Emergence of Quantum Phases in Novel Materials

Fermi liquid theory

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Outline

- ❑ The Fermi gas

- ❑ Why does band theory work? Concepts: Adiabaticity and quasiparticles

  - Definition of adiabaticity and quasiparticles
  - Quasiparticle weight
  - Electronic distribution function
  - Quasiparticle decay and spectral function

- ❑ Energy as a functional of the number of quasiparticles. Measurable quantities.

  - Renormalized mass
    - Specific heat
    - Mass in ARPES
  - Interaction parameters
    - Spin susceptibility
    - Resistivity

- ❑ Fermi liquid behavior and instabilities of the Fermi liquid

Some references

- Introduction to many body physics. Piers Coleman. Cambridge University Press.
- Theory of Quantum Liquids. Phillipe Nozieres, David Pines. Advanced Books Classics.
- A guide to Feynmann Diagrams in the Many Body problem. E.D. Mattuck. Dover Books on Physics
- Metal-insulator transitions. M. Imada, A. Fujimori, Y. Tokura. Rev. Mod. Phys. 70, 1039 (1998)

Basic concepts: the Fermi gas

- Focus on electrons. Work at fixed μ
- Electrons are fermions: Fermi-Dirac Statistics

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/K_B T}+1}$$

energy

chemical potential

Zero temperature

Step function

States filled up to the Fermi Energy ε_F

Fermi surface

(inside Fermi surface filled outside empty)

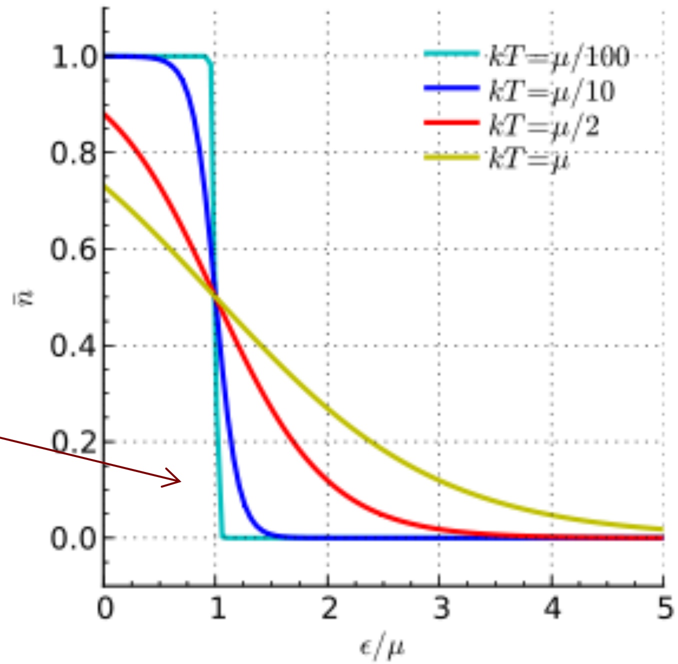

Fig: Wikipedia

High temperatures

(classical gas)

$K_B T \gg \mu$

$f(\varepsilon) \propto e^{-\varepsilon/K_B T}$

Maxwell-Boltzmann distribution function

Basic concepts: the Fermi gas

	$K_BT << \mu$	$K_BT >> \mu$
	(Fermi-Dirac: step)	(Maxwell-Boltzmann)
○ Specific heat		
Free energy $C_v = \left.\frac{\partial F}{\partial T}\right\vert_\mu$	$C_v = \gamma\, T$ Linear in temperature	$C_v =$ independent of T
○ Spin susceptibility		
Magnetization $\chi_s = \frac{\partial M}{\partial H}$	$\chi_s = \mu_B^2 N(\varepsilon_F)$ independent of T Pauli susceptibility	$\chi_s \propto \frac{1}{T}$ Curie law

Non-interacting fermions are correlated due to Fermi statistics & Pauli principle

Basic concepts: the Fermi gas

Different temperature dependence
of measurable quantities
in the degenerate (quantum)
and classical limits

- Low temperatures
  (quantum gas)
  $K_B T << \mu$
- High temperatures
  (classical gas) $K_B T >> \mu$

Fermi temperature $T_F = \varepsilon_F / K_B$

Temperature below which quantum effects are important

In metals

$\varepsilon_F \sim 3$ eV

$T_F \sim 2400$ K

In TBG & doped semiconductors

$\varepsilon_F \sim$ few meV

$T_F \sim 50\text{-}100$ K

Why does an independent electron model work at all?

Band theory:

Basis of our understanding of solids

- Successful description
- Metals and insulators
- Dependence on temperature of measurable quantities (Cv, χ, ..)

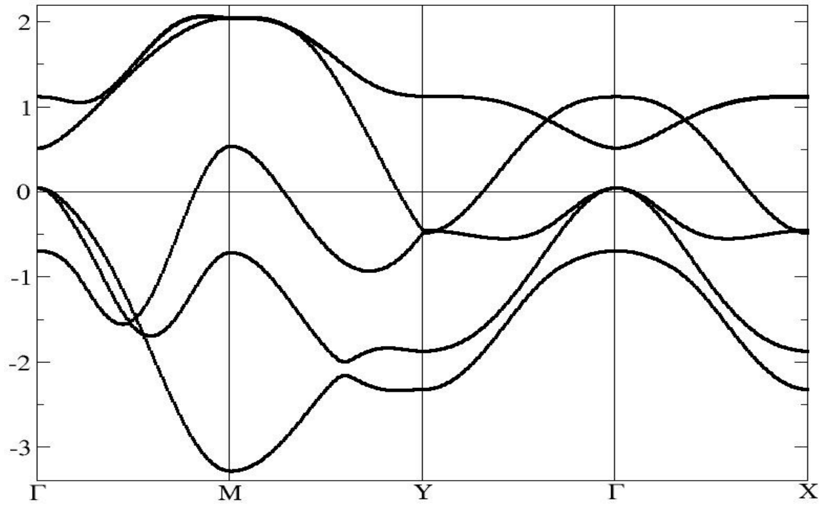

Fig: Calderón et al, PRB, 80, 094531 (2009)

Interactions:

Electrons interact between them (and with the lattice)

Interactions not included in the band picture beyond simple mean field

Interaction not small vs kinetic energy

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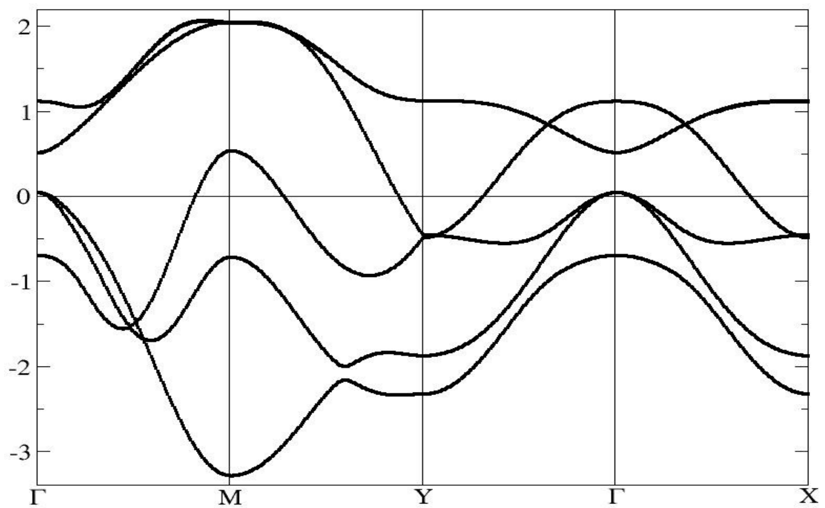

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Interactions:

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Interactions not included in the band picture beyond simple mean field

Interaction not small vs kinetic energy

Why does band theory work?

Fermi liquid theory

Does it always work?

NO (Mott physics, Luttinger liquids …)

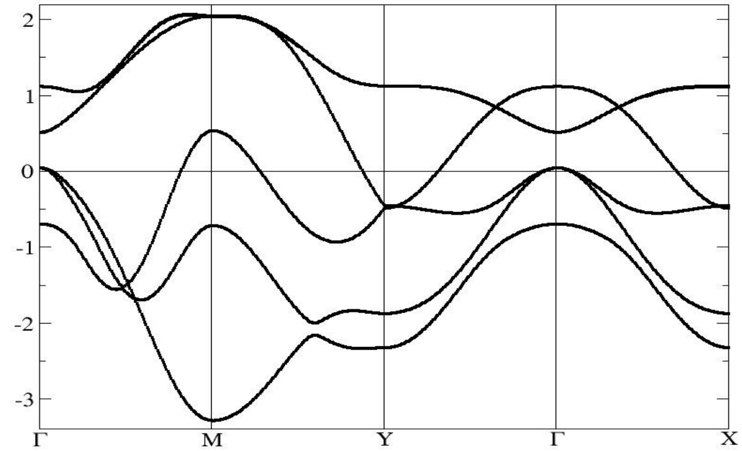
What does it mean that the "independent" electron model works?

Band theory:

- Well defined eigenstates with momentum k E(k)
- E(k) filled following Fermi-Dirac. Fermi surface.
- If we add electron with momentum k to ground state. The new electron occupies an eigenstate E(k).
- Energy levels are not modified, just filled. Rigid Band shift

Description in terms of

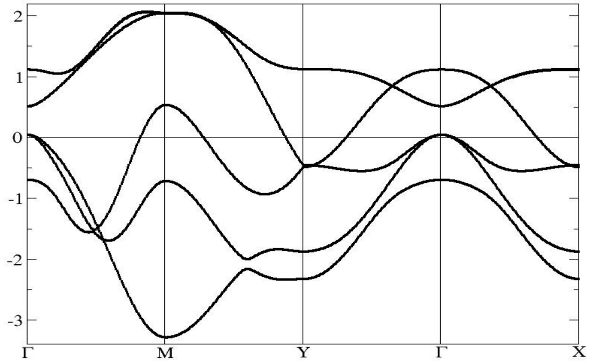
single particle states



What does it mean that the "independent" electron model works?

Independent electron model (Band theory):

- Well defined "single particle states" E(k)
- Rigid band shift.



Interacting system:

- If we add electron with momentum k to ground state of a Fermi sea we expect the energy levels E(k) to be modified due the interactions, not just filled

Non rigid band shift expected

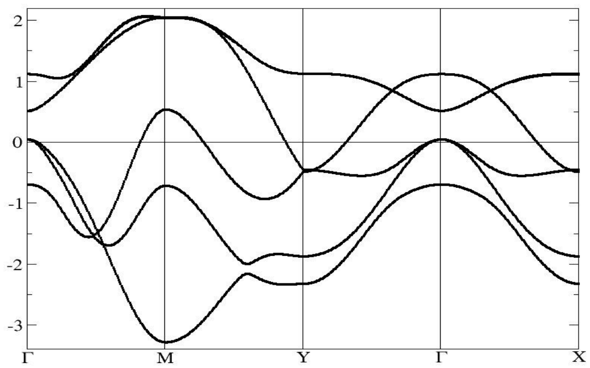
Electronic state E(k) expected to decay in excitations of the system

Description in terms of single particle states "does not seem possible".

What does it mean that the “independent” electron model works?

Band theory:

Rigid band shift. Single particle states



Interacting system:

Description in terms of single particle states does not seem possible.

Electronic state expected to decay in excitations of the system.

Fermi liquid theory

- Pauli principle restricts the phase space for decay of excitations
- Description based on elementary excitations not on ground state

Fermi liquid theory: general idea

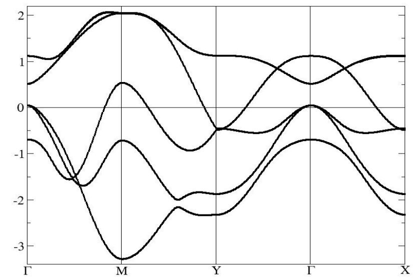
- It justifies the success of band theory
- Focus not in ground state but in low energy excitations. Introduces the concept of quasiparticles. Valid only at low energies and temperatures.
- Theory written in terms of parameters. It goes beyond particular models
- Phenomenological theory, but it can be justified with perturbation theory
- Perturbative theory, but not restricted to weak interactions.
- Proposed for 3-He: isotropic, no charge, short range interactions, in the continuum limit but it can be generalized to describe electrons in a metal
- Sometimes it fails. Correlated electron systems. Non-Fermi liquid behavior

Fermi liquid theory: Summary

❑ Band theory. Does band theory work? Why?

Well defined energy levels for a given k $\varepsilon(k)$

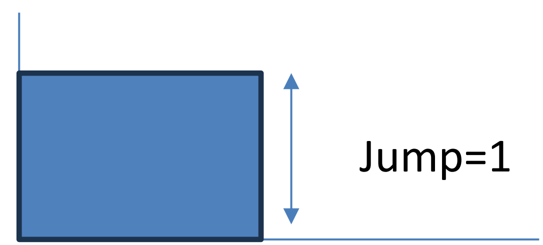
Rigid Band Shift (upon addition of particles (real electrons $c^{+}_{k\sigma}$)
with momentum k the levels are filled not changed)



❑ Introducing quasiparticles

- Assuming adiabaticity (perturbative effect of interactions) the system has elementary excitations with momentum k, spin ½ and charge **e** called quasiparticles $a^{+}_{k\sigma}$

  - The quasiparticles follow a Fermi-Dirac distribution at T=0 (step). Fermi surface



  - The quasiparticle energies $\varepsilon^{*}(K)$ well defined at small T and small energy (with respect to Fermi level) $\Gamma << \varepsilon^{*}(K)$ in 3D and 2D. But not in 1D.

Away from T=0 and Fermi Surface $\tau^{-1}=\Gamma$ is finite (excitation decay)

Fermi liquid theory: Summary

❑ **Connecting real electrons and quasiparticles**

$$c^{+}_{k\sigma}\,|\Psi*_g>$$

STM

real electron

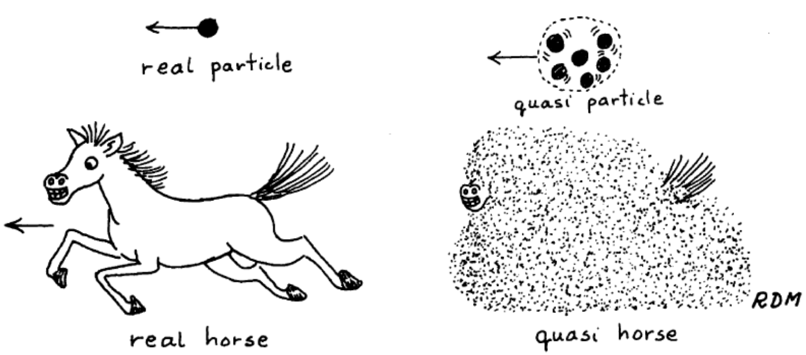
$$c_{k\sigma}\,|\Psi*_g>$$

ARPES

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\,a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\,a^{\dagger}{}_{\mathbf{k}_4\sigma_4}a^{\dagger}{}_{\mathbf{k}_3\sigma_3}a_{\mathbf{k}_2\sigma_2} + \dots$$

Quasiparticle weight Z ∈[0,1]
Needs to be finite for the "single-particle" to be meaningful

Part of the electron decays in quasiparticle electron-hole excitations (**continuum or incoherent part**)



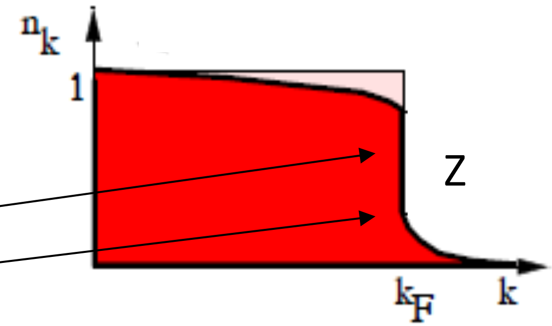
Fermi liquid theory: Summary

❑ Connecting Fermi liquid description with experiments

Similarities and differences with band theory

- Z>0 Band theory is meaningful in a range of temperatures and energies but with some differences

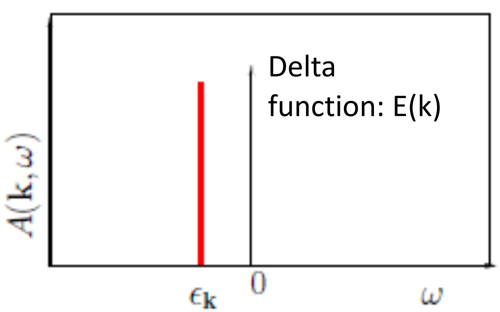
- Distribution Function of real electrons



Fermi Surface for real electrons.
But step at T=0 with height Z not 1
Leakage

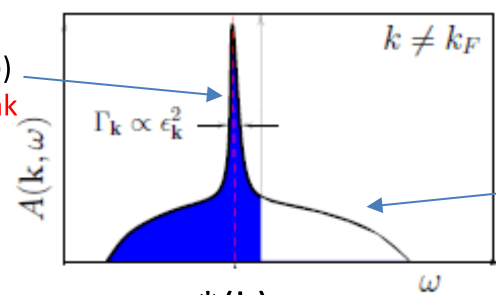
- Spectral function (distribution of excitations when we add or remove an electron with momentum k (measures the bands))

Non-interacting electrons



Interacting electrons

Broadened peak (broadened E*(K))
Quasiparticle peak
Broadening Associated to Quasiparticle decay

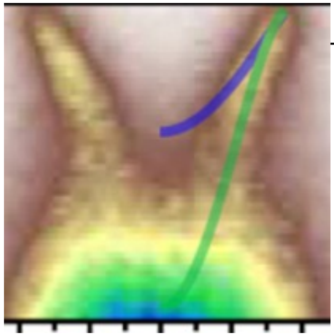


ε*(k)

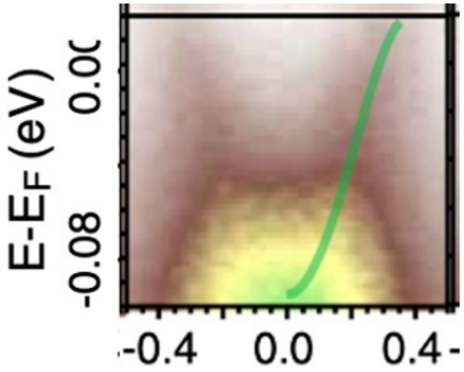
Incoherent Spectrum (decay of the electron)

Fermi liquid theory: Summary

❑ Connecting Fermi liquid description with experiments



Broadened band

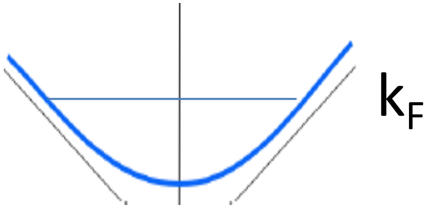


❑ ε*(k) Renormalized parameters

Non-interacting electrons

$$\varepsilon_{k\sigma} = k^2/2m - \mu = (k^2 - k_F^2)/2m$$

$$\varepsilon_{k\sigma} \sim \frac{k_F}{m}(k-k_F)$$



Linearization of energy **very close** to Fermi surface

Interacting electrons

By analogy

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

We do not know the spectrum, but **very close** to the Fermi Surface we can write an expression like this (Taylor expansion in terms of $(k-k_F)$)

$$m^* = \frac{m}{Z}$$

Definition of renormalized mass

Fermi liquid theory: Summary

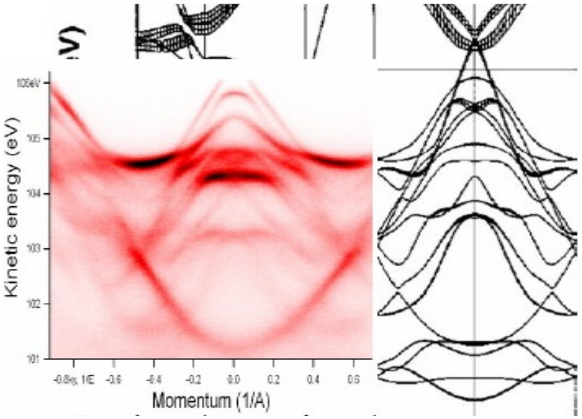
❑ Connecting Fermi liquid description with experiments

We want to determine m*

How large is m*/m?

Experimental ↗ ↖ LDA

Angle resolved Photoemission (ARPES)



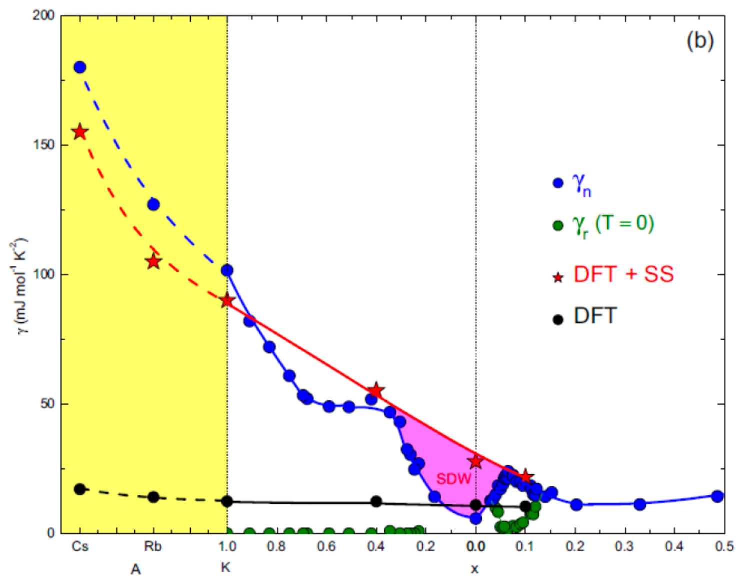
Specific heat

Non-interacting electrons

$C_v=\gamma\, T \qquad \gamma=(\pi/3)K_B^2 N(E_F) \propto m$

Non-interacting electrons

$C_v=\gamma^*\, T \qquad \gamma \propto m^*$



Fermi liquid theory: Summary. Fermi liquid behavior

- **Bands observed in photoemission**

Reduction in bandwidth as compared to LDA estimate of m*/m

- **Specific heat linear in temperature.**

Enhancement of γ estimate of m*/m

Careful in materials with multiple Fermi pockets

$$C^*_v=\gamma^* T$$

- **Temperature independent spin susceptibility**

Enhancement: mass renormalization + Stoner enhancement

- **Resistivity quadratic in temperature**

Interactions enhance resistivity as $(m^*)^2$

$$\rho=\rho_0 + AT^2$$

Correlated electrons: frequently non-Fermi liquid behavior

Adiabaticity and quasiparticles

Assume N interacting fermions

$$H = H_0 + V$$

H Interacting electronic Hamiltonian

(we do not know how to solve it)

Ground state $|\Psi^*_g>$

H_0 : Non interacting electronic Hamiltonian

(we know how to solve it)

Ground state $|\Psi_g>$

Interactions

See Coleman's book

Adiabaticity and quasiparticles

Assume N interacting fermions

$$H=H_0 + V$$

H Interacting electronic Hamiltonian

(we do not know how to solve it)

Ground state $|\Psi^*_g\rangle$

H_0 : Non interacting electronic Hamiltonian

(we know how to solve it)

Ground state $|\Psi_g\rangle$

Interactions

Assume adiabaticity

$|\Psi^*_g\rangle$ and $|\Psi_g\rangle$ adiabatically connected ⇒ Effect of interactions V is perturbative (No phase transition)

See Coleman's book

Adiabaticity, quasiparticles and electrons

Assume adiabaticity

- Introduce quasiparticles: elementary excitations of the system $a^{+}_{k\sigma}$ with momentum k, charge e and spin ½ which follow Fermi-Dirac statistics

- $a^{+}_{k\sigma}$ (quasiparticles) well defined at low temperatures and energies

(except in 1D)

Real electron → $c^{+}_{k\sigma} \neq a^{+}_{k\sigma}$ ← Quasiparticle

Adiabaticity, quasiparticles and electrons

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In an experiment:

$c^+_{k\sigma}\,|\Psi^*_g>$ ← real electron

STM

$c_{k\sigma}\,|\Psi^*_g>$

ARPES

- Write $c^+_{k\sigma}$ in terms of $a^+_{k\sigma}$ → Quasiparticle weight

Adiabaticity

- Interacting Hamiltonian $H=H_0+V$ with ground state $|\Psi^*_g>$ for N particles

- Non interacting Hamiltonian H_0 with ground state $|\Psi_g>$ for N particles

- Imagine we start at $t=-\infty$ in $|\Psi_g>$ and switch on interactions slowly on time

$$H(t)=H_0+\lambda(t)V \qquad \lambda(t)=e^{-\delta|t|}$$

δ arbitrarily small

Interaction completely switched on at t=0

- Follow the evolution of the ground state from $|\Psi_g>$ to $|\Psi^*_g>$

$$|\Psi_g^*(t)>= V^{ev}|\Psi_g>$$

Adiabaticity: $|\Psi^*_g>$ and $|\Psi_g>$ are perturbatively connected

See Coleman's book

Adiabaticity and quasiparticles

- Consider the non-interacting state of H_0 with N+1 particles (fermions).

$$|k\sigma\rangle = C^{+}_{k\sigma}\,|\Psi_g\rangle$$

Eigenstate (Clean system)

The new particle added to an excited state with momentum **k** charge e and spin σ.

Ground state **non-interacting** system with N particles

- Switch on interactions and look at interacting state $H = H_0 + V$ with N+1 particles

$$|k\sigma *\rangle = V^{ev}|k\sigma\rangle = V^{ev}\,C^{+}_{k\sigma}\,|\Psi_g\rangle$$

See Coleman's book

Adiabaticity and quasiparticles

Same charge,
spin and momentum

$$|k\sigma*\rangle = V^{ev}|k\sigma\rangle$$

Not the same as

$$C^{+}_{k\sigma}\,|\Psi*_{g}\rangle$$

Interactions conserve
charge, momentum and spin
(Assumption: no spin-orbit coupling)

$$|k\sigma\rangle = C^{+}_{k\sigma}\,|\Psi_{g}\rangle$$

Particle: Elementary excitation
of the non-interacting system
Charge e, spin 1/2, momentum k

See Coleman's book

Adiabaticity and quasiparticles

Same charge,
spin and momentum

$$|k\sigma*\rangle = V^{ev}|k\sigma\rangle$$

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Particle: Elementary excitation of the non-interacting system
Charge e, spin 1/2, momentum k

$$|k\sigma*\rangle = a^{+}_{k\sigma}|\Psi*_g\rangle$$

Quasiparticle $\longrightarrow$ Elementary excitation of the interacting system with momentum k, spin 1/2, and charge e

See Coleman's book

Adiabaticity and quasiparticles

Same charge, spin and momentum

$$|k\sigma*\rangle = V^{ev}|k\sigma\rangle$$

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Quasiparticle $\longrightarrow$ Elementary excitation of the interacting system with momentum k, spin 1/2, and charge

Different operators for particle (electron) & quasiparticle

$$a^{\dagger}_{k\sigma} = V^{ev} c^{\dagger}_{k\sigma} V^{ev\dagger}$$

See Coleman's book

Adiabaticity and quasiparticles. Fermi surface

$|\Psi_g> \longrightarrow |\Psi*_g>$ Adiabatically connected

Existence and definition of quasiparticles

$$|k\sigma*> = a^{+}_{k\sigma} |\Psi*_g>$$

Quasiparticles and real electrons
are not the same

Elementary excitation
of the interacting system
with momentum k,
spin 1/2, and charge

↓

Fermionic excitations

Fermi-Dirac distribution
Fermi surface

Adiabaticity and quasiparticles. Fermi surface

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Existence and definition of quasiparticles

$$|k\sigma^*> = a^+_{k\sigma}\,|\Psi^*_g>$$

Elementary excitation of the interacting system with momentum k, spin 1/2, and charge

↓

Fermionic excitations

Quasiparticles and real electrons are not the same

Fermi-Dirac distribution

Fermi surface

$C^\dagger_{k\sigma}|\Psi^*_g> \longrightarrow a^\dagger_{k\sigma}$ Quasiparticle

$C_{k\sigma}|\Psi^*_g> \longrightarrow a_{k\sigma}$ Quasihole

Adiabaticity and quasiparticles. Fermi surface and decay

$|\Psi_g>$ $\longrightarrow$ $|\Psi^*_g>$ Adiabatically connected

Non-interacting system

$|k\sigma>=C^+_{k\sigma}\,|\Psi_g>$ **Eigenstate**

Interacting system

$|k\sigma^*>=a^+_{k\sigma}\,|\Psi^*_g>$ **Not an Eigenstate** $\longrightarrow$ **Finite lifetime** $\tau=\Gamma^{-1}$ (width level)

Stability of the quasiparticle requires	$\Gamma << \varepsilon$	Decay rate of the quasiparticle Much smaller than its energy

Quasiparticle decay

A quasiparticle with $k>k_F$ cannot decay into an **occupied** state below the FS with $k<k_F$

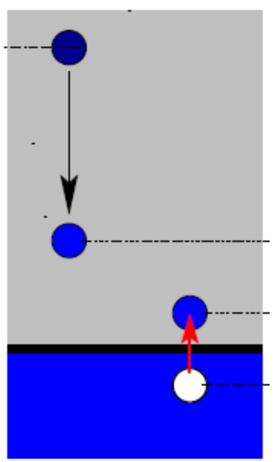
Decay of quasiparticles conserves momentum, charge & spin

$a^\dagger_{k\sigma}$

2 particles and 1 hole → $a^\dagger_{k1\sigma}a^\dagger_{k2\sigma}a_{k3\sigma}$

3 particles and 2 holes → $a^\dagger_{k1\sigma}a^\dagger_{k2\sigma}a^\dagger_{k4}a_{k3\sigma}a_{k3\sigma}$

…



Decay of one quasiparticle
in a 3-body process

At low temperatures and energies
Pauli principle restricts
the decay processes allowed

Figs. Coleman's book

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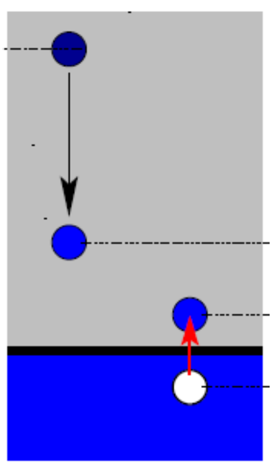

Decay of one quasiparticle
3-body process

$$\tau^{-1} = \frac{\varepsilon^{*2}+\pi^2 T^2}{E^*_F}$$

$\varepsilon^* = E^* - E^*_F$

Energy from Fermi Surface

Estimate in 3D based
on phase space considerations
(Pauli principle) & 3 body decay

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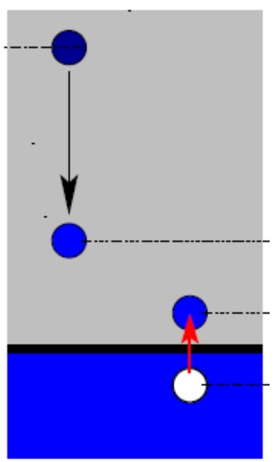

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Energy from Fermi Surface

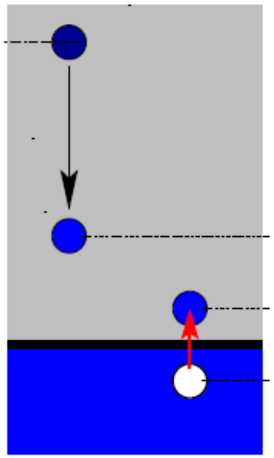
Estimate in 3D based on phase space considerations (Pauli principle) & 3 body decay

Decay of one quasiparticle
3-body process

Quasiparticles well defined at low energies & temperatures

Infinite lifetime at the Fermi surface at zero temperature

Quasiparticle decay



Decay of one quasiparticle
3-body process

$$\frac{\Gamma}{\varepsilon} \sim \varepsilon$$ Estimate in 3D

$$\frac{\Gamma}{\varepsilon} \sim \varepsilon \,\mathrm{Ln}\, \varepsilon$$ Estimate in 2D

$$\frac{\Gamma}{\varepsilon} \sim \mathrm{const}$$ Estimate in 1D

$\varepsilon^* = E^* - E^*_F$

Energy from
Fermi Surface

Quasiparticles well defined at low energies & temperatures

Infinite lifetime at the Fermi surface at zero temperature

in 3D and 2D but not 1D. Fermi liquid theory fails in 1D

Giuliani PRB 26, 4428 (1982)

Quasiparticle decay

3 dimensions

2 dimensions

©Michael Paraskevas

1 dimension

Interactions are never perturbative in 1 dimension

©Carlos Valenzuela

Quasiparticle decay

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Decay of quasiparticles conserves momentum, charge & spin

$a^{\dagger}_{k\sigma}$

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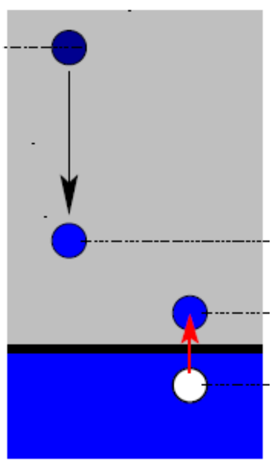
...



Decay of one quasiparticle
3-body process

$$\tau^{-1}=\frac{\varepsilon^{*2}+\pi^2T^2}{E^*_F}$$

$\varepsilon^*=E^*-E^*_F$

Energy from Fermi Surface

Estimate in 3D based on phase space considerations (Pauli principle) & 3 body decay

Energy which control the range of temperatures and energy at which the quasiparticle is well defined
It can be very small!

Adiabaticity, quasiparticles and electrons

Assume adiabaticity

- Introduce quasiparticles: elementary excitations of the system $a^+_{k\sigma}$ with momentum k, charge e and spin ½ which follow Fermi-Dirac statistics

- $a^+_{k\sigma}$ (quasiparticles) well defined at low temperatures and energies

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Real electron → $c^+_{k\sigma} \neq a^+_{k\sigma}$ ← Quasiparticle

In an experiment:

$c^+_{k\sigma} \,|\Psi^*_g>$ $\quad$ $c_{k\sigma} \,|\Psi^*_g>$

real electron → $c^+_{k\sigma}$

STM $\quad$ ARPES

- Write $c^+_{k\sigma}$ in terms of $a^+_{k\sigma}$ → Quasiparticle weight

Adiabaticity, quasiparticles and electrons

- Write the electron operator $c^{\dagger}_{k\sigma}$ in terms of the elementary excitations of the interacting system, the quasiparticles $a^{\dagger}_{k\sigma}$

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma) a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \dots$$

Generic expression which conserves charge, momentum & spin

If different from zero the electron decays into Particle-hole quasiparticle excitations

higher order decay processes

Decay of the electron into electron-hole quasiparticles excitations (not to be confused with the decay of the quasiparticle)

See Coleman's book

Adiabaticity, quasiparticles and electrons

- Write the electron operator $c^{\dagger}_{k\sigma}$ in terms of the elementary excitations of the interacting system, the quasiparticles $a^{\dagger}_{k\sigma}$

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)a^{\dagger}{}_{\mathbf{k}_4\sigma_4}a^{\dagger}{}_{\mathbf{k}_3\sigma_3}a_{\mathbf{k}_2\sigma_2} + \dots$$

A single particle description of $C^{\dagger}_{k\sigma}|\Psi^*_g>$ "makes sense" if Z_k is finite

Quasiparticle weight

Measurable

See Coleman's book

Adiabaticity and quasiparticles

$c^{\dagger}_{k\sigma}|\Psi^*_g>$

- Write the electron operator $c^{\dagger}_{k\sigma}$ in terms of the excitations of the interacting system $a^{\dagger}_{k\sigma}$

Same charge, momentum & spin

$$c^{\dagger}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}a^{\dagger}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma) a^{\dagger}_{\mathbf{k}_4\sigma_4} a^{\dagger}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

higher order decay processes

$$0 \leq Z_k \leq 1$$

Quasiparticle weight

Z_k Measures the strength of the correlations & & the validity of the Fermi liquid description

- $Z_k=1$ non interacting system
- $Z_k=0$ Fermi liquid theory not applicable
- $Z_k>0$ ensures one to one correspondence between electron and quasiparticle.

See Coleman's book

Adiabaticity and quasiparticles

$C^{\dagger}_{k\sigma}|\Psi^*_g>$

❑ Write the electron operator $c^{\dagger}_{k\sigma}$ in terms of the excitations of the interacting system $a^{\dagger}_{k\sigma}$

Same charge, momentum & spin

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma) a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

A single particle description of $C^{\dagger}_{k\sigma}|\Psi^*_g>$ makes sense if Z_k is finite

higher order decay processes

$$Z_k=|<\Psi^*_g|a_{k\sigma}c^{\dagger}_{k\sigma}|\Psi^*_g>|^2>0$$

Quasiparticle weight

Overlap between the state reached adding an electron to the interacting system and the state which results from adding an elementary excitation to the interacting system

See Coleman's book

Adiabaticity and quasiparticles

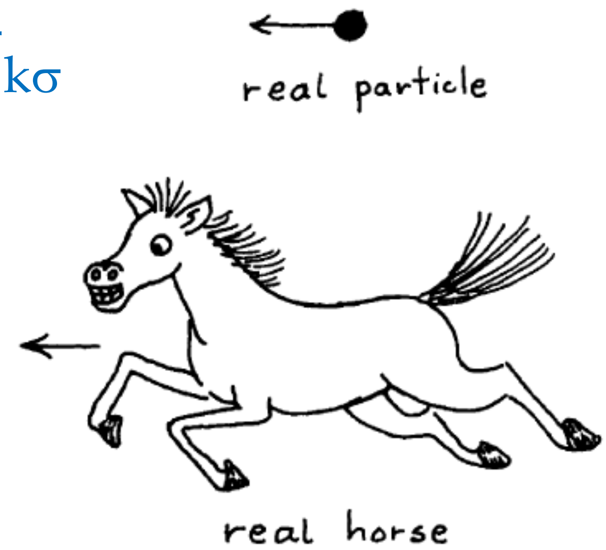
Interacting system, **if Z_k is finite**

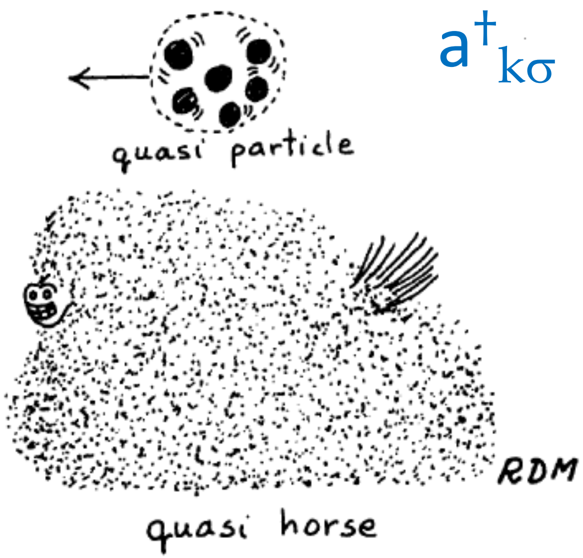
Elementary excitations:

quasiparticles with charge e and spin ½ and momentum k

-The quasiparticles are not electrons but there is a one-to-one correspondence with an electron

$c^{\dagger}_{k\sigma}$





$a^{\dagger}_{k\sigma}$

Mattuck's book

Z_k gives the part of the excitation $c^{\dagger}_{k\sigma}|\Psi*_g>$ which is in the quasiparticle state

(how much of the quasi-horse is really the horse)

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Electronic distribution. Jump at Fermi Surface

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma) a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

$$n_k = \langle \Psi^*{}_g | C^{\dagger}{}_{k\sigma} C_{k\sigma} | \Psi^*{}_g \rangle = Z_k \langle \Psi^*{}_g | a^{\dagger}{}_{k\sigma} a_{k\sigma} | \Psi^*{}_g \rangle + \text{continuum}$$

Distribution of real electrons

Fermi Dirac statistics

At T=0 the distribution of **quasiparticles** has jump of height 1 at the Fermi surface

$$\Theta(-\varepsilon_k)$$

Continuum or incoherence: Amount of the Electronic excitation which is **not in the quasiparticle state**

also called incoherent part

Do not confuse this continuum with the continuum limit vs lattice

Electronic distribution. Jump at Fermi Surface

$$c^\dagger_{\mathbf{k}\sigma} = \sqrt{Z_\mathbf{k}}\, a^\dagger_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^\dagger_{\mathbf{k}_4\sigma_4} a^\dagger_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

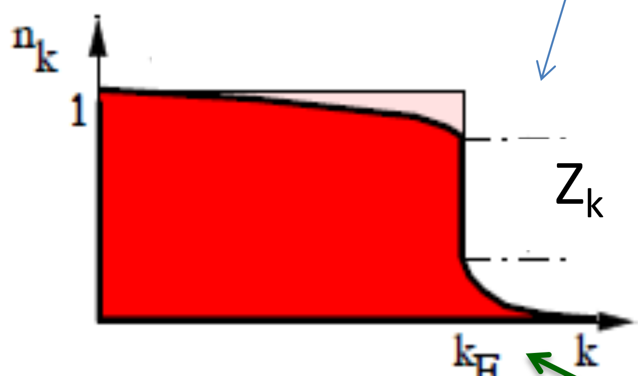
$$n_k = <\Psi^*_g | C^\dagger_{k\sigma} C_{k\sigma} | \Psi^*_g> = Z_k <\Psi^*_g | a^\dagger_{k\sigma} a_{k\sigma} | \Psi^*_g> + \text{continuum}$$

Fermi Dirac statistics

Continuum:
Amount of the Electronic excitation which is **not in the quasiparticle state**

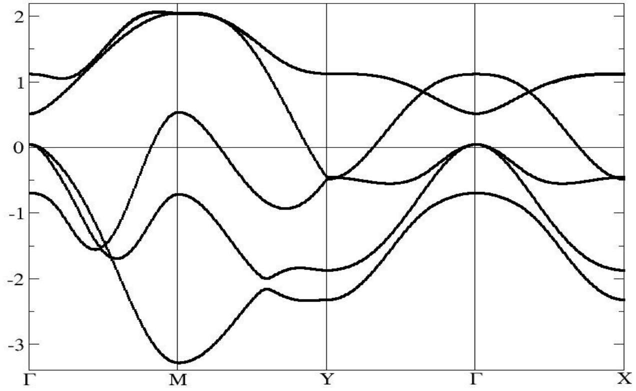
$$Z_k \; \Theta(-\varepsilon_k)$$

At T=0 the distribution of **electrons** has a jump of height $\mathbf{Z_k}$ (not 1) at the Fermi level



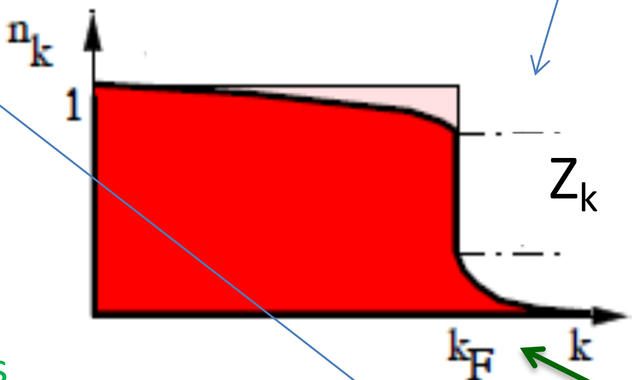
Leakage of electrons away from the Fermi surface (stronger correlations larger leakage)

Does band theory work?



At T=0 the distribution of **electrons** has a jump of height **Z_k** (not 1) at the Fermi level

$Z_k\ \Theta(-\varepsilon_k)$



Z_k
Strength of correlations

Simpler models:
Z_k independent of k

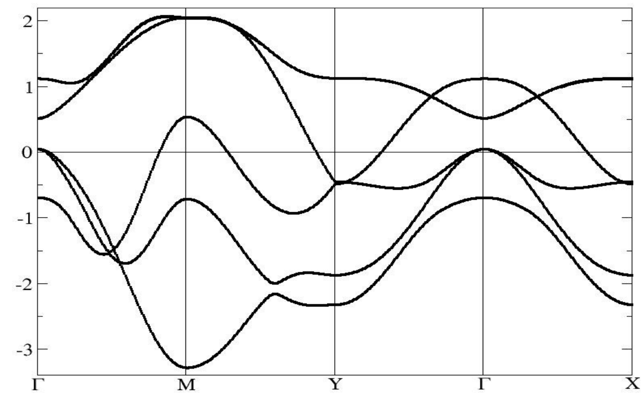
Leakage of electrons away from the Fermi surface
even at T=0
(stronger correlations larger leakage)

Deviation from band structure predictions

(remember: Normal state)

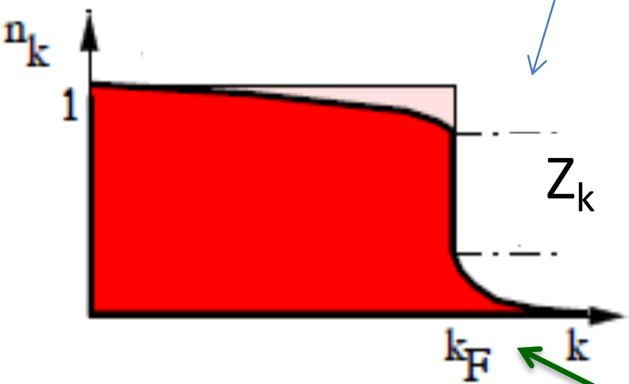
E. Bascones leni.bascones@csic.es

Does band theory work?



At T=0 the distribution of **electrons** has a jump of height $\mathbf{Z_k}$ (not 1) at the Fermi level

$Z_k \; \Theta(-\varepsilon_k)$



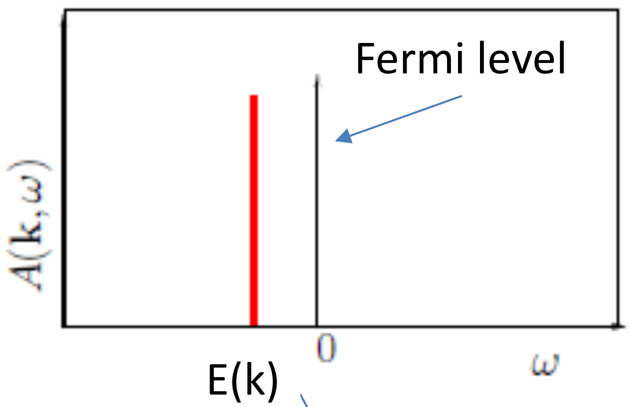
Leakage of electrons away from the Fermi surface even at T=0 (stronger correlations larger leakage)

El real electron $C^{\dagger}_{k\sigma}$ decays → E(k) not an eigenvalue → Bands may become badly defined

Spectral function $A(k,\omega)$

Distribution of excitations created when an electron (a real electron) with momentum k is added or removed from the system

- Non interacting system



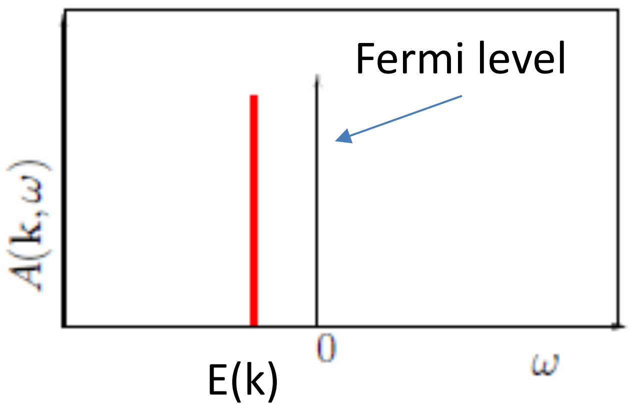
Eigenstate, well defined energy for a given momentum.
Infinite lifetime

The spectral function “measures the bands”

Spectral function A(k,ω)

Distribution of excitations created when
an electron (a real electron) with momentum k
is added or removed from the system

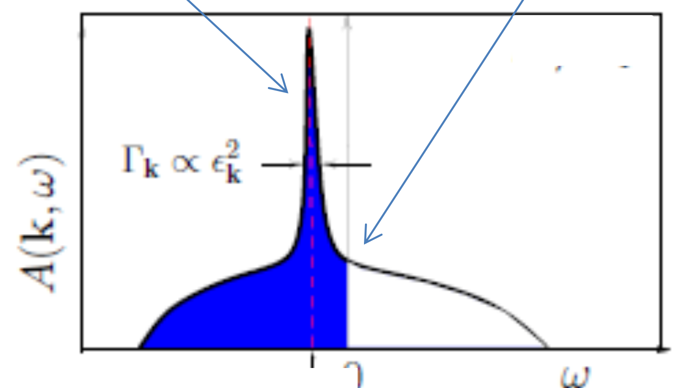
- Non interacting system



- Interacting system

Quasiparticle peak:
Part of the electronic excitation which is in the quasiparticle state
Coherent part

Incoherent part:
Amount of the Electronic excitation which is **not in the quasiparticle state**
(also called continuum)



Not the same energy
as in the non-interacting limit

Figs. Coleman's book

Spectral function $A(k,\omega)$

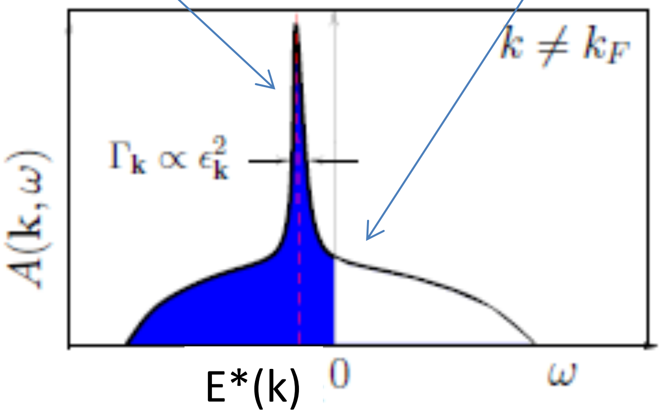
Distribution of excitations created when
a real electron is added or removed from the system

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

Quasiparticle peak:
Part of the electronic excitation which is in The quasiparticle state
Coherent part

Incoherent part:
Amount of the Electronic excitation which is not in the quasiparticle state
(also called continuum)

- Interacting system



Figs. Coleman's book

Spectral function A(k,ω)

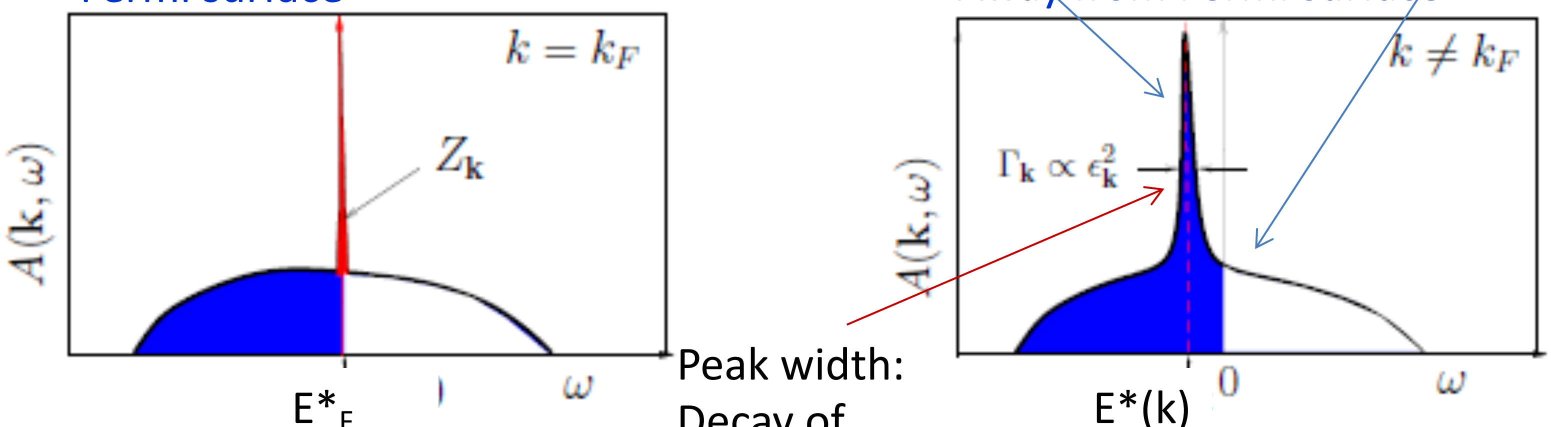
Distribution of excitations created when
an electron is added or removed from the system

Quasiparticle peak
Infinitely narrow
at the Fermi surface

Quasiparticle peak:
Part of the electronic
excitation which is in
The quasiparticle state
Coherent part

Fermi surface

Away from Fermi surface



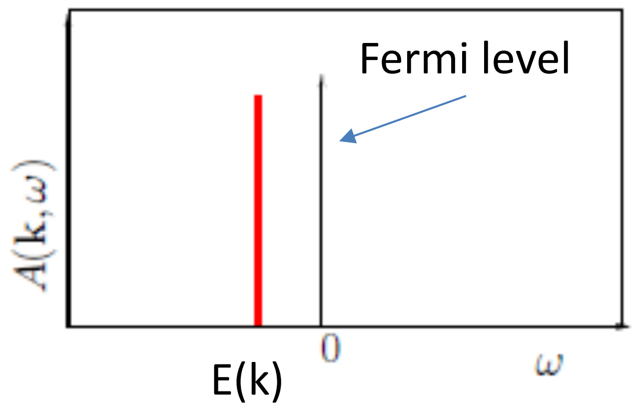
Peak width:
Decay of
the quasiparticle

Figs. Coleman's book

Spectral function A(k,ω)

Distribution of excitations created when **an electron** is added or removed from the system

- ❑ Non interacting system

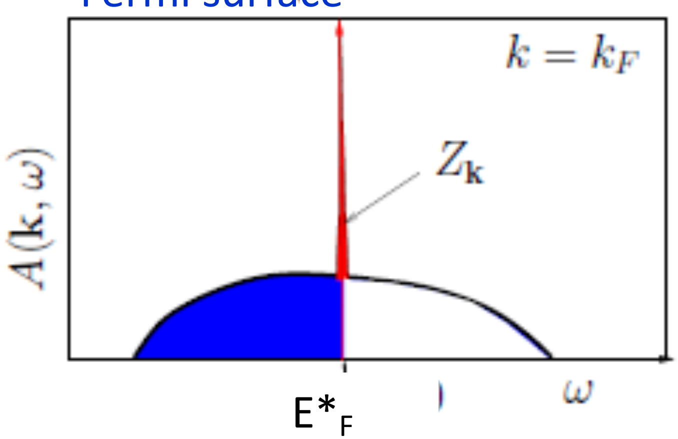


Larger similarities at the Fermi Surface and if the quasiparticle weight Z is close to one

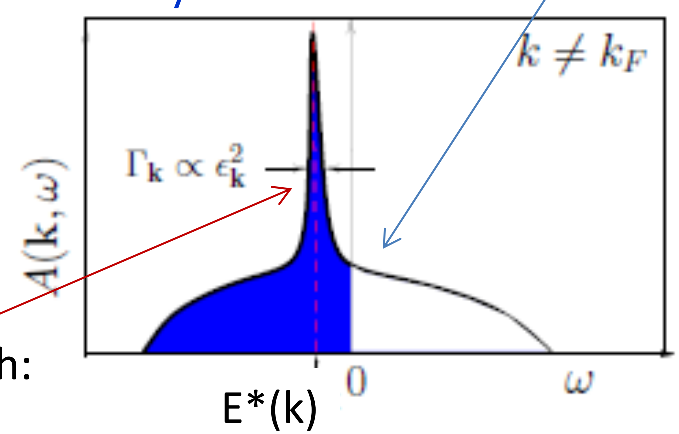
- ❑ Interacting system

Incoherence

Fermi surface



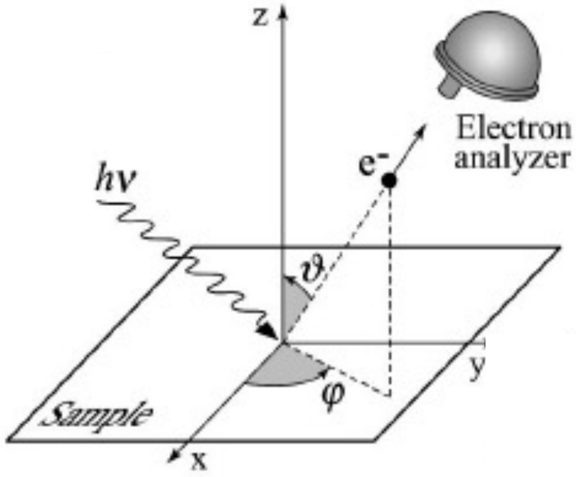
Away from Fermi surface

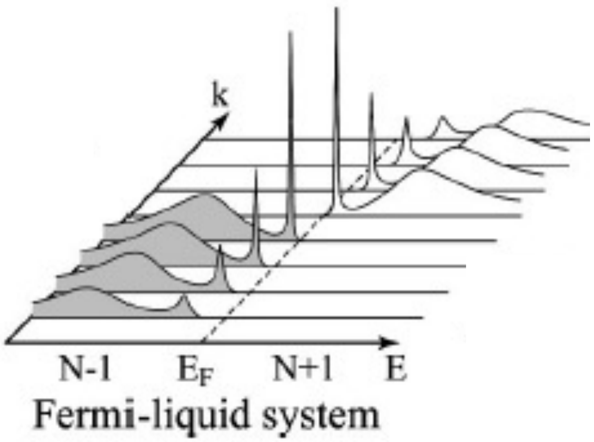


Peak width:
Decay of
the quasiparticle

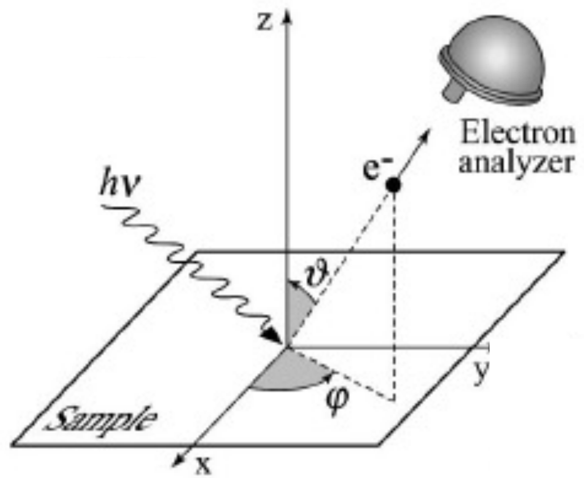
Figs. Coleman's book

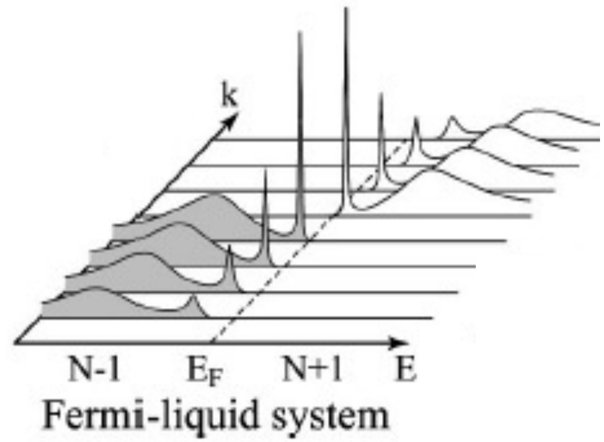
Spectral function $A(k,\omega)$ and ARPES (angle resolved photoemission)



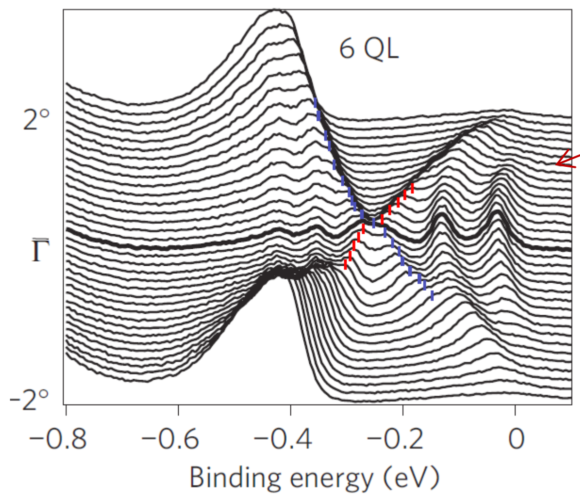


Spectral function A(k,ω) and ARPES





Bi_2Se_3



Well defined
Quasiparticle
peaks

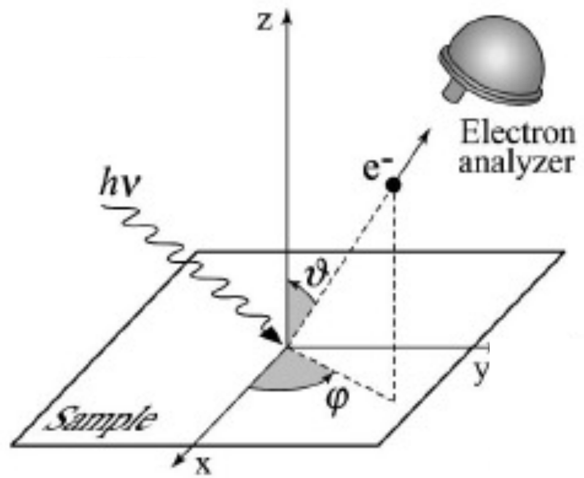
Weakly correlated system

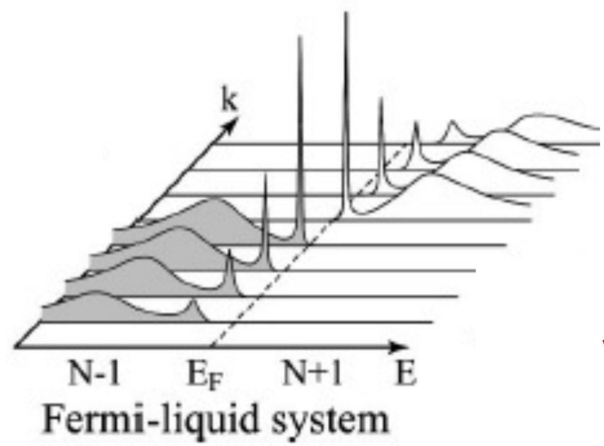
Possibility to map the bands

Figs:

Zhang et al, Nat Phys. 2010

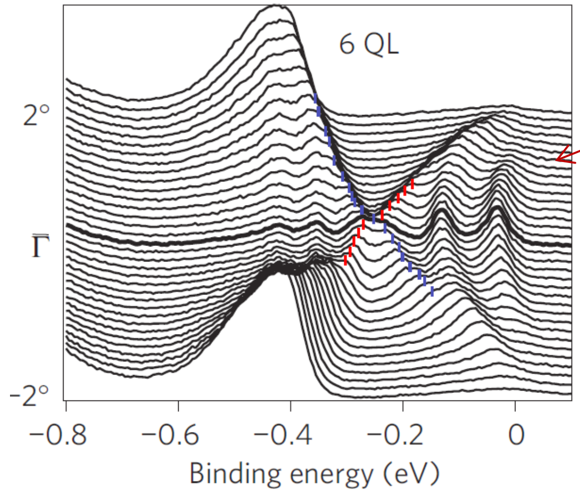
Spectral function A(k,ω) and ARPES





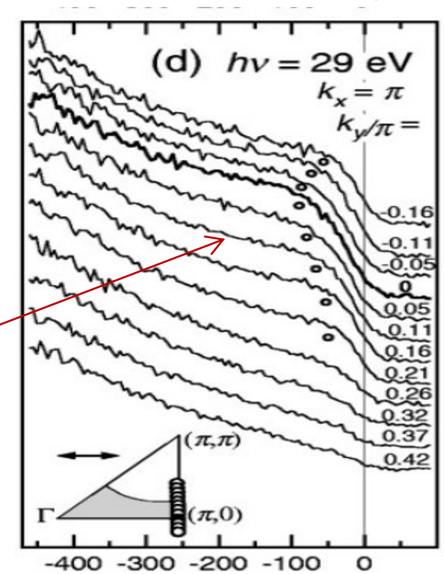
Very strongly correlated system

Bi_2Se_3



Well defined Quasiparticle peaks

Cuprate



Badly defined quasiparticle peak

Possibility to map the bands

Figs: Damascelli et al, RMP 2003
Zhang et al, Nat Phys. 2010

Spectral function A(k,ω) and ARPES

Weakly correlated system

Well defined bands

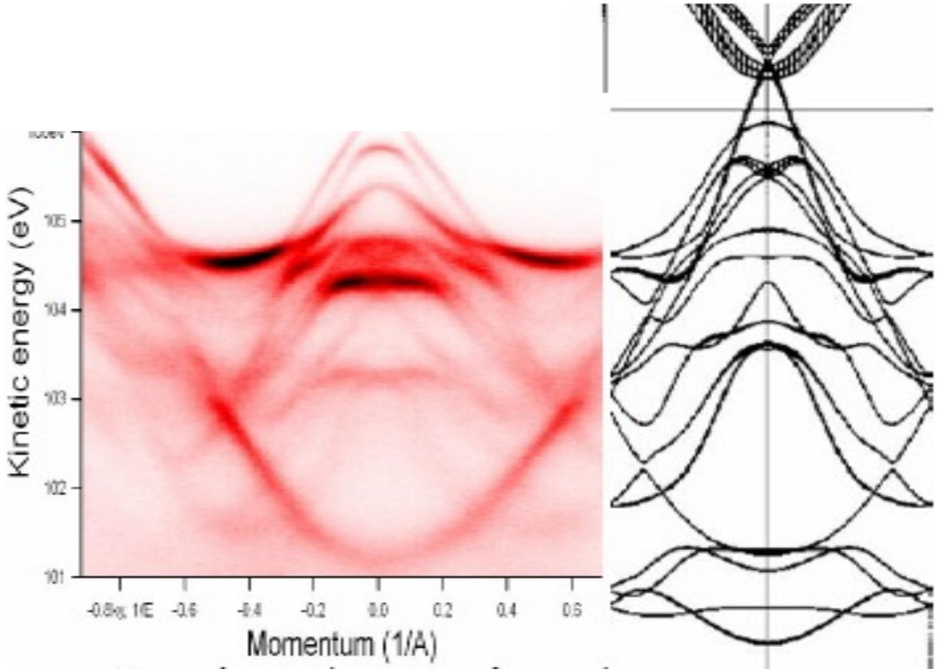
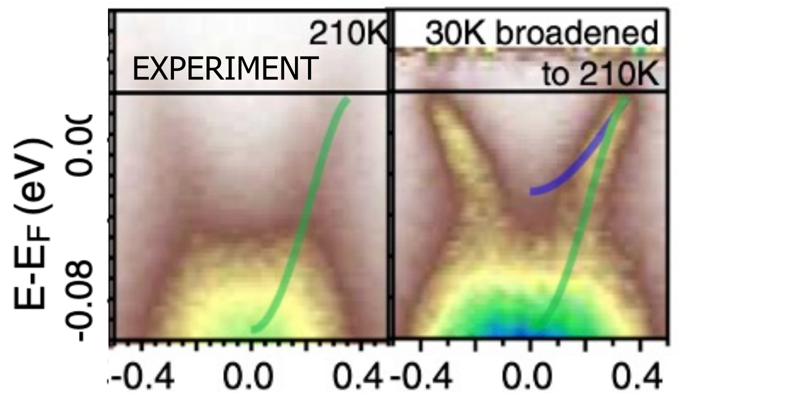


Fig. : Evtushinksy lectures

Strongly correlated system

Blurred spectrum and
anomalous temperature behavior

Iron chalcogenide



PRL 110, 067003 (2013

Fermi liquid theory

- ❑ Assume adiabaticity: the interacting ground state is perturbatively connected

- ❑Quasiparticles: elementary excitations of the interacting system have a one to one correspondence with elementary excitations of the non-interacting one. Implies quasiparticle weight Z_k finite.

- ❑ Quasiparticles in a metal: fermionic excitations with spin ½ and charge e. Well defined Fermi surface

- ❑ Quasiparticle decay rate in 3D: $$\tau^{-1}= \frac{(\varepsilon^*)^2+\pi^2T^2}{E^*_F}$$

- ❑ Description of the system in terms not of the ground state of the system but in terms of the low energy excitations$\rightarrow$ deviation from equilibrium δn_p.

- ❑ Energy functional $F[\delta n_p]$ in terms of parameters which can be measured experimentally.

Description in terms of excitations (quasiparticles)

- Consider non-interacting ground state

$$H = H_0 - \mu N = \sum_{\sigma} (E_{\mathbf{k}} - \mu) c^{\dagger}{}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} \qquad |\psi_g\rangle = \prod_{|\mathbf{k}|<k_F,\ \sigma} c^{\dagger}{}_{\mathbf{k}\sigma}|0\rangle$$

- Define the transformation

$$d^{\dagger}{}_{\mathbf{k}\sigma} = \begin{cases} c^{\dagger}{}_{\mathbf{k}\sigma} & (k > k_F) \quad \text{particle} \\ \mathrm{sgn}(\sigma) c_{-\mathbf{k}-\sigma} & (k > k_F) \quad \text{hole} \end{cases}$$

- Rewrite the hamiltonian

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} |(E_{\mathbf{k}} - \mu)| d^{\dagger}{}_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + F_g$$

Ground state

Excitations

Deviation from equilibrium occupation

Fermionic excitations
Charge e
Spin σ

Fermi liquid theory: description in terms of quasiparticles

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} |(E_{\mathbf{k}} - \mu)\, d^{\dagger}_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + F_g$$

Excitations (equilibrium occupation of states included in F_g)

$\delta n_{k\sigma} = d^{\dagger}_{k\sigma} d_{k\sigma}$ → Density of excitations

$$H_0 - \mu N = F_g + \Sigma_{k\sigma} \varepsilon_{k\sigma} \delta n_{k\sigma} = F\,[\delta n_{k\sigma}]$$

Excitation energy measured with respect to the ground state

Energy written as a functional of the density of excitations

Fermi liquid theory: description in terms of quasiparticles

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} |(E_{\mathbf{k}} - \mu)\, d^{\dagger}_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + F_g$$

Excitations (equilibrium occupation of states included in F_g)

$$\delta n_{k\sigma} = d^{\dagger}_{k\sigma} d_{k\sigma}$$ → Density of excitations

$$H_0 - \mu N = F_g + \Sigma_{k\sigma} \varepsilon_{k\sigma} \delta n_{k\sigma} = F[\delta n_{k\sigma}]$$

Excitation energy measured with respect to the ground state

Energy written as a functional of the density of excitations

- Landau's idea: Energy of the interacting system is a functional of the density of quasiparticles $F[\delta n_{k\sigma}]$. Expansion around equilibrium. Small $\delta n_{k\sigma}$

$$\delta n_{k\sigma} = a^{\dagger}_{k\sigma} a_{k\sigma}$$

Fermi liquid theory: description in terms of quasiparticles

- Landau's idea: Energy of the interacting system is a functional of the density of quasiparticles $F\,[\delta n_{k\sigma}]$. Expansion around equilibrium. Small $\delta n_{k\sigma}$

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

First order in the expansion

second order in the expansion

$$\varepsilon^*_{k\sigma} = \left.\frac{\partial F}{\partial(\delta n_{k\sigma})}\right|_{\delta n_{k\sigma}=0}$$

$$f_{k\sigma k'\sigma'} = \left.\frac{\partial^2 F}{\partial(\delta n_{k\sigma})\partial(\delta n_{k\sigma})}\right|_{\delta n_{k\sigma}=0}$$

Fermi liquid theory: description in terms of quasiparticles

- Landau's idea: Energy of the interacting system is a functional of the density of quasiparticles $F\,[\delta n_{k\sigma}]$. Expansion around equilibrium. Small $\delta n_{k\sigma}$

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

Residual interactions between the quasiparticles (responsable for quasiparticle decay and instabilities)

Energy of a quasiparticle in the absence of other quasiparticles (renormalized with respect to $\varepsilon_{k\sigma}$)

No second order term in non interacting limit

$$H_0 - \mu N = F_g + \Sigma_{k\sigma}\varepsilon_{k\sigma}\delta n_{k\sigma} = F\,[\delta n_{k\sigma}]$$

Fermi liquid theory: Parameters of the model

- To first order:

Energy of a quasiparticle
in the absence of other quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma} \varepsilon^*_{k\sigma} \delta n_{k\sigma}$$

Fermi liquid theory: Parameters of the model

- To first order:

Energy of a quasiparticle
in the absence of other quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma} \varepsilon^*_{k\sigma} \delta n_{k\sigma}$$

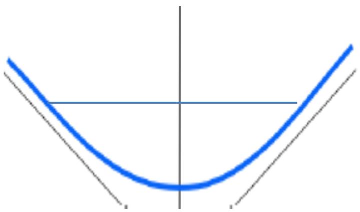
In a non-interacting state (continuum limit)

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma} \varepsilon_{k\sigma} \delta n_{k\sigma}$$

$$\varepsilon_{k\sigma} = k^2/2m - \mu = (k^2 - k_F^2)/2m$$

close to k_F we can linearize the
spectrum (first order expansion in $k-k_F$)

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$



k_F

Fermi liquid theory: Parameters of the model

- To first order:

Energy of a quasiparticle
in the absence of other quasiparticles

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma}$$

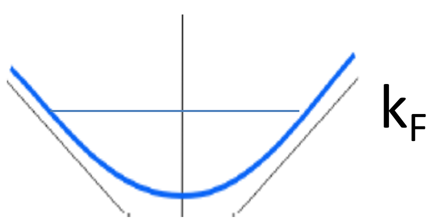
In a non-interacting state (continuum)

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon_{k\sigma}\delta n_{k\sigma}$$

$$\varepsilon_{k\sigma} = k^2/2m - \mu = (k^2 - k_F^2)/2m$$

close to k_F we can linearize the spectrum (first order expansion in $k-k_F$)

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$



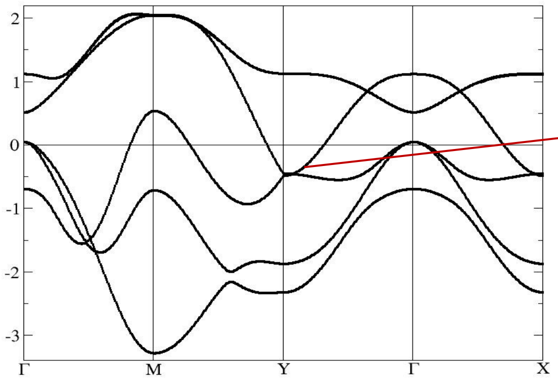
By analogy, in the interacting system, we linearize the spectrum close to k_F and **define m***

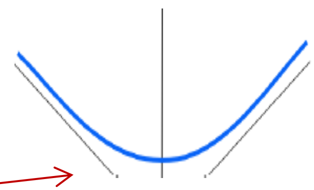
$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

Renormalized mass or quasiparticle mass

Fermi liquid theory: Parameters of the model

- Non-interacting system:





Band mass different to free electron mass
Modification due to ionic potential

$$m^{-1} = |\partial^2\varepsilon/\partial k^2|$$

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$

- Interacting system:

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

→ Quasiparticle mass → Different from m (larger) due to electronic interactions

In simple models

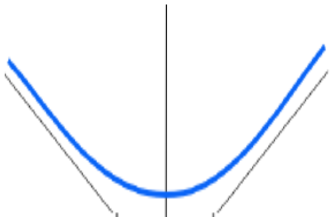
$$m^* = \frac{m}{Z}$$

If Z vanishes (no quasiparticle weight) the expansion does not converge
Fermi liquid theory is not applicable

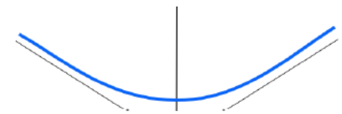
Fermi liquid theory: Measurable quantities

Bandwidth renormalization

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$

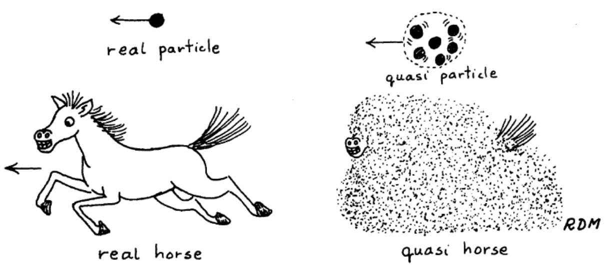


$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$



m*>m always

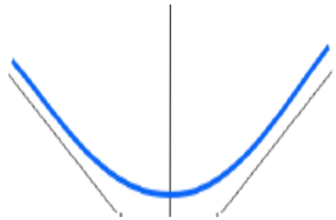
The backflow of the surrounding fluid enhances the mass



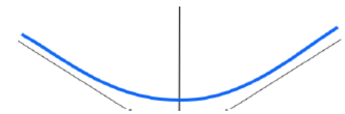
Fermi liquid theory: Measurable quantities

Bandwidth renormalization

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$



$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$



m*>m always

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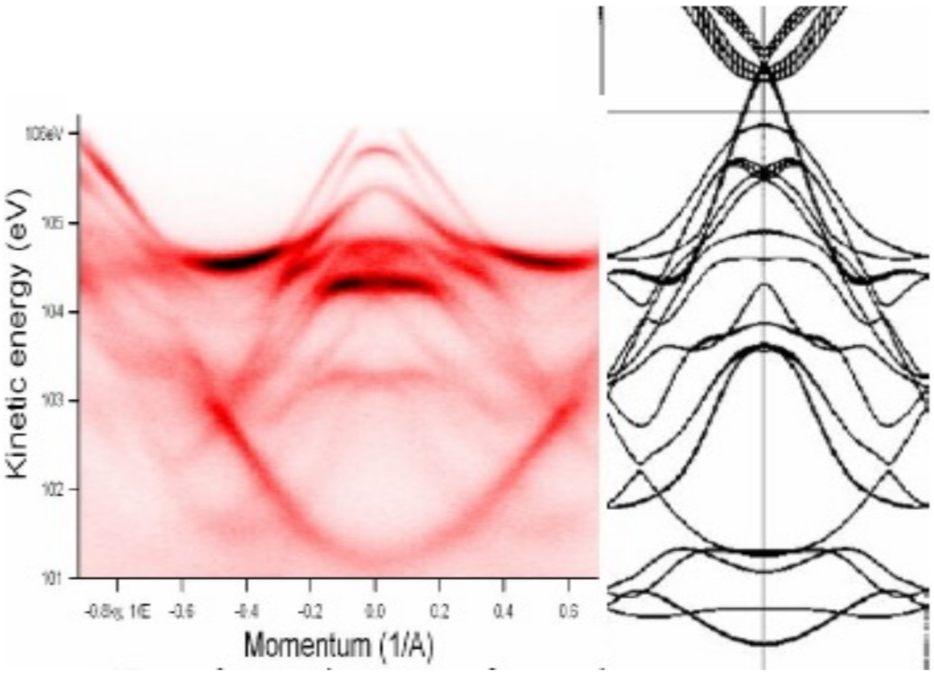


How large is m*/m?

Experimental ↗ ↖ LDA

Fig. : Evtushinksy lectures

Fermi liquid theory: Measurable quantities

Non-interacting Fermi gas: $K_B T << \mu$

From Fermi-Dirac statistics $$n(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/K_B T}+1}$$

- **Specific heat** $$C_v = \left.\frac{\partial F}{\partial T}\right|_{\mu}$$

$$C_v = \gamma T$$ **Linear in temperature** $$\gamma = (\pi/3) K^2_B N(E_F) \propto m$$

- **Spin susceptibility** $$\chi_s = \frac{\partial M}{\partial H}$$

$$\chi_s = \mu_B^2 N(E_F) \propto m$$ **independent of T**

Fermi liquid theory: Measurable quantities

Interacting Fermi liquid:

$$\delta n_{k\sigma}=n_{k\sigma}(T,\mu)-n_{k\sigma}(0,\mu)$$

$$n_{k\sigma} = \frac{1}{e^{-(\varepsilon_{k\sigma}'-\mu)/K_BT}+1}$$

Fermi-Dirac distribution for "interacting" quasiparticle energies

Quasiparticle weight Z_k not present

$$\varepsilon'_{k\sigma}= \varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'} \delta n_{k'\sigma'}$$

See Pines & Nozieres' and Coleman's books for details on the derivation

Fermi liquid theory: Measurable quantities

Specific heat $C_v = \left.\frac{\partial F}{\partial T}\right|_{\mu}$

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

F(T) $\propto T^2$ $\propto T^4$

$$C^*_v = \gamma^* T \qquad \gamma^* = (\pi/3)K^2_B N^*(E^*_F) \propto m^*$$

Same temperature dependence as in Fermi gas but with a renormalized proportionality constant γ^*

Fermi liquid theory: Measurable quantities

Specific heat $C_v = \left.\frac{\partial F}{\partial T}\right|_{\mu}$

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

F(T) $\qquad \propto T^2 \qquad \propto T^4$

$$C^*_v = \gamma^* T \qquad \gamma^* = (\pi/3)K^2_B N^*(E^*_F) \propto m^*$$

Same temperature dependence as in Fermi gas but with a renormalized proportionality constant γ^*

Check experimental dependence and measure γ
Can be also compared with Model calculations

Interacting — Non-Interacting

$$C^*_v = \frac{m^*}{m} C_v$$

Comparison with LDA like calculations

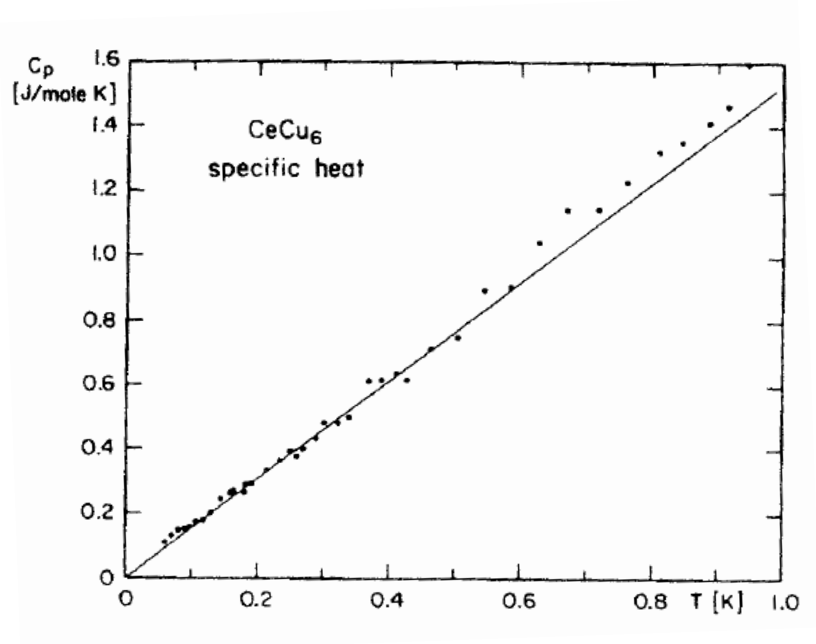
Fermi liquid theory: Measurable quantities

Specific heat

$C_v = \gamma^* T \qquad \gamma^* = (\pi/3) K_B^2 N^*(E^*_F) \propto m^*$

$\gamma^*_{Au,Ag} = 0.67$ mJ mol^{-1} K^{-1} (vs 0.63 mJ mol^{-1} K^{-1} in absence of interaction)

$\gamma^*_{CeCu6} = 1{,}5$ J mol^{-1} K^{-1} → **Heavy fermion**



Fermi liquid theory: Measurable quantities

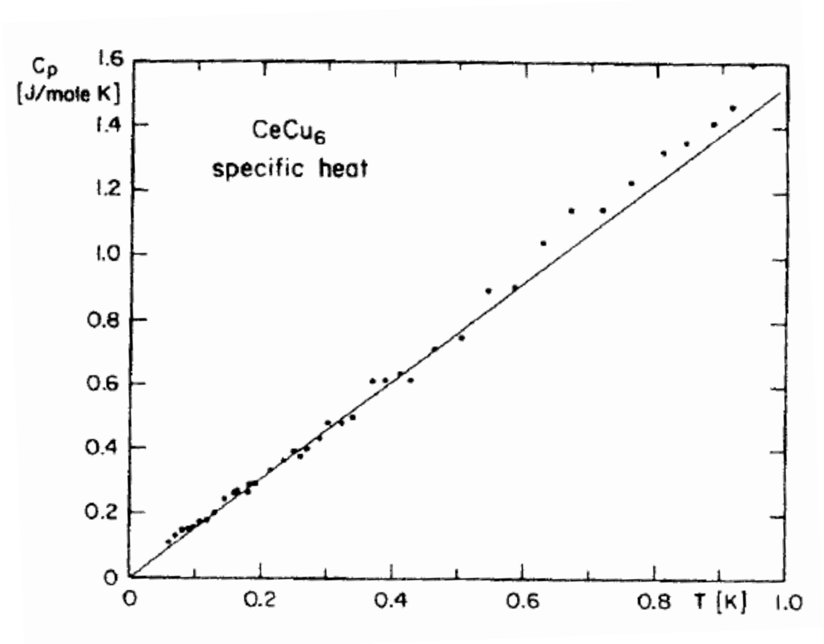
Specific heat

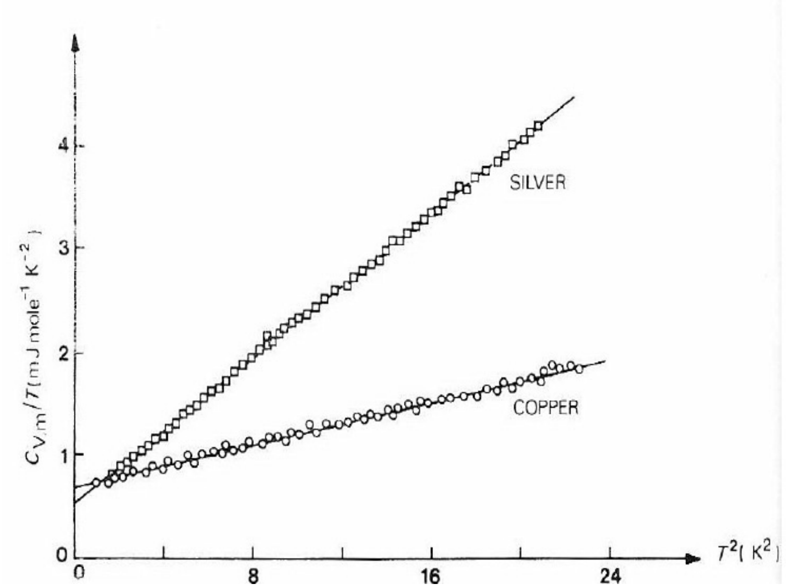
$C_v = \gamma^* T$ $\gamma^* = (\pi/3) K^2_B N^*(E^*_F) \propto m^*$

(note phonon contribution can dominate $C_v \propto T^3$)

$\gamma^*_{Au,Ag} = 0.67$ mJ mol^{-1} K^{-1} (vs 0.63 mJ mol^{-1} K^{-1} in absence of interaction)

$\gamma^*_{CeCu6} = 1{,}5$ J mol^{-1} K^{-1} → **Heavy fermion**





Fermi liquid theory: Interactions between quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

Energy of a quasiparticle in the absence of other quasiparticles

Interaction between quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}(\varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k'\sigma'})\delta n_{k\sigma}$$

The energy of a quasiparticle Is modified by the presence of other quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon'_{k\sigma}\delta n_{k\sigma}$$

$$\varepsilon'_{k\sigma} = \varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k'\sigma'}$$

Non-rigid band shift

Fermi liquid theory: Interaction parameters

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

Remember assumption of isotropic system

Focus on the Fermi surface $|k|=|k'|=K_F$

Time-reversal invariance (no magnetic field) $f_{k\sigma,k'\sigma'}=f_{-k-\sigma,-k'-\sigma'}$

Fermi surface invariant under reflection $k \to -k$ $f_{k\sigma,k'\sigma'}=f_{k-\sigma,k'-\sigma'}$

If spin is conserved in general the dependence on spin enters only via their relative orientation

$$f_{k\sigma,k'\sigma'}=f_{kk'}^{s}+f_{kk'}^{a}\sigma.\sigma'$$

Spin symmetric

Spin antisymmetric (exchange)

Pines & Nozieres and Coleman's books

Fermi liquid theory: Interaction parameters

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

$f_{kk'}^{s}$ $\quad$ $f_{kk'}^{a}$

Focus on the Fermi surface $|k|=|k'|=K_F$. Isotropic system

$f_{kk'}^{s,a}$: dependence only on the angle ξ between k and k' → Expansion in Legendre Polynomials

$$f_{kk'}^{s,a} = \Sigma_{l=0}^{\infty} \mathbf{f_l^{s,a}} P_l(\cos\xi)$$

Dimensionless parameters $\quad N^*(E^*_F)\, f_l^{s,a} = \mathbf{F_l^{s,a}}$

Density of quasiparticle states at the Fermi level In the interacting system

Interaction parameters

$F_0^s, F_0^a, F_1^s, F_1^a \ldots$

Can be extracted from experiment and model calculations

Fermi liquid theory: Interaction and parameters. Summary

Expansion of the free energy in terms of the quasiparticle density

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

Energy of a quasiparticle in the absence of other quasiparticles

Residual interactions between the quasiparticles

Linearized dispersion around K_F

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

Renormalized mass or quasiparticle mass

$$f_{kk'}^{s,a} = \Sigma_{l=0}^{\infty} f_l^{s,a} P_l(\cos \xi)$$

$$N^*(E^*_F)\, f_l^{s,a} = F_l^{s,a}$$

Small number of parameters l=0,1 …

Fermi liquid theory: Measurable quantities

$$\chi_s = \frac{\partial M}{\partial H}$$

- **Spin susceptibility**

$$\chi^*_s = \frac{\mu_B{}^2 N^*(E_F)}{1 + F_0{}^a} = \frac{(m^*/m)}{1 + F_0{}^a}\,\chi_s$$

independent of T as in Fermi gas

interacting

Non-interacting

Susceptibility
Spin dependent
Isotropic response

Fermi liquid theory: Measurable quantities

- **Spin susceptibility**

$$\chi^*_s = \frac{\mu_B^2 N^*(E_F)}{1 + F_0^a} = \frac{(m^*/m)}{1 + F_0^a} \chi_s$$

independent of T as in Fermi gas

interacting

Non-interacting

Susceptibility
Spin dependent
Isotropic response

Wilson ratio or Stoner enhancement factor

$$W = \frac{(\gamma^*/\chi^*_s)}{(\gamma/\chi_s)} = \frac{1}{1 + F_0^a}$$

experiment

LDA

Constant

$C_v = \gamma^* T$ $\gamma^* \propto m^*$

Deviation from unity, signature of electronic correlations

A way to obtain F_0^a

Fermi liquid theory: Measurable quantities. Resistivity

Band theory: classification into metals and insulators

Metal ρ
increases with T

Not changed by interactions in a Fermi liquid

Insulator ρ
decreases with T

Adiabaticity

Fermi liquid theory: Measurable quantities. Resistivity

Band theory: classification into metals and insulators

Metal ρ increases with T

Not changed by interactions in a Fermi liquid

Insulator ρ decreases with T

Drude conductivity $\sigma = ne^2\tau/m^*$

$$\tau^{-1} = \frac{\pi^2 T^2}{E^*_F} \propto m^* T^2$$

Fermi liquid theory: Measurable quantities. Resistivity

Band theory: classification into metals and insulators

Metal ρ increases with T

Not changed by interactions in a Fermi liquid

Insulator ρ decreases with T

Drude conductivity $\sigma = ne^2\tau/m^*$ $\tau^{-1} = \frac{\pi^2 T^2}{E^*_F} \propto m^* T^2$

$$\rho = \rho_0 + AT^2 \qquad A \propto (m^*)^2$$

High resistivity due to interactions

disorder

Quadratic dependence on T

Constant strongly dependent on interactions

Phonon resistivity $\propto T^5$ can dominate in simple metals

Fermi liquid theory: Measurable quantities. Resistivity

$$\rho=\rho_0 + AT^2 \qquad A \propto (m^*)^2$$

The resistivity does not seem to saturate

High-Tc superconducting cuprates

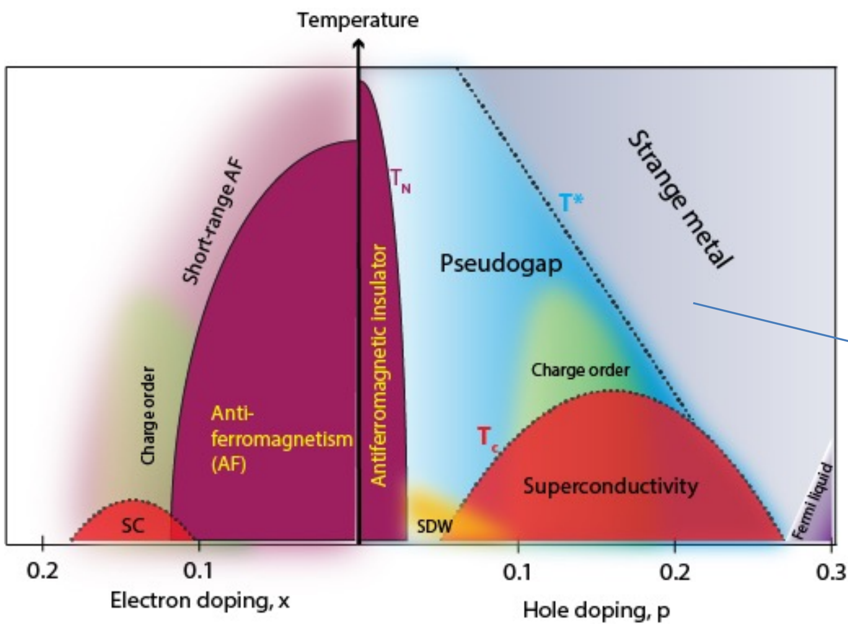


Fig: cme.physics.ucdavis.edu

Strange metal

$$\rho \sim T$$

Observed in many correlated systems
Not the behavior expected from quasiparticles (or phonons)

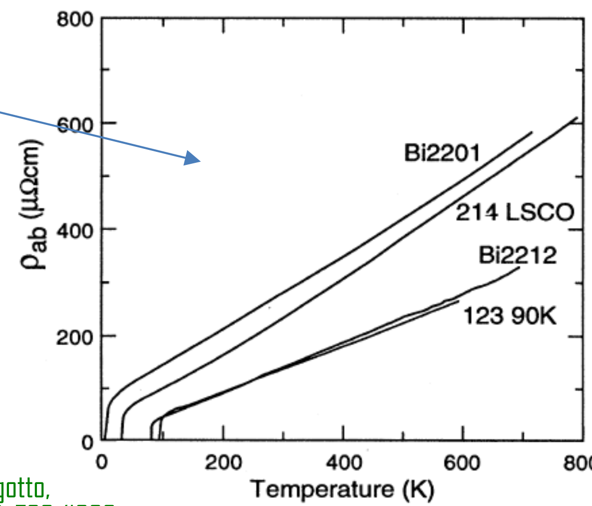


Fig: Dagotto, RMP 66, 763 (1993)

Fermi liquid theory: Measurable quantities

$C^*_v = \gamma^* T \qquad \gamma^* \propto m^*$

$\rho = \rho_0 + AT^2 \qquad A \propto (m^*)^2$

$$\frac{A}{(\gamma^*)^2} \sim \text{constant}$$

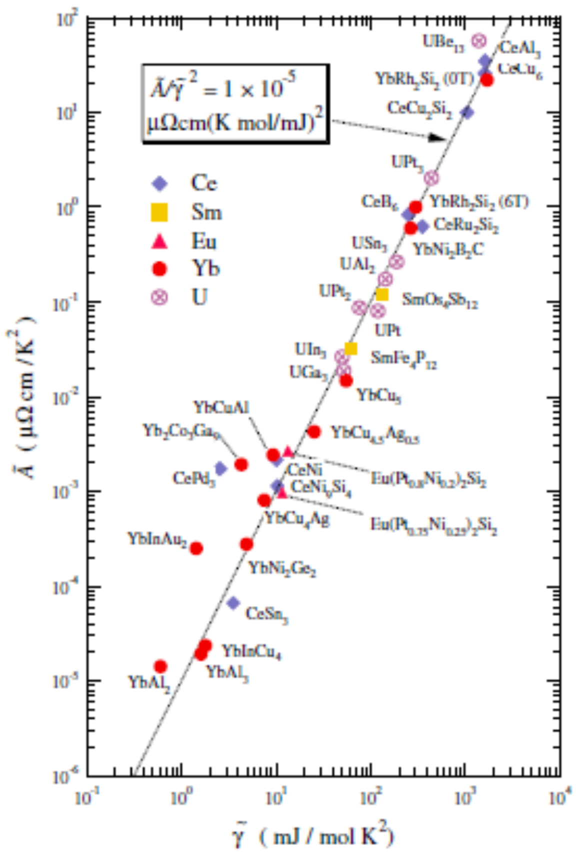
Kadowaki –Woods ratio

Fermi liquid theory: Measurable quantities

$C^*_v = \gamma^* T \qquad \gamma^* \propto m^*$

$\rho = \rho_0 + AT^2 \qquad A \propto (m^*)^2$

$$\frac{A}{(\gamma^*)^2} \sim \text{constant}$$



Kadowaki –Woods ratio

A and γ corrected by degeneracy

Fig: Tsuji et al, PRL 94, 057201 (2005)

Fermi liquid theory: Measurable quantities

Mass renormalization

Looking at the quasiparticle current

$$m^* = m\,(1 + F_1^s) = \frac{m}{1 - N(E_F) f^1_s}$$

Spin independent
But directional response

Coleman's book

Fermi liquid behavior

- **Bands observed in photoemission**

Reduction in bandwidth as compared to LDA estimate of m*/m

- **Specific heat linear in temperature.**

$C^*_v = \gamma^* \, T$

Enhancement of γ estimate of m*/m

Careful in materials with multiple Fermi pockets

- **Temperature independent spin susceptibility**

Enhancement: mass renormalization + Stoner enhancement

- **Resistivity quadratic in temperature**

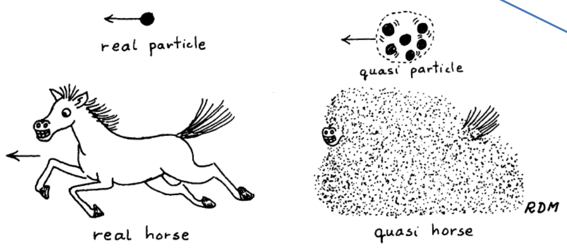
Interactions enhance resistivity as $(m^*)^2$

$\rho = \rho_0 + AT^2$

Correlated electrons: frequently non-Fermi liquid behavior

Strong correlations: the role of quasiparticle weight

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}} a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma) a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$



Renormalized parameters
m^* and E^*_F, g^*, A (resistivity)

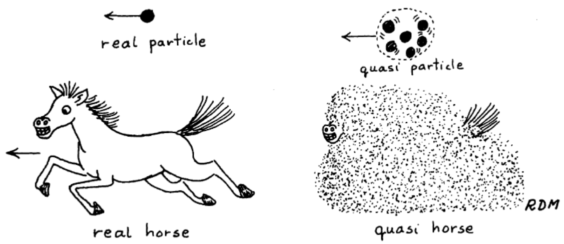
$Z \in [0,1]$

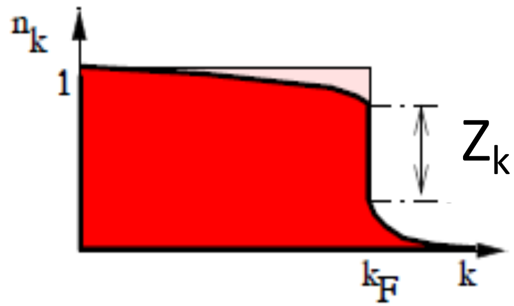
No quasiparticle description

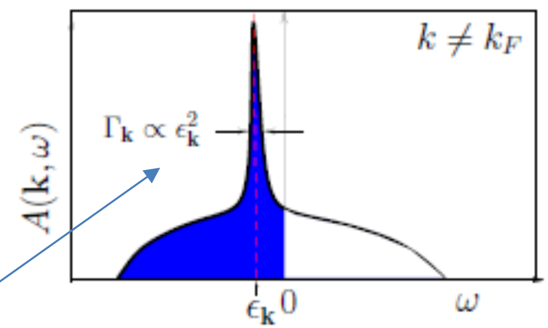
Non-interacting

Strong correlations: the role of quasiparticle weight

$$c^{\dagger}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^{\dagger}_{\mathbf{k}_4\sigma_4} a^{\dagger}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \dots$$







$Z \in [0,1]$

No quasiparticle description

Non-interacting

Amount of spectral weight at the quasiparticle peak

(blurred bands)

Renormalizes the energies (via the mass $m^*=m/Z$)

Value of specific heat, magnetic susceptibility, bandwidth, resistivity …

Key role in range of temperatures and energies with Fermi liquid behavior (anomalour T behavior)

$$\tau^{-1} = \frac{\varepsilon^{*2} + \pi^2 T^2}{E^*_F}$$

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Emergence of Quantum Phases in Novel Materials

Interactions between many particles (electrons)

Correlated Quantum States
in many different and novel materials

Symmetry breaking states

Formation of local moments
(and other effects)

Strong correlations
Z << 1

Fermi liquid instabilities. Phase transitions

$$m^*=m\,(1+F_1^s)=\frac{m}{1-N(E_F)f^1_s}$$

It could diverge for large enough interactions (Localization. Metal-insulator transition)

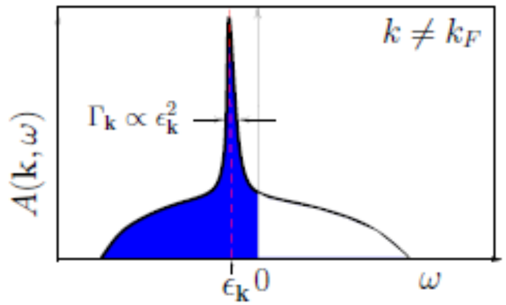
$$\chi^*_s=\frac{\mu_B^2 N^*(E_F)}{1+F_0^a}$$

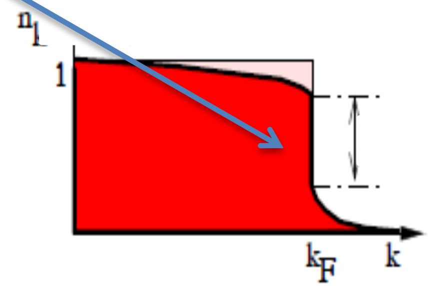
It diverges if $F^0_a=-1$

Fermi liquid theory: Summary

- Adiabaticity and well defined quasiparticles assumed ($Z_k > 0$). Quasiparticles spin ½ and charge e and momentum k. Fermi surface.

- In the 3D continuum quasiparticle decay ε^{*2}. Quasiparticles well defined at low energies





Z_k

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k - k_F)$$

$$N^*(E^*_F)\, f_l^{s,a} = F_l^{s,a}$$

- Fermi liquid behavior $\rho^* = \rho^*_0 + AT^2$ $C^*_v = \gamma^* T$ χ^*_s Bands in ARPES

- Role of Quasiparticle weight. $Z = (m/m^*)$ Measurable experimentally!

- The Fermi liquid can be unstable

Strength of correlations

- And more: transport equation, collective modes, …