

$$\boldsymbol{J}_s \sim e \int (d\boldsymbol{k}) D_s \dot{\boldsymbol{x}} f_{1s}$$

$$D_s \dot{\boldsymbol{k}} \cdot \boldsymbol{v}_{\boldsymbol{k}} \frac{\partial f_0}{\partial \varepsilon} + (\boldsymbol{v}_{\boldsymbol{k}} \times \boldsymbol{B}) \cdot \partial_{\boldsymbol{k}} f_{1s} = \sum_{s'} \int d\boldsymbol{k}' W_{\boldsymbol{k}\boldsymbol{k}'}^{ss'} (f'_{1s'} - f_{1s})$$

3. Transport properties: Negative magnetoresistance

Out of equilibrium properties: apply an electric field

$$D_s \dot{\boldsymbol{x}} \sim e(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})\boldsymbol{B} \qquad D_s \dot{\boldsymbol{k}} \cdot \boldsymbol{v}_{\boldsymbol{k}} \sim e^2 \boldsymbol{E} \cdot \boldsymbol{B}(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})$$

CME precursor — Chiral anomaly precursor

$$\boldsymbol{J}_s \sim e \int (d\boldsymbol{k}) D_s \dot{\boldsymbol{x}} f_{1s}$$

$$D_s \dot{\boldsymbol{k}} \cdot \boldsymbol{v}_{\boldsymbol{k}} \frac{\partial f_0}{\partial \varepsilon} + (\boldsymbol{v}_{\boldsymbol{k}} \times \boldsymbol{B}) \cdot \partial_{\boldsymbol{k}} f_{1s} = \sum_{s'} \int d\boldsymbol{k}' W^{ss'}_{\boldsymbol{k}\boldsymbol{k}'} (f'_{1s'} - f_{1s})$$

Isotropy $\boldsymbol{E} // \boldsymbol{B}$

3. Transport properties: Negative magnetoresistance

Out of equilibrium properties: apply an electric field

$$D_s \dot{\boldsymbol{x}} \sim e(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})\boldsymbol{B}$$

CME precursor

$$D_s \dot{\boldsymbol{k}} \cdot \boldsymbol{v}_{\boldsymbol{k}} \sim e^2 \boldsymbol{E} \cdot \boldsymbol{B}(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})$$

Chiral anomaly precursor

$$\boldsymbol{J}_s \sim e \int (d\boldsymbol{k}) D_s \dot{\boldsymbol{x}} f_{1s}$$

$$\left(e\boldsymbol{E} \cdot \boldsymbol{v}_{\boldsymbol{k}} + e^2 \boldsymbol{E} \cdot \boldsymbol{B}(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})\right) \frac{\partial f_0}{\partial \varepsilon} = \sum_{s'} \int d\boldsymbol{k}' W^{ss'}_{\boldsymbol{k}\boldsymbol{k}'} (f'_{1s'} - f_{1s})$$

3. Transport properties: Negative magnetoresistance

Out of equilibrium properties: apply an electric field

$$\boldsymbol{J}_s \sim e \int (d\boldsymbol{k}) D_s \dot{\boldsymbol{x}} f_{1s} \qquad D_s \dot{\boldsymbol{x}} \sim e(\boldsymbol{\Omega}_s \cdot \boldsymbol{v_k})\boldsymbol{B}$$

$$f_{1s} \sim \tau^* \frac{\partial f_0}{\partial \varepsilon} \left(e\boldsymbol{E} \cdot \boldsymbol{v_k} + e^2 (\boldsymbol{E} \cdot \boldsymbol{B}) \boldsymbol{\Omega}_s \cdot \boldsymbol{v_k} \right)$$

$$\boldsymbol{J}_s \sim \tau^* e^3 \boldsymbol{B}(\boldsymbol{B} \cdot \boldsymbol{E}) \int (d\boldsymbol{k}) \frac{\partial f_0}{\partial \varepsilon} \left(\boldsymbol{\Omega}_s \cdot \boldsymbol{v_k} \right)^2$$

3. Transport properties: Negative magnetoresistance

Out of equilibrium properties: apply an electric field

$$\boldsymbol{J}_s \sim e \int (d\boldsymbol{k}) D_s \dot{\boldsymbol{x}} f_{1s} \qquad D_s \dot{\boldsymbol{x}} \sim e(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}})\boldsymbol{B}$$

$$f_{1s} \sim \tau^* \frac{\partial f_0}{\partial \varepsilon} \left(e\boldsymbol{E} \cdot \boldsymbol{v}_{\boldsymbol{k}} + e^2(\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}}\right)$$

$$\sigma \sim \tau^* \boldsymbol{B}^2 \int (d\boldsymbol{k}) \left(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}}\right)^2 \delta(\varepsilon_F - \varepsilon_{\boldsymbol{k}})$$

$$T = 0$$

3. Transport properties: Negative magnetoresistance

Out of equilibrium properties: apply an electric field

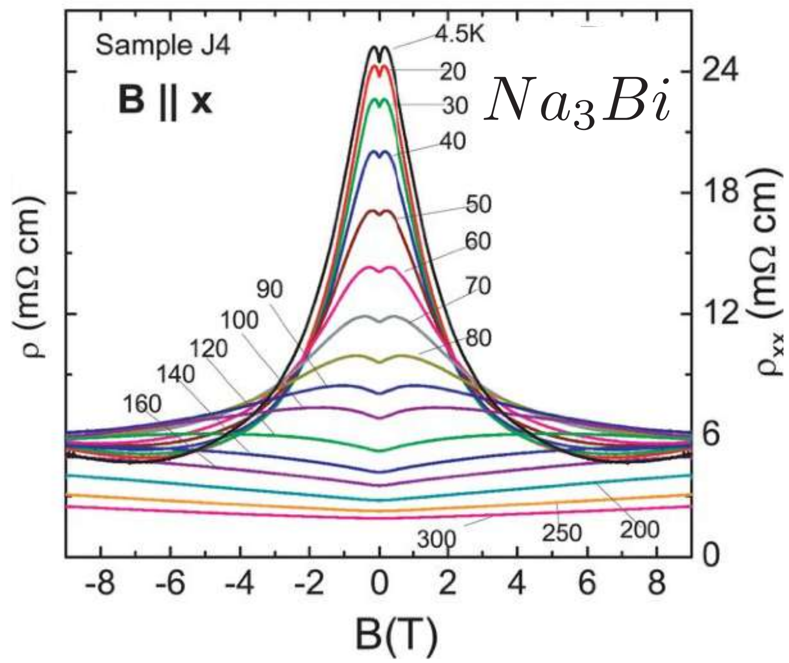
$$\sigma \sim \tau^* \boldsymbol{B}^2 \int (d\boldsymbol{k}) \left(\boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}}\right)^2 \delta(\varepsilon_F - \varepsilon_{\boldsymbol{k}})$$

$$\varepsilon_{\boldsymbol{k}} = vk \qquad\qquad \boldsymbol{\Omega}_s \cdot \boldsymbol{v}_{\boldsymbol{k}} = s\frac{v}{2k^2}$$

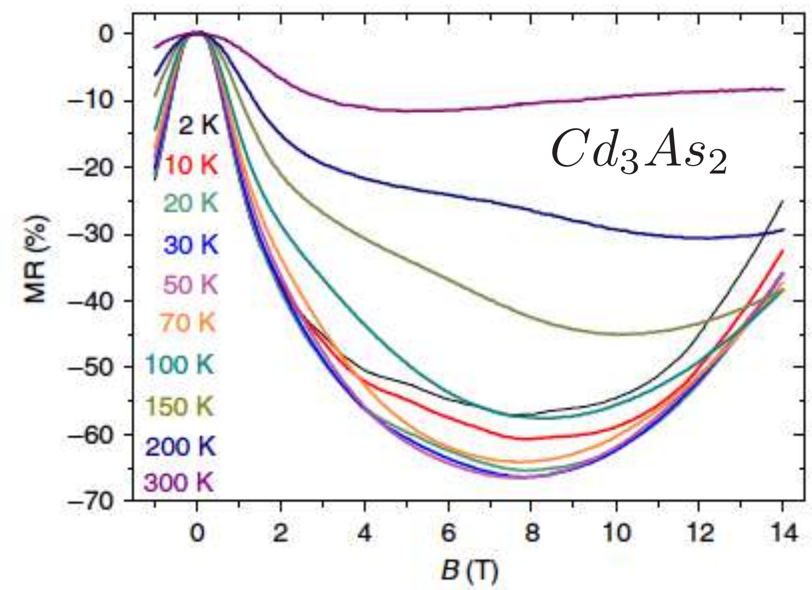
$$\sigma \sim e^3 \tau^* \frac{v^3}{\varepsilon_F^2} B^2$$

$$T = 0$$

3. Transport properties: Negative magnetoresistance

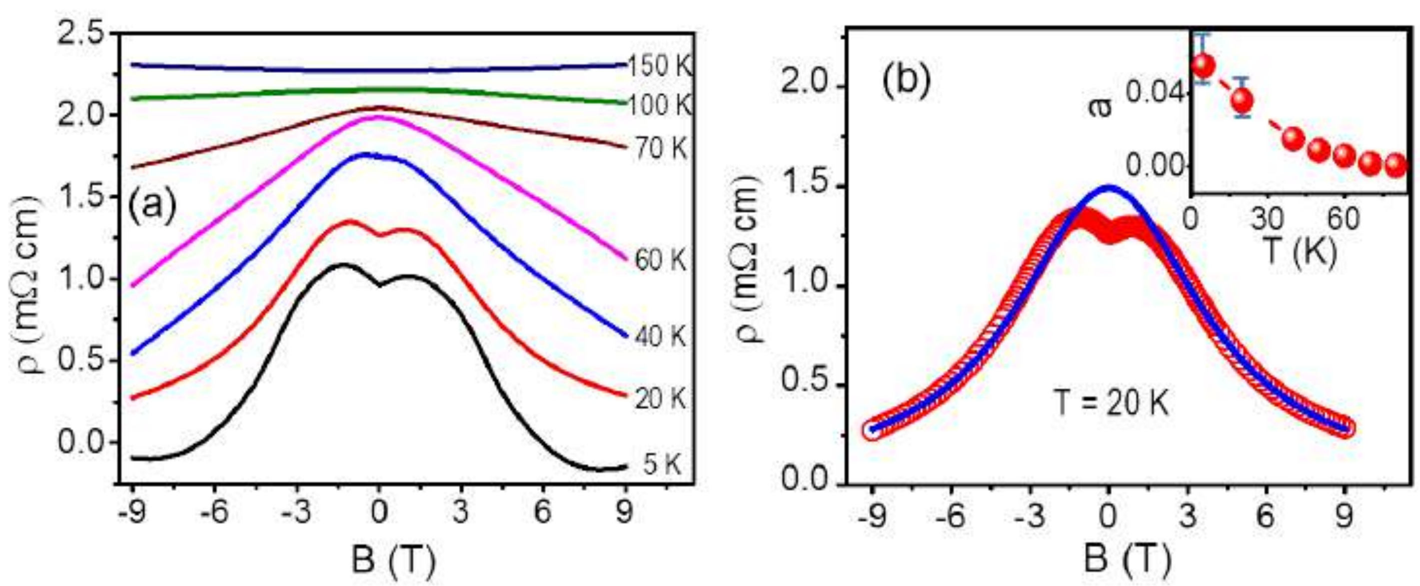


J Xiong et al. Science 23, 413 (2015)



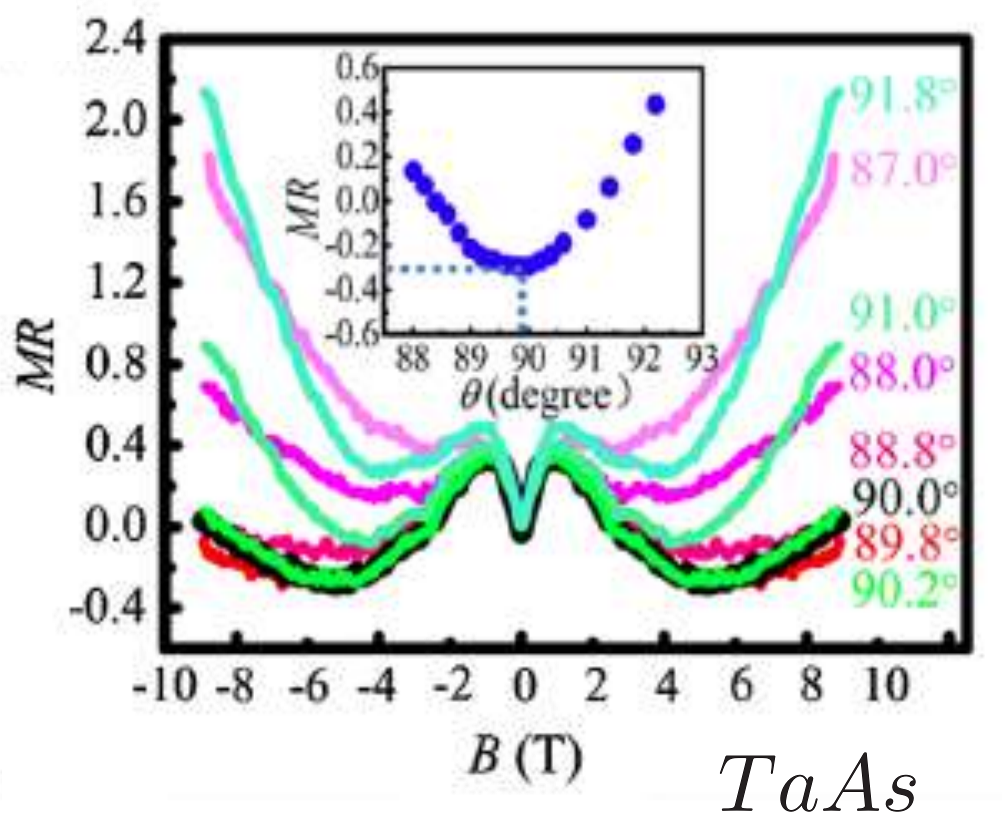
H Li et al. Nat. Comm. 7, 10301 (2016)

$ZnTe_5$

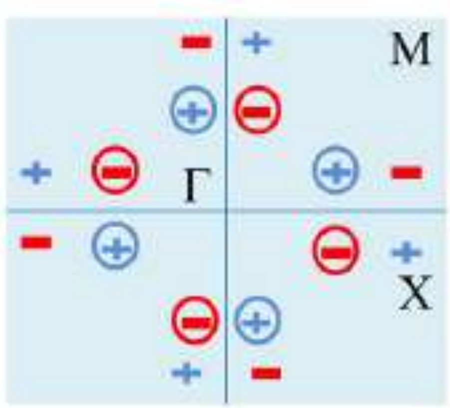


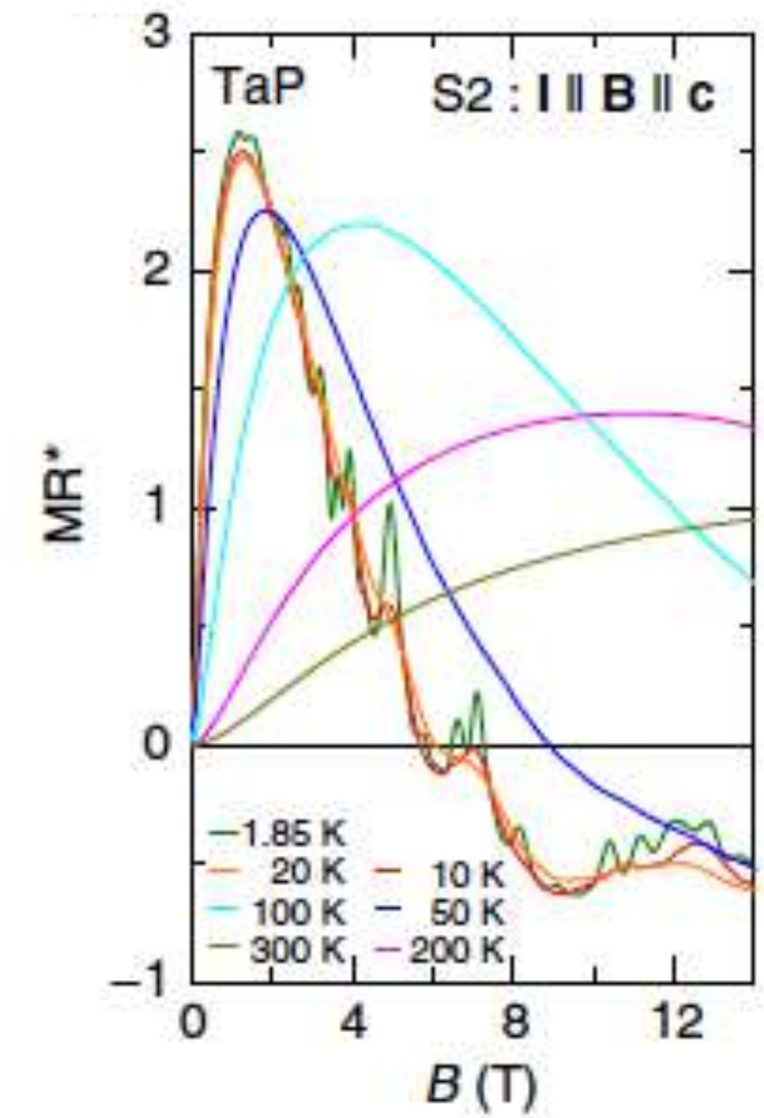
Q Li et al. Nat. Phys. 12, 550 (2016)

3. Transport properties: Negative magnetoresistance



X Huang et al. PRX 5, 03023 (2015)

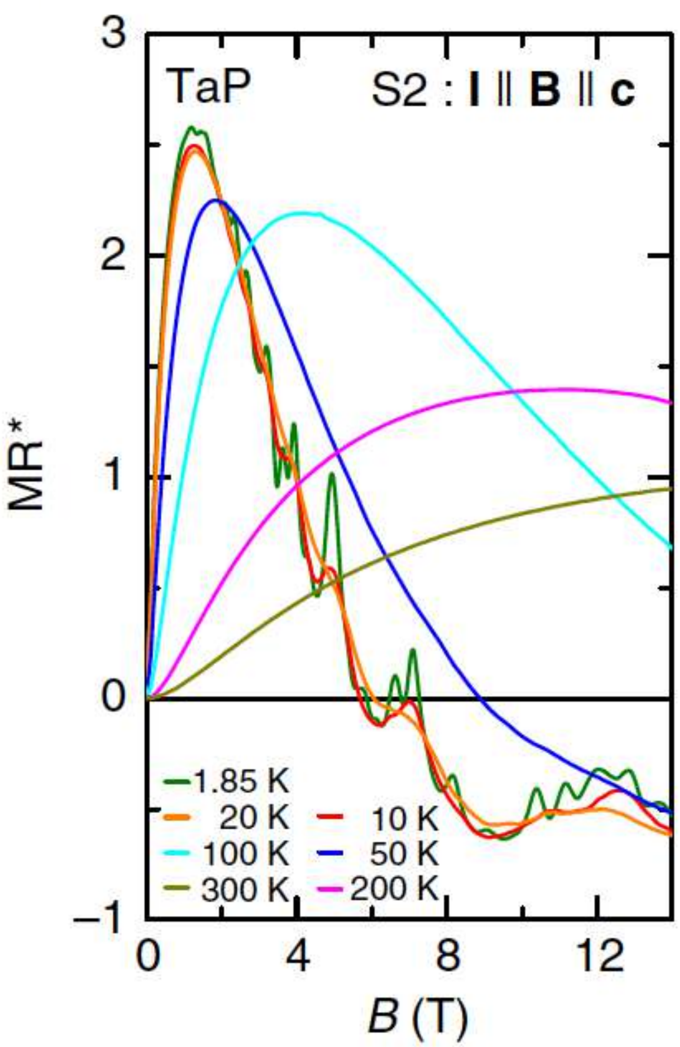


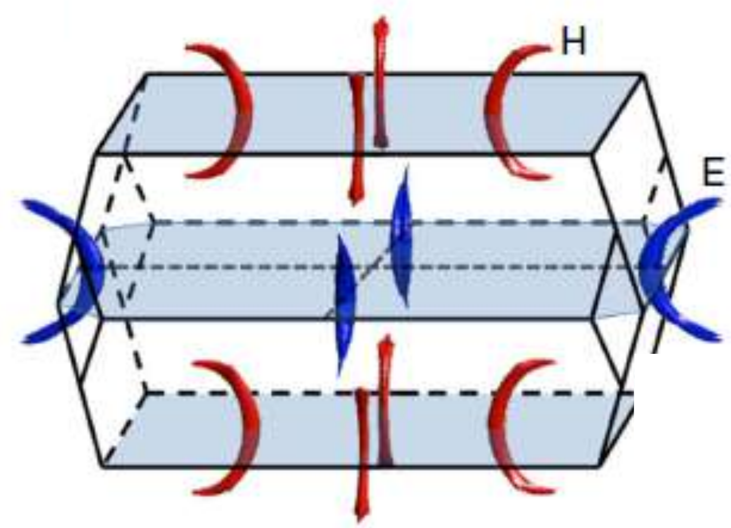


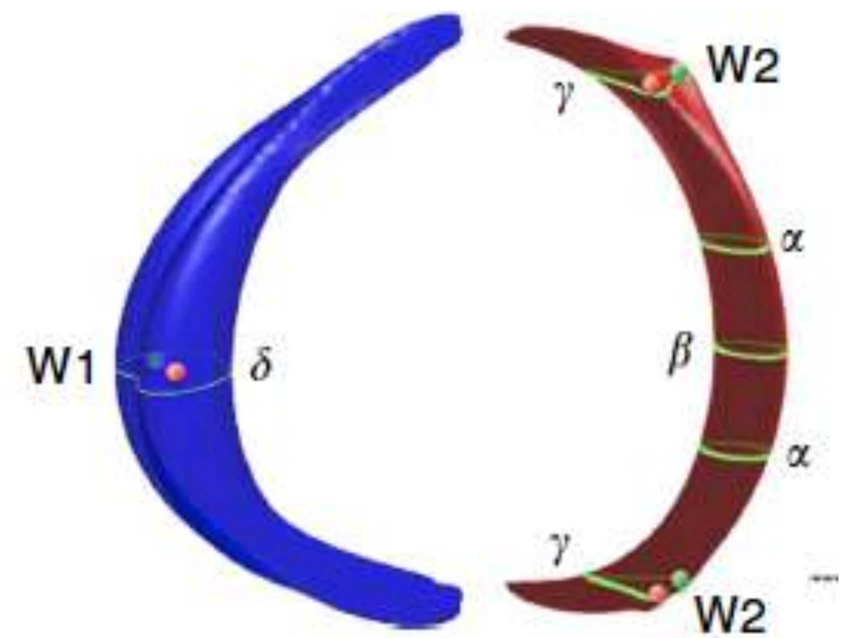
Arnold et al. Nat. Comm. 7, 11615 (2016)

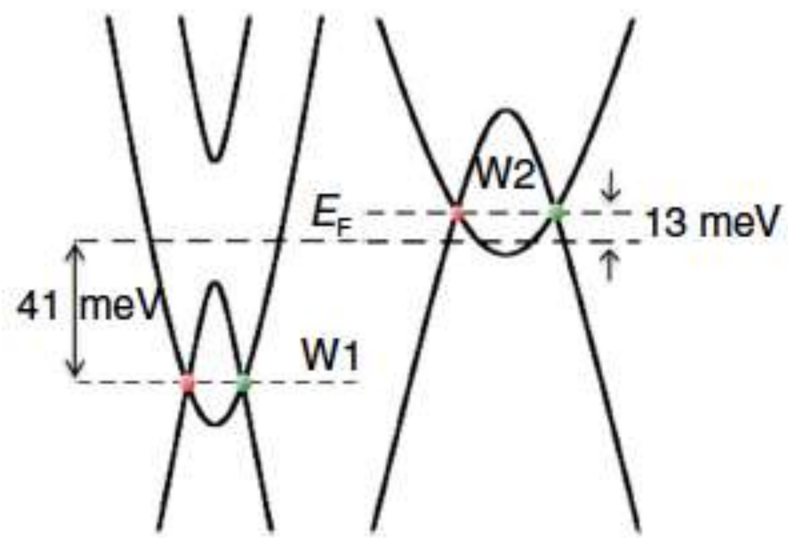
3. Transport properties: Negative magnetoresistance

Arnold et al. Nat. Comm. 7, 11615 (2016)



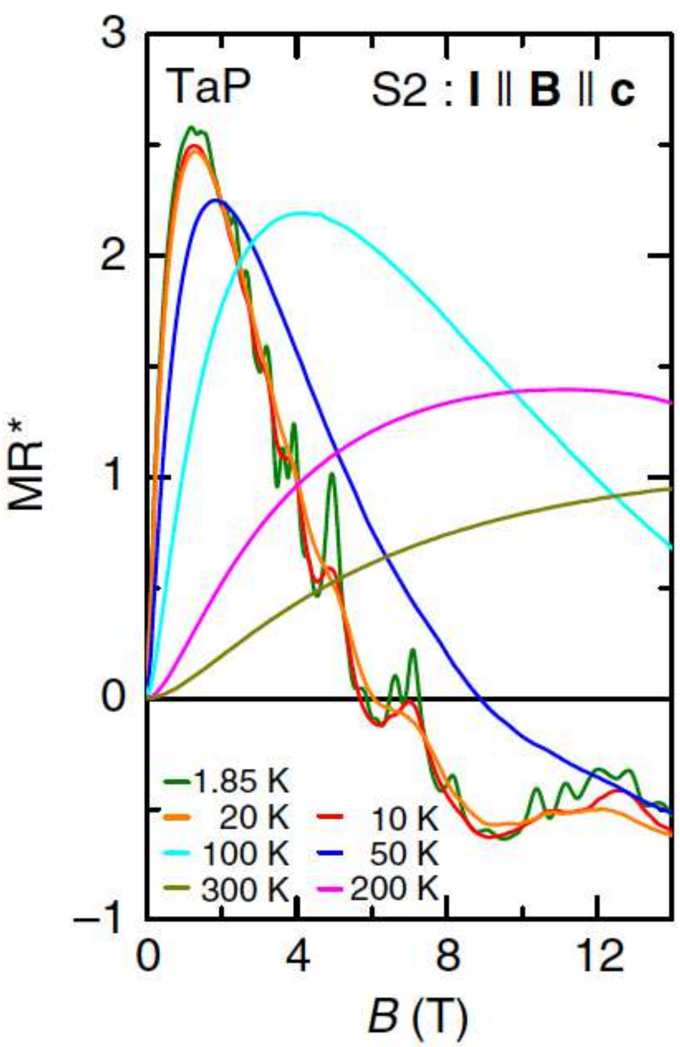


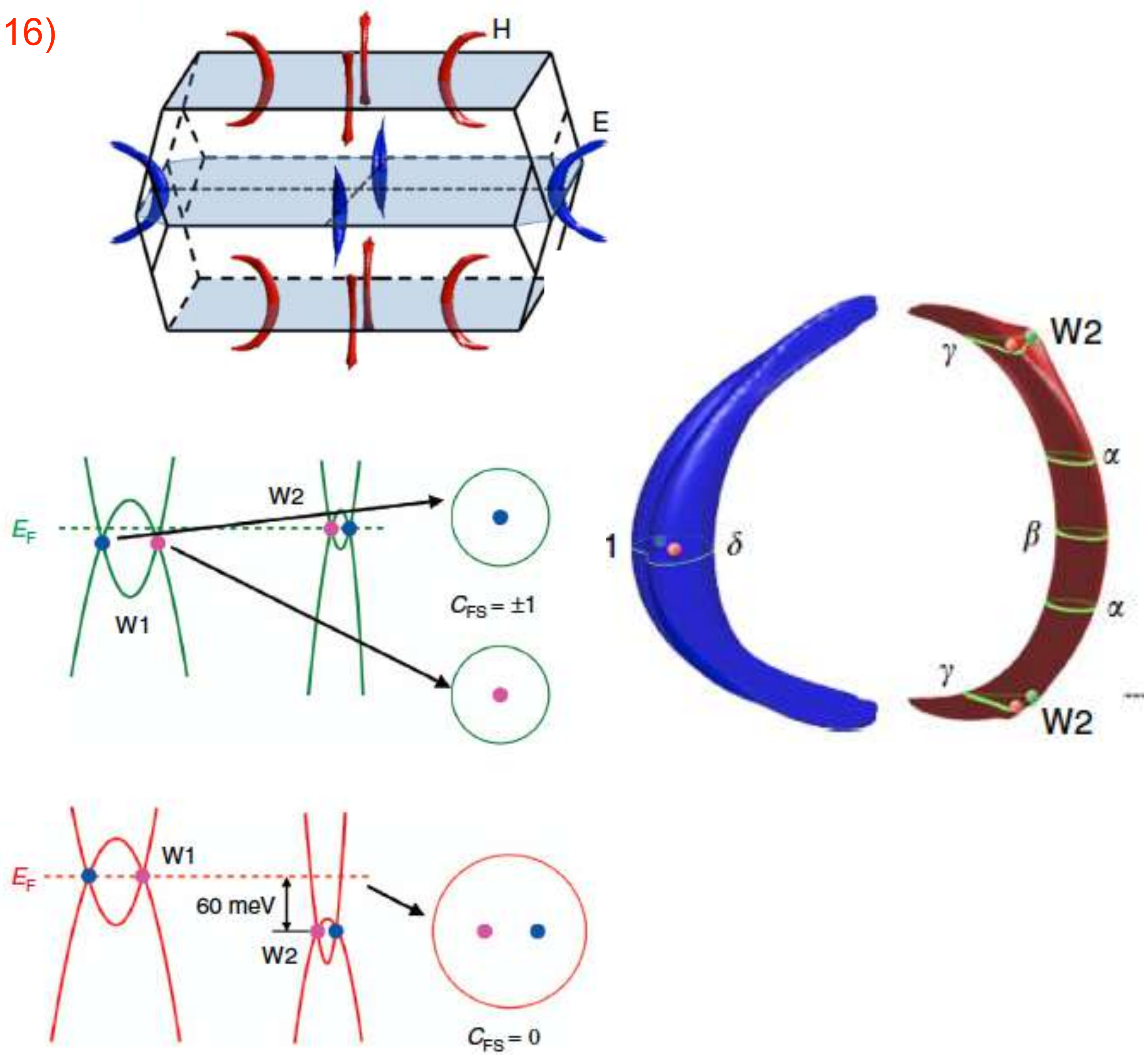




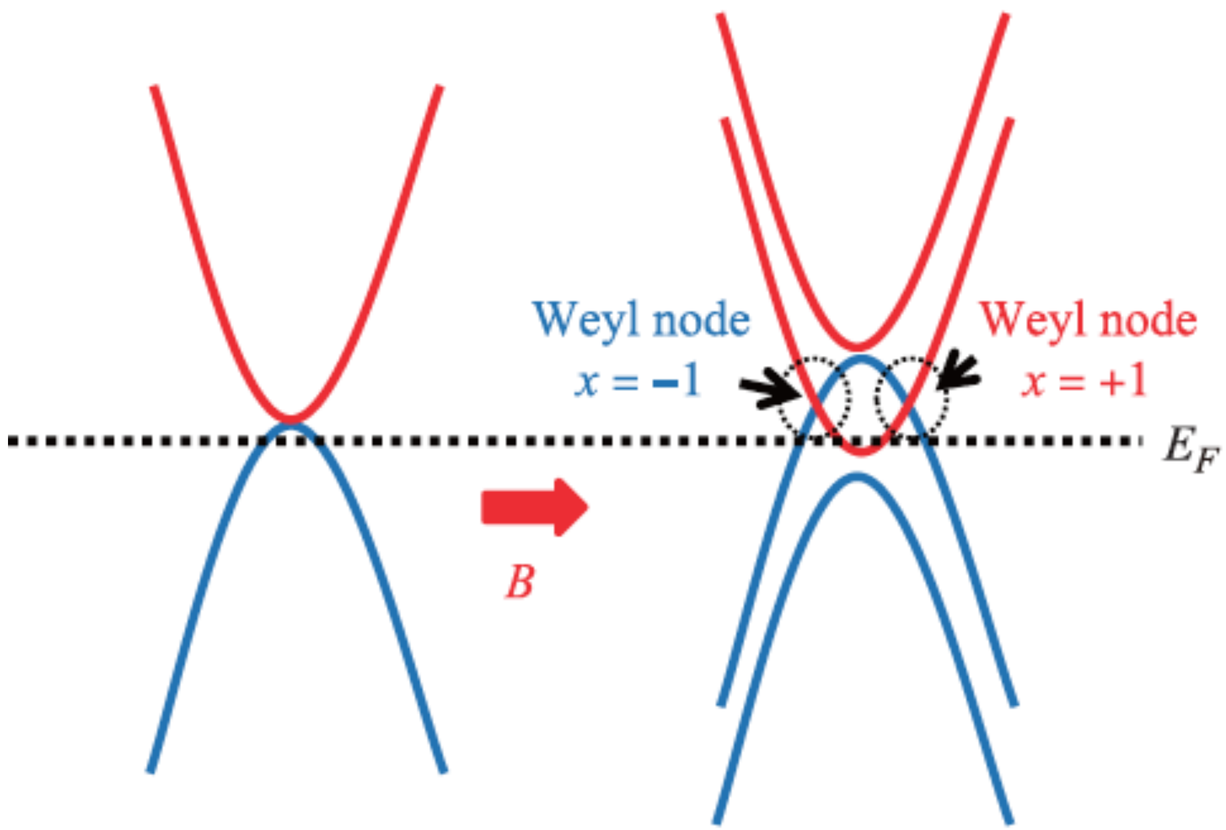
3. Transport properties: Negative magnetoresistance

Arnold et al. Nat. Comm. 7, 11615 (2016)

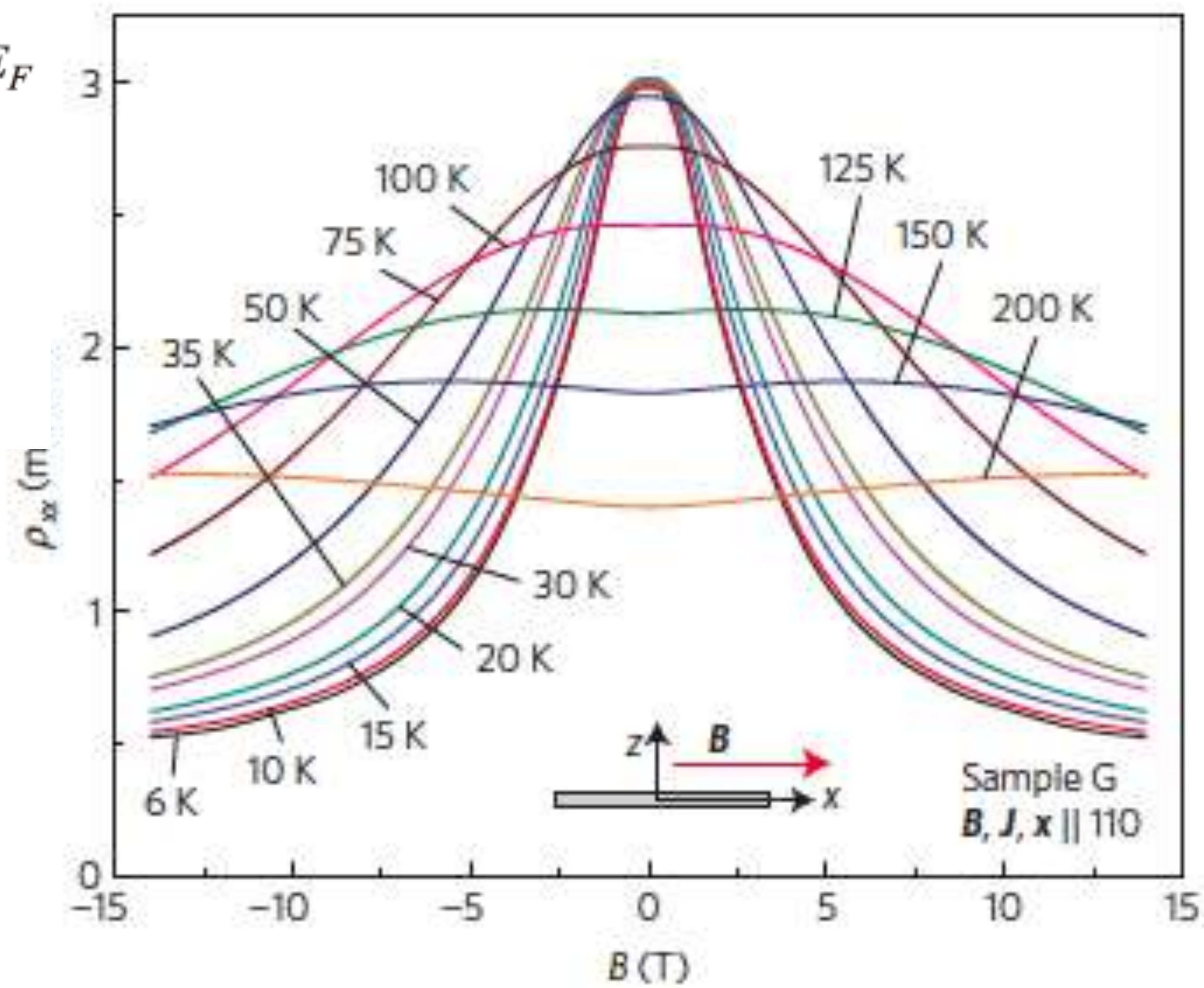




3. Transport properties: Negative magnetoresistance



$GdPtBi$

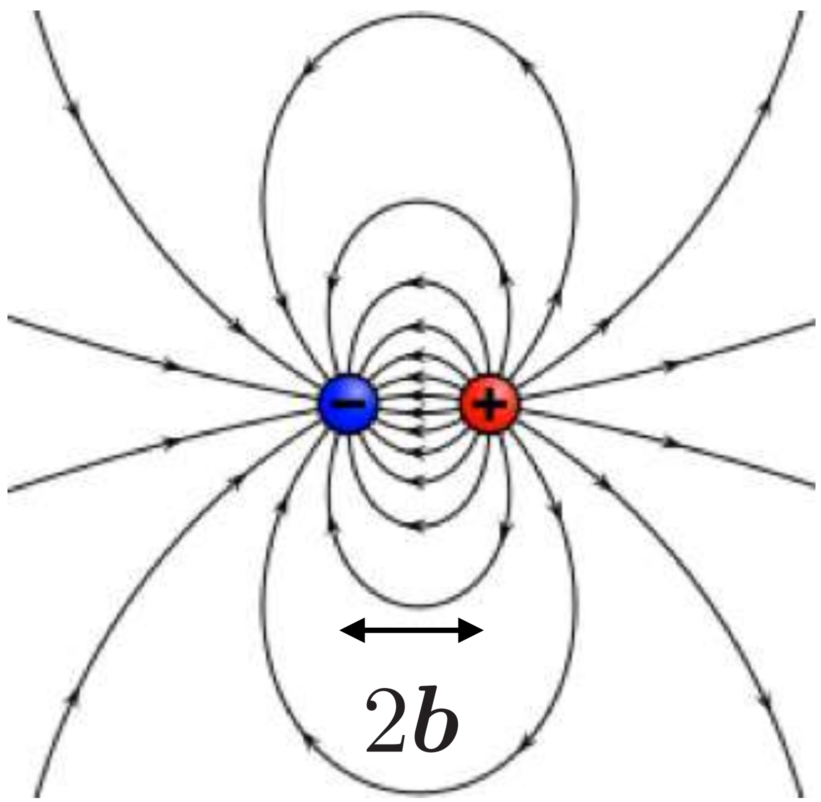


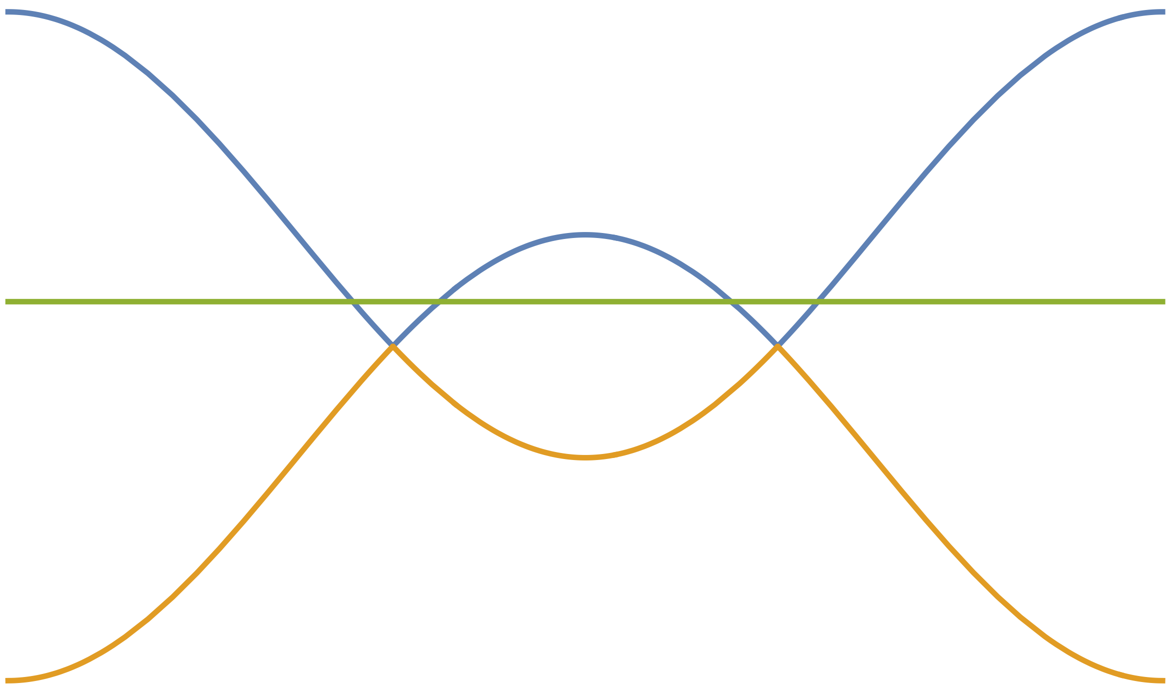
$$\boldsymbol{b} \sim g\boldsymbol{B}$$

M Hirschberger et al. Nat. Mat. 15, 1161 (2016)

3. Transport properties: Beyond the monopole paradigm

Nielsen-Ninomiya theorem





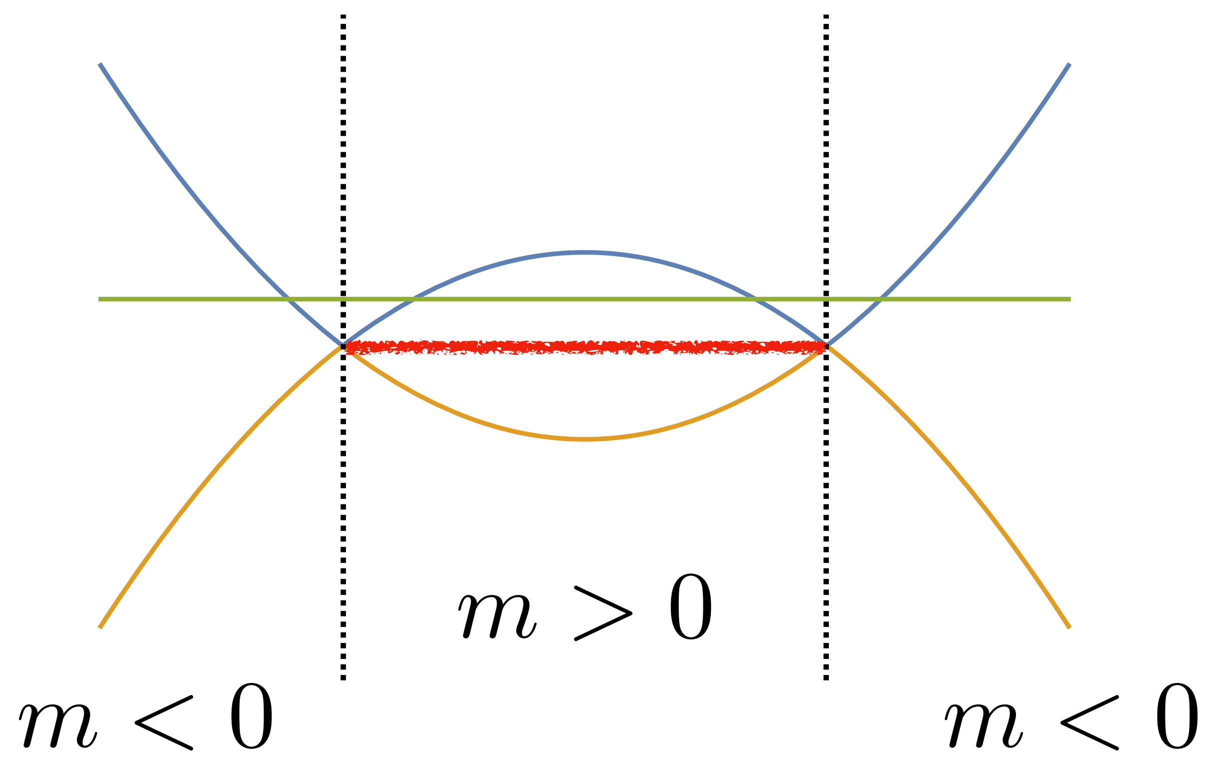
the Berry curvature is more complex
than the sum of monopoles

Nielsen, Ninomiya, NucPhysB 185, 20 (1981)

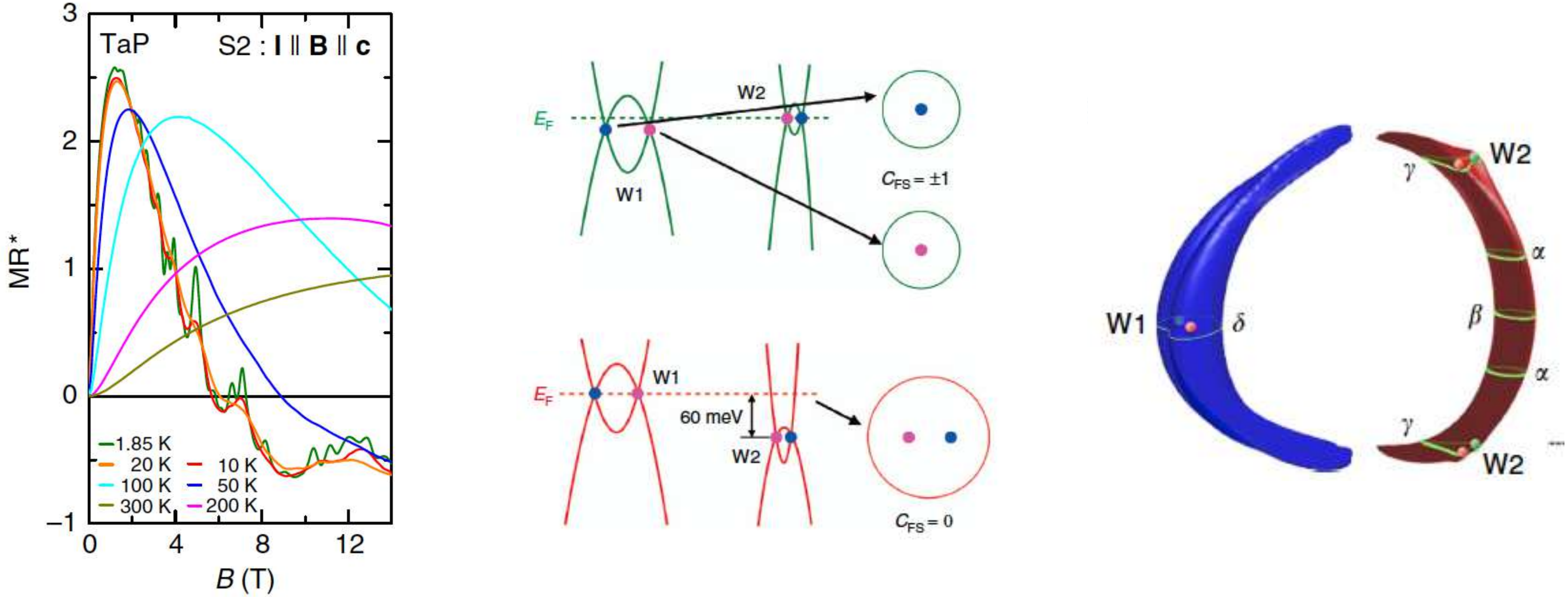
3. Transport properties: Beyond the monopole paradigm

Fermi arcs

$$H(\boldsymbol{k}) = \boldsymbol{\sigma} \cdot \boldsymbol{k}_\perp + \sigma_3 m(\boldsymbol{k}) \qquad m = m_0 - \beta k_3^2 - \beta |\boldsymbol{k}_\perp|^2$$

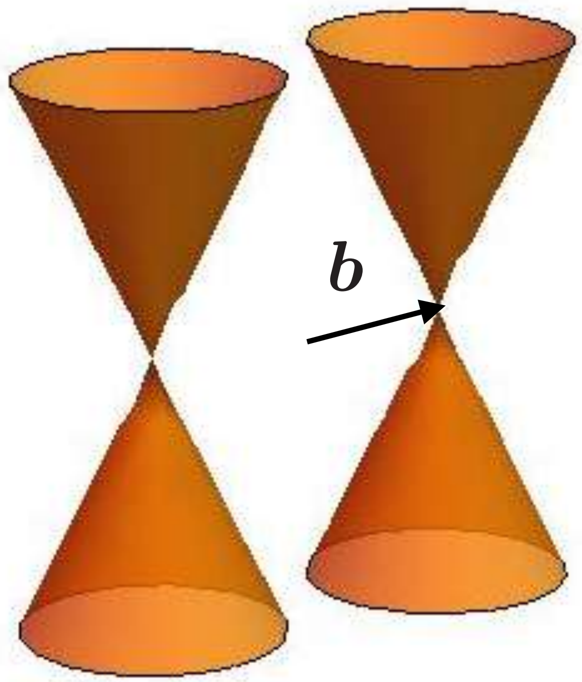


3. Transport properties: Beyond the monopole paradigm



In condensed matter physics, it is natural to find departures from the strict linear behaviour

3. Transport properties: Beyond the monopole paradigm

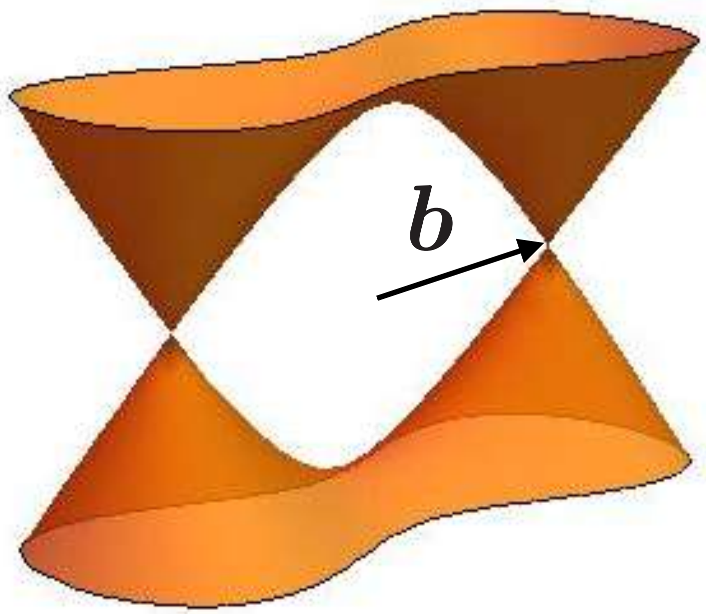


$$\dot{\rho}_5 + \nabla \cdot \boldsymbol{J}_5 = \frac{e^3}{2\pi^2} \boldsymbol{B} \cdot \boldsymbol{E}$$

strict linear (and unbounded) spectrum

LH Karsten, J Smit, Nuc. Phys. B 183, 103 (1981)

3. Transport properties: Beyond the monopole paradigm



$$\dot{\rho}_5 + \nabla \cdot \boldsymbol{J}_5 = \frac{e^3}{2\pi^2} \boldsymbol{B} \cdot \boldsymbol{E} + \mathcal{O}(a, \hat{\boldsymbol{k}}, \boldsymbol{A})$$

⇧

they vanish in the long wavelength limit

$$a \to 0$$

$$H(\boldsymbol{k}) = \boldsymbol{\sigma} \cdot \boldsymbol{k}_\perp + \sigma_3 m(\boldsymbol{k})$$

$$\boldsymbol{b}_\pm = (\boldsymbol{0}_\perp, \pm b_3) \qquad b_3 = \sqrt{\frac{m_0}{\beta}}$$

$$m(k_3) = m_0 - \beta k_3^2$$

$$m(k_3) \approx \pm v_3 \delta k_3 + \frac{1}{2} \frac{v_3}{b_3} \delta k_3^2$$

LH Karsten, J Smit, Nuc. Phys. B 183, 103 (1981)

3. Transport properties: Beyond the monopole paradigm

$$\dot{\boldsymbol{k}} \cdot \boldsymbol{v}^0 \sim e\boldsymbol{E} \cdot \boldsymbol{v}^0 + e^2(\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega} \cdot \boldsymbol{v}^0$$

$$f_1 \sim \tau^* \frac{\partial f_0}{\partial \varepsilon} D\dot{\boldsymbol{k}} \cdot \boldsymbol{v}^0$$

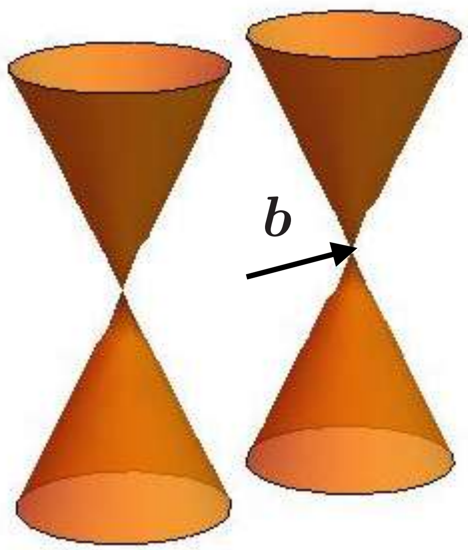
$$D\dot{\boldsymbol{r}} \sim \boldsymbol{v}^0 + e(\boldsymbol{\Omega} \cdot \boldsymbol{v}^0)\boldsymbol{B}$$

$$\boldsymbol{J}_3 = e^2 \boldsymbol{B} \int (d\boldsymbol{k}) f(\boldsymbol{\Omega}_{\boldsymbol{k}} \cdot \boldsymbol{v}_{\boldsymbol{k}})$$

$$\boldsymbol{J} = -\tau e^3(\boldsymbol{B} \cdot \boldsymbol{E}) \int (d\boldsymbol{k}) \frac{\partial f_0}{\partial \epsilon} (\boldsymbol{\Omega} \cdot \boldsymbol{v}^0)\boldsymbol{v}^0 - \\ -\tau e^3 \boldsymbol{B} \int (d\boldsymbol{k}) \frac{\partial f_0}{\partial \epsilon} (\boldsymbol{\Omega} \cdot \boldsymbol{v}^0)(\boldsymbol{v}^0 \cdot \boldsymbol{E})$$

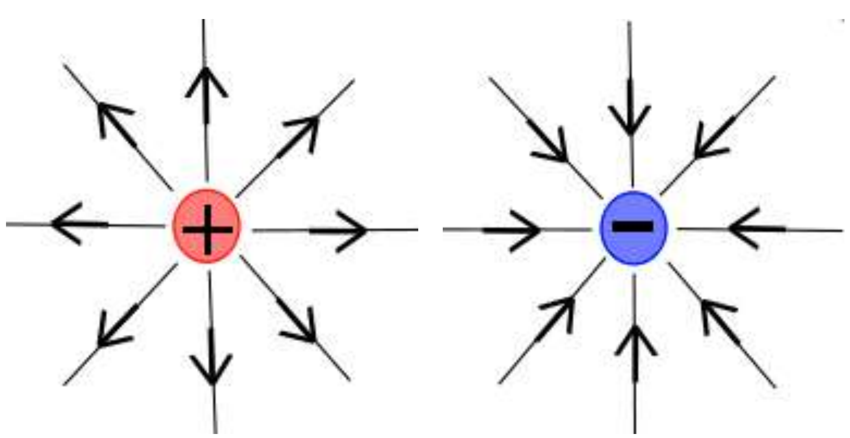
chiral kinetic theory does not forbid linear-in-B terms

3. Transport properties: Beyond the monopole paradigm



$$\boldsymbol{J} = -\tau e^3(\boldsymbol{B}\cdot\boldsymbol{E})\int(d\boldsymbol{k})\frac{\partial f_0}{\partial\epsilon}(\boldsymbol{\Omega}\cdot\boldsymbol{v}^0)\boldsymbol{v}^0 - \\ -\tau e^3\boldsymbol{B}\int(d\boldsymbol{k})\frac{\partial f_0}{\partial\epsilon}(\boldsymbol{\Omega}\cdot\boldsymbol{v}^0)(\boldsymbol{v}^0\cdot\boldsymbol{E})$$

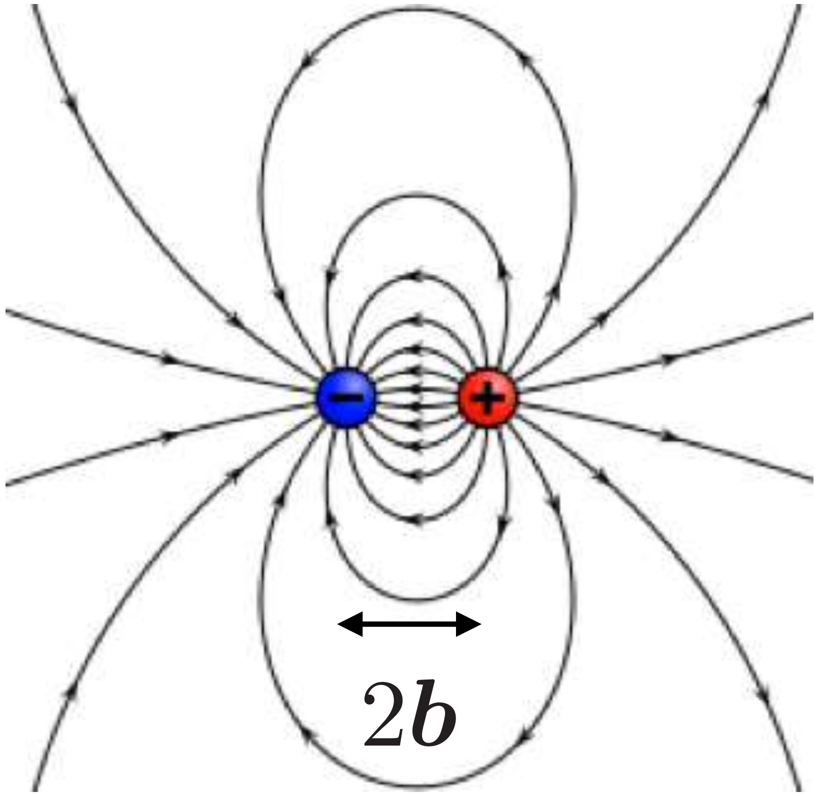
$$\boldsymbol{J}_1 = \boldsymbol{J}_1^+ + \boldsymbol{J}_1^- \qquad \boldsymbol{v}^0 = \frac{\partial\epsilon_{\boldsymbol{k}}}{\partial\boldsymbol{k}}$$



$$\boldsymbol{\Omega} \to \boldsymbol{\Omega}^{\pm}$$

$$\boldsymbol{J}_1 = 0$$

3. Transport properties: Beyond the monopole paradigm



multipole expansion as
in electromagnetism

$$\boldsymbol{\Omega} \sim \boldsymbol{\Omega}^{+} + \boldsymbol{\Omega}^{-} + 2\boldsymbol{b} \cdot \partial_{\boldsymbol{k}} \boldsymbol{\Omega} + \ldots$$

in the near field regime $|\boldsymbol{k}| \ll \boldsymbol{b}$

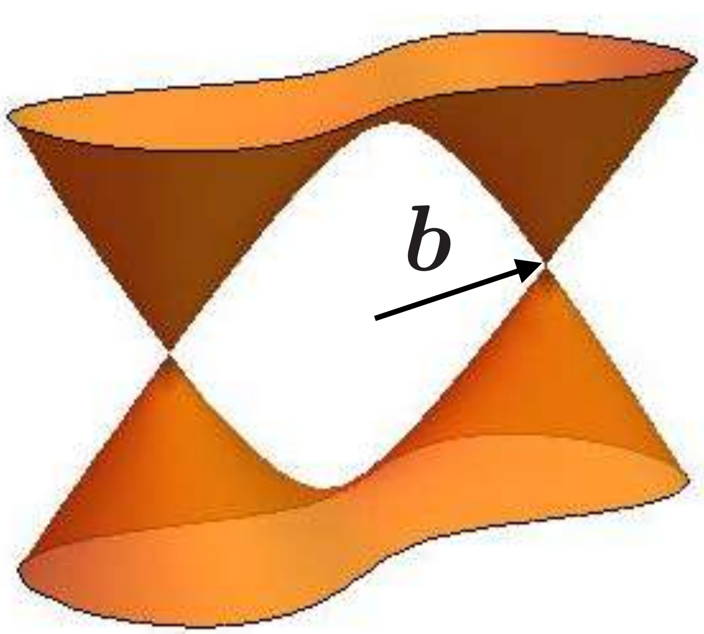
$$\boldsymbol{\Omega} \sim \boldsymbol{\Omega}^{\pm} \mp \boldsymbol{b} \cdot \partial_{\boldsymbol{k}} \boldsymbol{\Omega} + \ldots$$

in the far field regime $|\boldsymbol{k}| \gg \boldsymbol{b}$

$$\boldsymbol{\Omega} \sim 2\boldsymbol{b} \cdot \partial_{\boldsymbol{k}} \boldsymbol{\Omega} + \ldots$$

3. Transport properties: Beyond the monopole paradigm

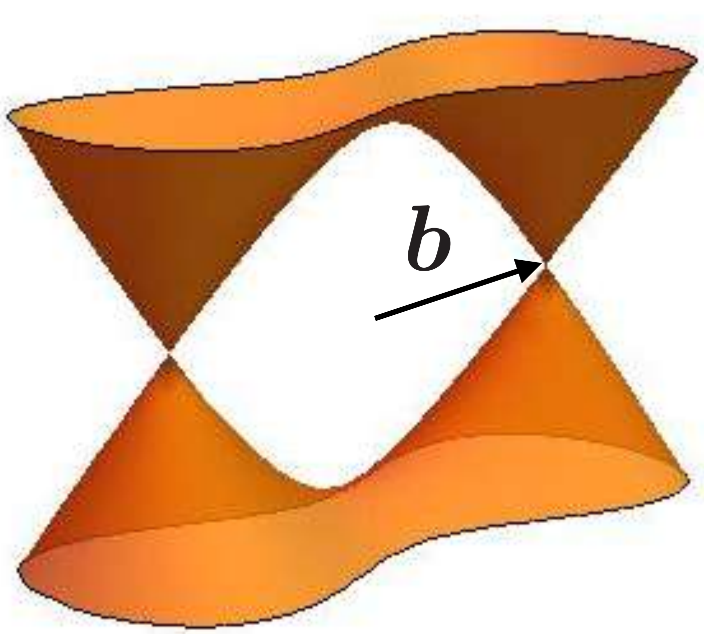
$$\boldsymbol{J}_1 \sim \tau e^3(\boldsymbol{E}\cdot\boldsymbol{B})\int (d\boldsymbol{k})\frac{\partial f_0}{\partial \varepsilon}\boldsymbol{v}^0(\boldsymbol{\Omega}_m + \boldsymbol{b}\cdot\partial\boldsymbol{\Omega})\boldsymbol{v}^0$$



A Cortijo, PRB 94, 241105(R) (2016)

3. Transport properties: Beyond the monopole paradigm

$$\boldsymbol{J}_1 \sim \tau e^3 (\boldsymbol{E}\cdot\boldsymbol{B}) \int (d\boldsymbol{k}) \frac{\partial f_0}{\partial \varepsilon} \boldsymbol{v}^0 (\cancel{\boldsymbol{\Omega}_m} + \boldsymbol{b}\cdot\partial\boldsymbol{\Omega}) \boldsymbol{v}^0$$



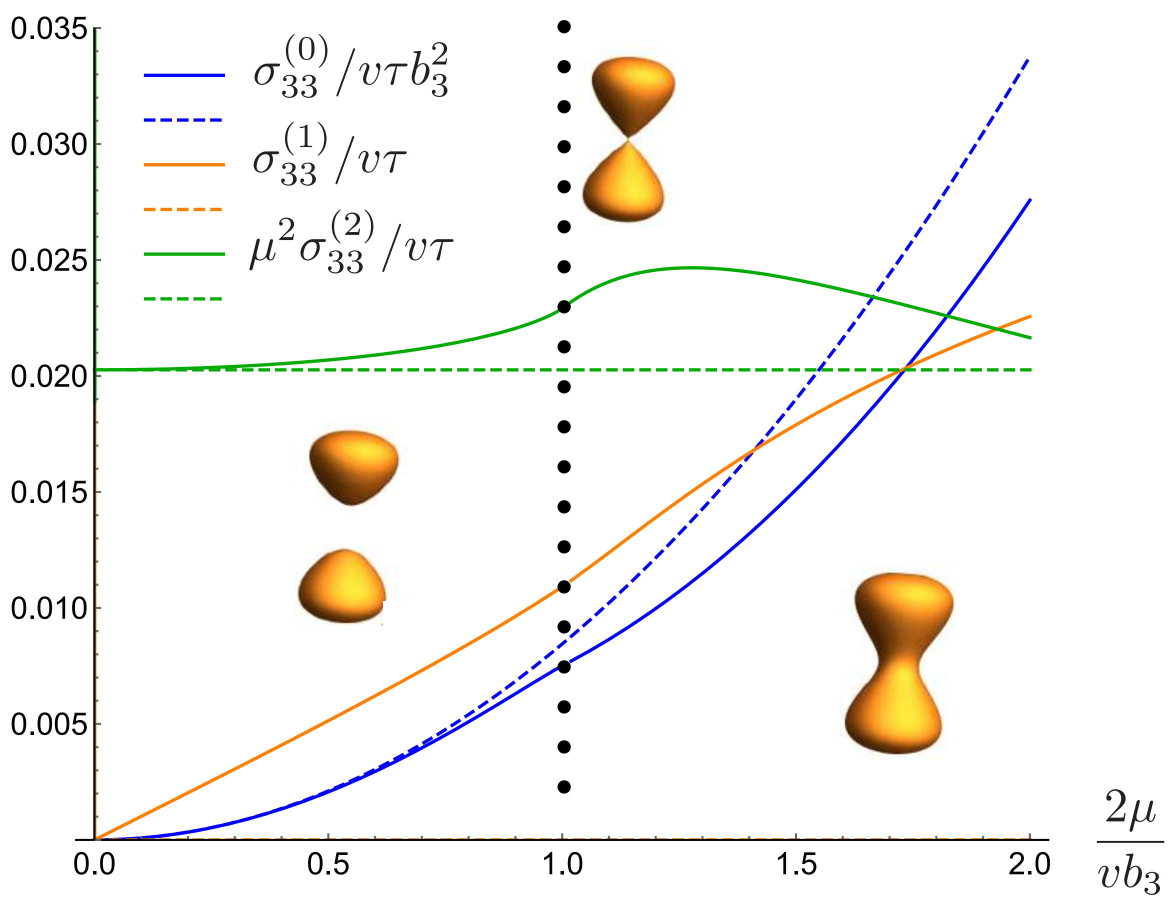
$$\boldsymbol{J}_1 \sim (\boldsymbol{E}\cdot\boldsymbol{b})\boldsymbol{B} + (\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{b}$$

A Cortijo, PRB 94, 241105(R) (2016)

3. Transport properties: Beyond the monopole paradigm

$$\boldsymbol{J} = -\tau e^3(\boldsymbol{B}\cdot\boldsymbol{E})\int (d\boldsymbol{k})\frac{\partial f_0}{\partial \epsilon}(\boldsymbol{\Omega}\cdot\boldsymbol{v}^0)\boldsymbol{v}^0 -$$

$$-\tau e^3\boldsymbol{B}\int (d\boldsymbol{k})\frac{\partial f_0}{\partial \epsilon}(\boldsymbol{\Omega}\cdot\boldsymbol{v}^0)(\boldsymbol{v}^0\cdot\boldsymbol{E})$$

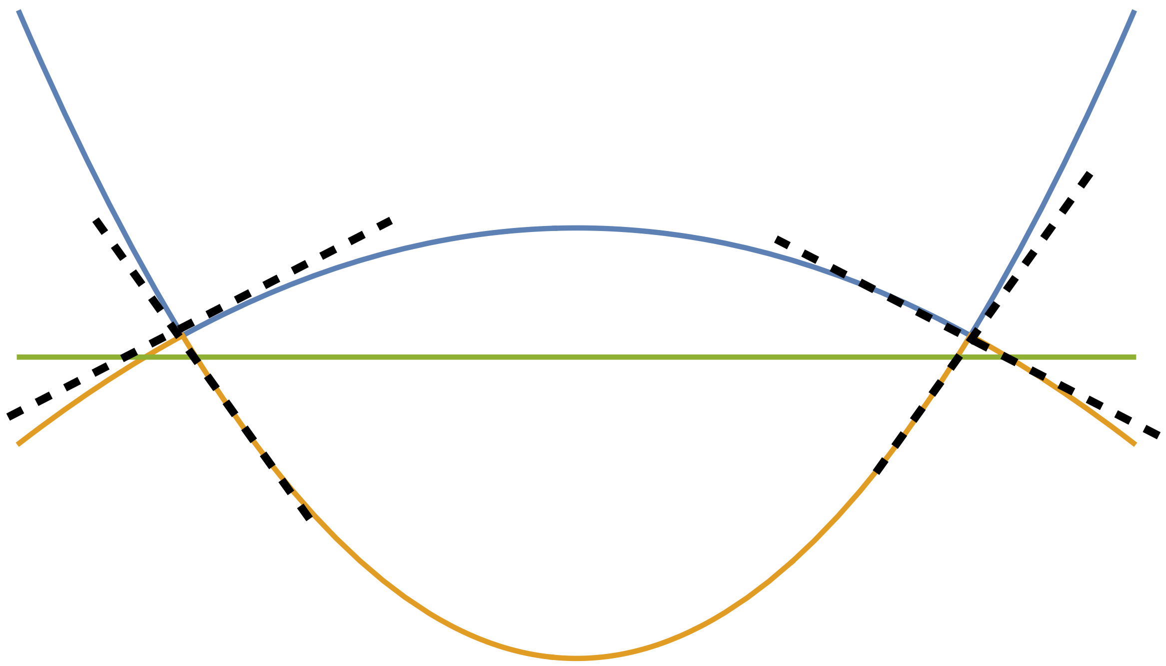


for the same token, we can put the Fermi level above the VH energy

$$\sigma_{33}^{[2]} \sim \frac{v^2 v_3 \tau}{\mu^2}\left(\frac{\mu}{2v_3|\boldsymbol{b}|}\right)^{\frac{1}{2}} B^2$$

A Cortijo, PRB 94, 241105(R) (2016)

3. Transport properties: Beyond the monopole paradigm



$$H(\boldsymbol{k}) = s\sigma_0(\boldsymbol{C}\cdot\boldsymbol{k}) + s\boldsymbol{\sigma}\cdot\boldsymbol{k}$$

Tilted Weyl semimetals

$$\boldsymbol{J}_1 \sim \tau_V(\frac{1}{4}(\boldsymbol{E}\cdot\boldsymbol{C})\boldsymbol{B} + \frac{1}{9}(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{C})$$

VA Zyuzin, PRB 95, 245128 (29017)

4. Conclussions

1. Weyl semimetals constitute a new family of topological materials.

2. The Berry physics manifests itself both at the level of the spectrum (Fermi arcs) and in transport properties.

3. Dissipative quantities (transport and optical) are modified by the presence of geometrical quantities, Berry curvature and orbital magnetic moment, through the chiral anomaly and (the precursor of) the chiral magnetic effect.

4. In the linear model, the Berry curvature takes the form of a monopole in momentum space, leading to a positive quadratic magnetoconductivity.

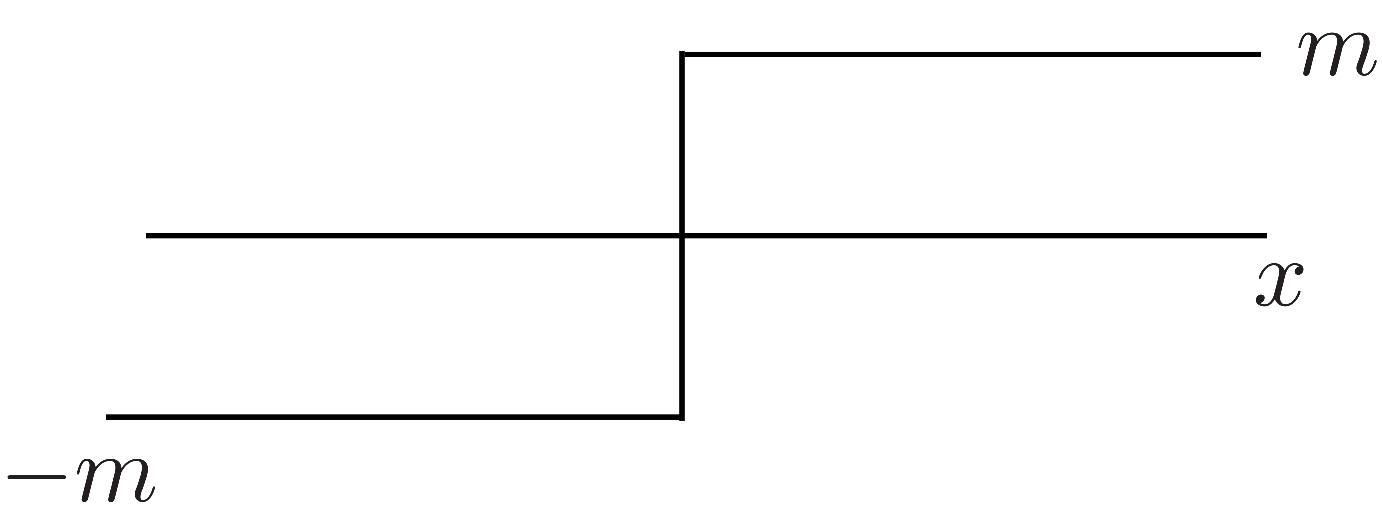
5. New physical phenomena appear when we consider corrections to the Berry monopole.

THANK YOU FOR LISTENING!

V. Edge/surface states.

$$\begin{pmatrix} E - v\boldsymbol{\sigma}\cdot\boldsymbol{k} & m(\boldsymbol{k}) \\ m(\boldsymbol{k}) & E + v\boldsymbol{\sigma}\cdot\boldsymbol{k} \end{pmatrix}\begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} = 0$$

$$H(\boldsymbol{k}) = H_0(-i\partial_1) + \Delta H(\boldsymbol{k})$$



$$H_0(-i\partial_1) = -is\sigma_1\partial_1 + m(x)\sigma_3$$

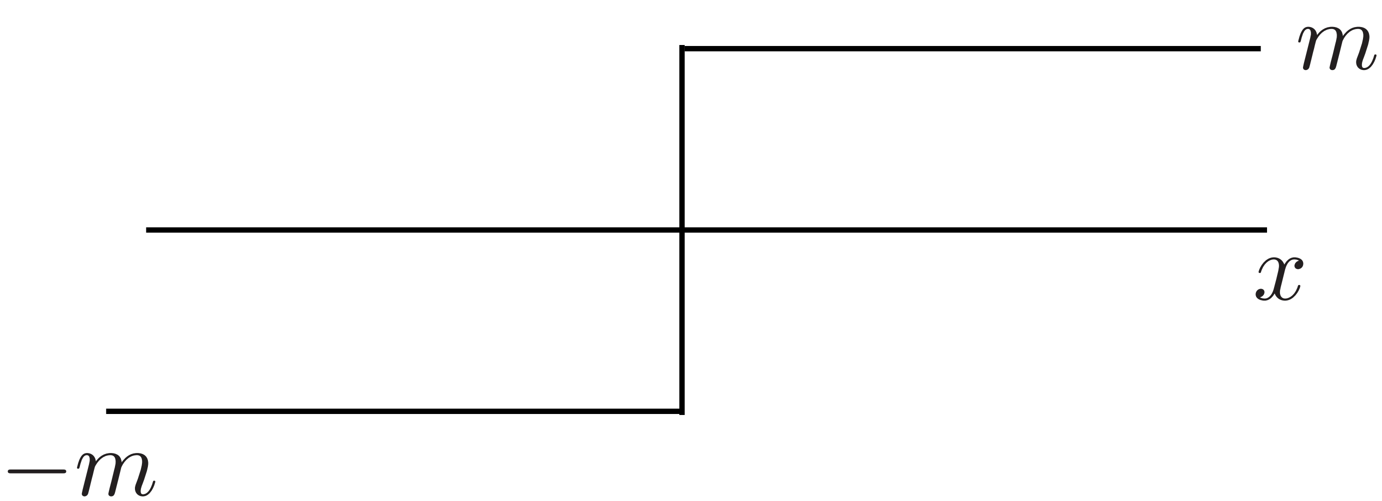
$$m(x) = \begin{cases} m, x > 0 \\ -m, x < 0 \end{cases}$$

$$\partial_1 \left|\psi\right\rangle = -sm(x)\sigma_2 \left|\psi\right\rangle$$

V. Edge/surface states.

$$\begin{pmatrix} E - v\boldsymbol{\sigma}\cdot\boldsymbol{k} & m(\boldsymbol{k}) \\ m(\boldsymbol{k}) & E + v\boldsymbol{\sigma}\cdot\boldsymbol{k} \end{pmatrix}\begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} = 0$$

$$H(\boldsymbol{k}) = H_0(-i\partial_1) + \Delta H(\boldsymbol{k})$$



$$H_0(-i\partial_1) = -is\sigma_1\partial_1 + m(x)\sigma_3$$

$$m(x) = \begin{cases} m, x > 0 \\ -m, x < 0 \end{cases}$$

$$\partial_1 |\psi\rangle = -sm(x)\sigma_2 |\psi\rangle$$

$$|\psi\rangle = f(x)\,|s\rangle$$

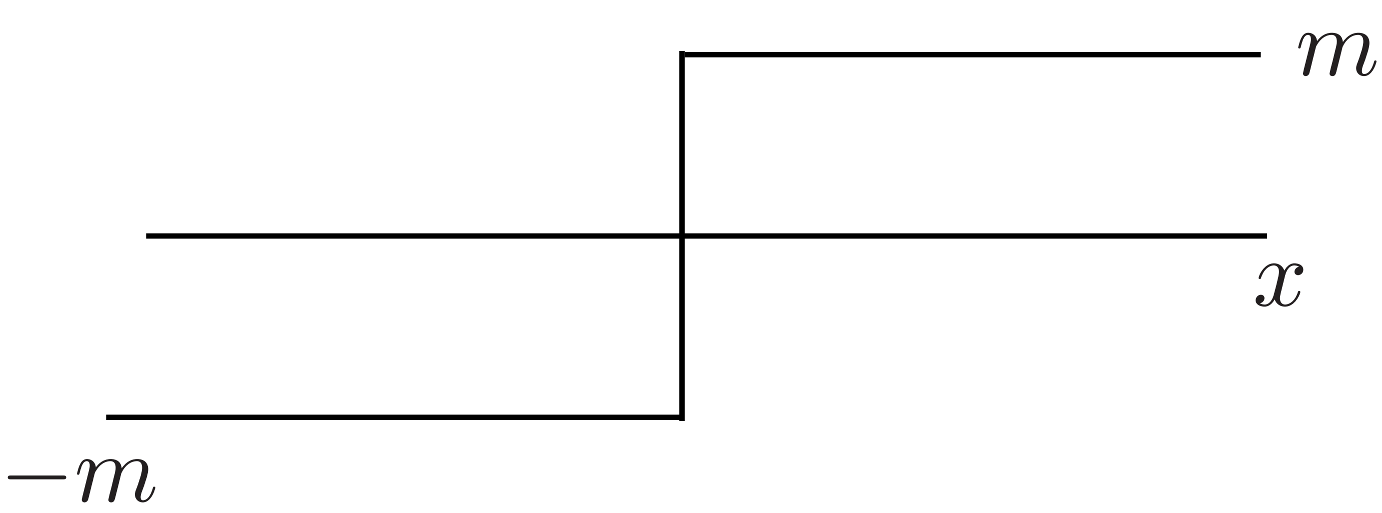
$$\sigma_2 |s\rangle = s\,|s\rangle$$

$$\partial_1 f(x)\,|s'\rangle = -ss'm(x)f(x)\,|s'\rangle$$

V. Edge/surface states.

$$\begin{pmatrix} E - v\boldsymbol{\sigma}\cdot\boldsymbol{k} & m(\boldsymbol{k}) \\ m(\boldsymbol{k}) & E + v\boldsymbol{\sigma}\cdot\boldsymbol{k} \end{pmatrix} \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} = 0$$

$$H(\boldsymbol{k}) = H_0(-i\partial_1) + \Delta H(\boldsymbol{k})$$



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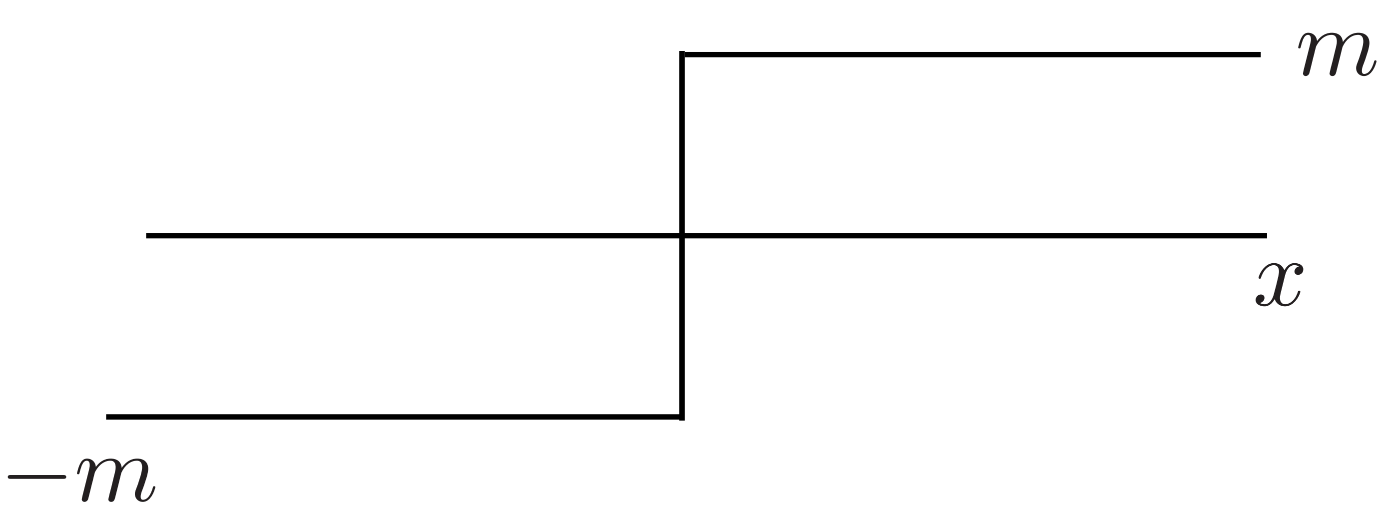

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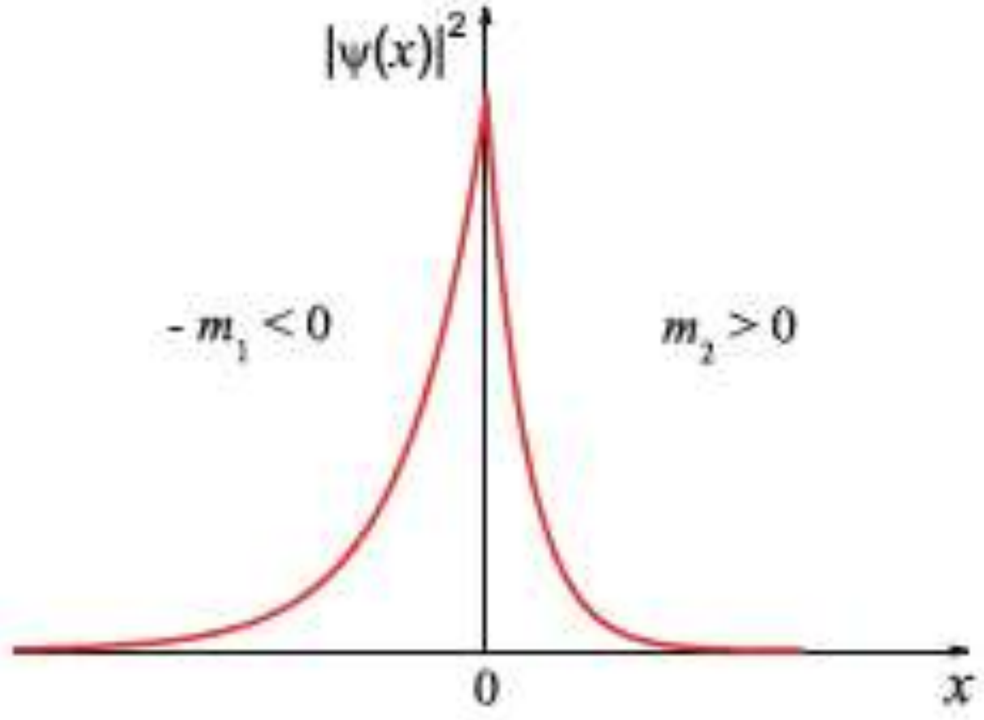


There is a zero energy mode localized at the boundary, with a well defined spin projection

V. Edge/surface states.

$$H(\boldsymbol{k}) = H_0(-i\partial_1) + \Delta H(\boldsymbol{k})$$

$$f(x) = Ae^{-ss|x|}$$

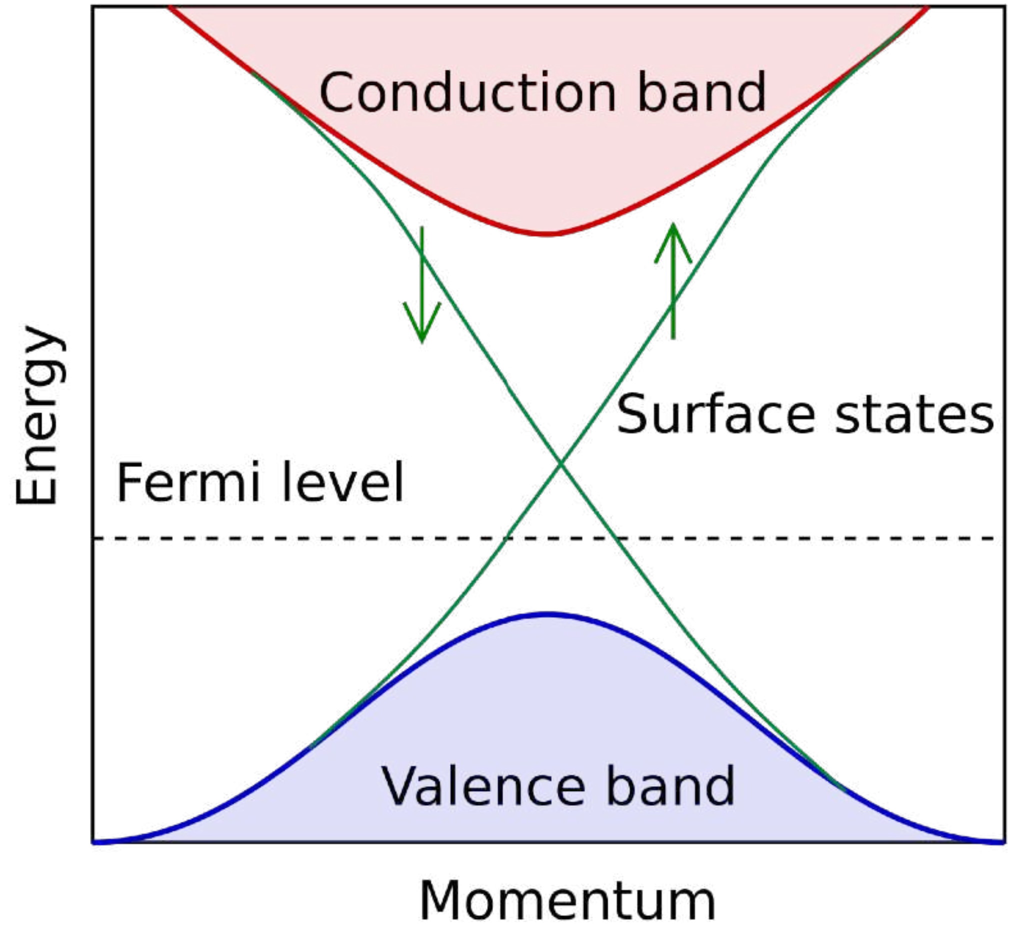
2D:

$$s = s'$$

$$H_{eff}(k_2) = \langle\psi|\,\Delta H(\boldsymbol{k})\,|\psi\rangle$$

$$\sigma_2\,|s\rangle = s\,|s\rangle$$

$$\psi = e^{-m|x|}\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix}$$



$$H_{eff}(k_2) = k_2\sigma_3$$

Helical edge states

V. Edge/surface states.

$$H(\boldsymbol{k}) = H_0(-i\partial_1) + \Delta H(\boldsymbol{k})$$

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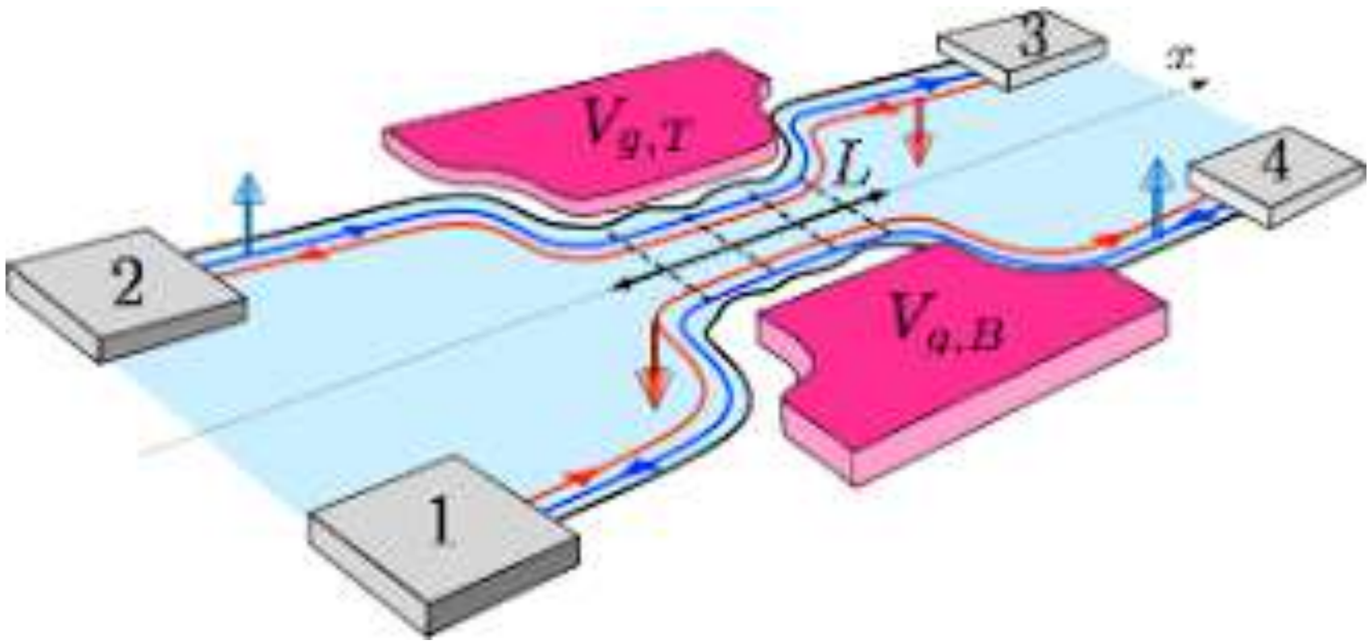

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Helical edge states