

Emergence of Quantum Phases in Novel Materials

VII Edition ICMM-CSIC Postgraduate Course

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Emergence



More is different

<<The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe...

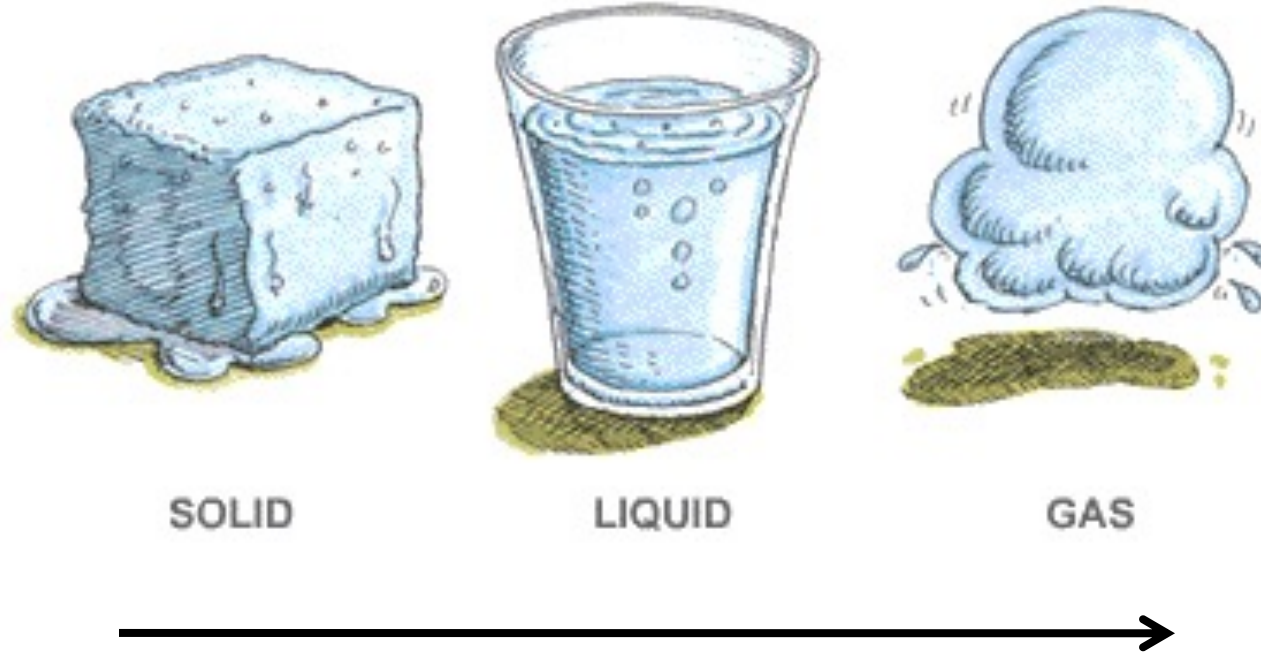
At each level of complexity, entirely new properties appear.>>

Anderson

Science 177, 4047 (1972)

New phenomena due to the interactions between particles

Phase transitions

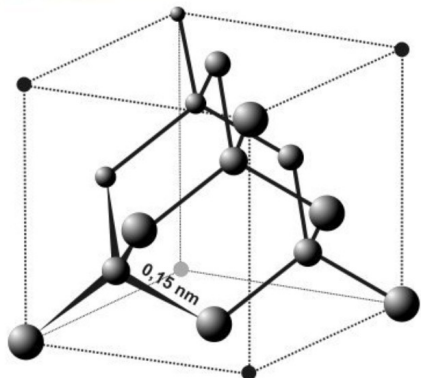
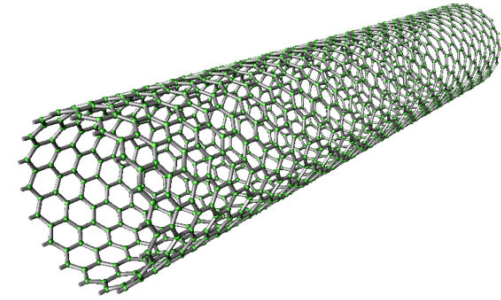


Temperature

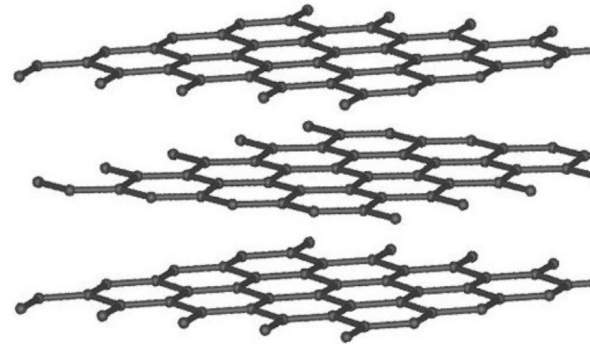
Changes also as a function of pressure ...

Possibility of detecting but also of controlling the state of the system

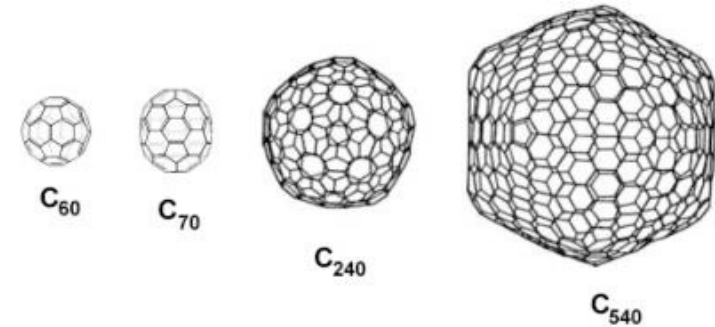
Crystalline order in solids



©Itub



Molecules

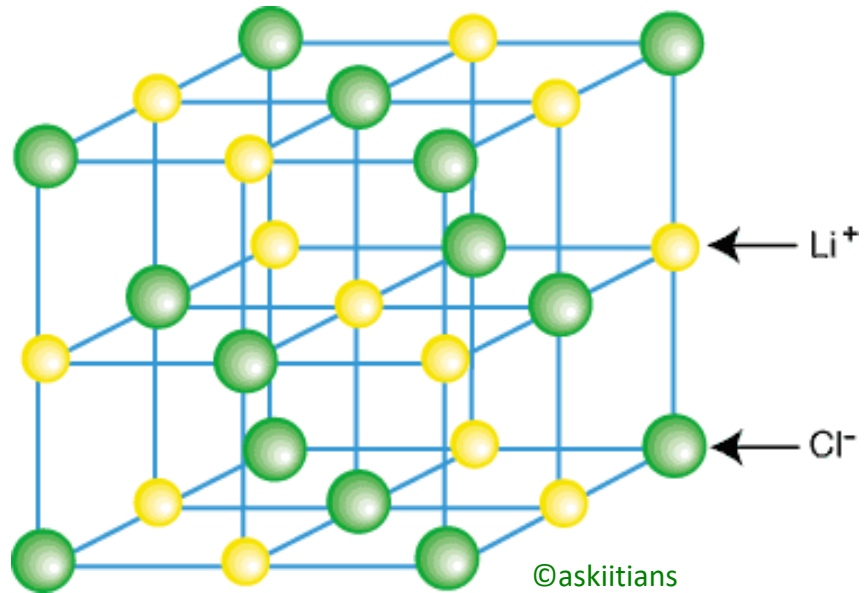


©Fig:web Belén Garrido

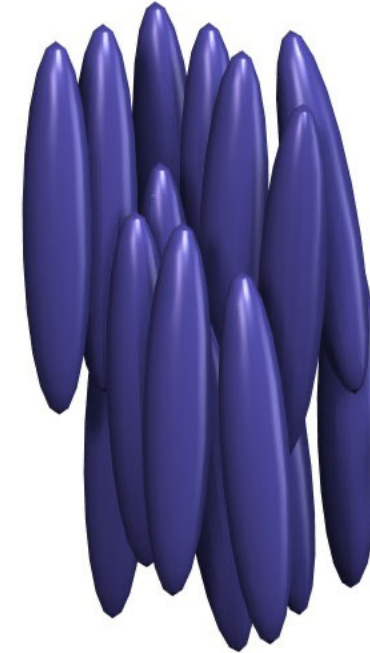
Symmetry breaking (translational symmetry), long-range order: ordered phases

Ordered phases and symmetry breaking

Translational symmetry breaking
(solid)

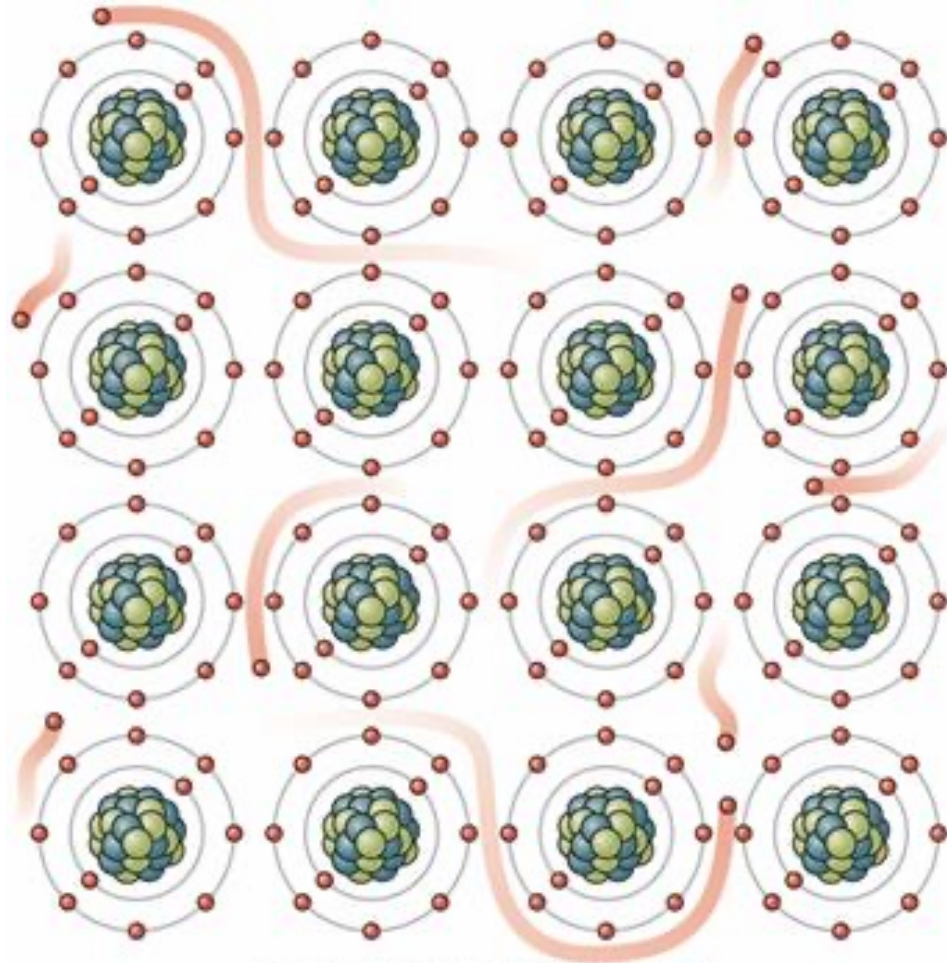


Rotational symmetry breaking
(nematic liquid crystal)



Different symmetries that can be broken

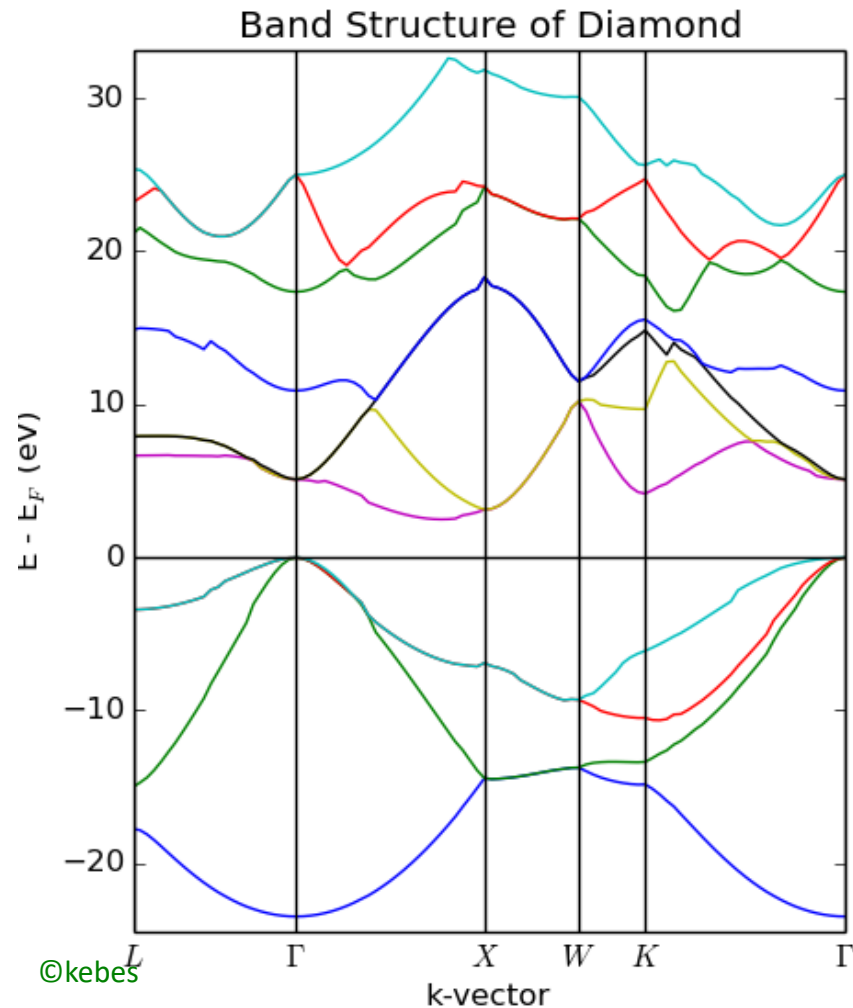
Emergence of quantum phases in novel materials



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Electrons interact among them
and with the environment

Band theory: the basis of our understanding of solids



Bloch states: electronic bands

Band structure $E(k)$

The electron moves in the average periodic potential of the solid

Density Functional Theory:
Ability to calculate the bands

Band theory: the basis of our understanding of solids

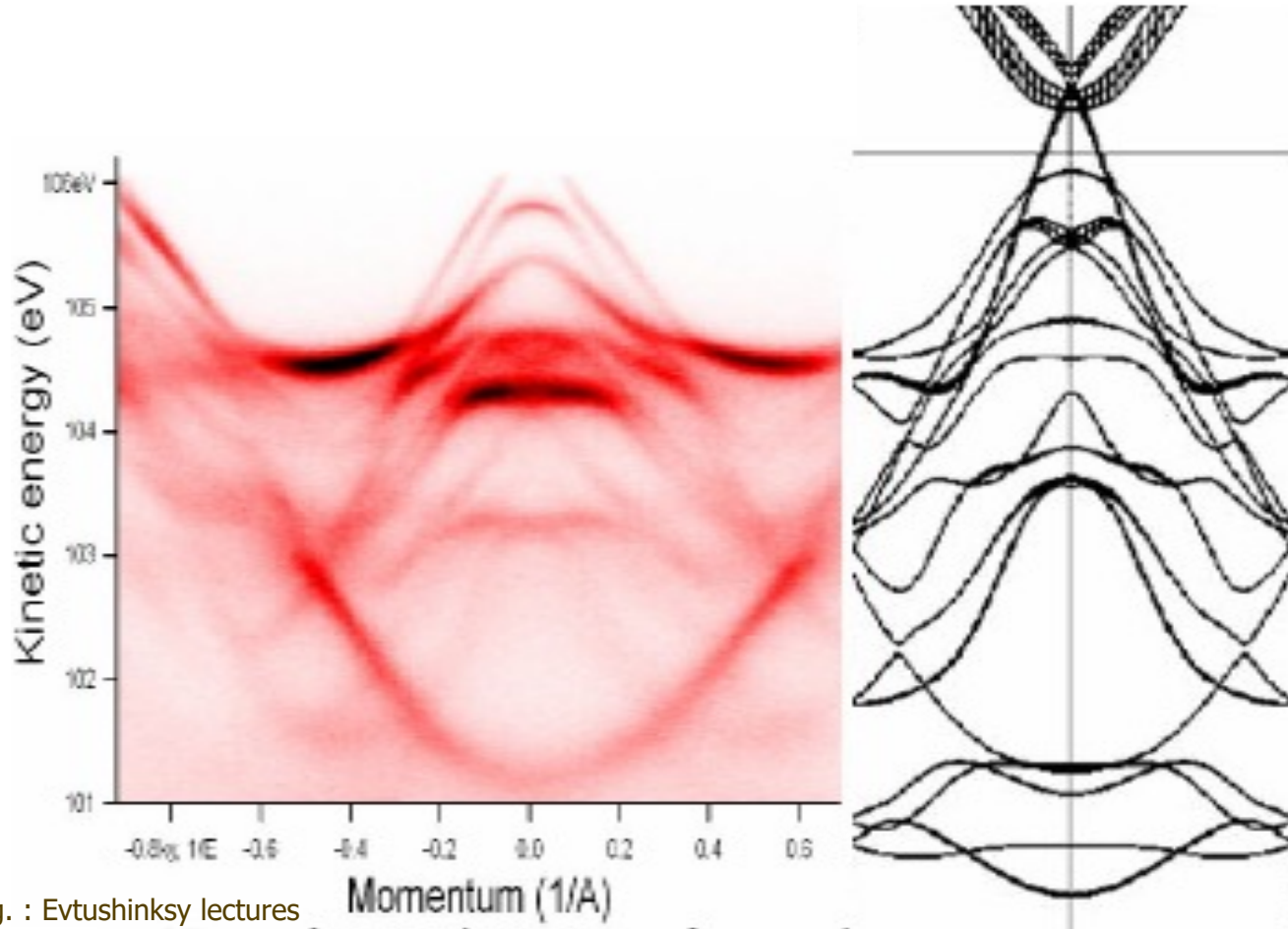
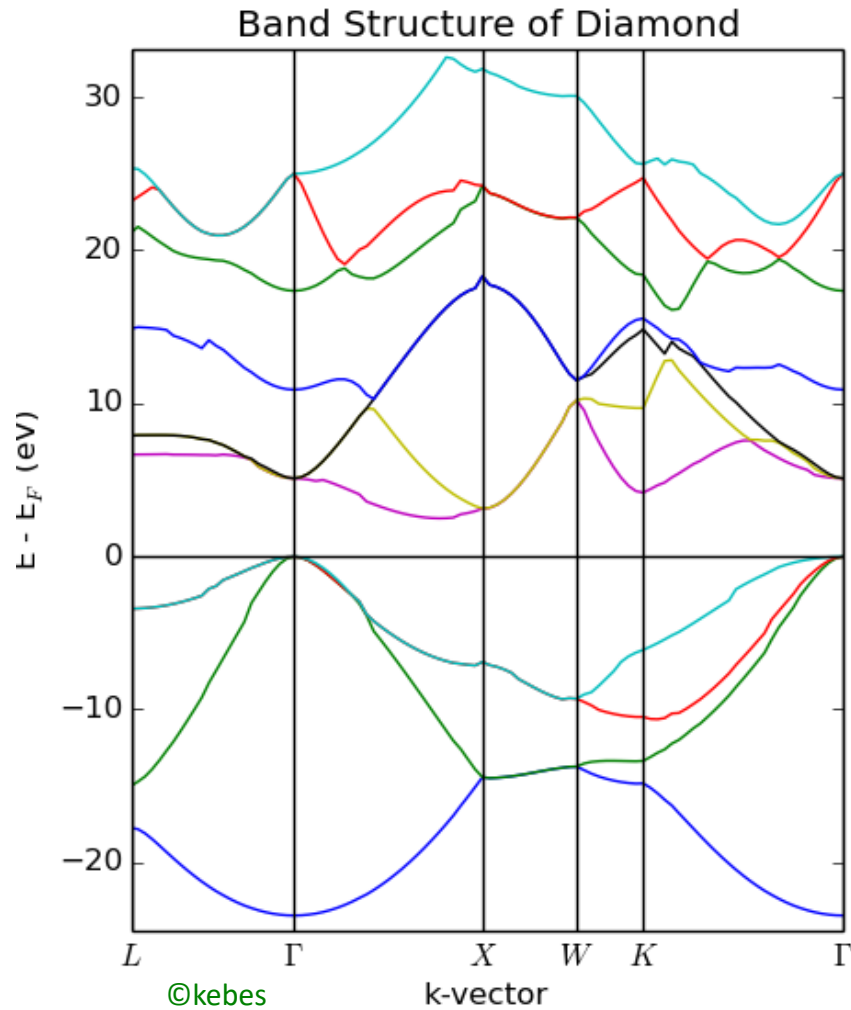


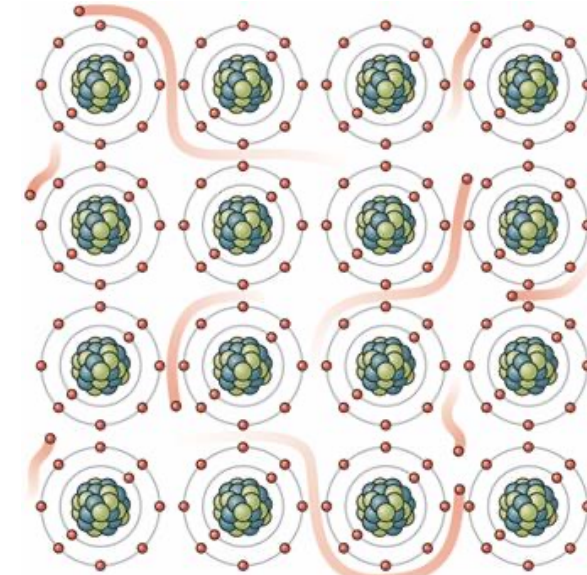
Fig. : Evtushinsky lectures

The bandstructure can be measured Experimentally with probes such as angle resolved photoemission (ARPES) and compared to theoretical predictions

Band theory: an “effective non-interacting” description



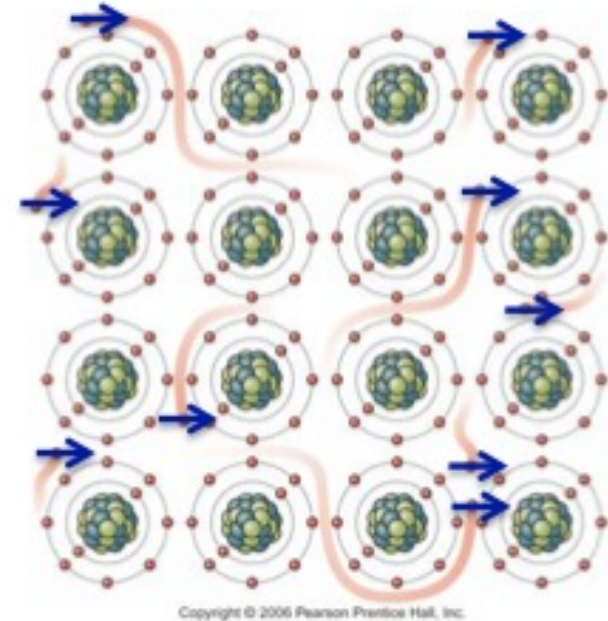
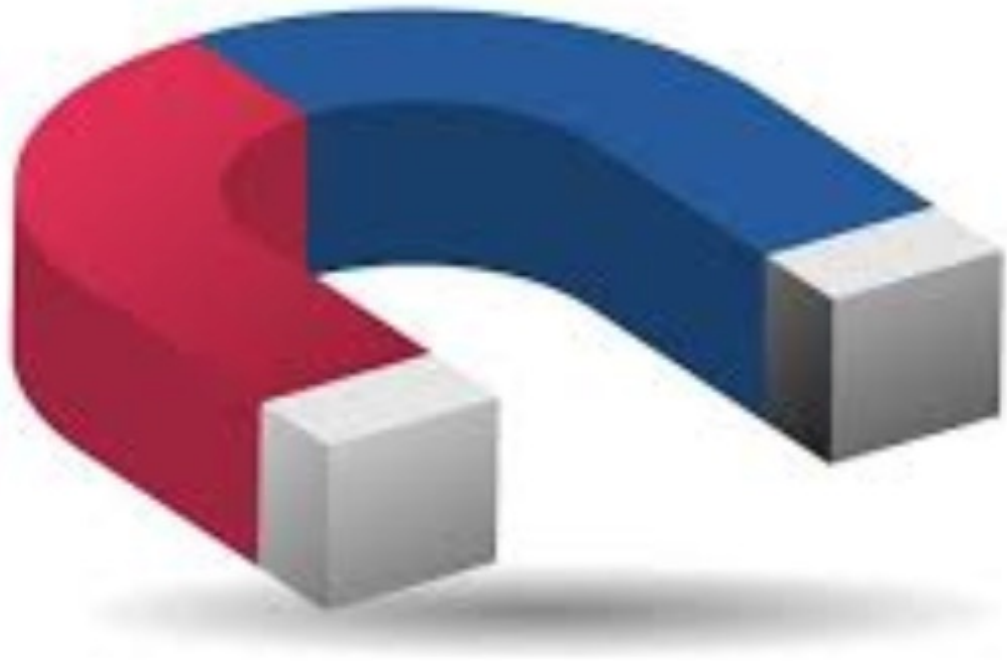
Metals and insulators



Bandstructure:

Effective “independent” electronic states including in an effective way the symmetry of the lattice and the interactions of the electron with the atomic lattice and with other electrons.

Electronic phases: symmetry breaking



Ferromagnetism:

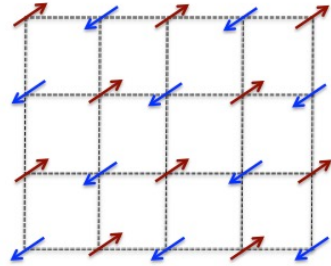
Spin order

It does not require lattice symmetry breaking

Electronic phases with lattice symmetry breaking

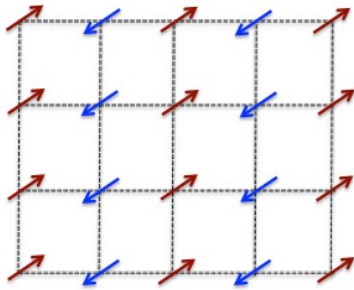
Spin Order

Lattice translational symmetry broken (density waves)



Neel antiferromagnet

Lattice translational and rotational symmetry broken (stripe-like phases)

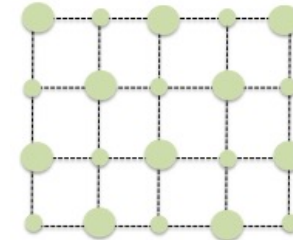


Stripe antiferromagnet

Lattice rotational symmetry broken (nematic states)

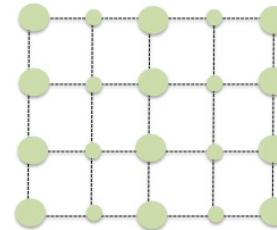
Spin nematic

Charge Order

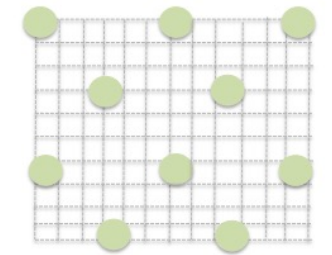


Charge density wave

Charge stripes

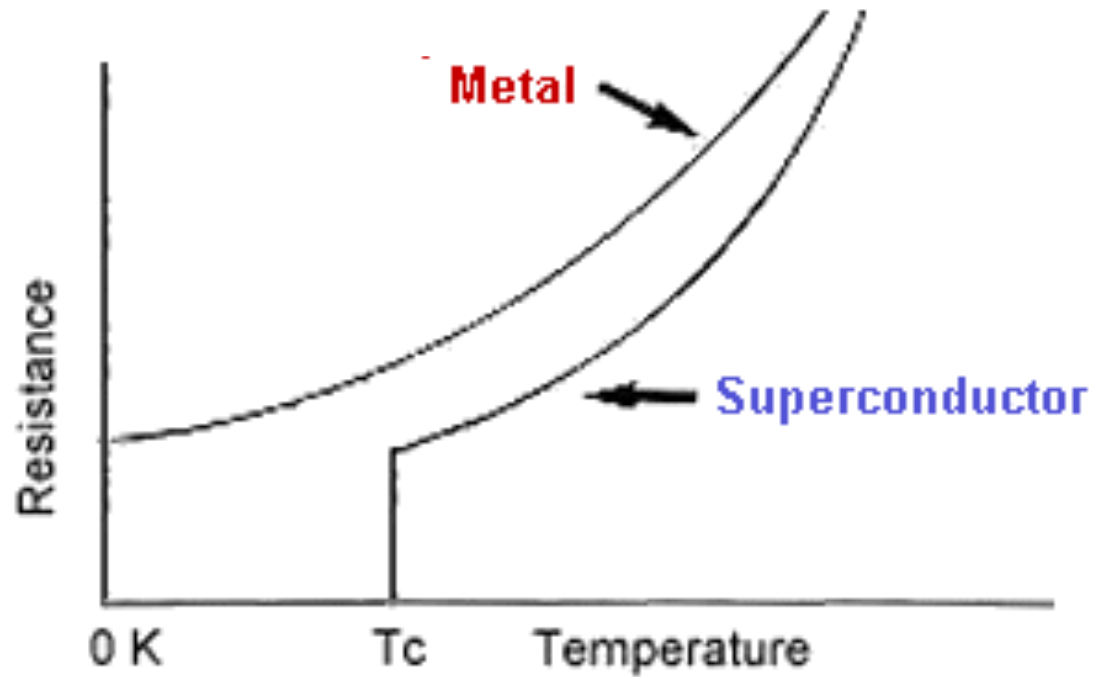


Wigner crystal



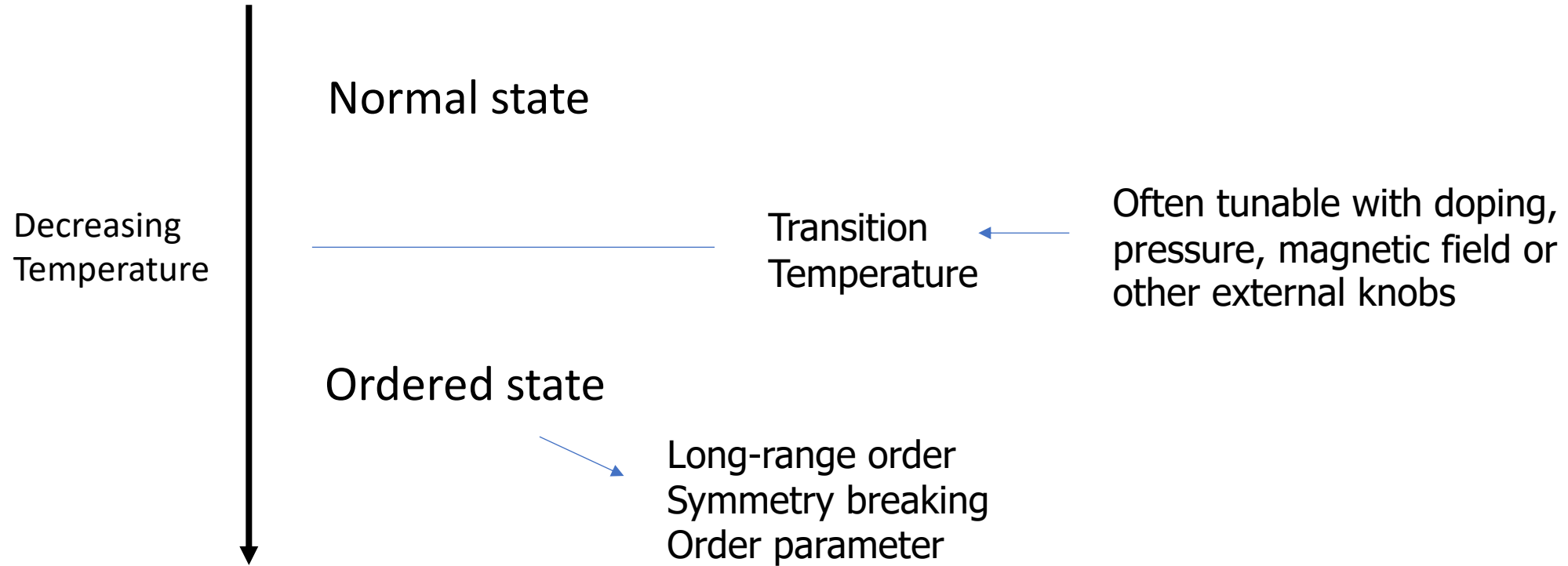
Charge nematic

Superconductivity



Breaking of gauge symmetry

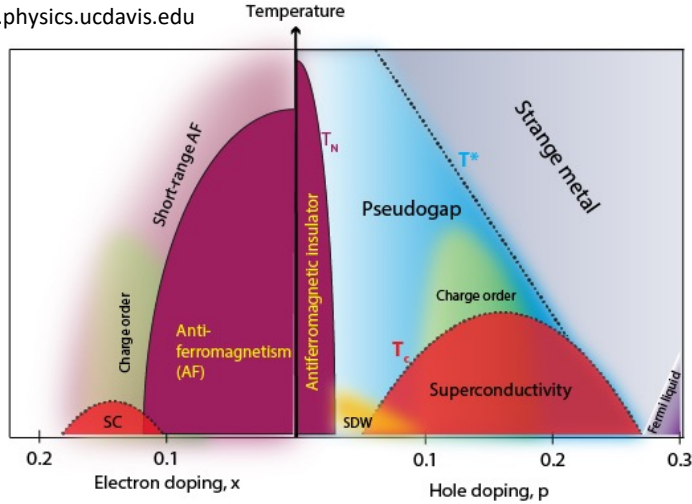
Normal vs ordered state



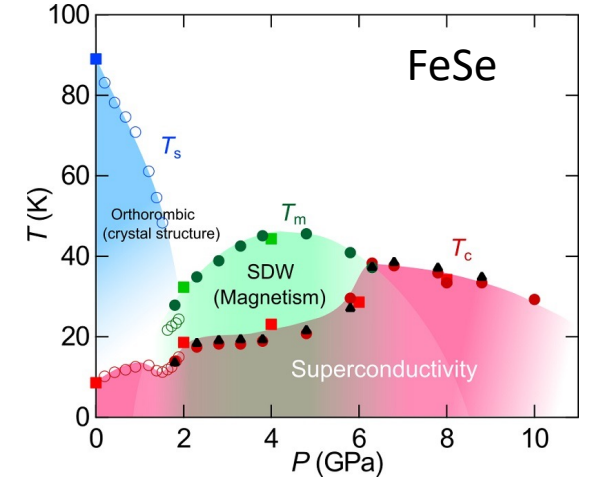
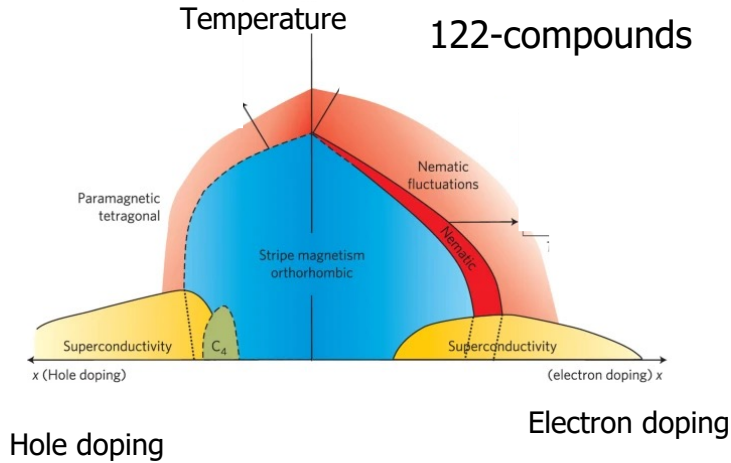
Strongly correlated electron systems: complex phase diagrams

High-Tc superconducting cuprates

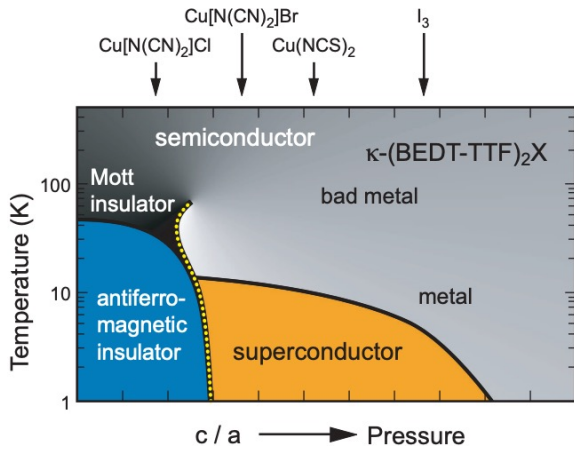
Fig: cme.physics.ucdavis.edu



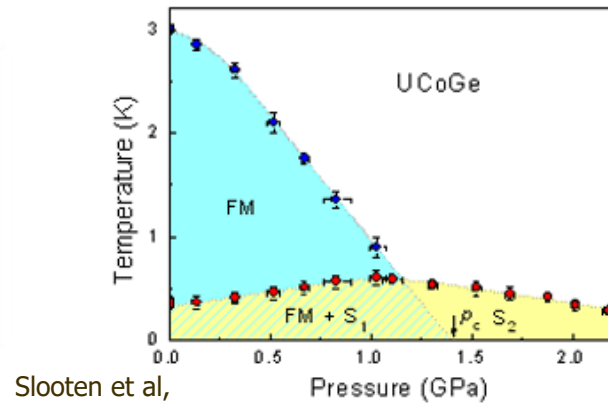
Iron superconductors



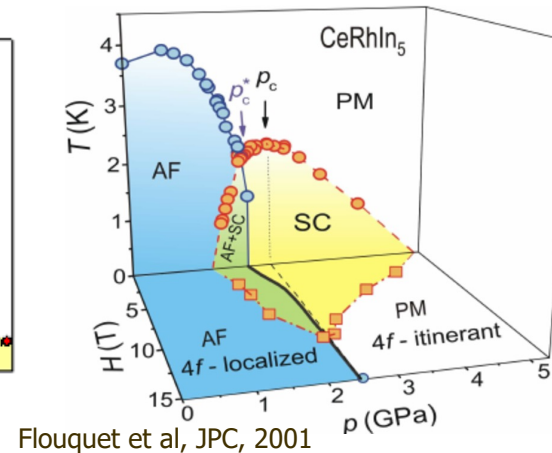
Organic superconductors



Heavy fermion compounds

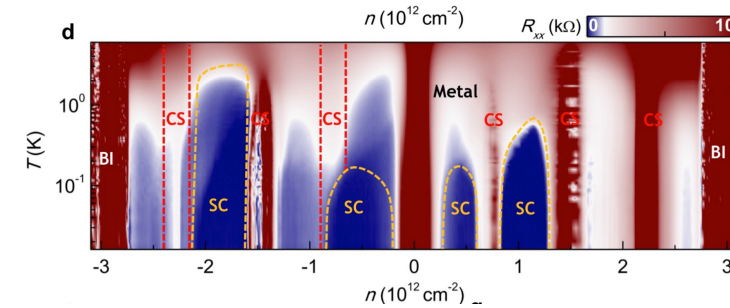


Slooten et al, PRL 103, 097003(2009)



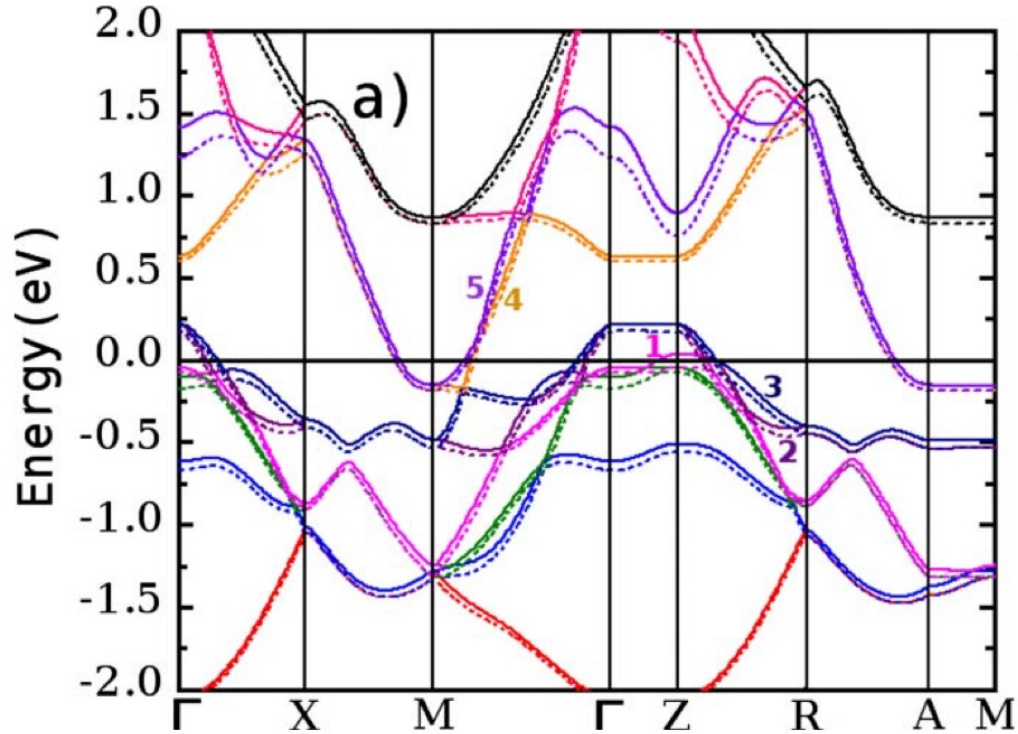
Flouquet et al, JPC, 2001

Twisted bilayer graphene

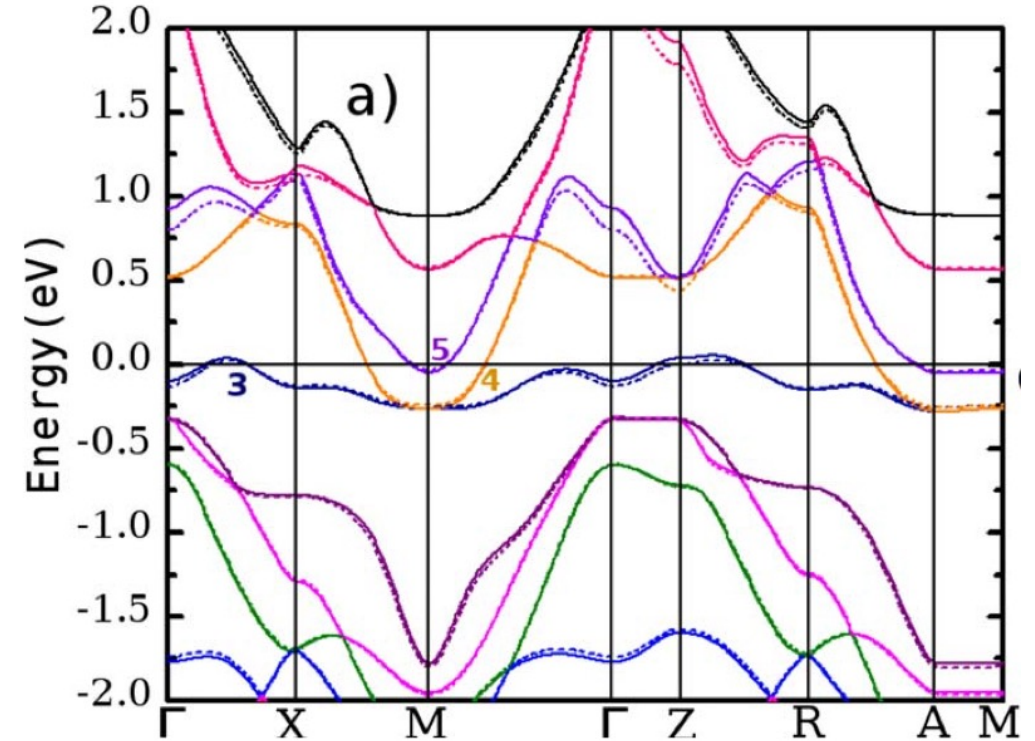


Band structure in normal vs ordered state.

Non-ordered "normal" state



Symmetry breaking:
ordered state at low temperature



Ma & Lu, PRB 78, 033111 (2008)

Band structure $E(k)$

Is it $E(k)$ always well defined? How good are DFT-like and static mean field descriptions?
Why and when does band theory work?

The "normal" state of strongly correlated systems

When the band picture does not work so well

Weakly correlated system

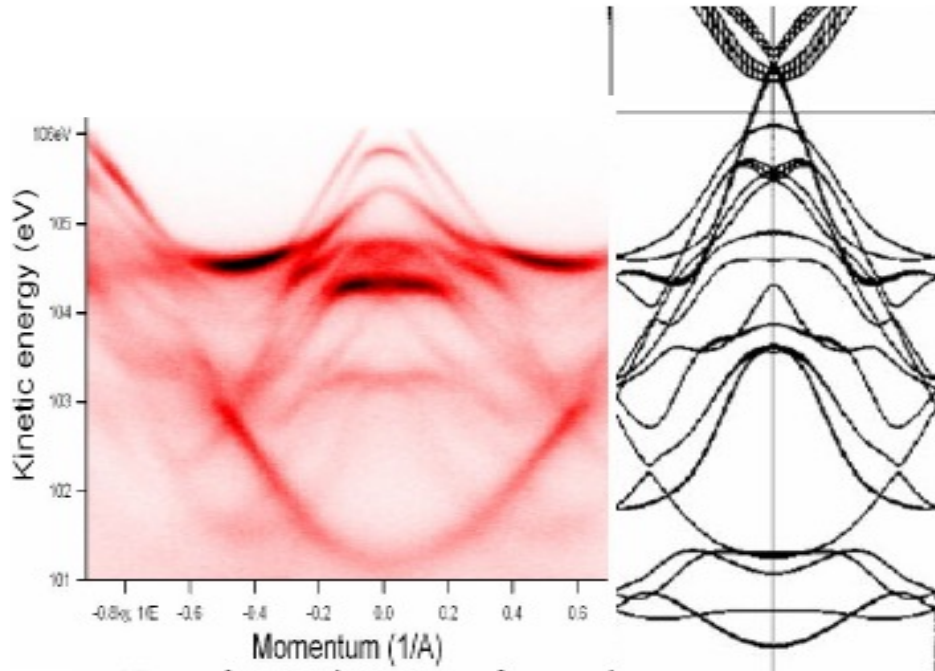


Fig. : Evtushinsky lectures

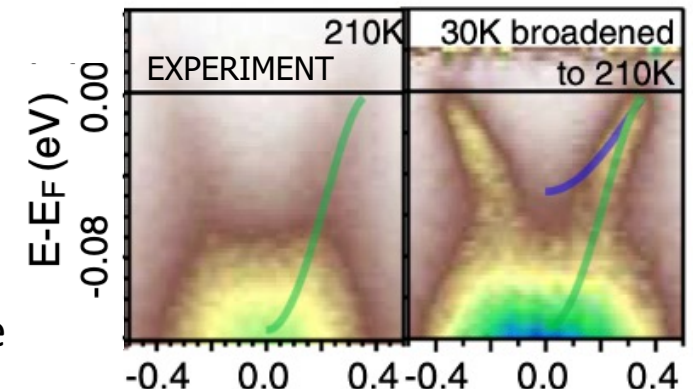
Strongly correlated system

Not just ordered phases and symmetry breaking states

Correlated metals

- ❑ Bandwidths which deviate from DFT predictions (larger mass, up to 1000 thousands times larger)
- ❑ Blurred spectrum and anomalous temperature behavior

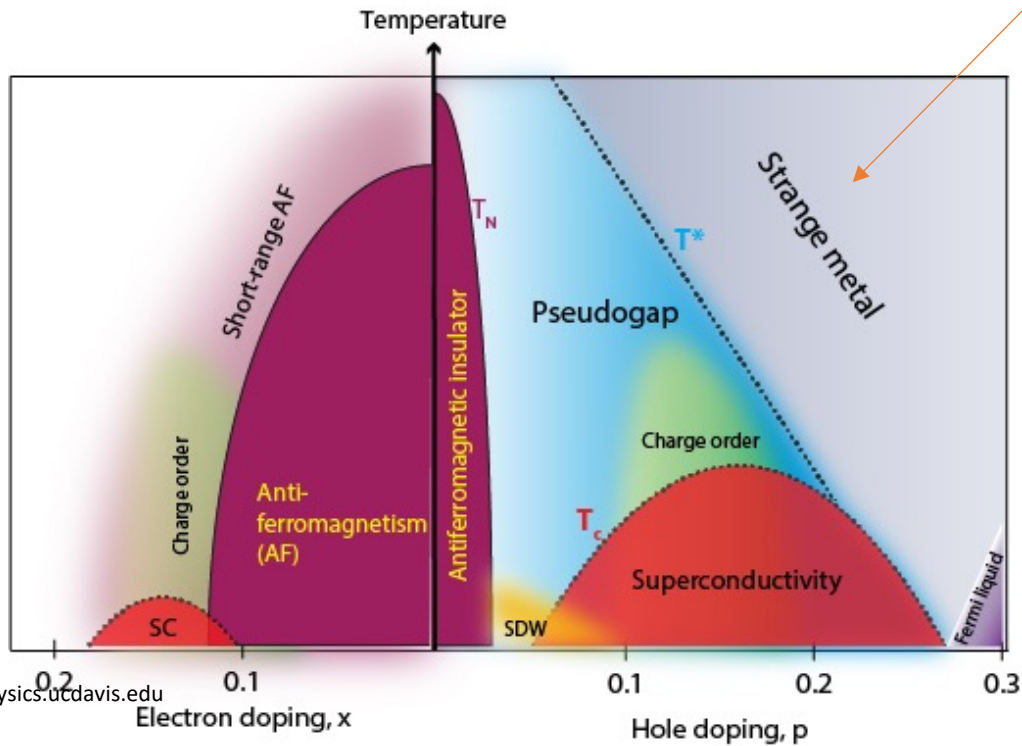
Iron chalcogenide



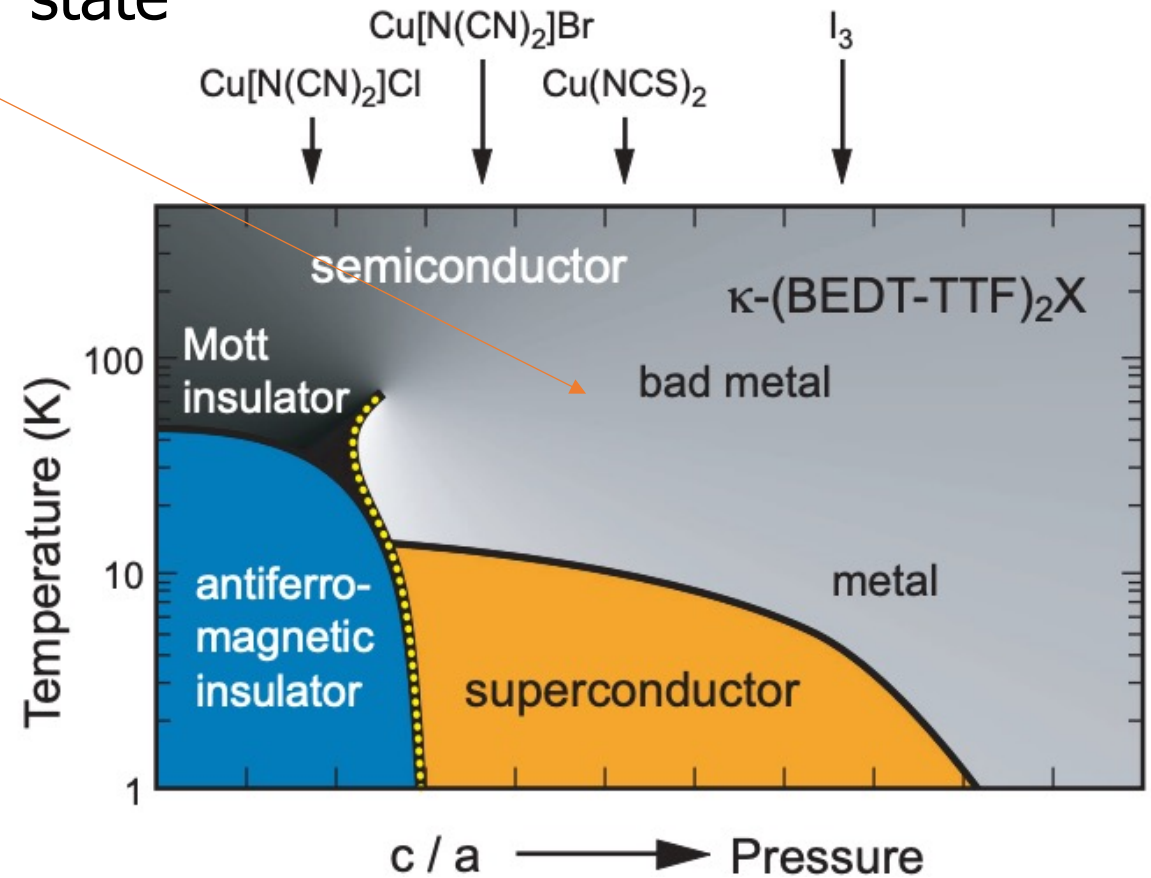
PRL 110, 067003 (2013)

Strongly correlated metals

“Normal” state



High- T_c superconducting cuprates

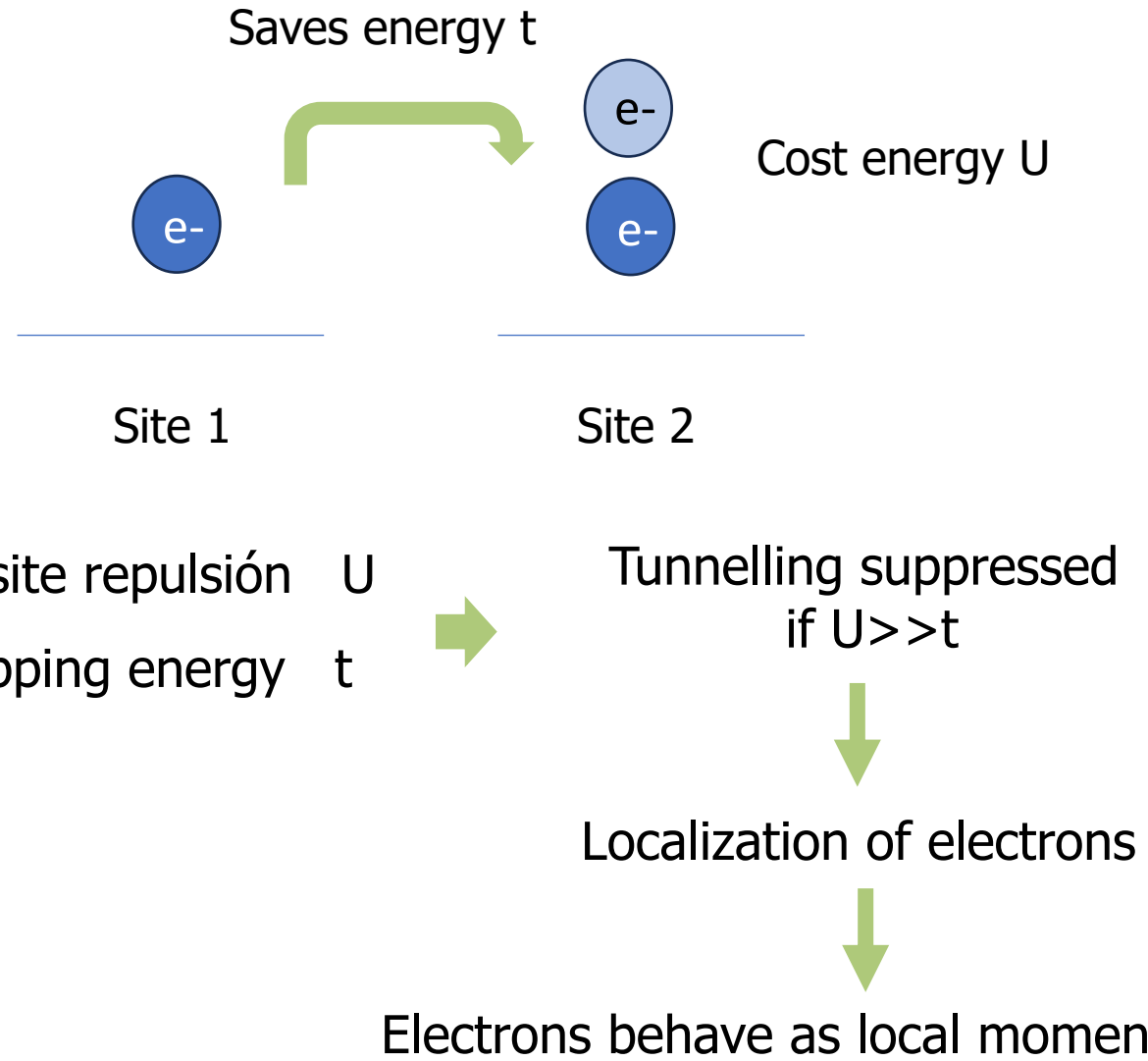
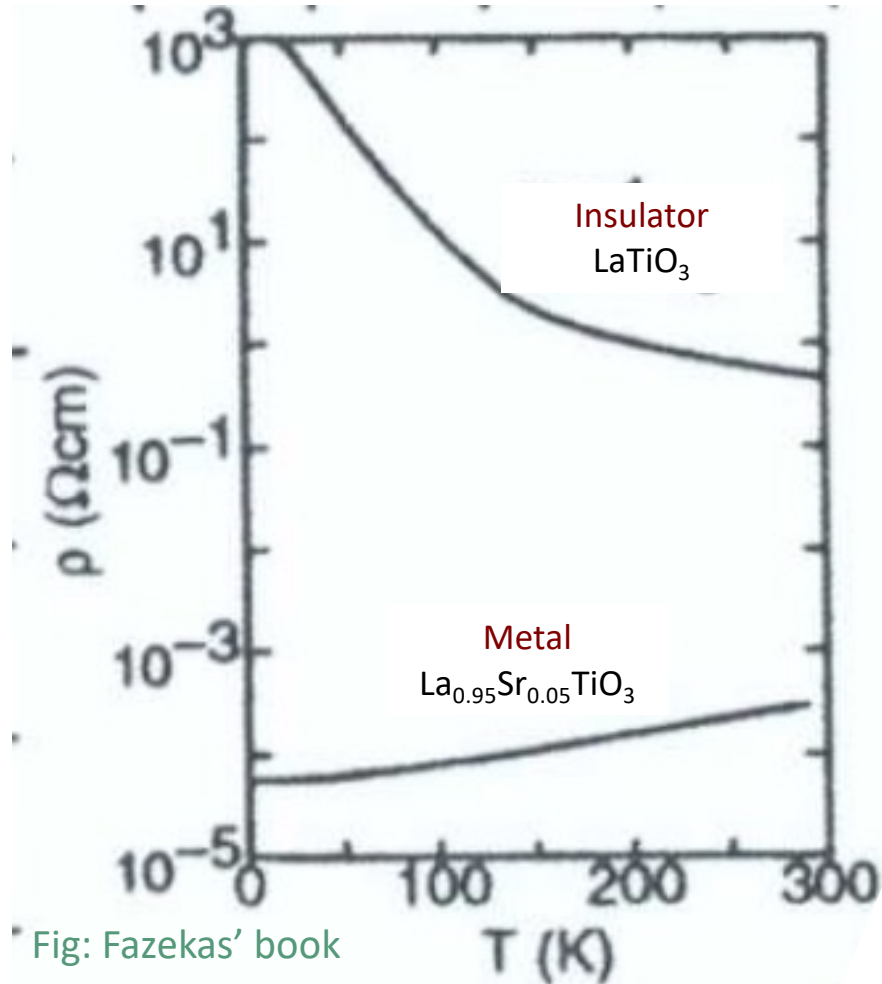


Organic superconductors

Many effects also in the Density of States measured by Scanning Tunnelling microscope (STM) and in other probes

Mott insulator and local moments

Having a metal when we expect an insulator (even without symmetry breaking)



Local moments and itinerant electrons

Weakly correlated system

Itinerant electrons

Momentum-space

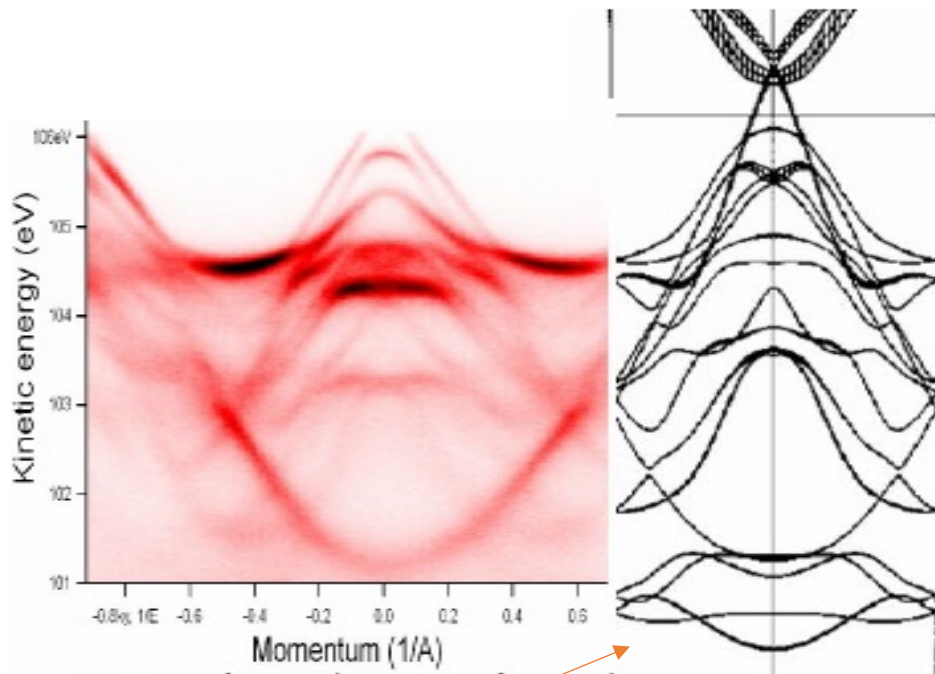


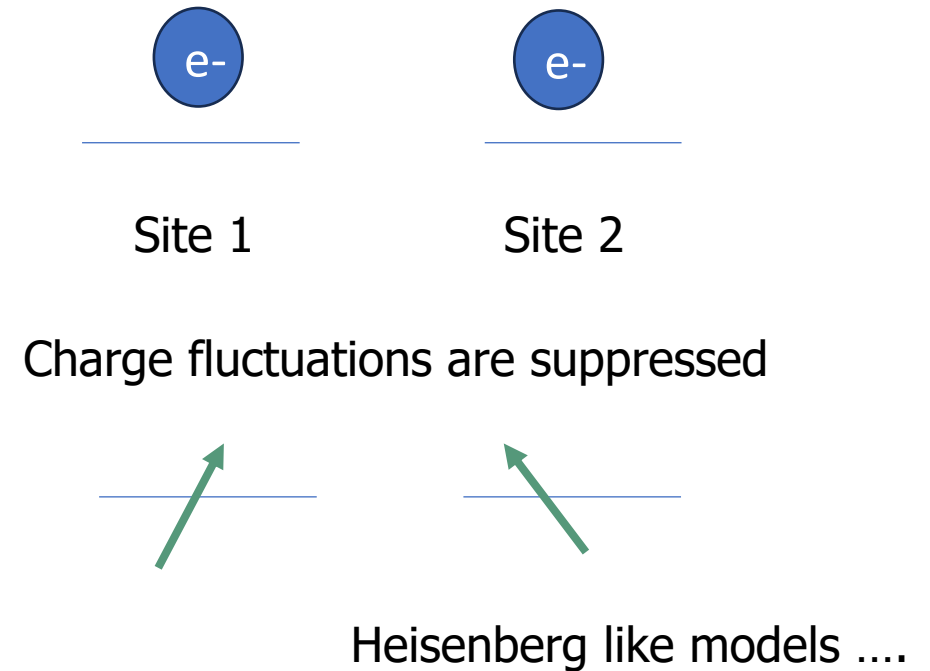
Fig. : Evtushinsky lectures

Band description

Strongly correlated system

Local moments

Real-space



Strong modification of the spectrum & not always a binary choice:
Strongly correlated (non-ordered) metals

When different types of electrons play a role

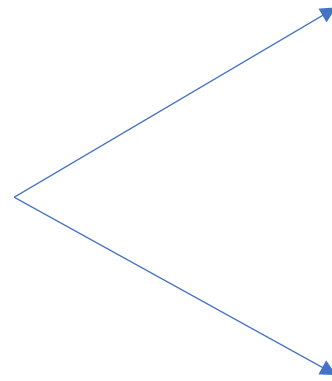
Itinerant electrons

less correlated (larger kinetic energy or smaller interactions)



Local electrons: local moments

More correlated (smaller kinetic energy or larger interactions)



Kondo effect

Screening of a local moment by a bath of itinerant electrons (macroscopic measurable effects)

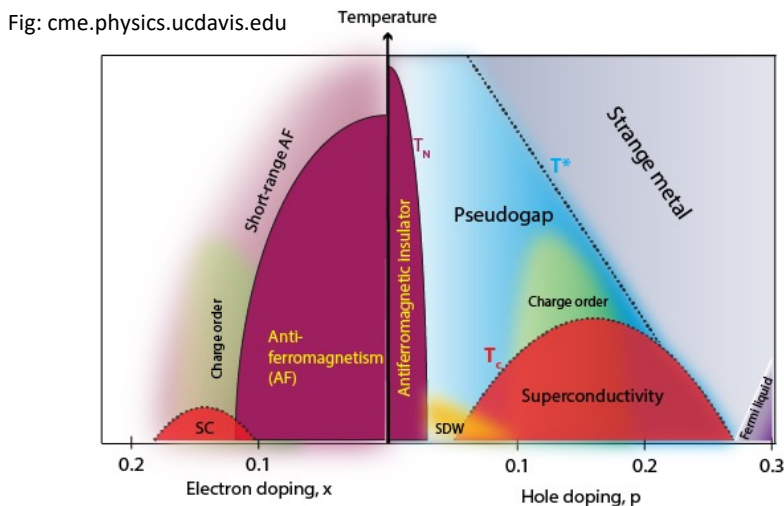
Heavy fermions

A lattice of local electrons coupled through tunneling to Itinerant electrons

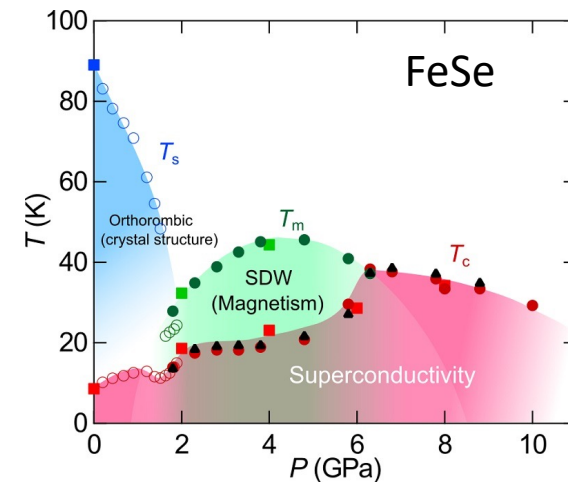
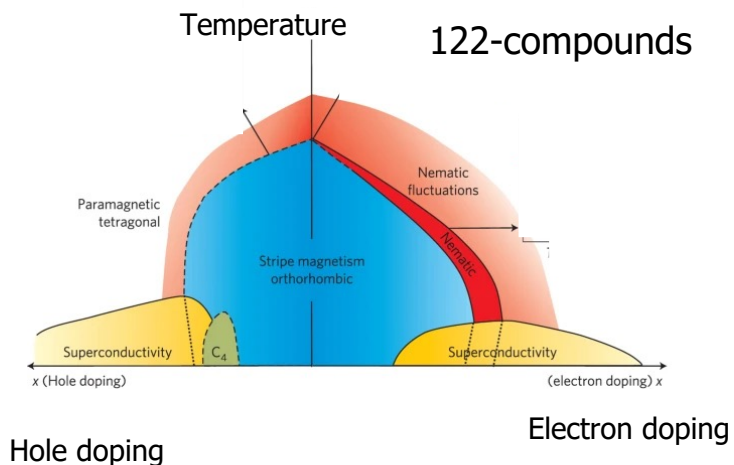
Also: Hund's physics in multi-orbital systems ...

Superconductivity in strongly correlated electron systems

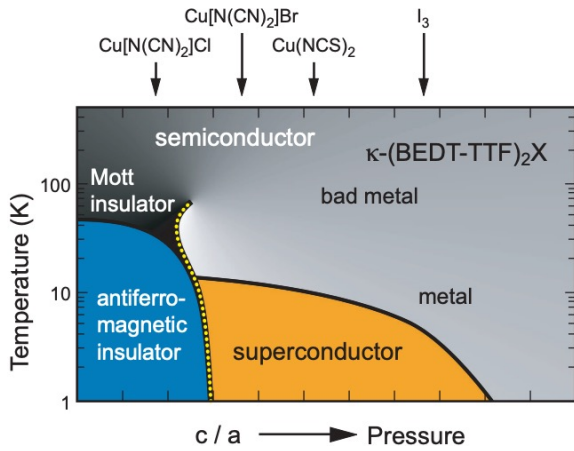
High-Tc superconducting cuprates



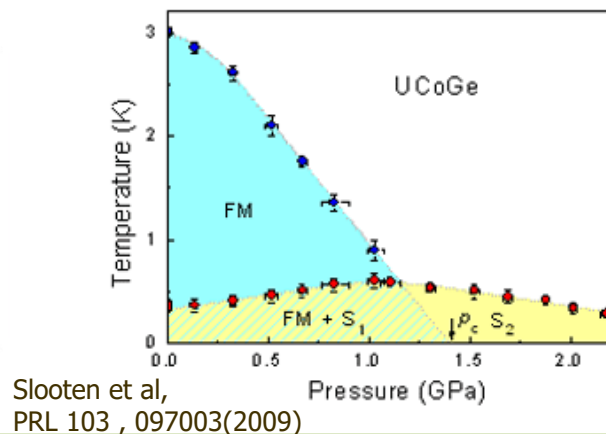
Iron superconductors



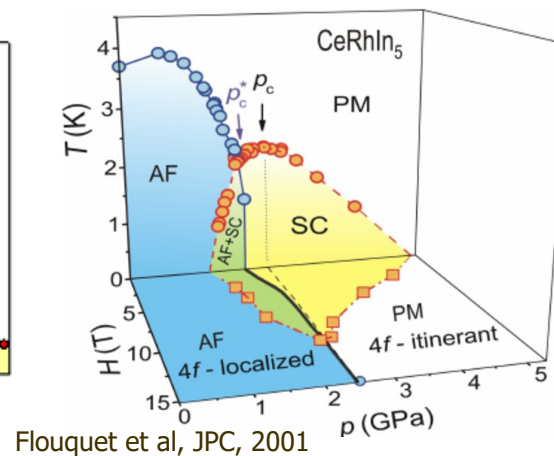
Organic superconductors



Heavy fermion compounds

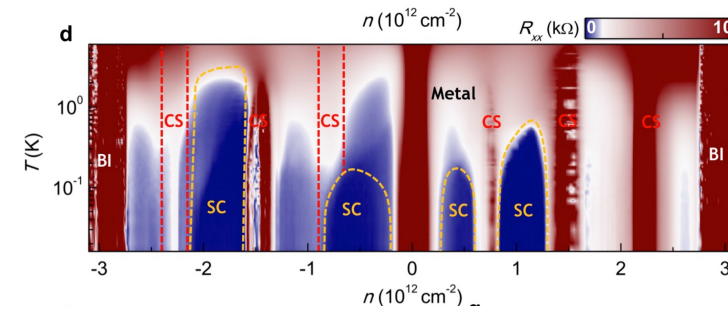


Slooten et al, PRL 103, 097003(2009)



Flouquet et al, JPC, 2001

Twisted bilayer graphene



From a Mott insulator to high-temperature superconductivity

High-T_c superconducting cuprates

Which is the role of Mott (local) physics and strong correlations in high temperature superconductivity?

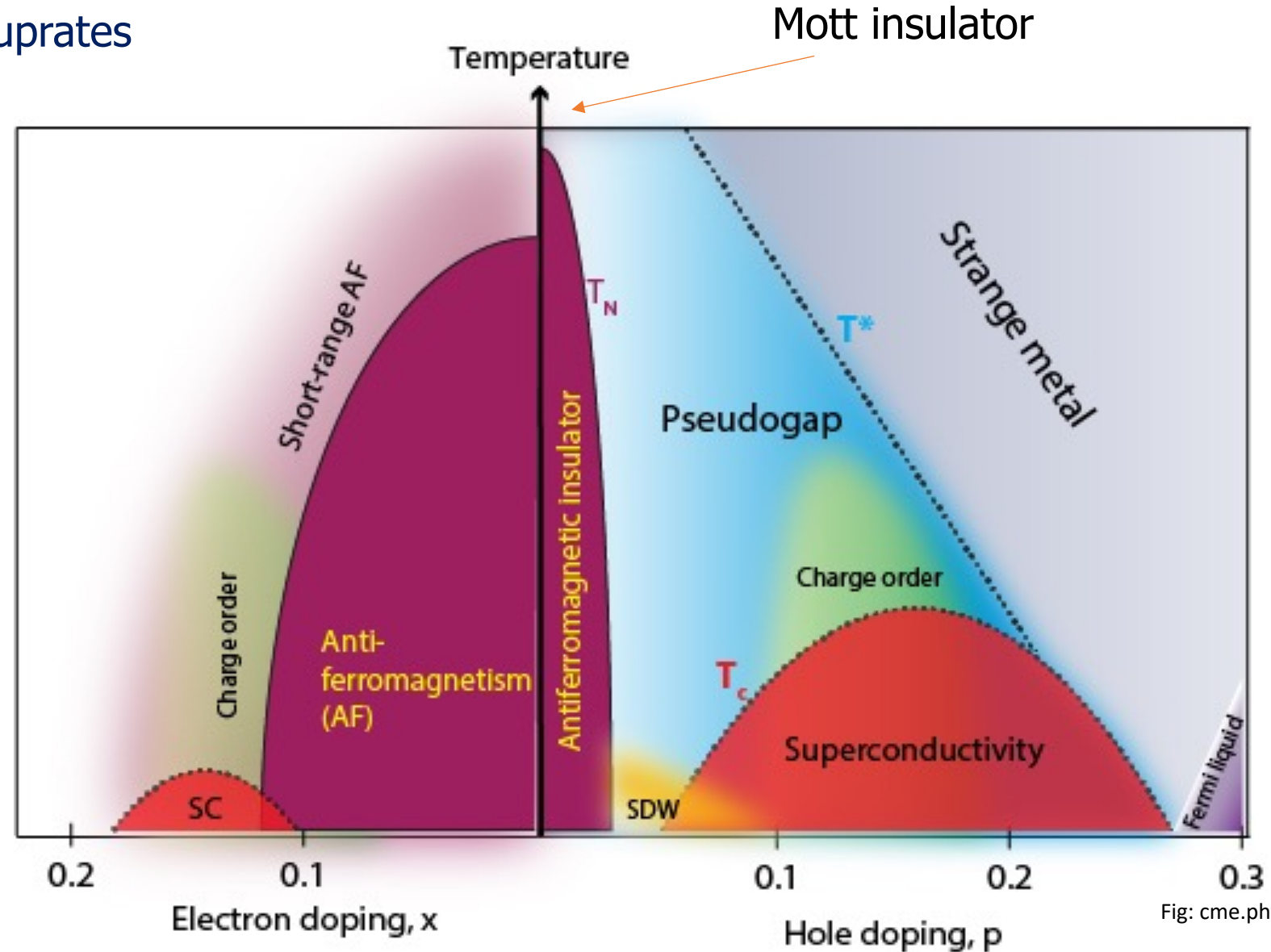


Fig: cme.physics.ucdavis.edu

Quantum materials: lattice and dimensionality

3 dimensions



©Michael Paraskevas

2 dimensions



©Sebastian Dubiel

1 dimension



©Carlos Valenzuela

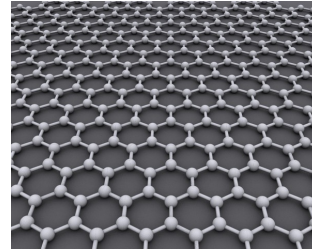
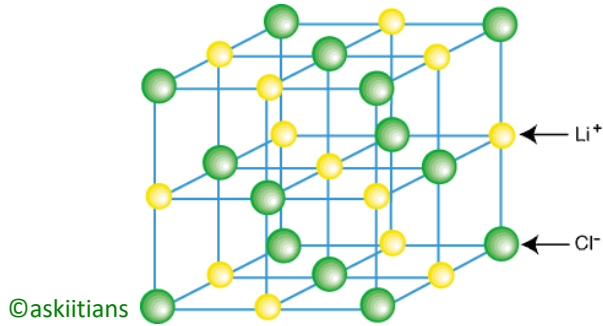
Interactions are never perturbative
in 1 dimension

Quantum materials: lattice and dimensionality

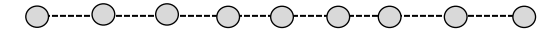
3 dimensions

2 dimensions

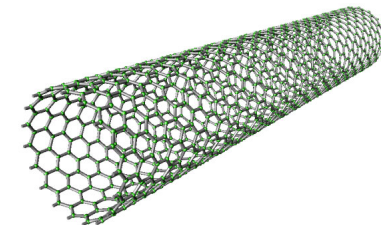
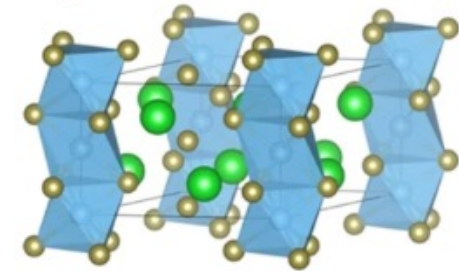
1 dimension



1 or few layers

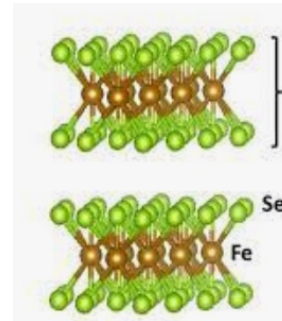
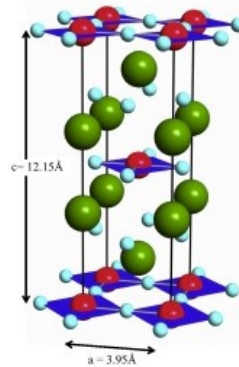


Quasi-1d materials

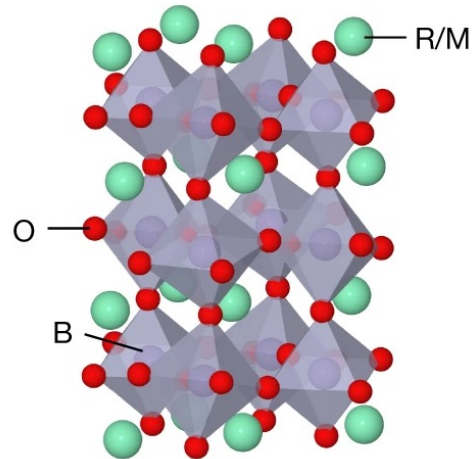
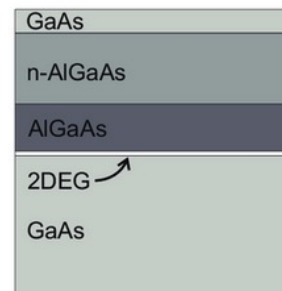


Interactions are never perturbative in 1 dimension

Layered quasi-2d materials



Interfaces

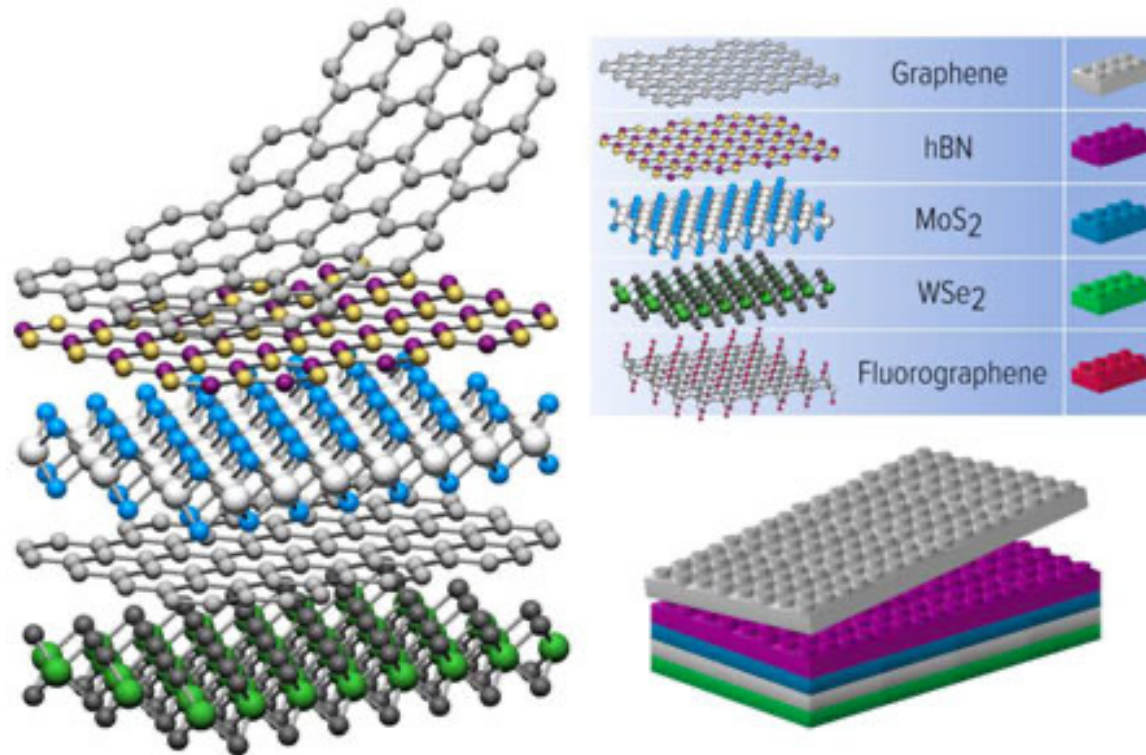


ABO₃ perovskite

Fig: Guzmán-Verri et al, Nature 576, 429 (2019)

Engineering 2d materials and heterostructures

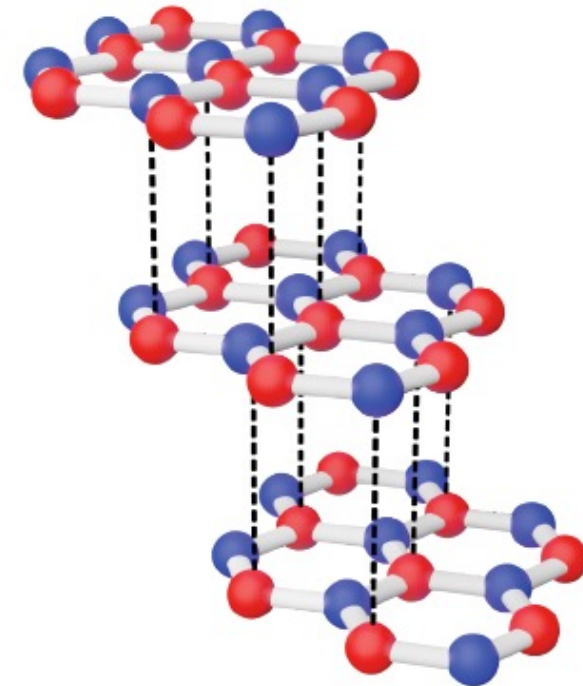
Stacking different 2d materials



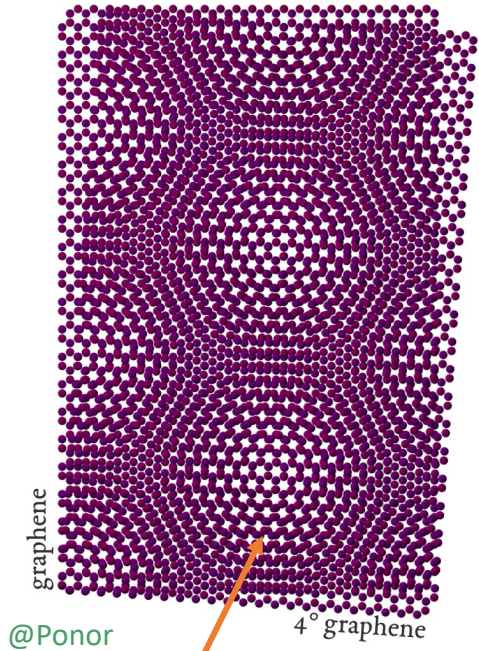
Geim & Grigorieva,
Nature 499, 419 (2013)

Semimetal, insulating,
metal, ferromagnetic ...

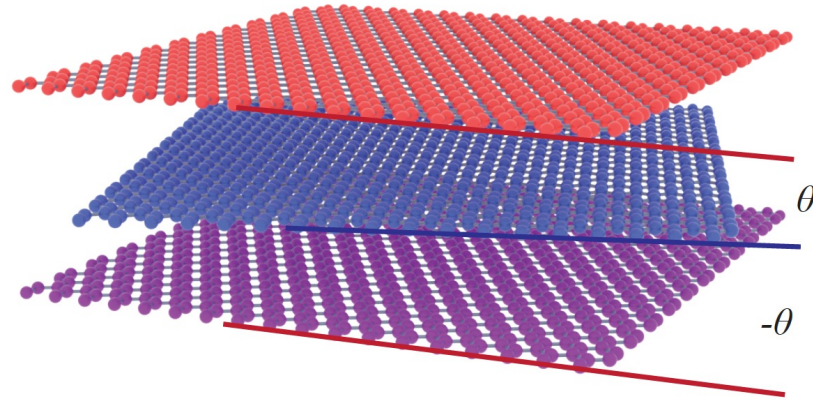
Multilayers with specific stacking



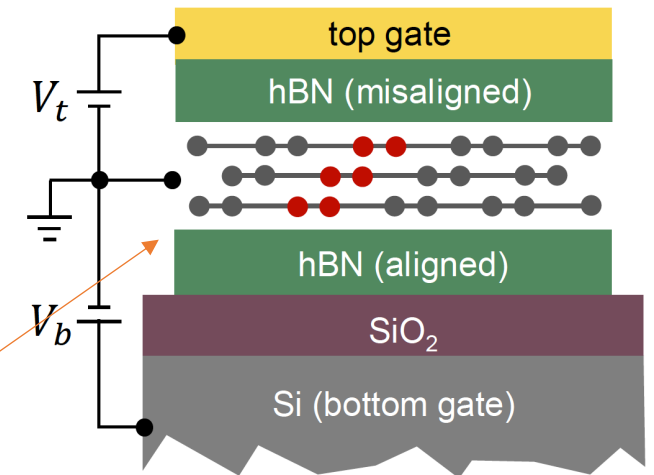
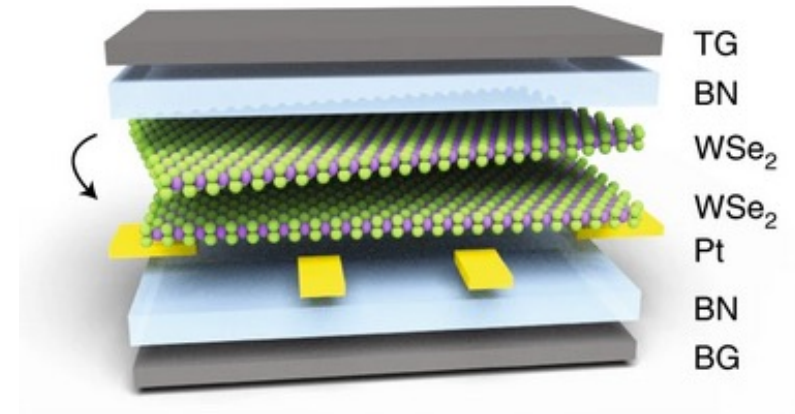
Engineering 2d moiré heterostructures. Tunability



Moiré pattern with a much larger unit cell

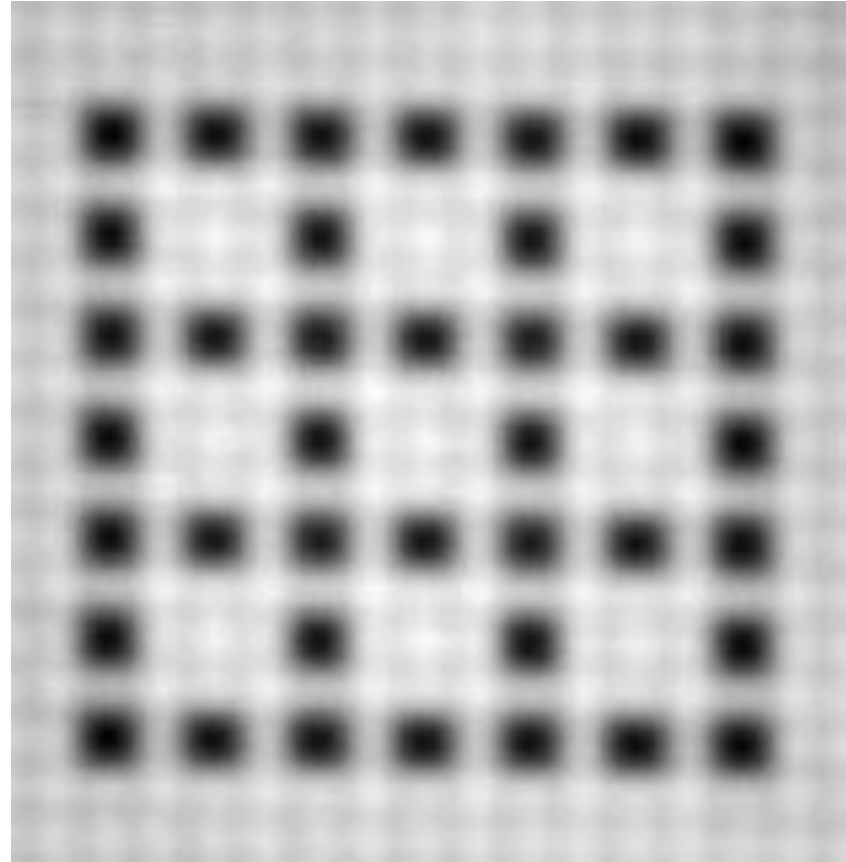


Variety of twisting arrangements with equal or different layers: graphene, dichalcogenides, magnetic 2D materials ...



Moiré also through slight mismatch or through ad-hoc engineered periodic potential

Engineering 2d lattices: atoms on surfaces



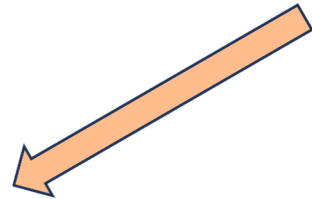
Yan & Liljeroth,
Advances in Physics X, 4, 2019

Emergence of Quantum Phases in Novel Materials

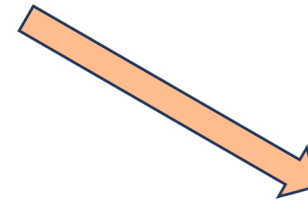
Interactions between many particles (electrons)



Correlated Quantum States
in many different and novel materials



Symmetry breaking states



Formation of local moments



Tuning parameters for correlated states

Interaction energy vs Kinetic energy

- Filling of electronic bands
- Electronic bands details
- Dimensionality
- Crystalline lattice
- Screening
- External knobs (pressure, ...)
- Interaction with lattice
- Other (spin-orbit, ...)

Active electrons for correlated states

- Ion= nucleus + core electrons
- Focus on electrons close to Fermi level

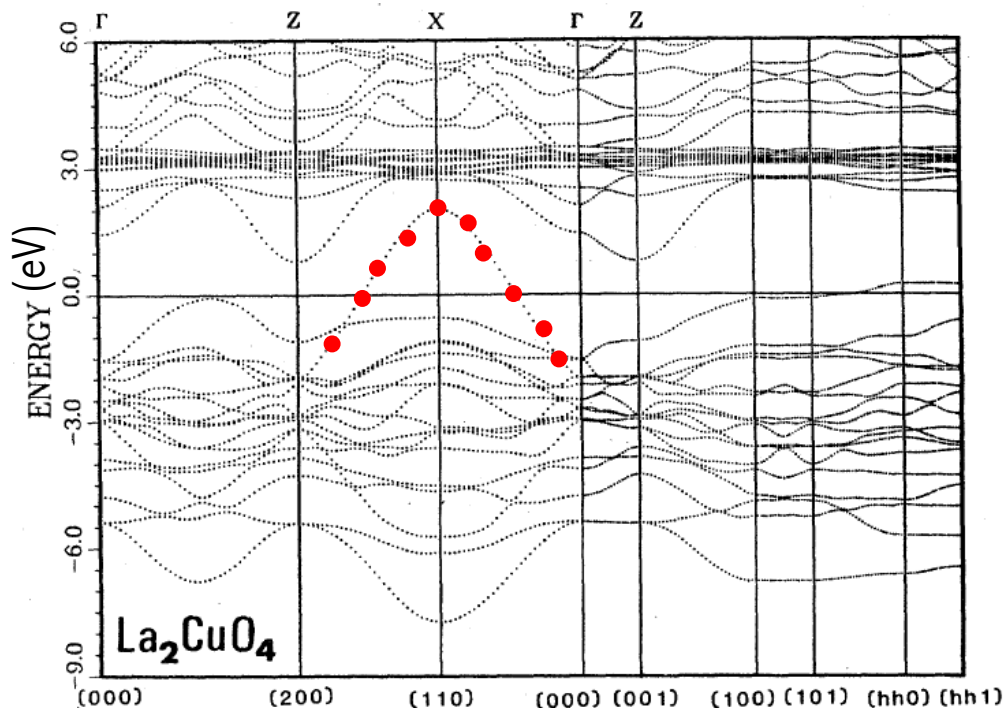
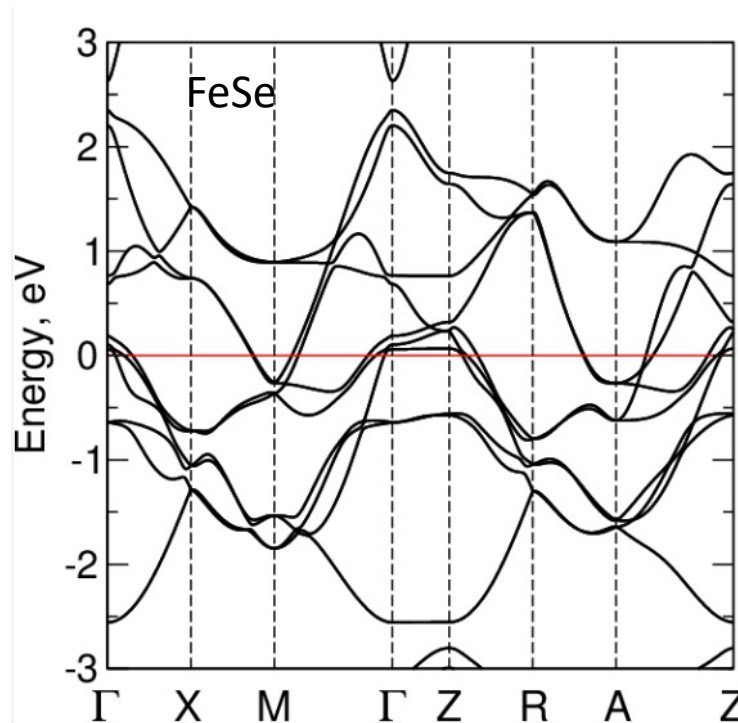
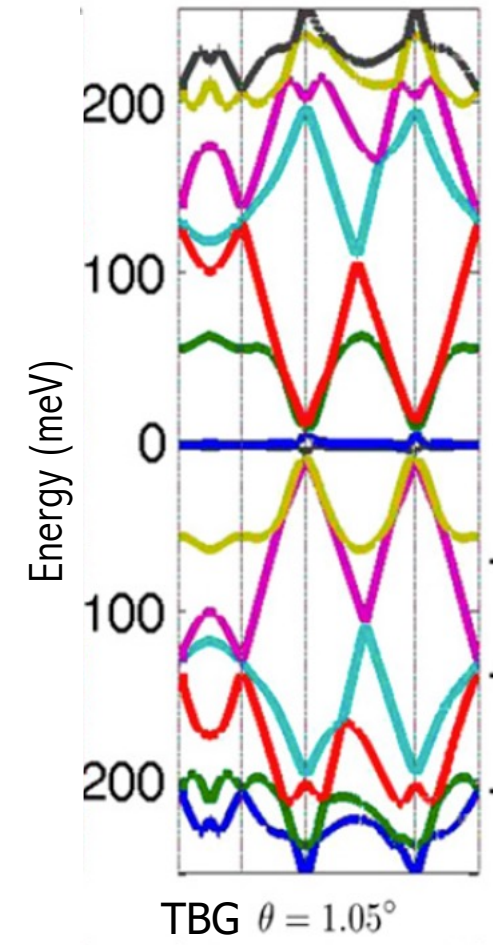


Fig: Pickett, RMP 61, 433 (1989)



Grechnev et al, Ukran. J. Phys 57, 171 (2012)



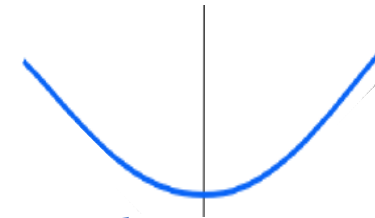
Bistritzer & MacDonald,
PNAS 108, 12233 (2011),

Kinetic energy: lattice versus continuum

Free space: Parabolic band

- Isotropic
- No length scale
- Momentum conserved

$$\text{Kinetic energy} = \frac{k^2}{2m}$$



$$m^{-1} = \left| \frac{\partial^2 \varepsilon}{\partial k^2} \right|$$

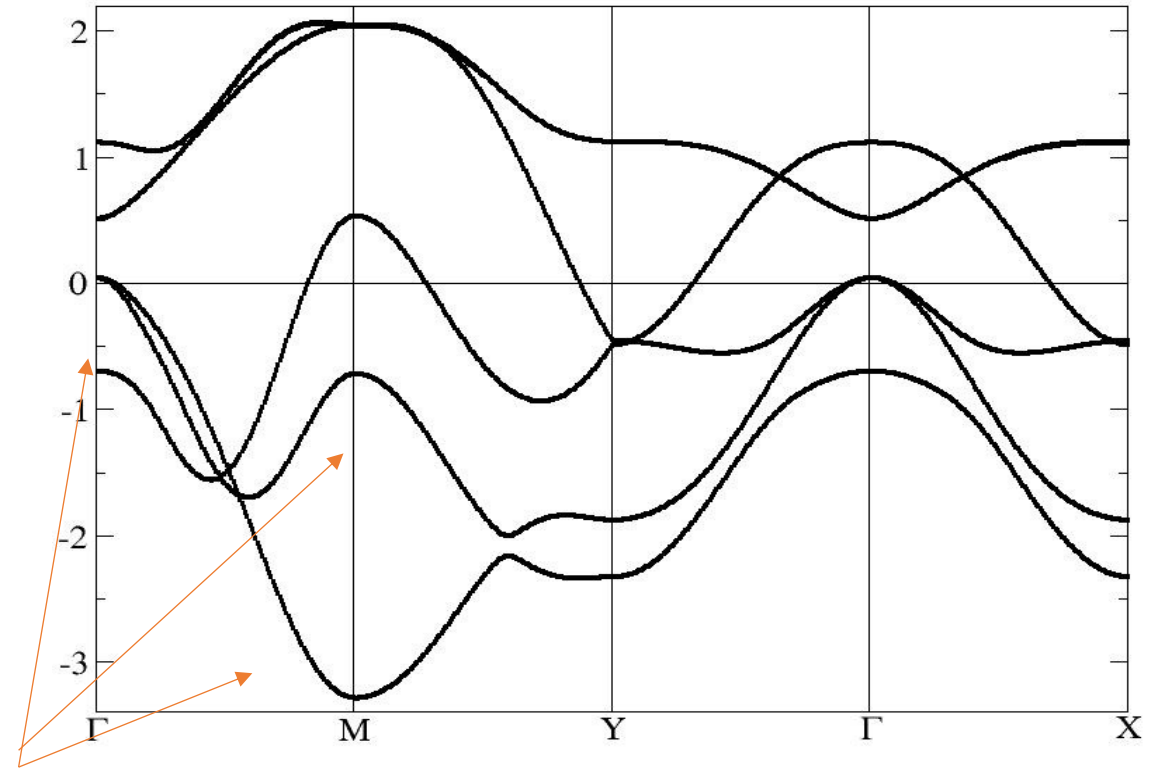
Inverse of second derivative
of the energy gives the
mass of the electron

Kinetic energy: lattice versus continuum

In a solid:

Periodic potential of the ionic lattice

- Length scale: the lattice constant \mathbf{a}
- Momentum conserved modulus $2\pi/\mathbf{a}$
- In general, anisotropic potential
- Energy bands filled up to ϵ_F . Fermi surface



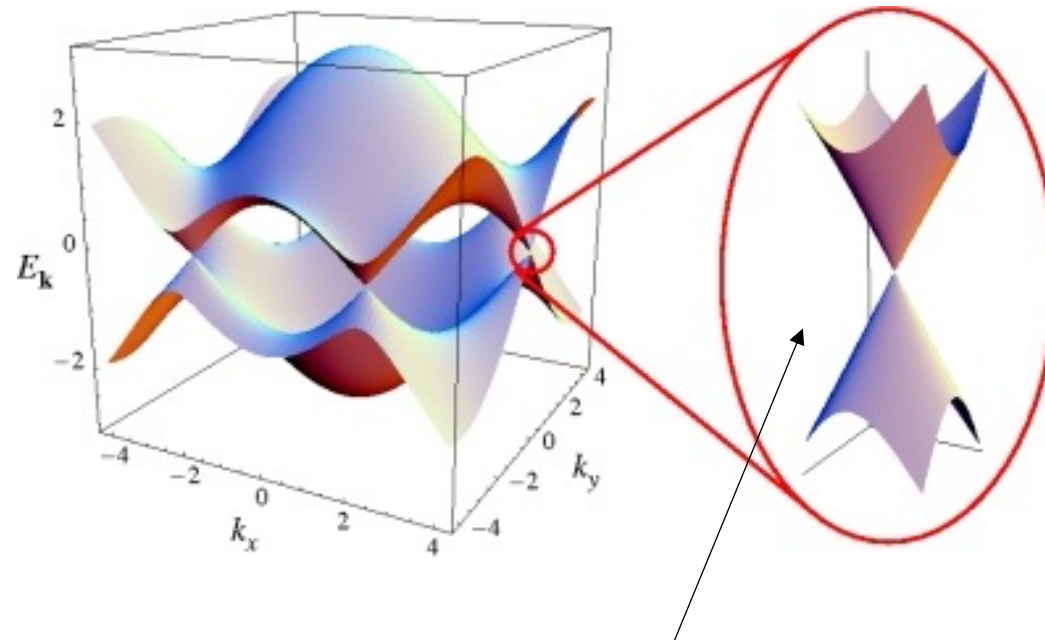
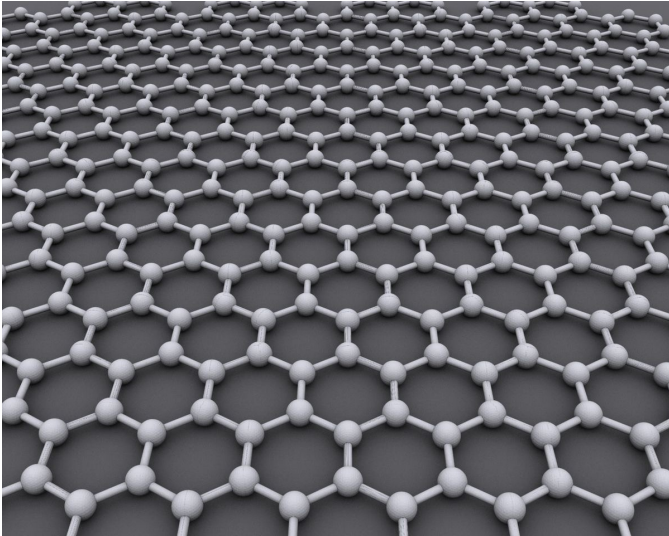
Approximated to parabolic band
very close to the bottom/top

Fig: Calderón et al,
PRB, 80, 094531 (2009)

$$m^{-1} = \left| \frac{\partial^2 \epsilon}{\partial k^2} \right|$$

Effective mass

Kinetic energy: massless fermions



Linear Dirac dispersion

$$m^{-1} = \left| \frac{\partial^2 \varepsilon}{\partial k^2} \right| = 0$$

Effective mass vanishes

massless fermions

Kinetic energy: lattice versus continuum

In a solid:

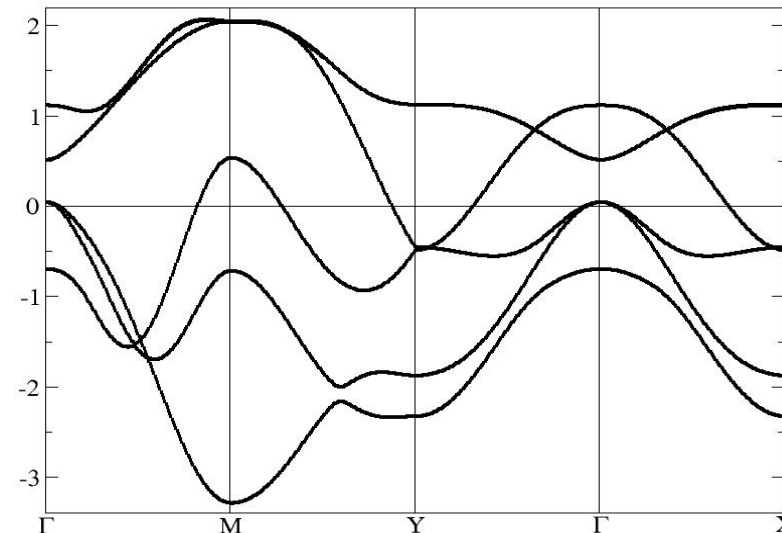
Periodic potential of the ionic lattice

- Length scale: the lattice constant **a**
- Momentum conserved modulus **$2\pi/a$**
- In general, anisotropic potential
- Energy bands filled up to ϵ_F . Fermi surface

Free space:

- Isotropic
- No length scale
- Momentum conserved

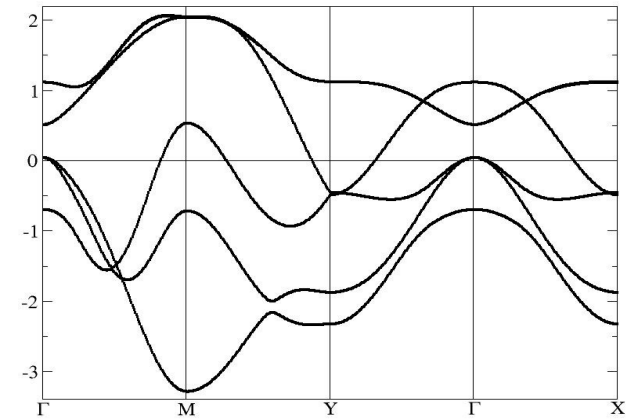
$$\frac{k^2}{2m}$$



Kinetic energy and bandwidth

Kinetic energy: $\int_{-\infty}^{\epsilon_F} \epsilon N(\epsilon) d\epsilon$

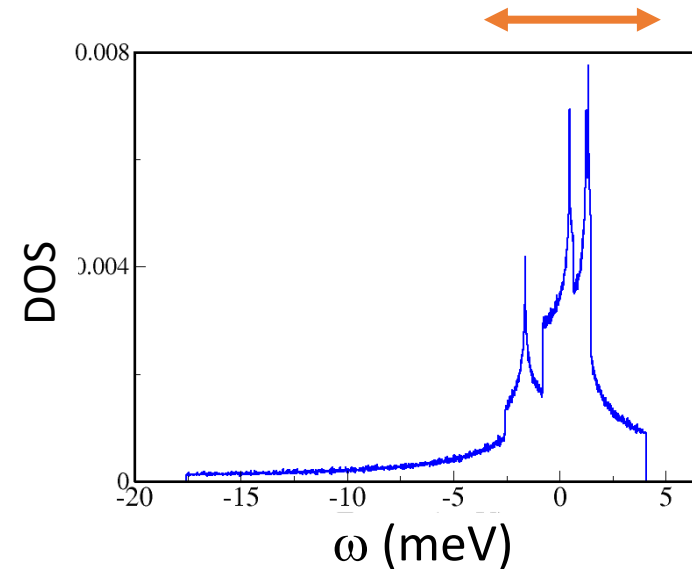
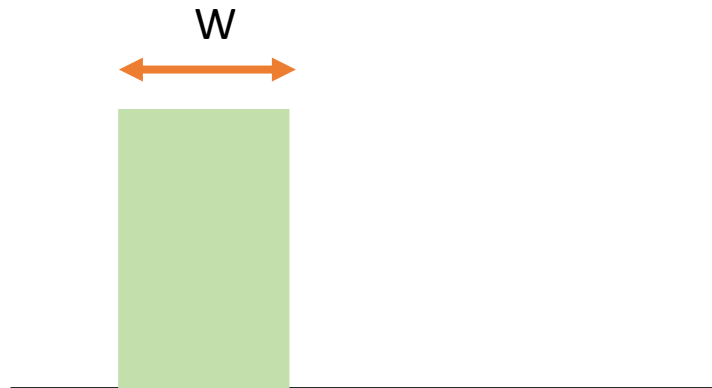
$N(\epsilon)$: density of states



$N(\epsilon)$: constant in $[-W/2, W/2]$, zero otherwise \longrightarrow Kinetic energy $\propto W$

W : bandwidth

Flat bands very sensitive to interactions



Kinetic energy and bandwidth

$$\text{Kinetic energy: } \int_{-\infty}^{\varepsilon_F} \varepsilon N(\varepsilon) d\varepsilon$$

$N(\varepsilon)$: density of states

$N(\varepsilon)$: constant in $[-W/2, W/2]$,
zero otherwise

W : bandwidth

Kinetic energy $\propto W$

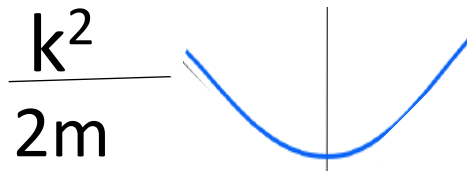
Flat bands
very sensitive
to interactions

Larger mass

Narrower band

Smaller bandwidth W

Smaller
Kinetic energy



Careful! Not applicable to Dirac dispersion

Kinetic energy and bandwidth

Kinetic energy \sim Bandwidth W

$W \sim$ eV in most materials

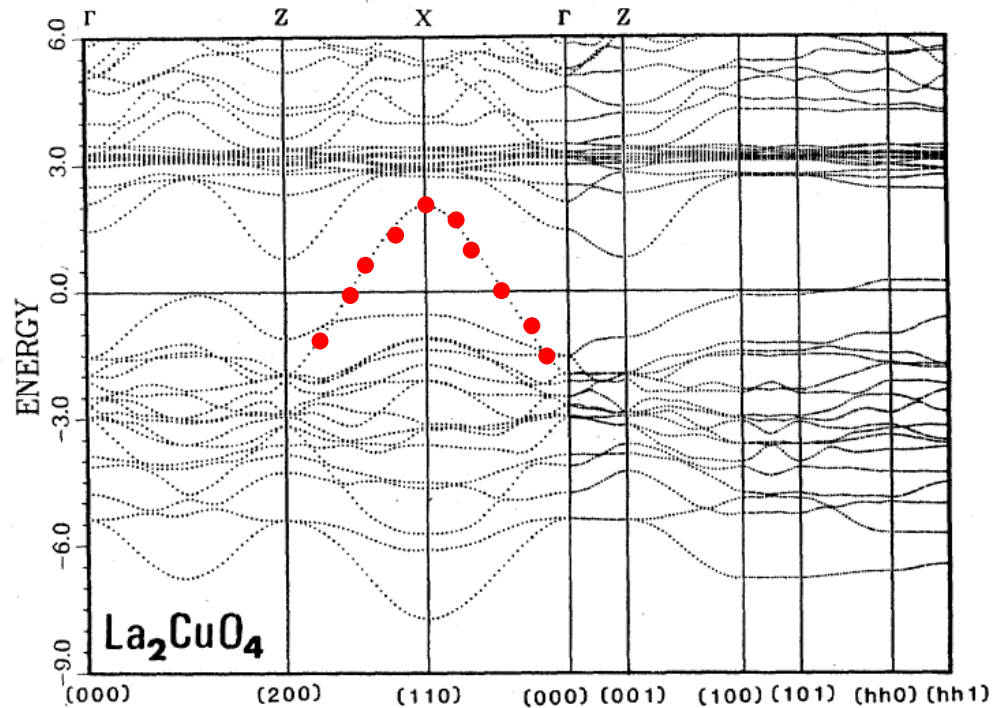
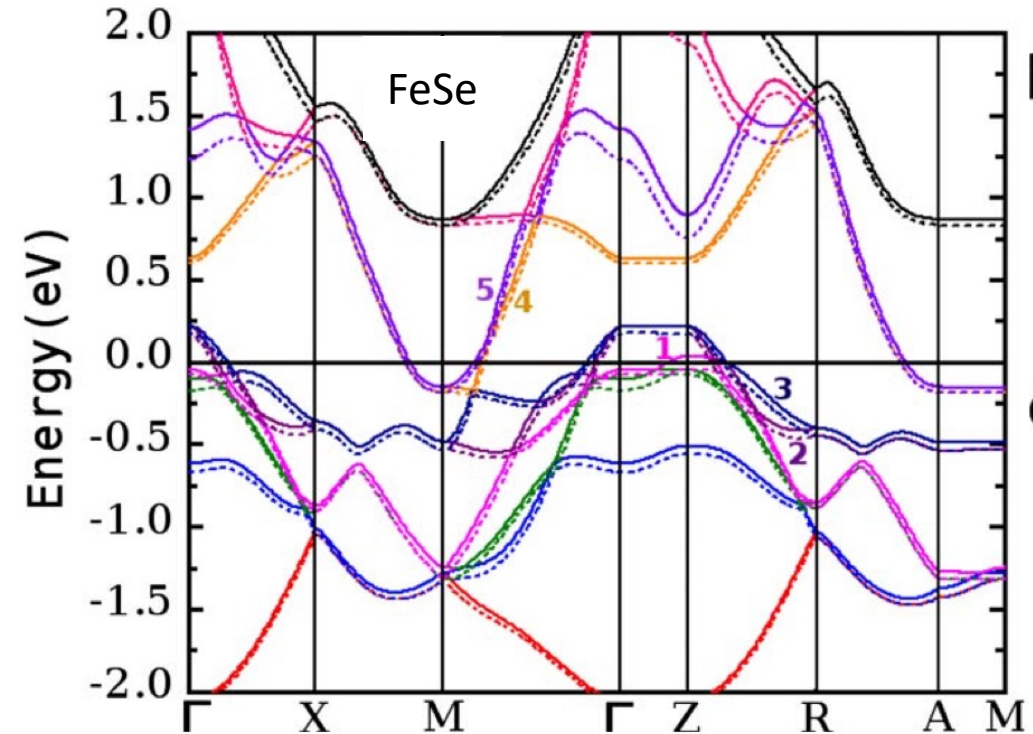


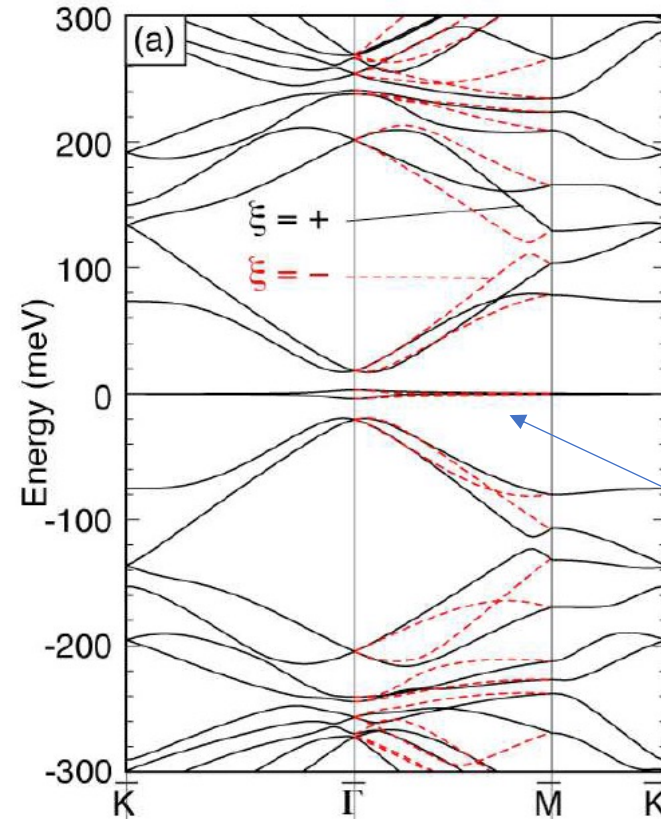
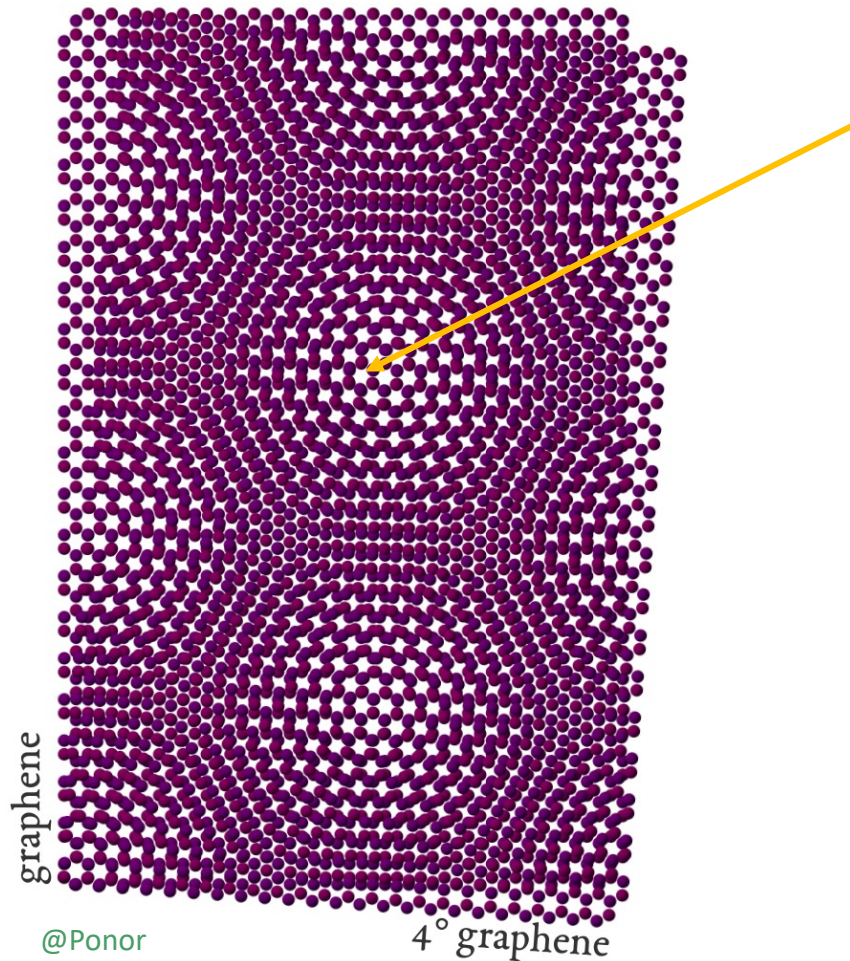
Fig: Pickett, RMP 61, 433 (1989)



Ma & Lu, PRB 78, 033111 (2008)

Moiré heterostructures: flat bands

Moiré unit cell ~ 10 nm lattice constant



Very narrow bands for certain twist angles (bandwidth 1-10 meV)

Very sensitive to interactions

Strength of interactions

□ Ratio between Interaction and Kinetic energies U/W

Kinetic energy \sim Bandwidth W

W \sim eV in most materials

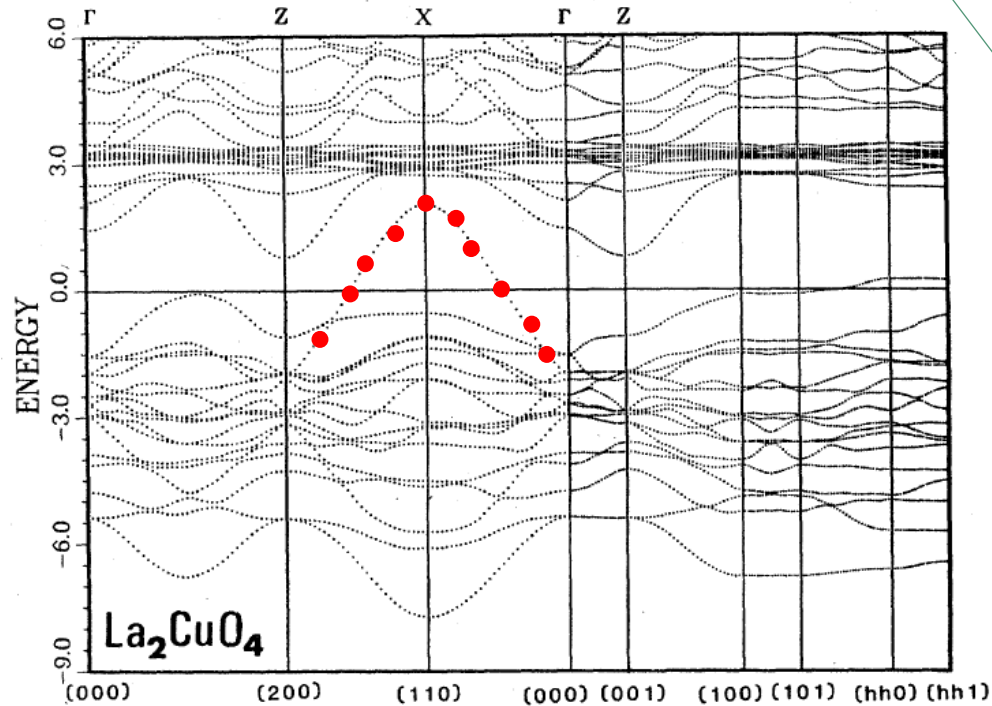


Fig: Pickett, RMP 61, 433 (1989)

Interaction energy:

$$U \sim \left(\frac{e^2}{4 \pi \epsilon_0} \right) \frac{1}{\epsilon_r r_0} = 14.37 \text{ eV } \text{\AA} \frac{1}{\epsilon_r r_0}$$

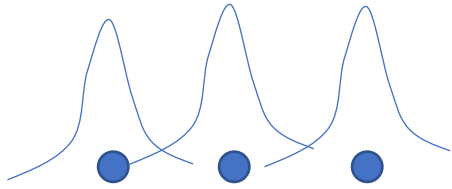
$r_0 \sim$ several \AA
in most materials

Similar energy scales

U/W not small

Strength of interactions

Bands: Tight binding model with atomic orbitals as a basis



Larger spread of wave function
(less localized orbitals, larger r_0)



Larger overlap between orbitals
in neighbouring sites
(larger kinetic energy & bandwidth)

Onsite repulsion
Two electrons in an atom less likely
to be found very close
(smaller effective interaction)

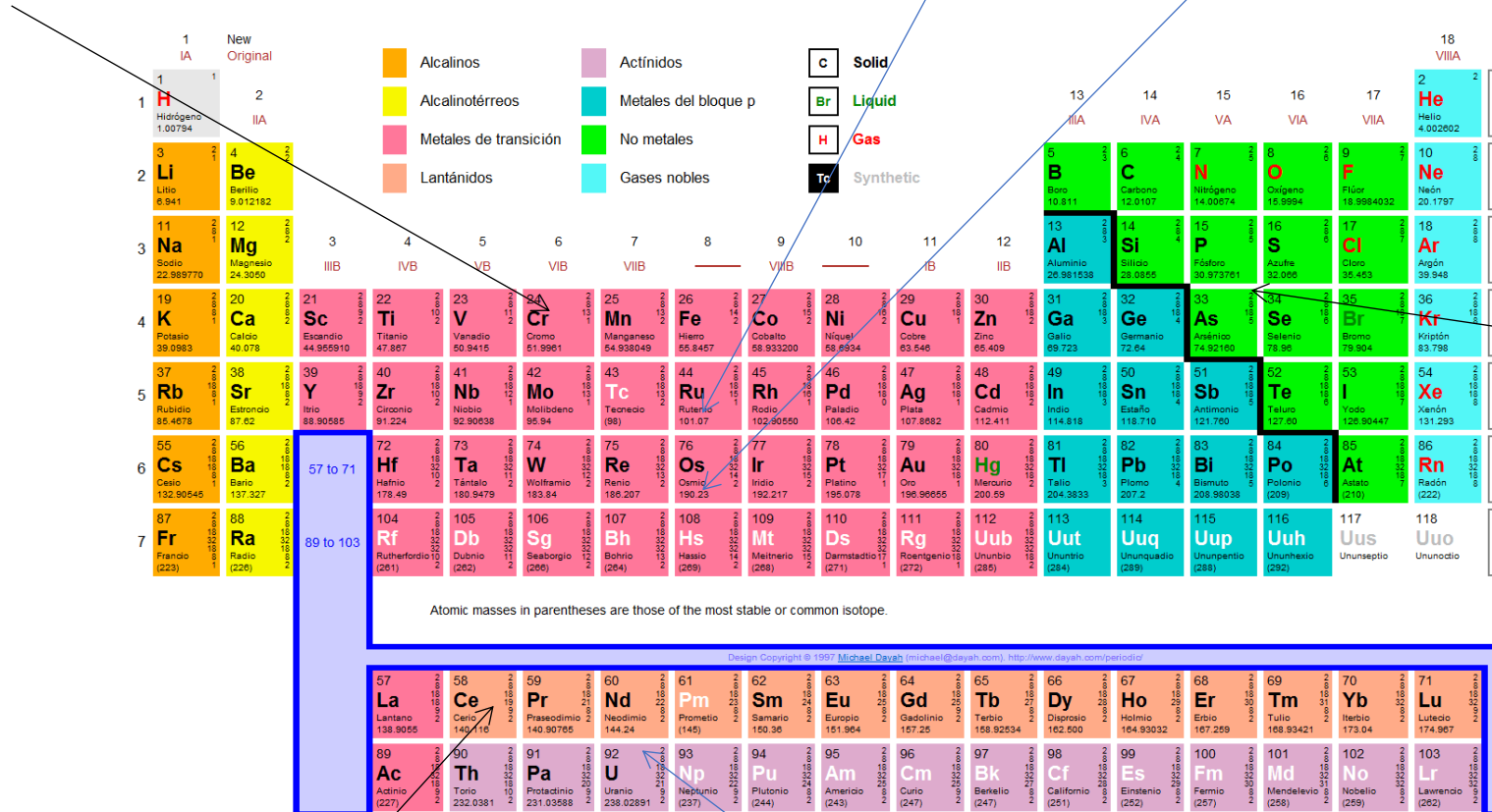
The type of atomic orbital matters

$$U \sim 14.37 \text{ eV} \frac{1}{\epsilon_r r_0}$$

Strength of interactions.

3d: competition between kinetic energy & interaction

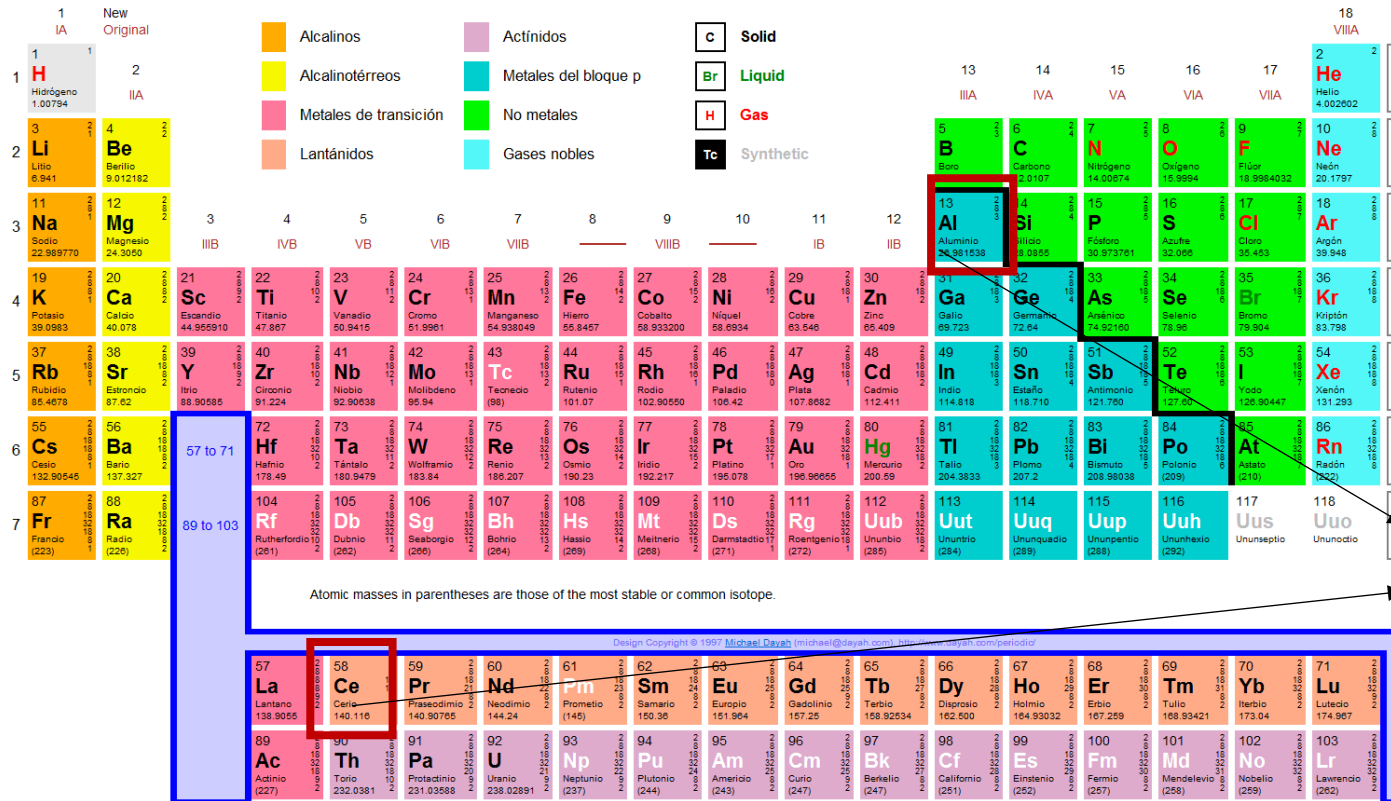
Interaction strength decreases in 4d & overall in 5d due to larger extension of the orbital



4f overall and 5f orbitals are quite localized on a site.
Very small kinetic energy. Interactions win

s & p electrons
kinetic energy
more important

Strength of interactions.



Itinerant + local electrons

Heavy fermion compound

Effective models

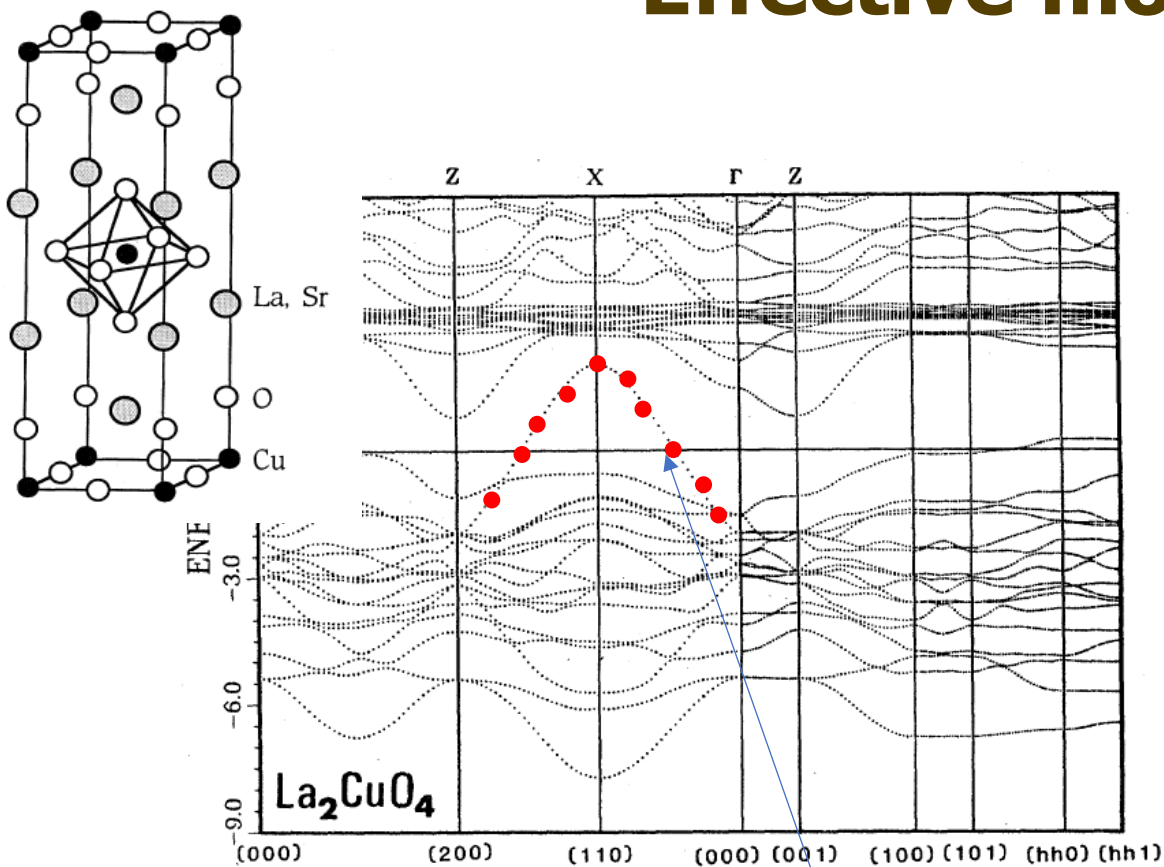


Fig: Pickett, RMP 61, 433 (1989)

Effective model for **Cu** electrons close to the Fermi level

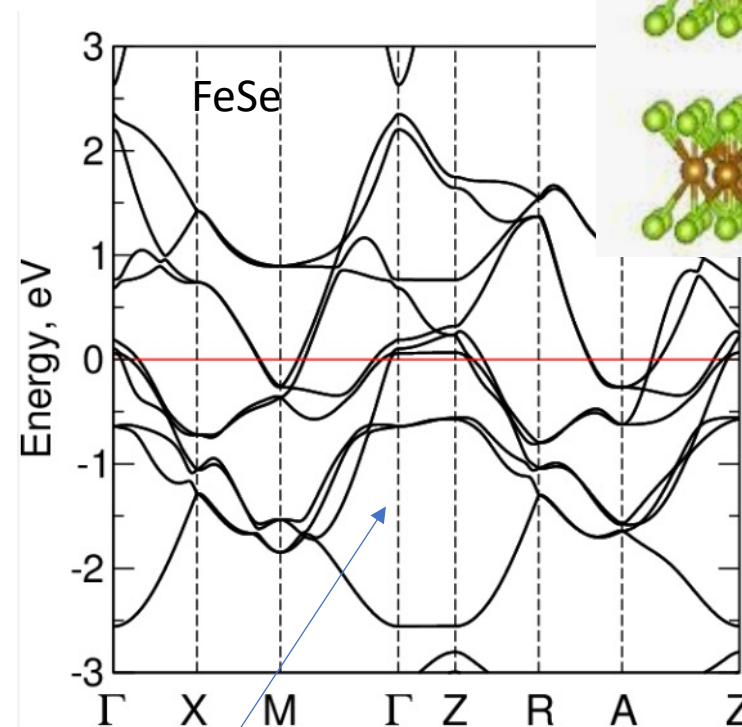
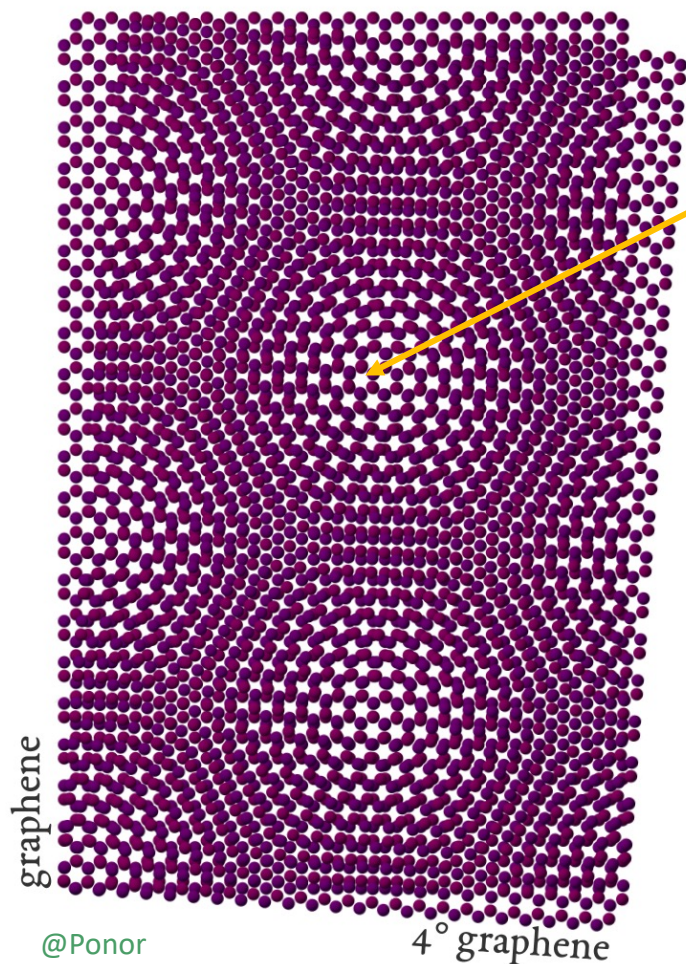


Fig: Grechnev et al, Ukran. J. Phys 57, 171 (2012)

Effective model for **Fe** electrons close to the Fermi level

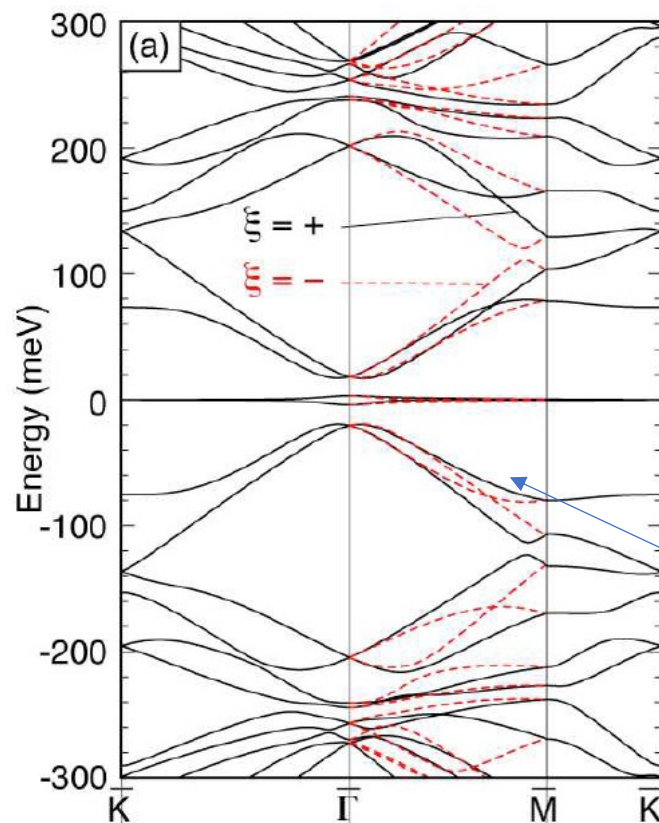
Moiré heterostructures



Moiré unit cell ~ 10 nm lattice constant

$U \sim 30$ meV

r_0 several nm



Very narrow bands for certain twist angles (bandwidth 1-10 meV)

Energy scales of the system

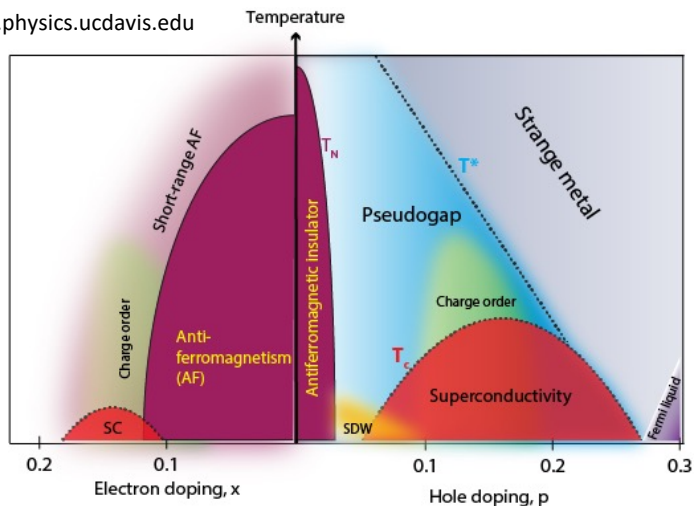


Temperature scale of correlated states

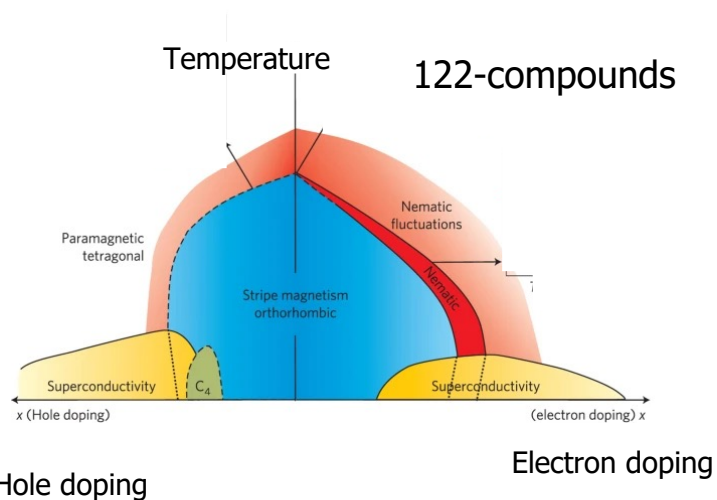
Superconductivity in strongly correlated electron systems

High-T_c superconducting cuprates

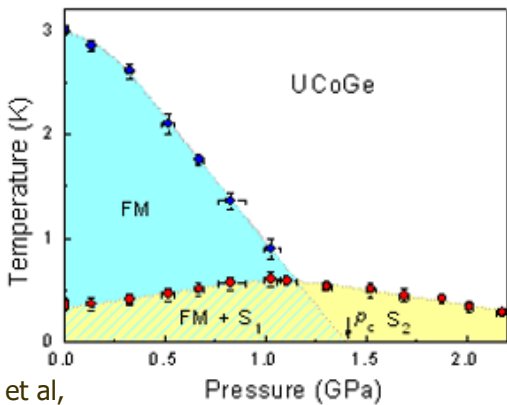
Fig: cme.physics.ucdavis.edu



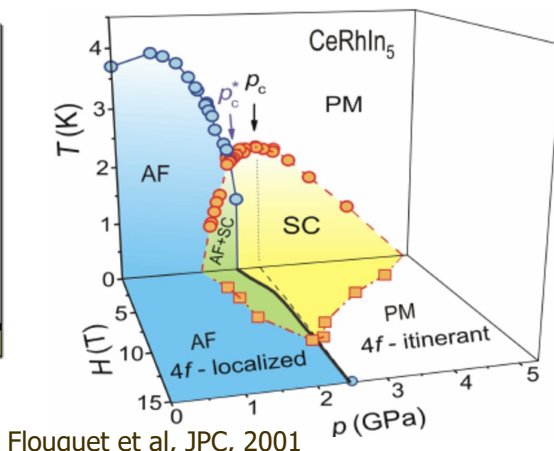
High-T_c iron based superconductors



Heavy fermion compounds

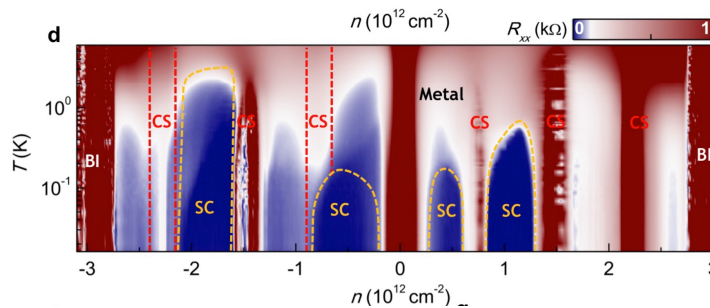


Slooten et al, PRL 103, 097003(2009)



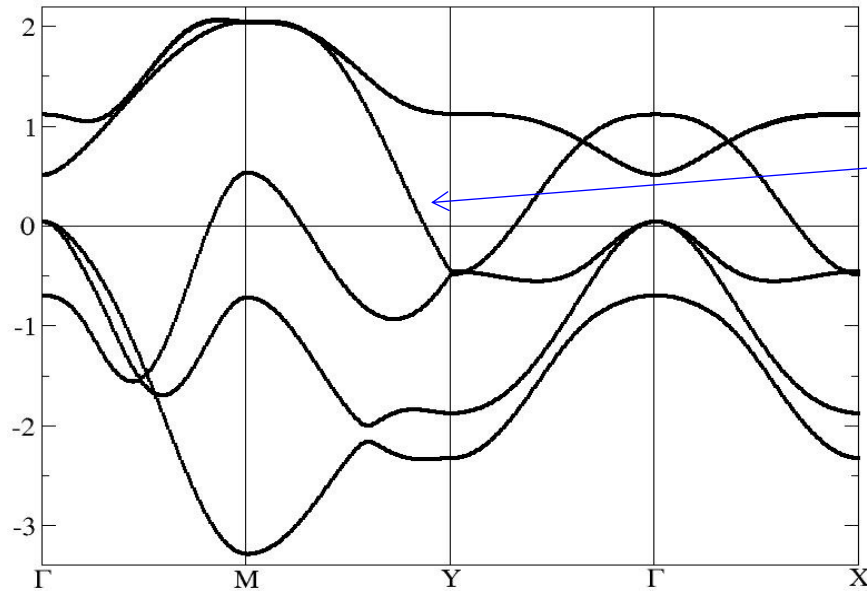
Flouquet et al, JPC, 2001

Twisted bilayer graphene



Critical temperatures not small with respect to the relevant energy scales

Metals and insulators



Metallicity
in clean systems
Bands crossing the Fermi level
(finite Density of States (DOS) at E_F)

Fig: Calderón et al, PRB, 80, 094531 (2009)

Insulating behaviour
in clean systems
Bands below
Fermi level filled

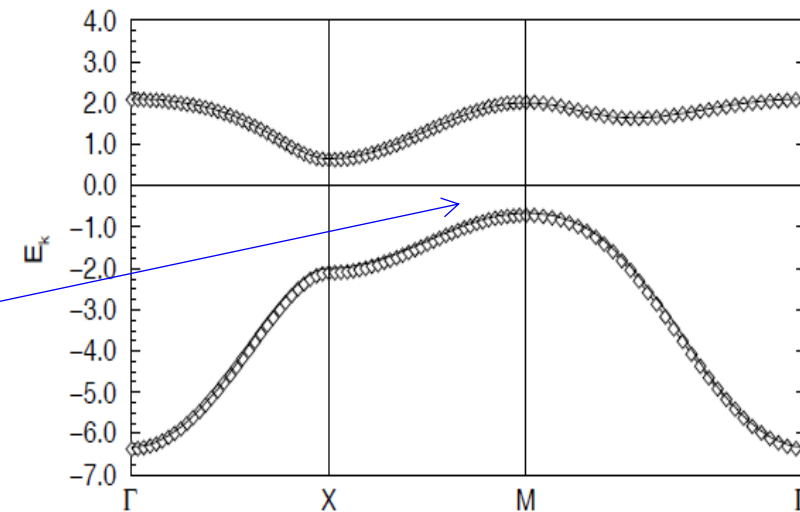


Fig: Hess & Serene, PRB 59, 15167 (1999)

Metals and insulators

Assume a material with spin degeneracy:
Each band can hold 2 electrons per unit cell

Odd number
of electrons
per unit cell



Metallic

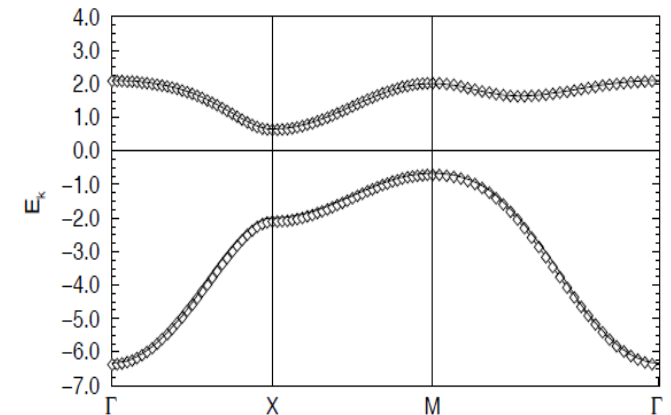
Even number
of electrons
per unit cell



Insulating

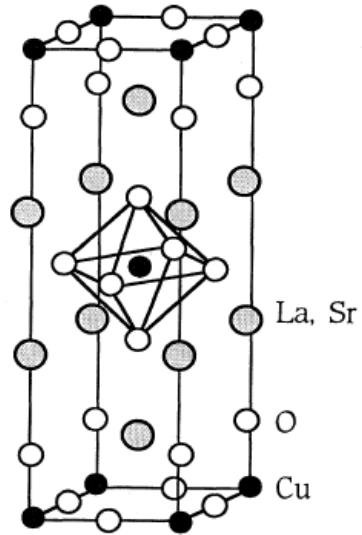


Metallic (in case
of band overlap)

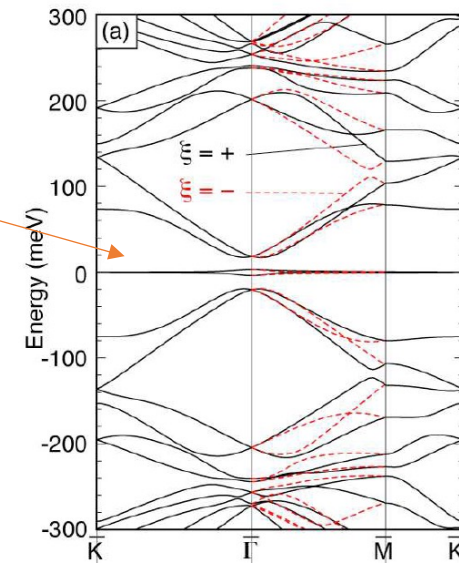
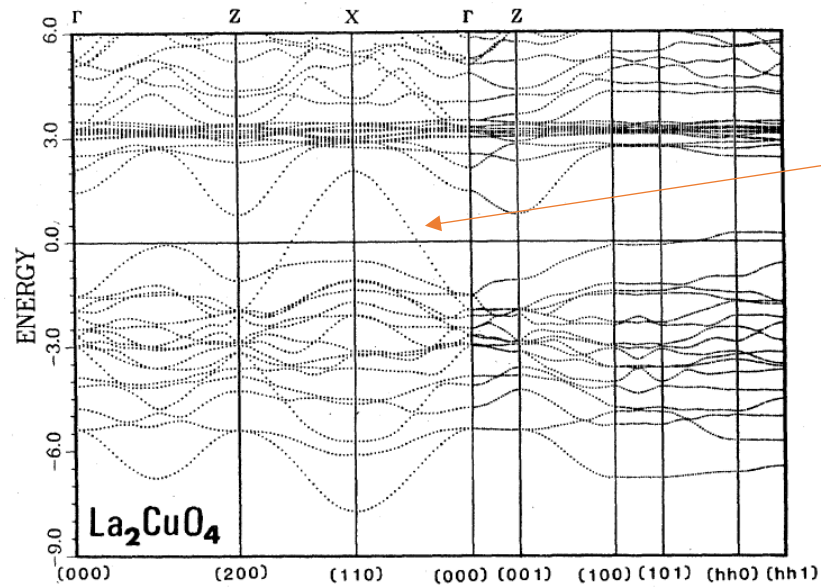
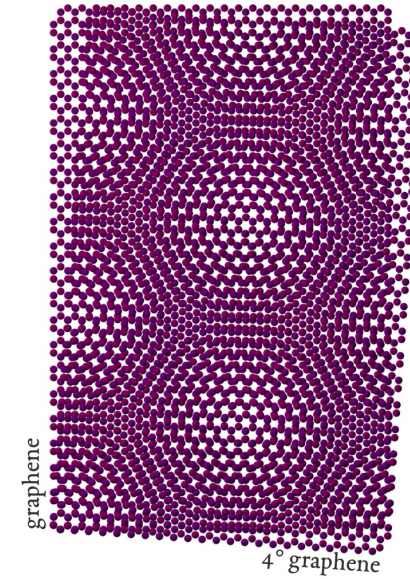


Correlated states and electronic filling

High-T_c
superconducting cuprates

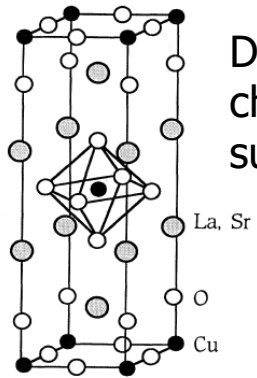


Twisted
bilayer graphene

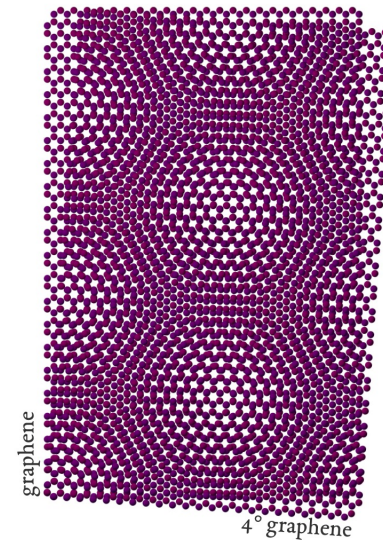
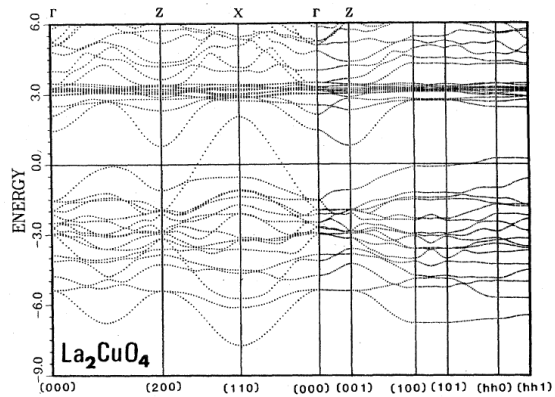


Correlated states and electronic filling

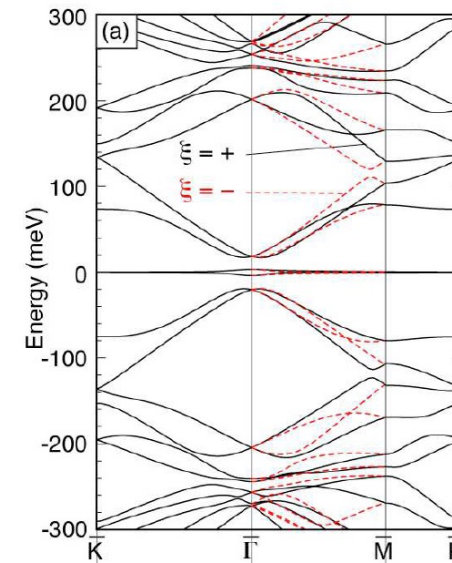
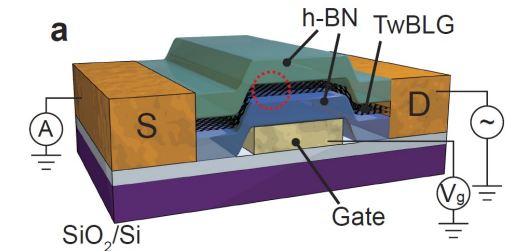
High-Tc superconducting cuprates



Doping through chemical substitution

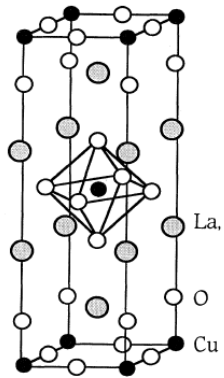


Twisted bilayer graphene

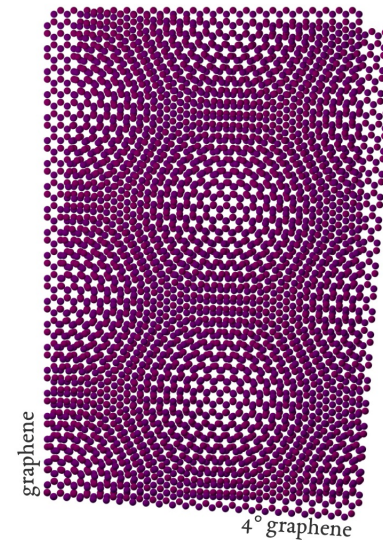
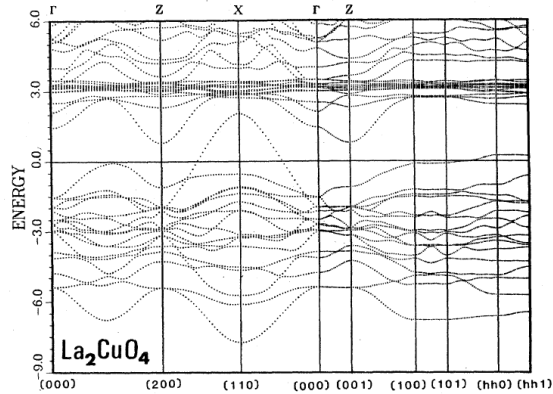


Correlated states and electronic filling

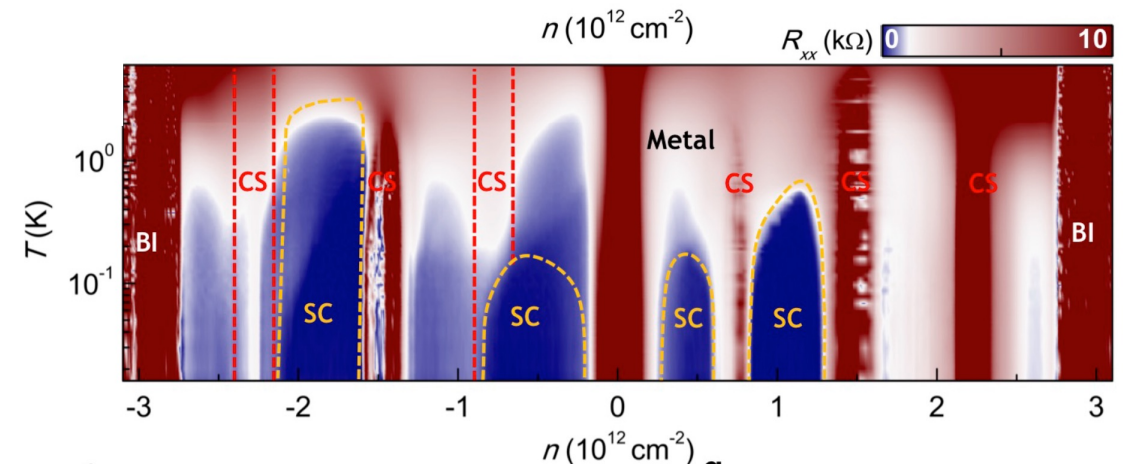
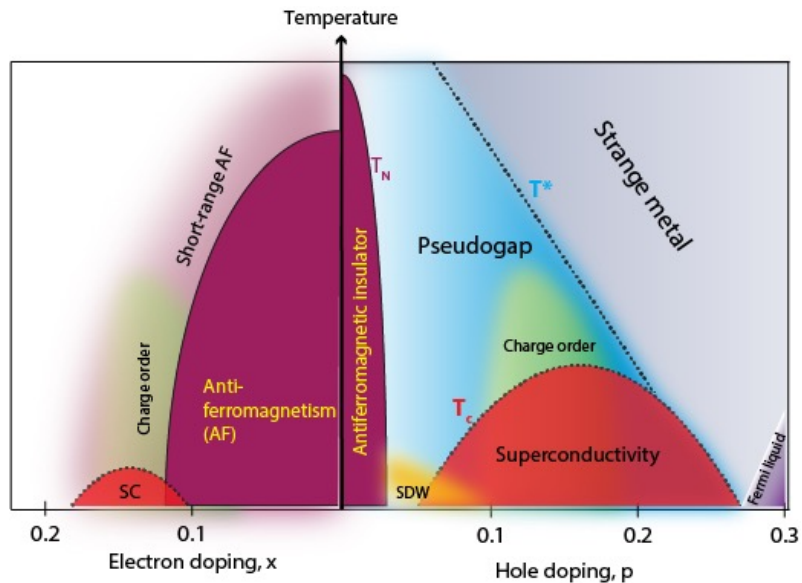
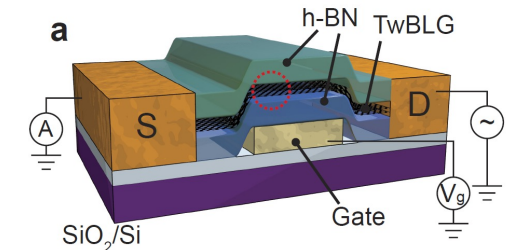
High-Tc superconducting cuprates



Doping through chemical substitution

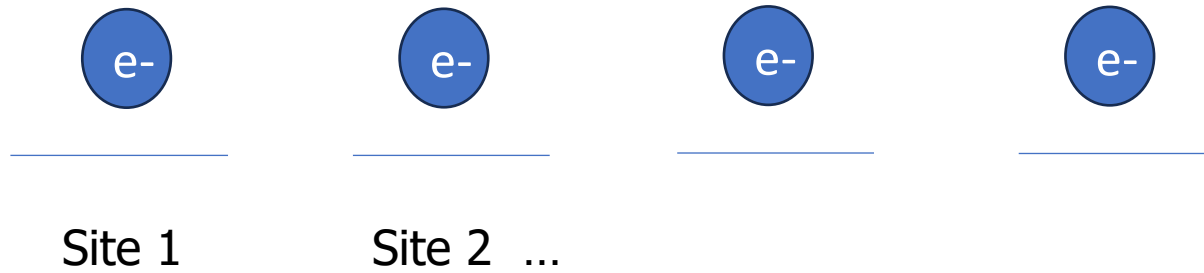


Twisted bilayer graphene



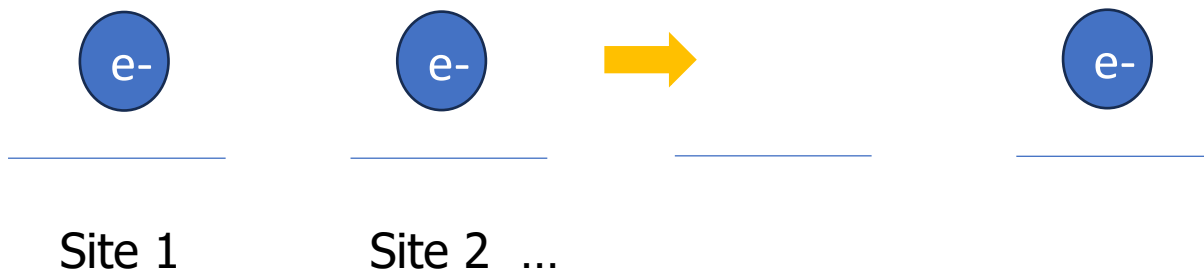
Correlated states and electronic filling

Onsite repulsión U
Hopping energy t



1 electron per site/atom

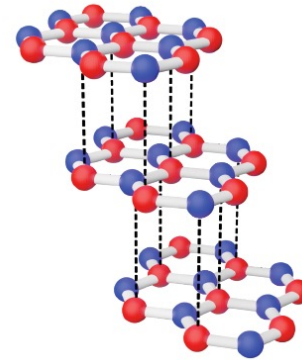
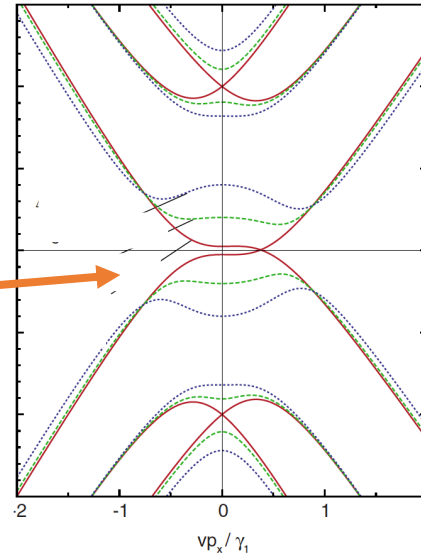
$U \gg t$ Metal insulator transition
due to localization of electrons



Partial electronic filling per site/atom
Always metallic (if no symmetry breaking)
But maybe correlated metal!

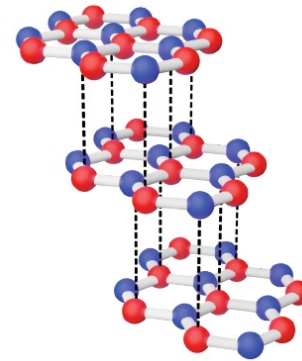
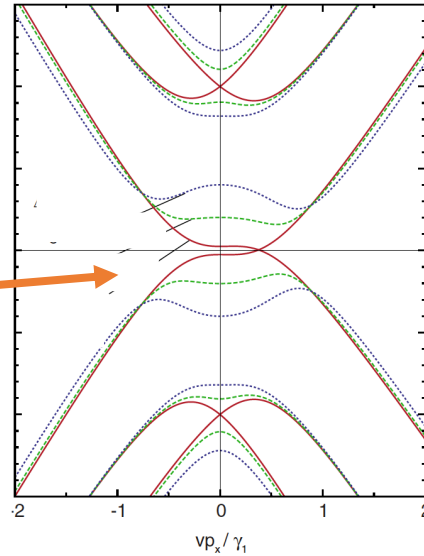
Band structure "details"

- ❑ Dimensionality
- ❑ Bandwidth W (flat bands)
- ❑ Flat band regions
(large density of states at E_F)

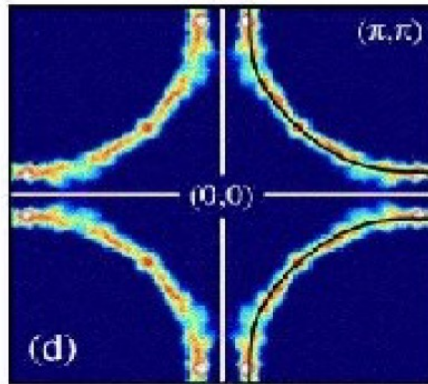
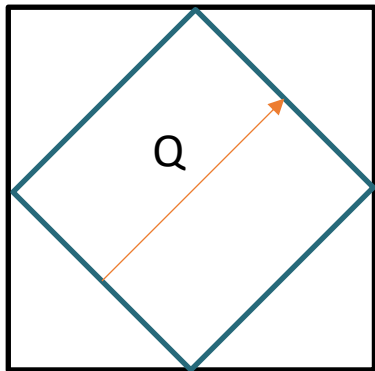


Band structure "details"

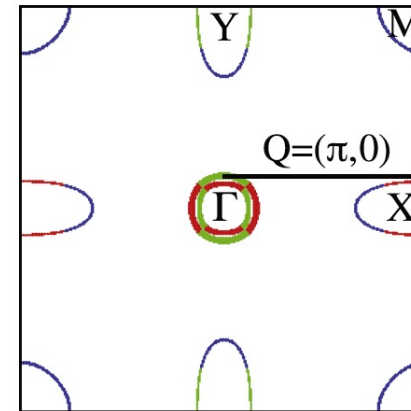
- ❑ Dimensionality
- ❑ Bandwidth W (flat bands)
- ❑ Flat band regions
(large density of states at E_F)
- ❑ Nesting with momentum Q
(always in 1D)



Fermi surfaces in 2D



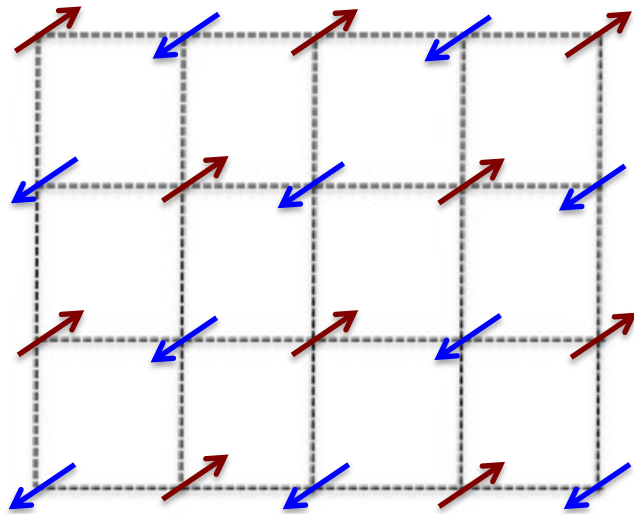
Fermi Surface of Cuprates



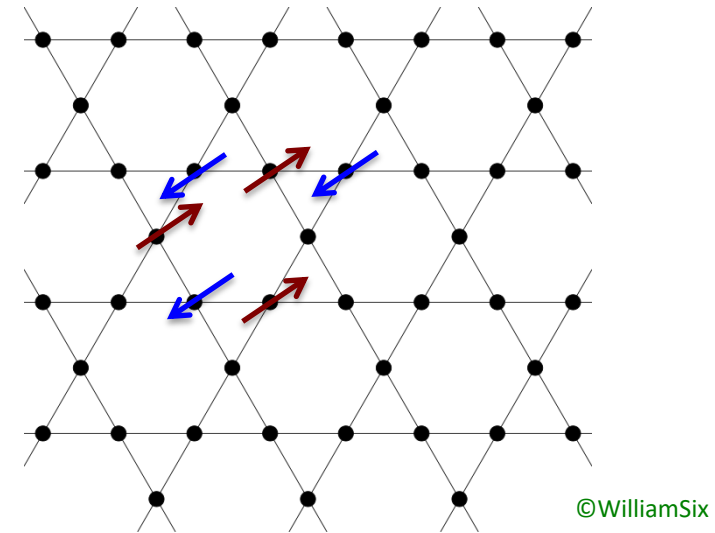
Fermi Surface of Iron superconductors

Dimensionality and crystalline lattice

Bipartite lattice

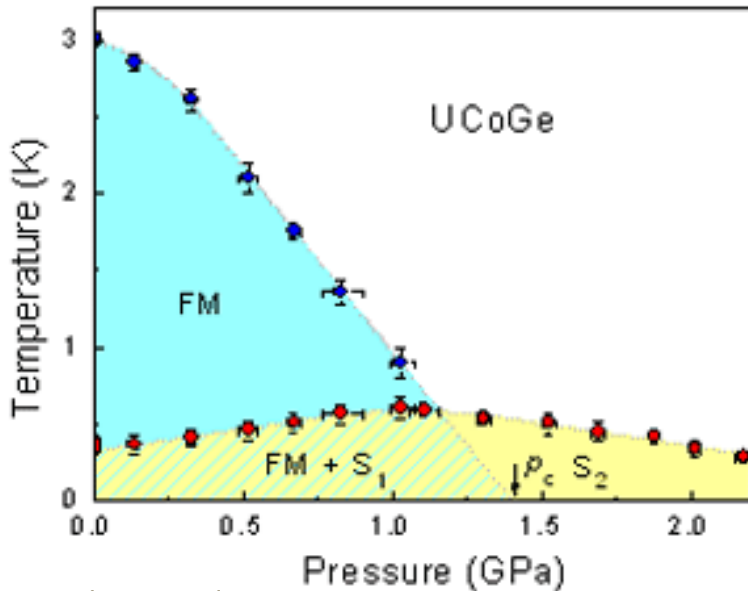
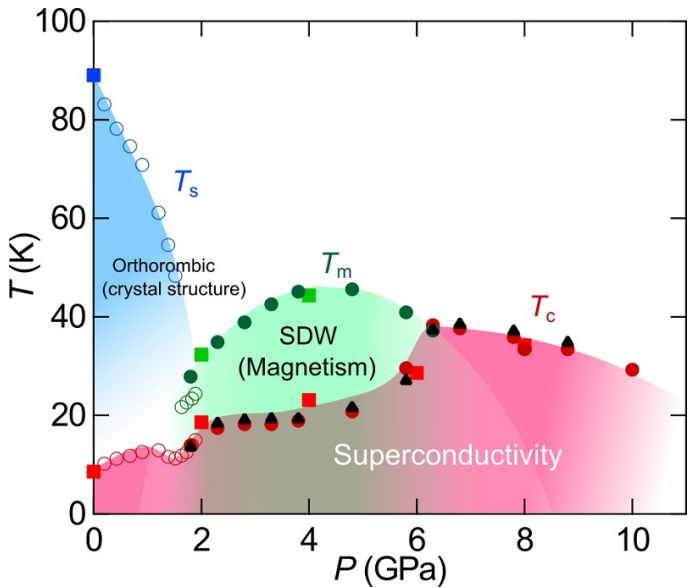


Frustrated lattice

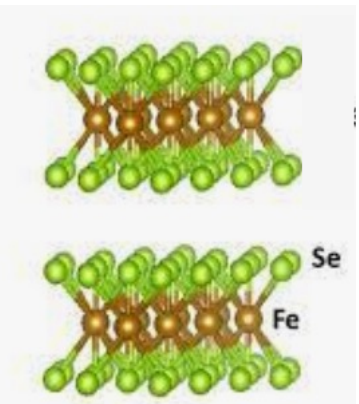
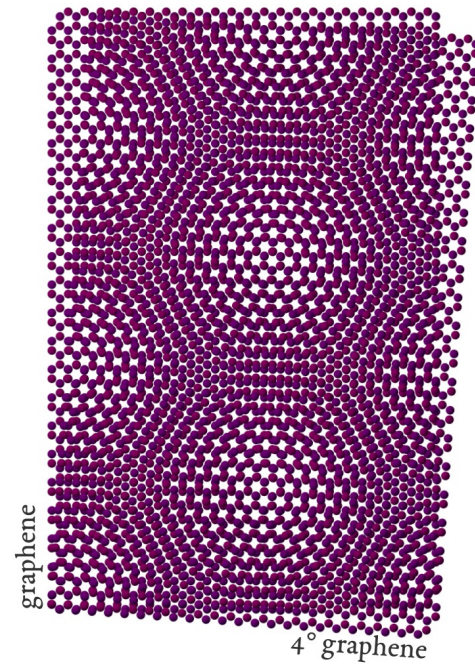


Gran variedad de estados electrónicos correlacionados

External knobs to control the correlated states: pressure, strain ...



Slooten et al, PRL 103 , 097003(2009)

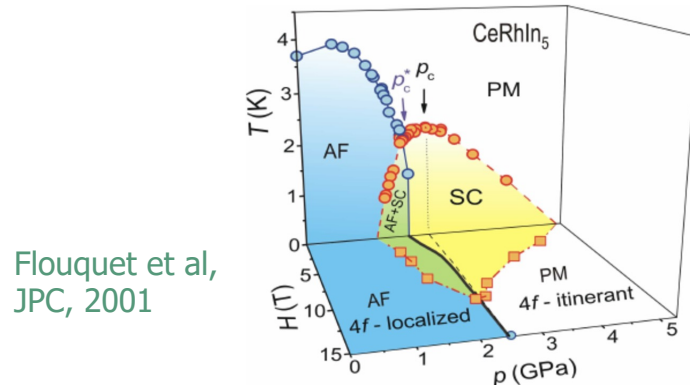


Modification of kinetic energy (lattice constant, Fermi Surface details ...)

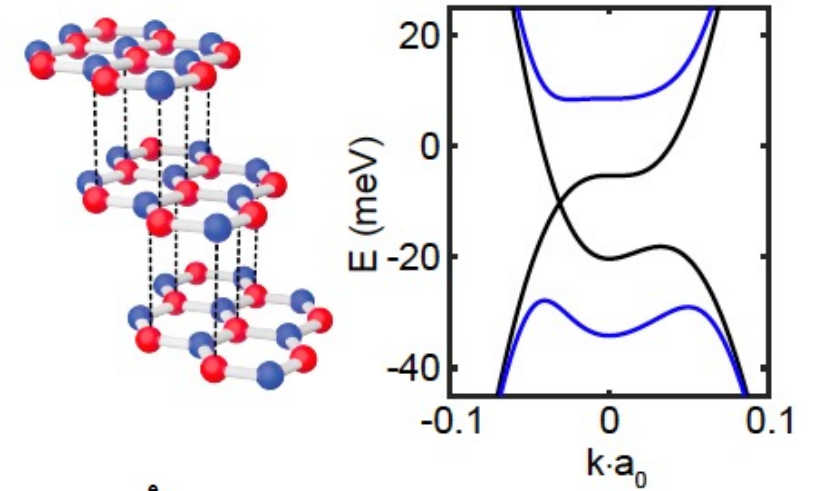
In a given material certain correlated states appear only under pressure

External knobs to control the correlated states: magnetic and electric field...

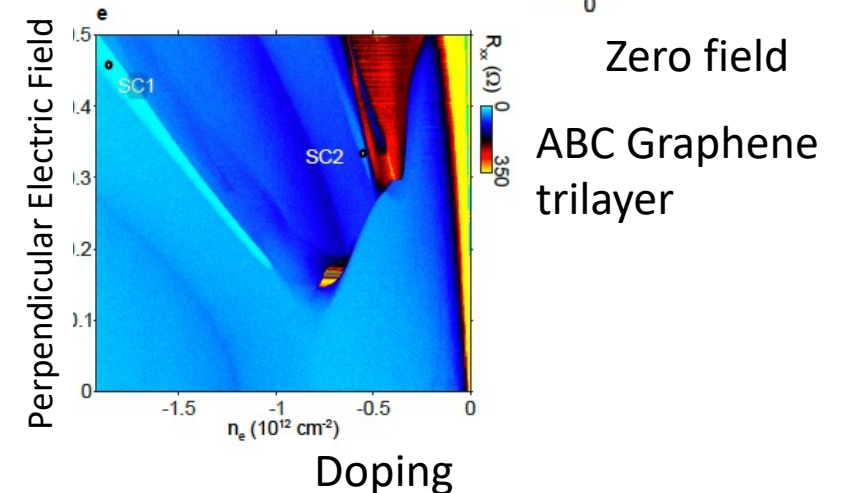
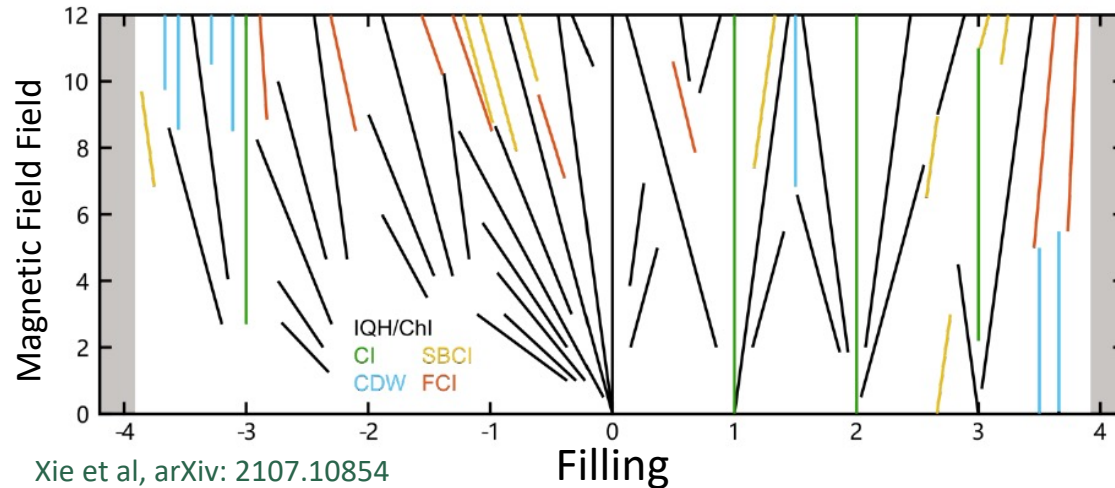
Magnetic field: spin and orbital effect



Perpendicular electric field



Twisted Bilayer Graphene (compressibility measurements)

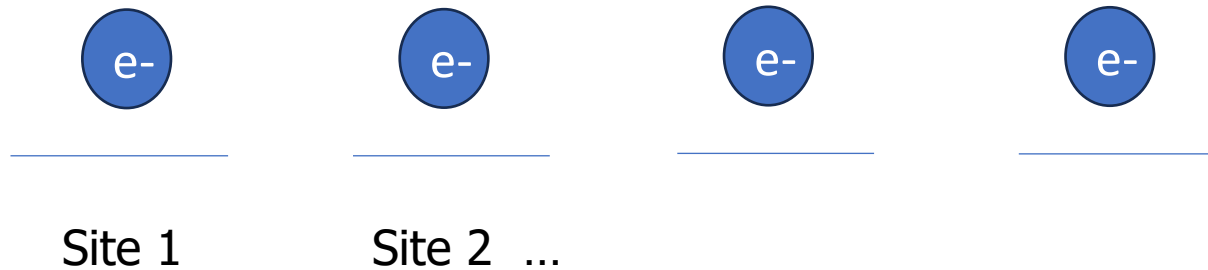


Different types of insulating correlated states

The role of screening: from Coulomb to Hubbard

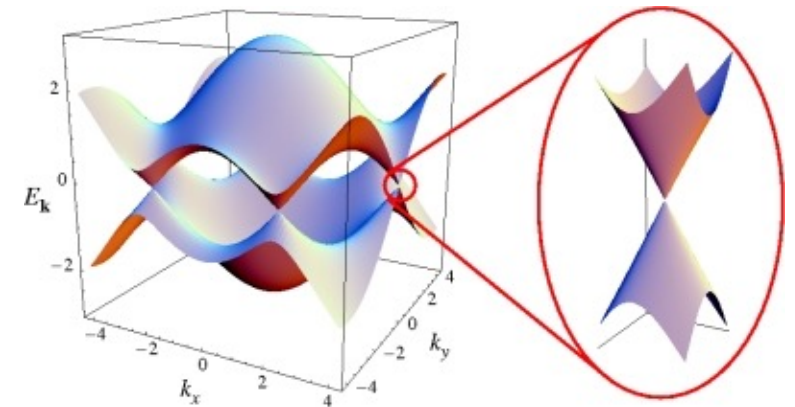
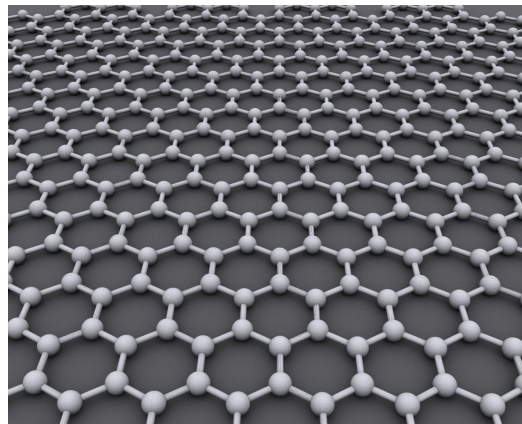
Coulomb interaction $\left(\frac{e^2}{4 \pi \epsilon_0} \right) \frac{1}{\epsilon_r r_0} = 14.37 \text{ eV } \text{\AA} \frac{1}{\epsilon_r r_0}$

Tuning ϵ_r and maximum interaction length r_0



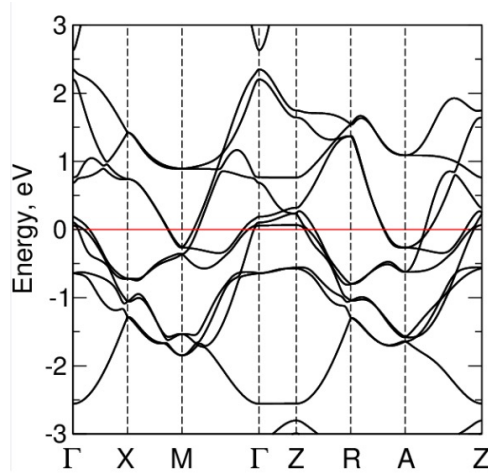
If not screened the electronic interaction decays a $1/r$

Graphene: $1/r$ interaction

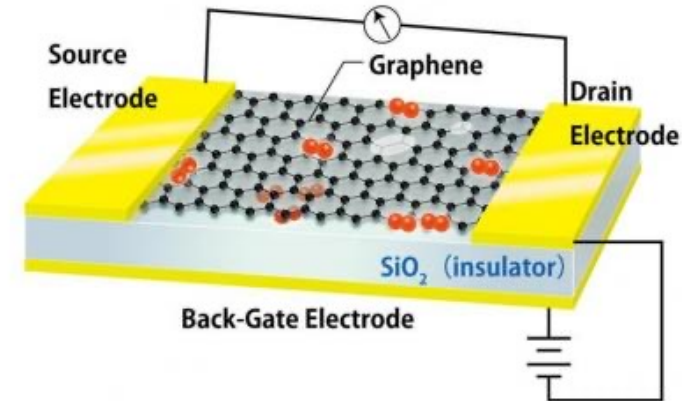


The role of screening: from Coulomb to Hubbard

Screening in metals



Screening with a gate



Coulomb interaction

$$\left(\frac{e^2}{4 \pi \epsilon_0} \right) \frac{1}{\epsilon_r r}$$



Screened interaction

$$\left(\frac{e^2}{4 \pi \epsilon_0} \right) \frac{1}{\epsilon_r} \frac{\text{Exp}(-r/\lambda_0)}{r}$$

The interaction decreases faster with distance



Tune interaction length

ϵ_r also determined by screening from deep bands, substrate, gates...:

The role of screening: from Coulomb to Hubbard

$$\left(\frac{e^2}{4 \pi \epsilon_0} \right) \frac{1}{\epsilon_r} \frac{\text{Exp}(-r/\lambda_0)}{r}$$

$\lambda_0 \sim$ lattice constant

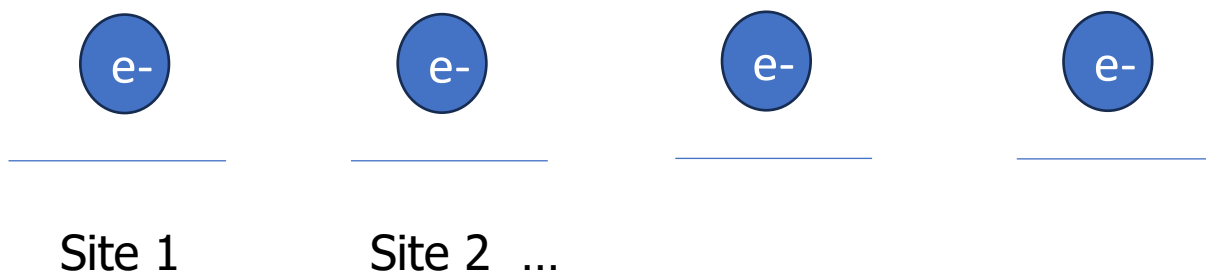
Only onsite repulsion U :

Hubbard model

$$\lambda_0 \rightarrow \infty$$

Coulomb $1/r$

Interaction decays slowly
and extends to all neighbors

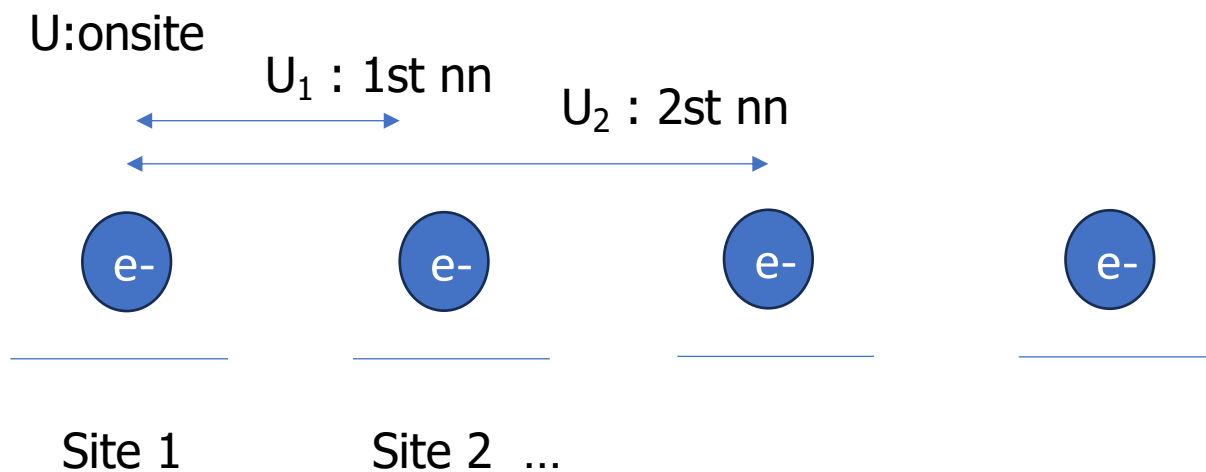


The role of screening: from Coulomb to Hubbard

$$\left(\frac{e^2}{4 \pi \epsilon_0} \right) \frac{1}{\epsilon_r} \frac{\text{Exp}(-r/\lambda_0)}{r}$$

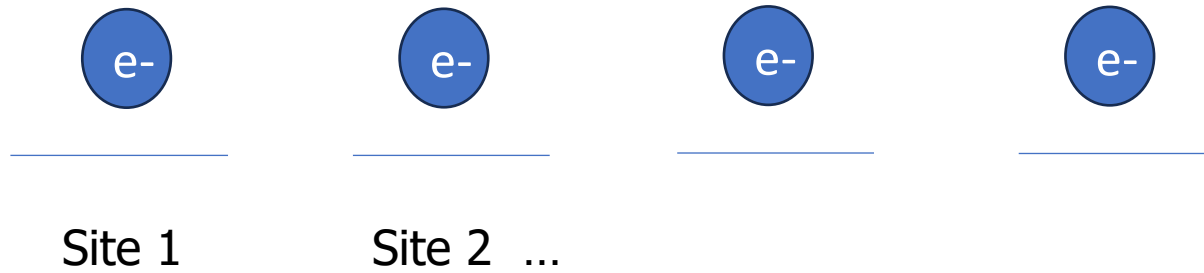
Extended Hubbard model

Interaction up to first or second neighbors



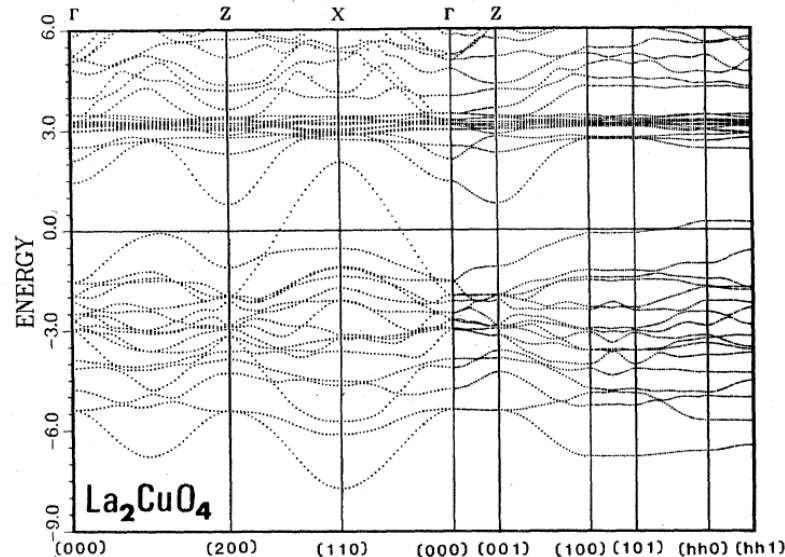
The role of screening: from Mott to Wigner

{ Onsite repulsión U
 { Hopping energy t



1 electron per site/atom

$U \gg t$ Metal insulator transition due to localization of electrons



Mott insulator (Mott-Hubbard)

Large density of electrons, onsite repulsion & role of lattice

The role of screening: from Mott to Wigner

Assume: { 3D, free space (continuum)
Coulomb $\propto \frac{e^2}{r}$

Electronic density

$$n = N_e / V$$

N_e : Number of electrons
 V : Volume

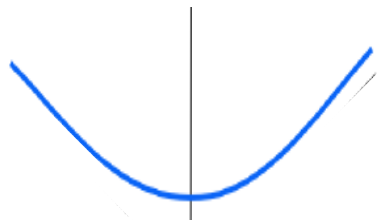
Kinetic energy

$$\int_{-\infty}^{\epsilon_F} \epsilon N(\epsilon) d\epsilon \propto n^{2/3}$$

density of states

Interaction energy

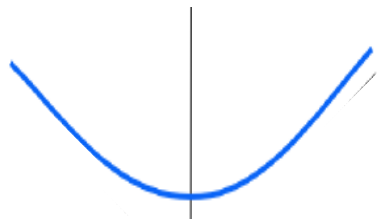
$$\propto \langle 1/r \rangle \cong n^{1/3}$$



$$\frac{\text{Interaction energy}}{\text{Kinetic energy}} \propto n^{-1/3}$$

← Larger for small density
Important in semiconductors

The role of screening: from Mott to Wigner



Interaction energy
Kinetic energy

$$\propto n^{-1/3}$$

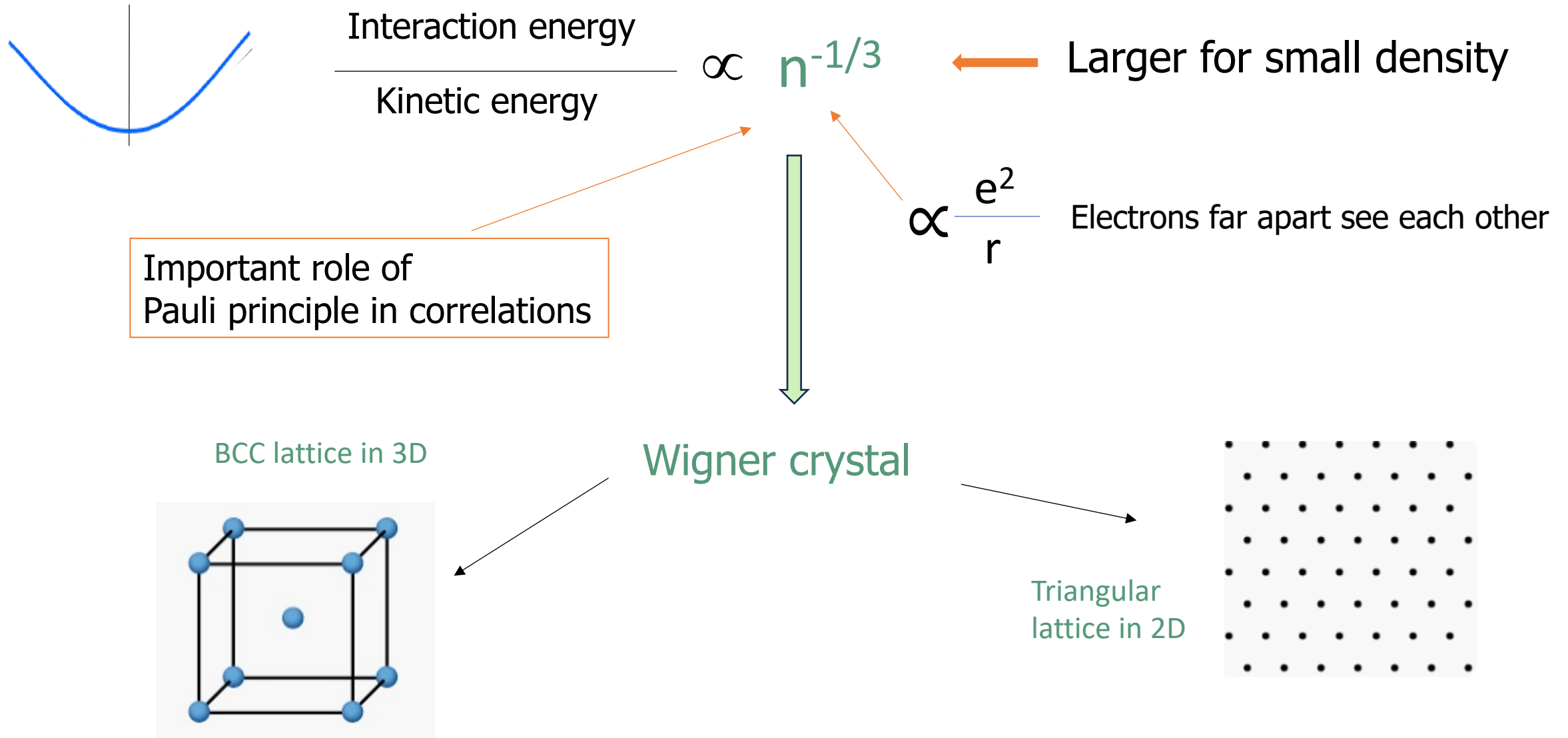
← Larger for small density

$$\propto \frac{e^2}{r}$$

Electrons far apart see each other

Important role of Pauli principle in correlations

The role of screening: from Mott to Wigner



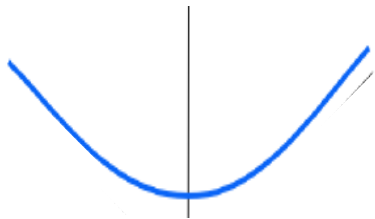
The role of screening: from Mott to Wigner

Wigner crystal

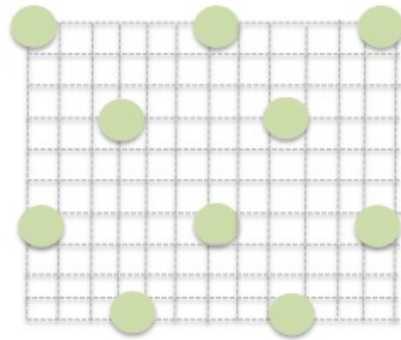
Free space (continuum)

$$\text{Coulomb interaction} \propto \frac{e^2}{r}$$

Insulating for small densities (incommensurate)



Close to the bottom of the band
The lattice less important



Breaks lattice symmetry

Mott insulator

Lattice (unit cell)

Onsite interaction U



Larger effect of interactions
at integer filling of the lattice sites
(commensurate filling)

Does not break
the lattice symmetry

Onsite repulsion only:
Electrons far apart
do not interact
(Pauli principle still matters)

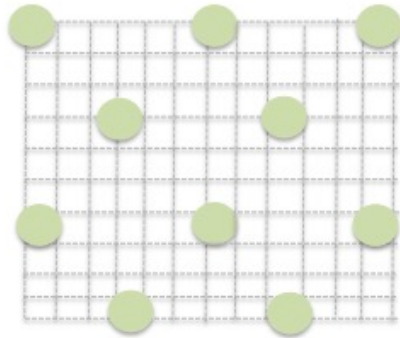
The role of screening: from Mott to Wigner

Wigner crystal

Free space (continuum)

$$\text{Coulomb interaction} \propto \frac{e^2}{r}$$

Insulating for small densities (incommensurate)



Mott insulator

Lattice (unit cell)

Onsite interaction U



Larger effect of interactions
at integer filling of the lattice sites

Charge localization (spin not invoked in insulating behavior)

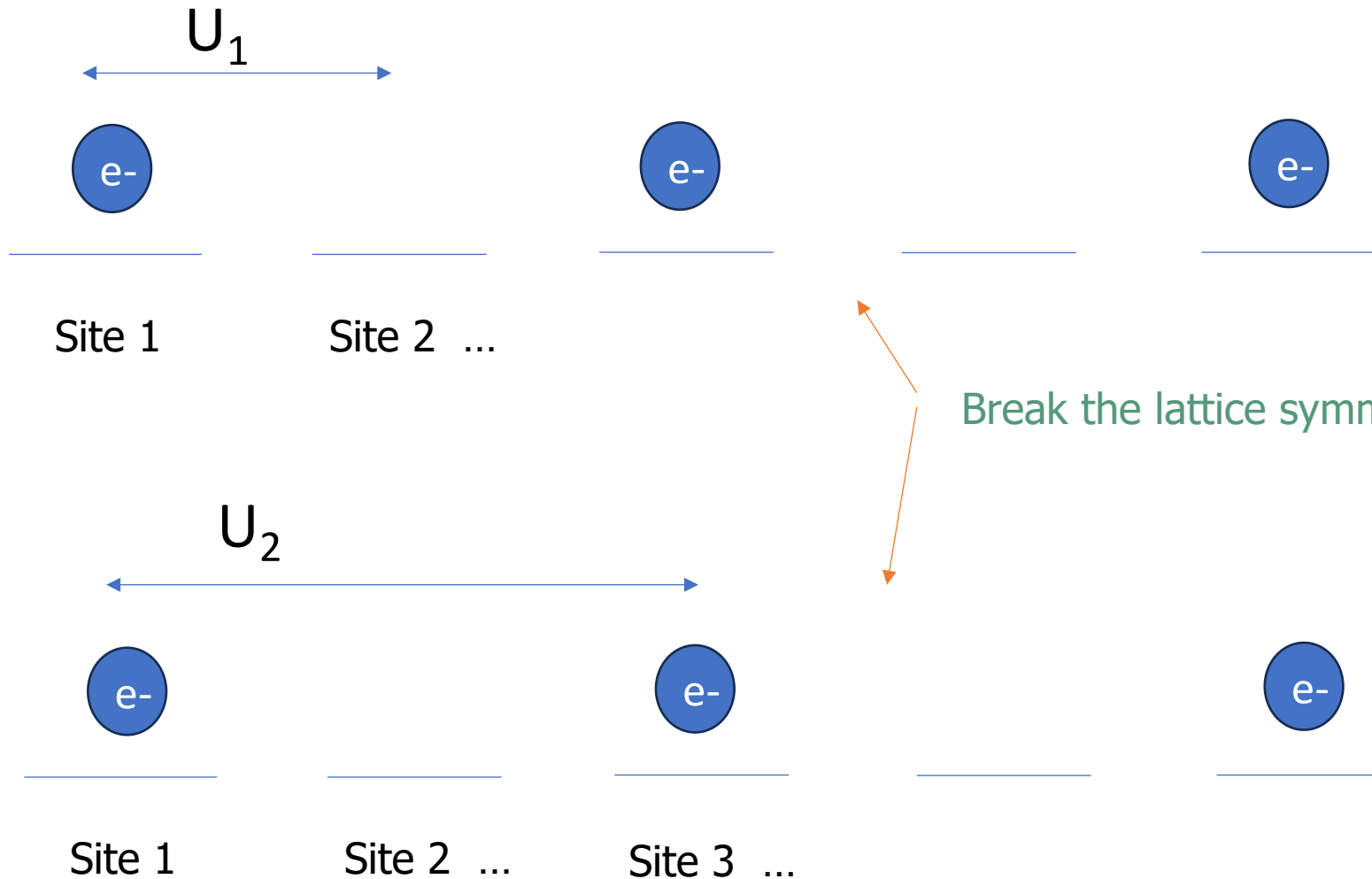
Generalized Wigner or Wigner-Mott

Extended Hubbard: finite range of interaction

Commensurate filling

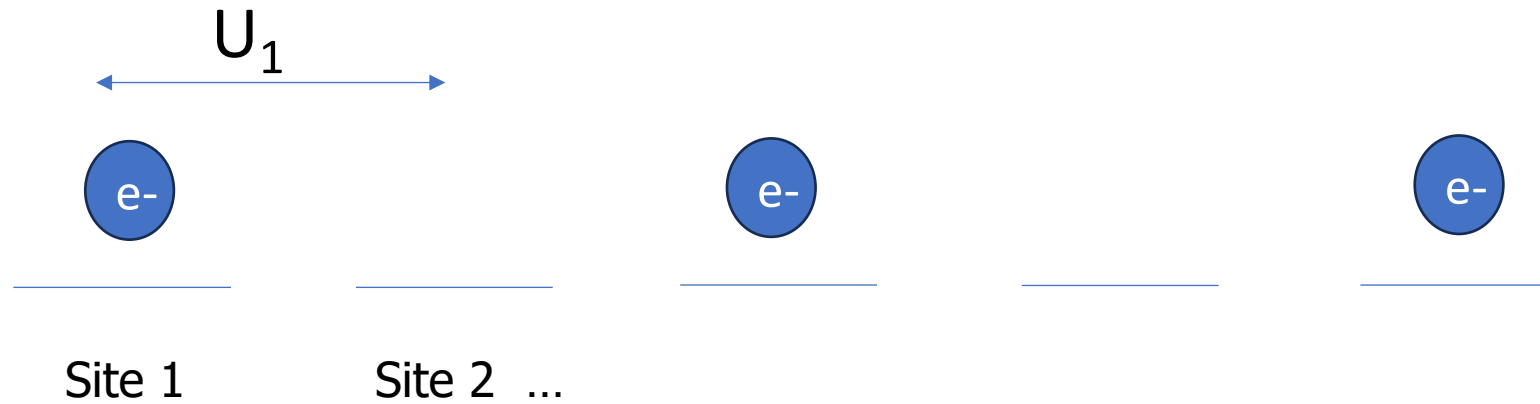
Repulsion up to
1st nearest neighbors

Repulsion up to
2nd nearest neighbors



Generalized Wigner or Wigner-Mott

Extended Hubbard: finite range of interaction



Commensurate filling

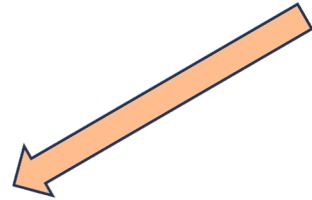
Repulsion up to
1st nearest neighbors

Emergence of Quantum Phases in Novel Materials

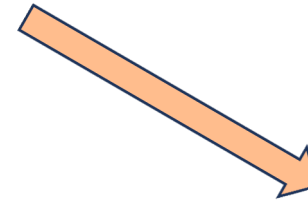
Interactions between many particles (electrons)



Correlated Quantum States



Symmetry breaking states



Formation of local moments



Tuning parameters for correlated states

Interaction energy vs Kinetic energy

- Filling of electronic bands
- Electronic bands details
- Dimensionality
- Crystalline lattice
- Screening
- External knobs (pressure, ...)
- Interaction with lattice
- Other (spin-orbit, ...)

40 years of intensive research effort and continuously new surprises appear

Emergence of quantum phases in novel materials

Basic ideas & generic concepts

Different kinds of phenomena
& why they happen

Theoretical description

Many different approaches and
analytical & numerical methods

Experimental signatures
of the quantum states

Materials and platforms
where these quantum phases appear