Kondo effect in metals and nanostructures

Ramón Aguado ICMM-CSIC

Bibliography:

- A. C. Hewson "The Kondo problem to heavy fermions", Cambridge University Press, 1997.

- P. Coleman, "Introduction to Many Body Physics" (chapters 16 and 17), Cambridge University Press , 2015

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Bibliography for quantum transport and mesoscopics (I will cite many papers along the lecture), some basic texts:

- Supriyo Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge 1997.
- Nazarov & Blanter, *Quantum Transport*, Cambridge 2009
- Bruus & Flensberg, *Many Body Theory in Condensed Matter* (chapters 7-10), Oxford 2004

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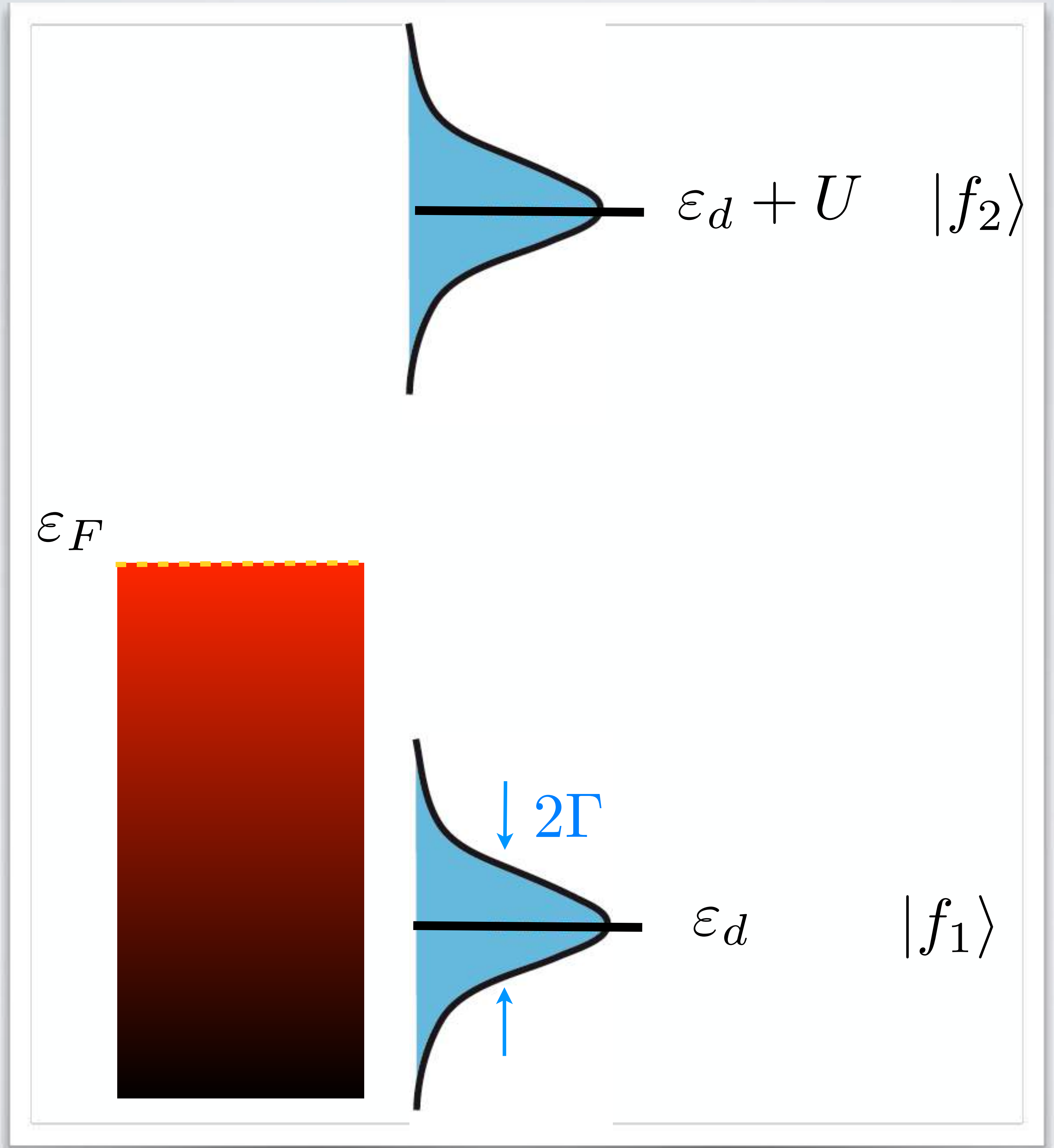
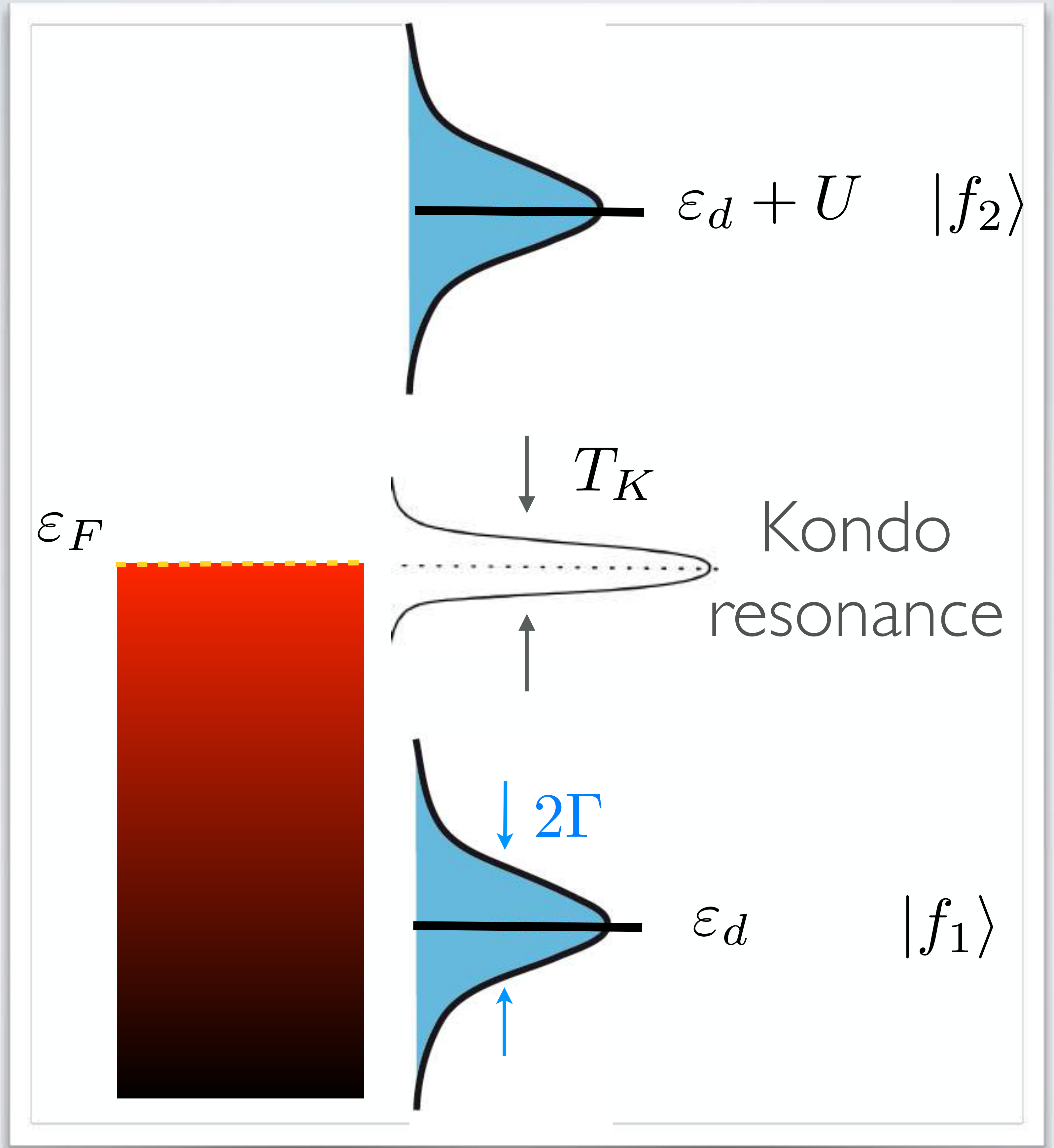
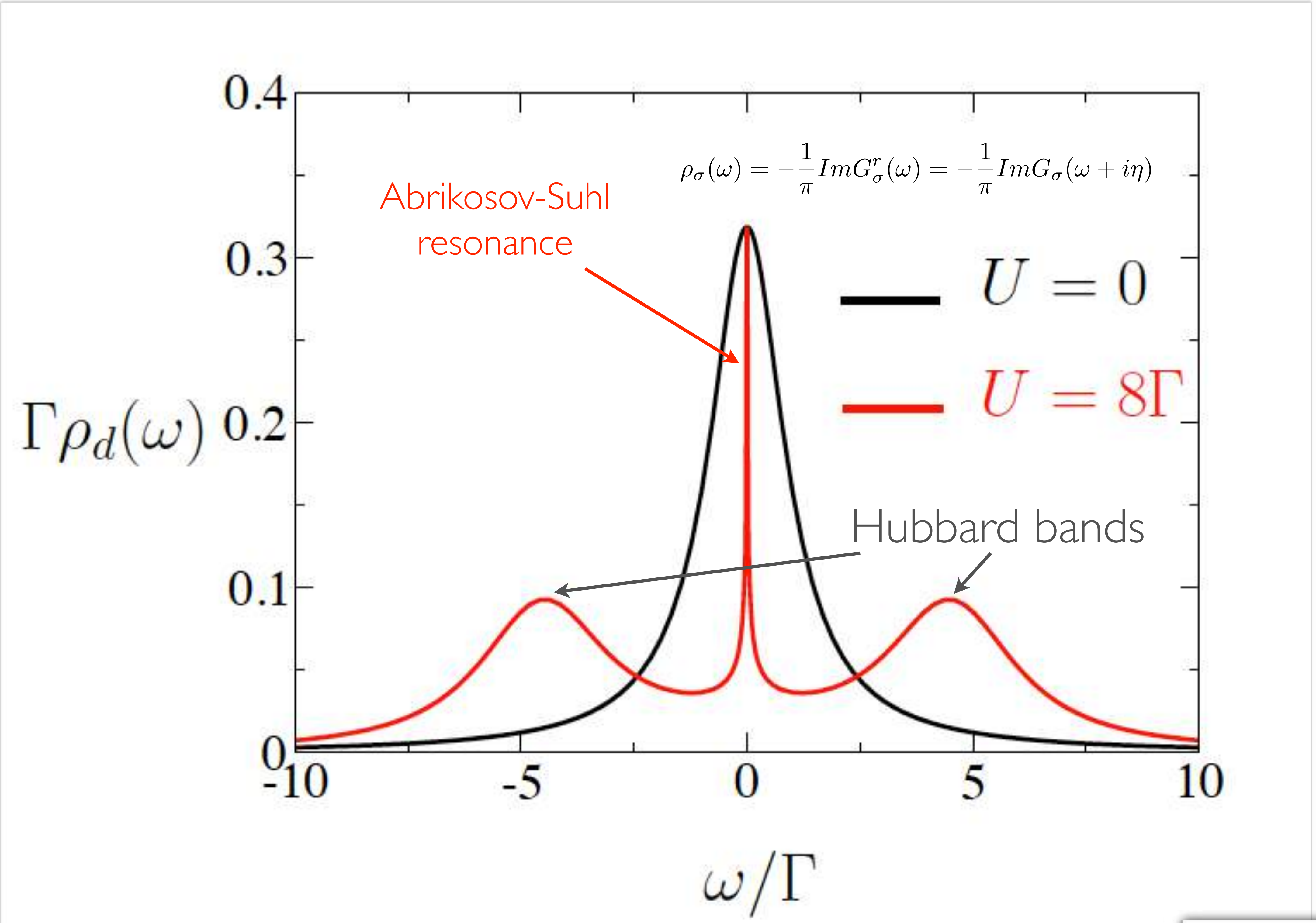


$\varepsilon_d + U$
$|f_2\rangle$
ε_F
2Γ
ε_d
$|f_1\rangle$

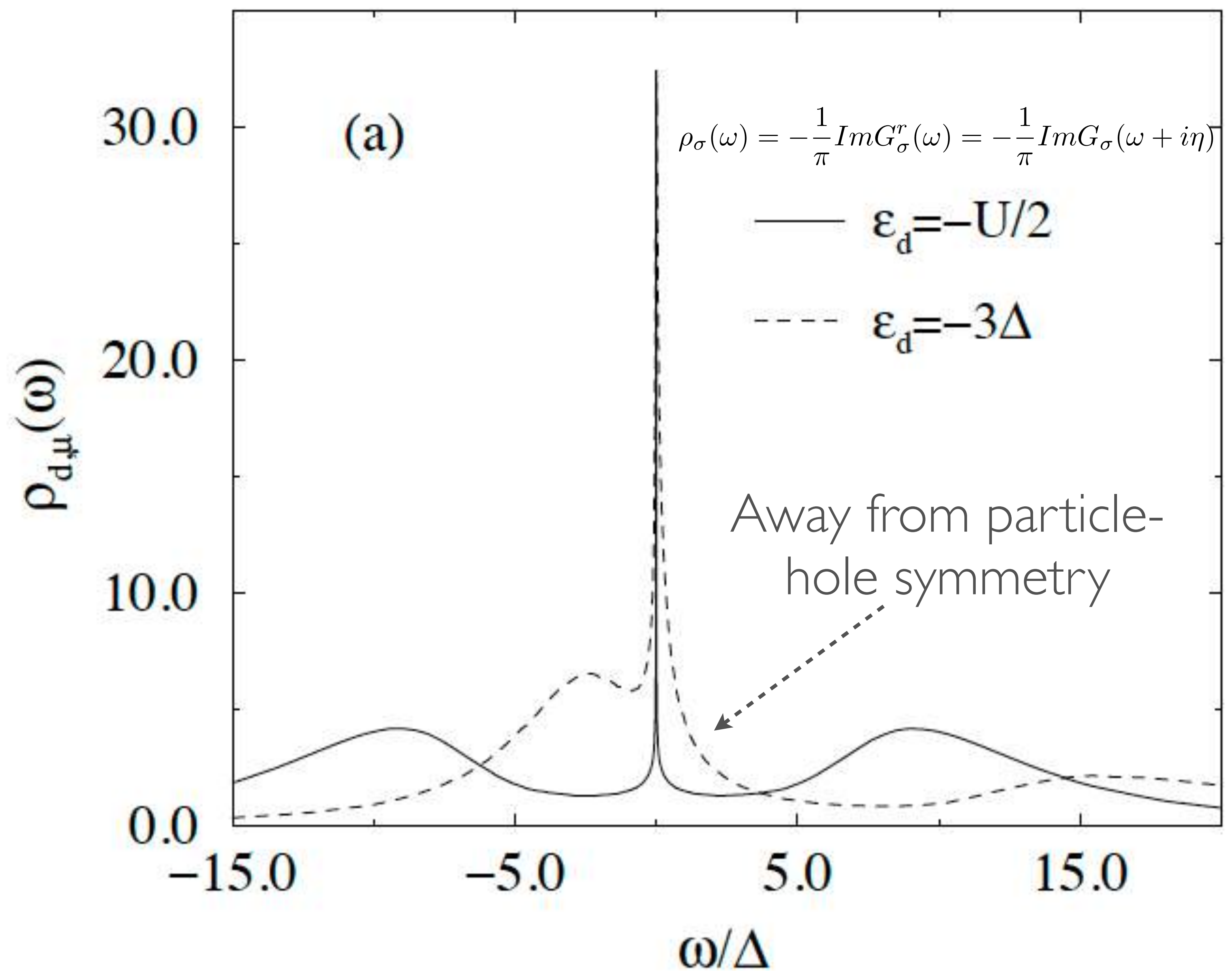

$\varepsilon_d + U$
$|f_2\rangle$
T_K
ε_F
Kondo resonance
2Γ
ε_d
$|f_1\rangle$

How about **dynamics**?



NRG calculations: Frota & Oliveira 86, Sakai, Shimizu & Kasuya 89, Costi & Hewson 90)

How about **dynamics**?



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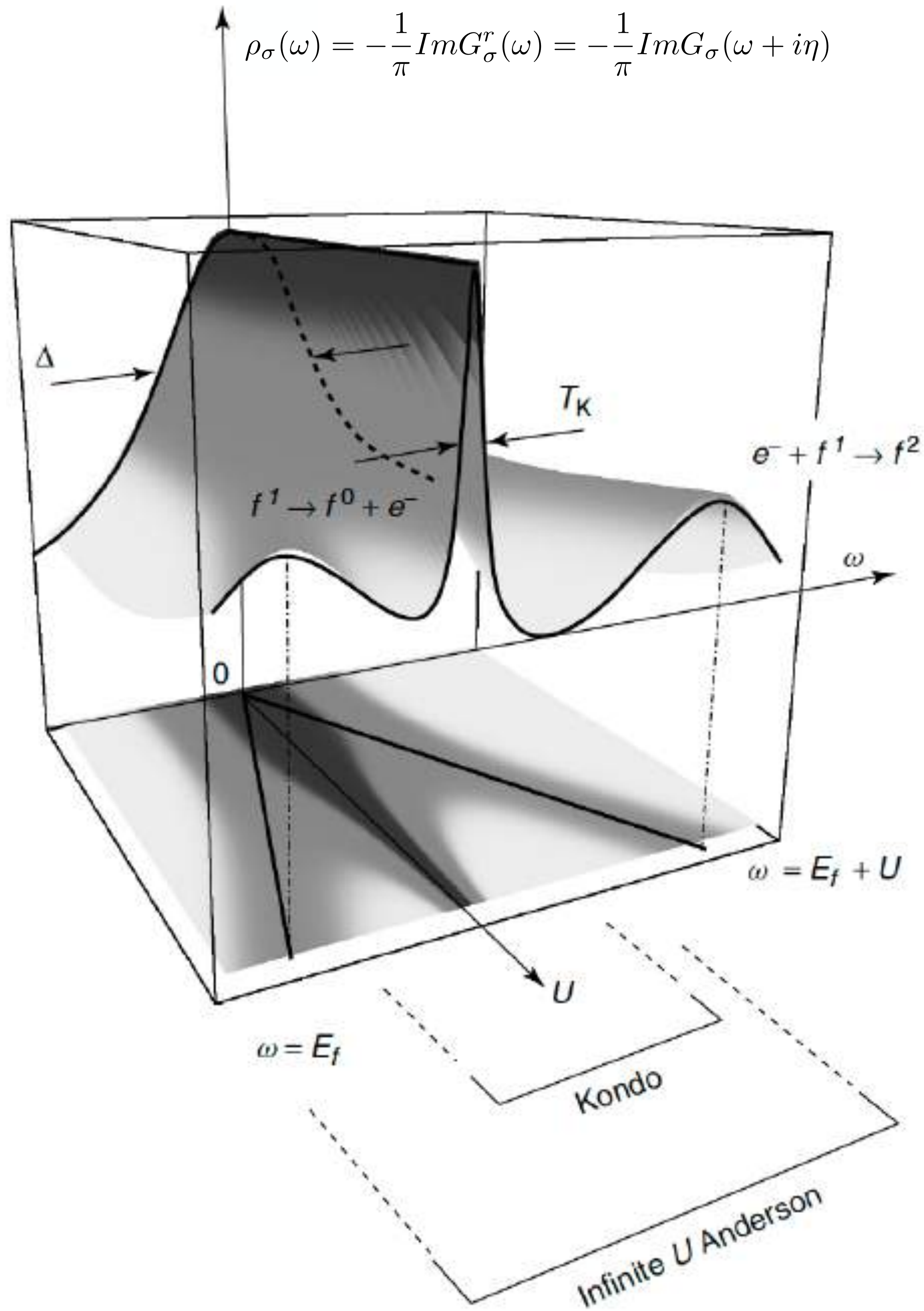


Image Piers Coleman, Rutgers

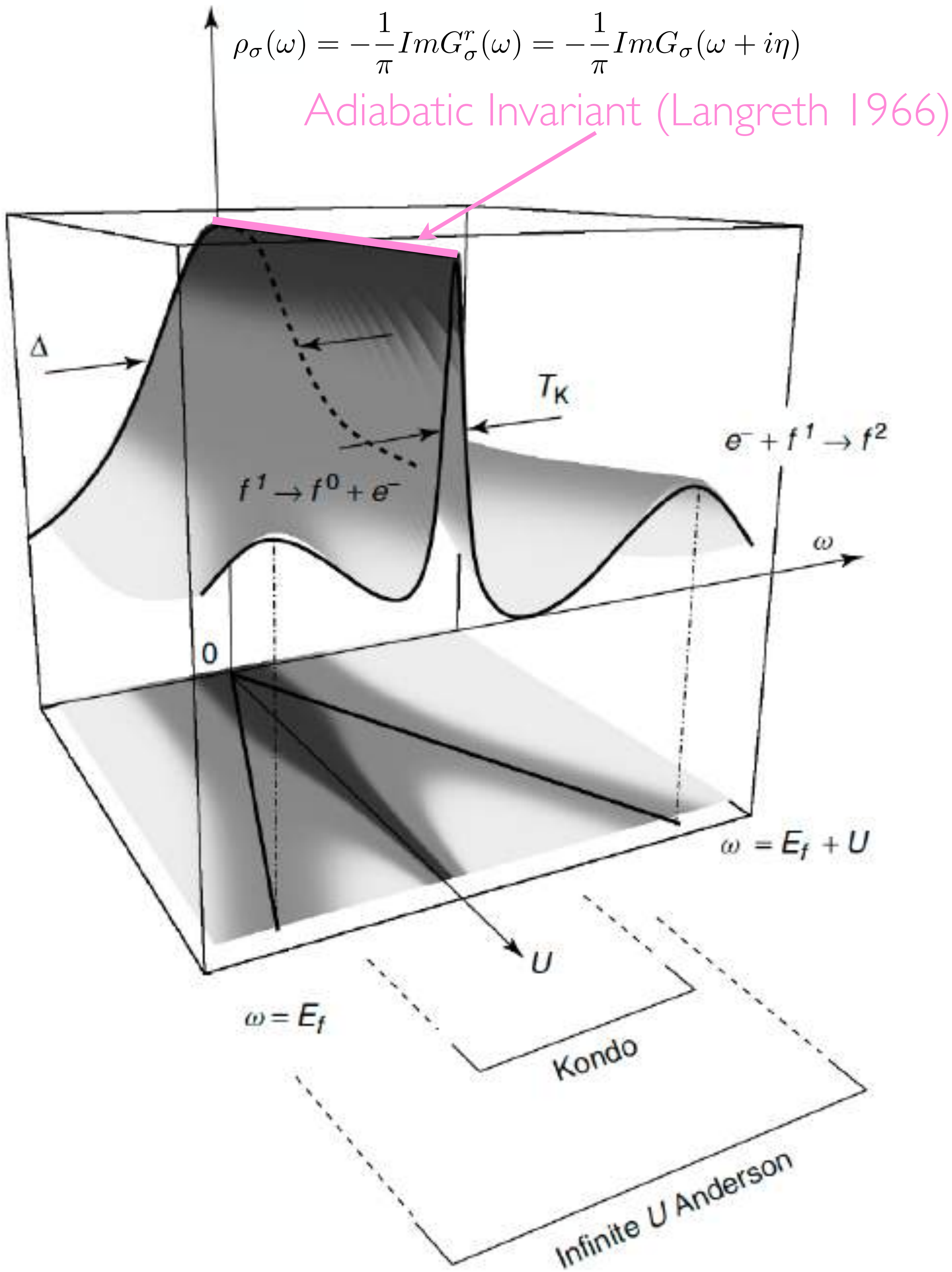
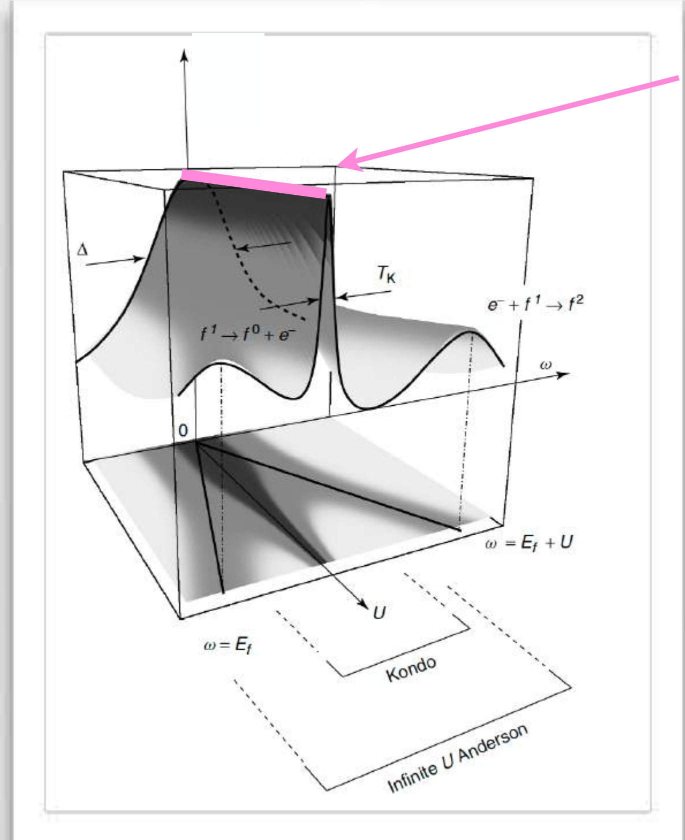


Image Piers Coleman, Rutgers

Adiabatic Invariant (Langreth 1966)



For large U we expect the two Hubbard bands, since the spectral weight is conserved each band should contribute 1/2.

$$\int A(\omega)d\omega = 1$$

Remarkably, the spectral function at $\omega = 0$ remains invariant as the interaction increases (always equal to the non-interacting value)

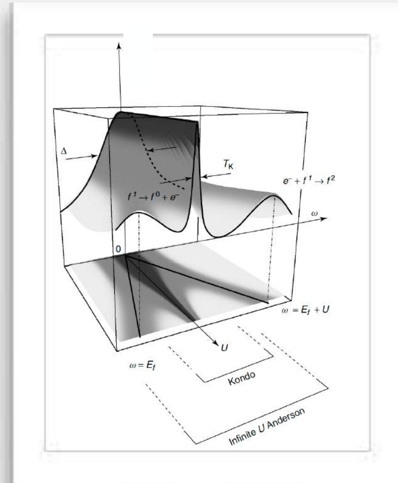
$$A(\omega = 0) = \frac{1}{\pi\Gamma} sin^2\delta$$

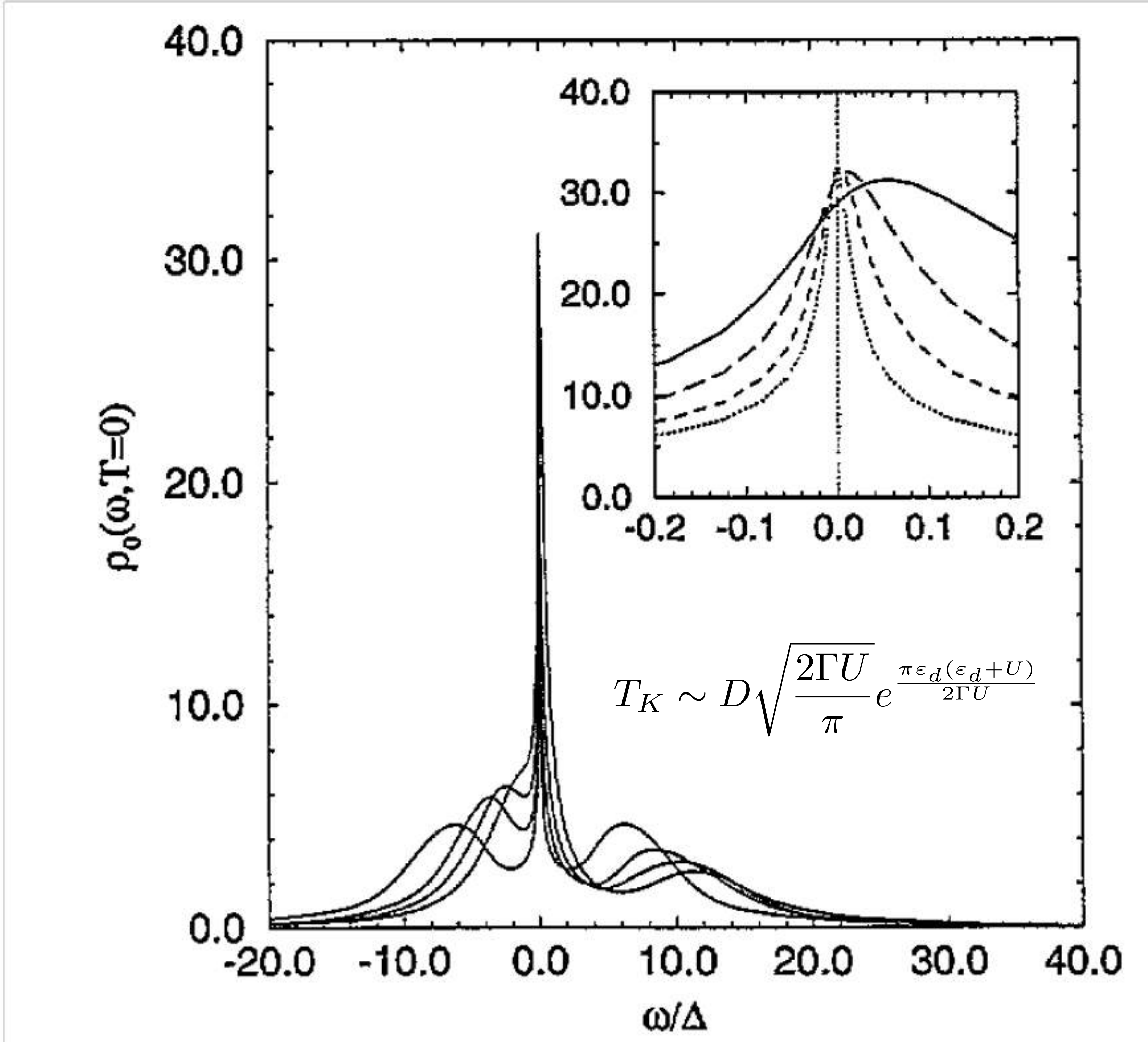
Deep result: the spectral function must always contain a peak of height $\frac{1}{\pi\Gamma} sin^2\delta$

and vanishingly small weight Z<<1 as U increases. This narrow resonance is a direct consequence of local Fermi liquid behaviour!

$$Z \sim \frac{T_K}{\Gamma}$$

Abrikosov-Suhl resonance

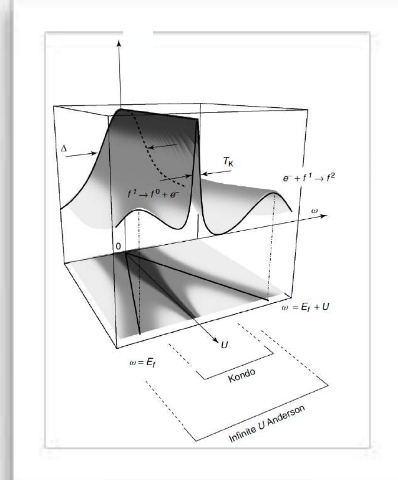


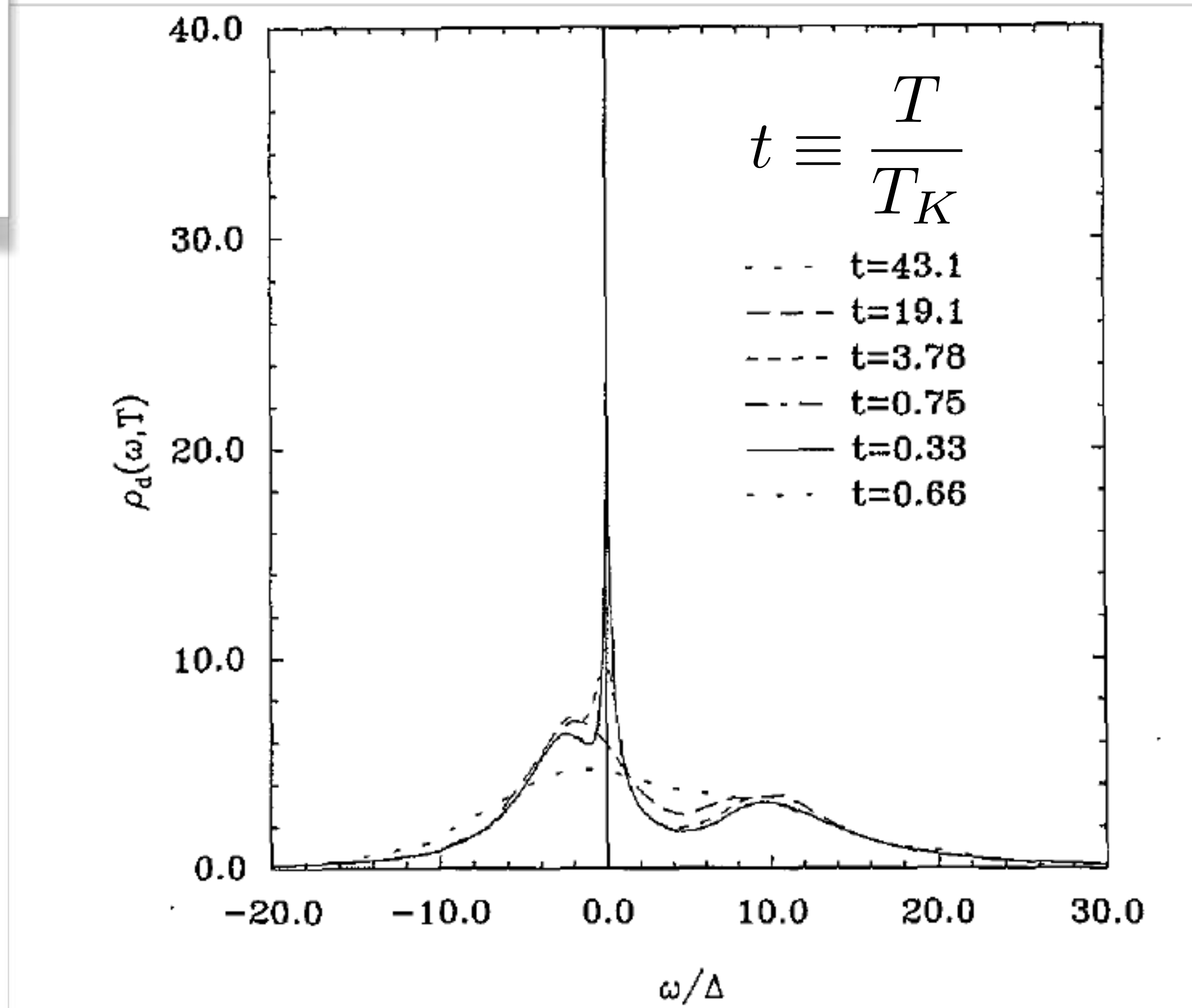


$$T_K \sim D\sqrt{\frac{2\Gamma U}{\pi}}e^{\frac{\pi\varepsilon_d(\varepsilon_d+U)}{2\Gamma U}}$$

Level position

Abrikosov-Suhl resonance





Temperature dependence

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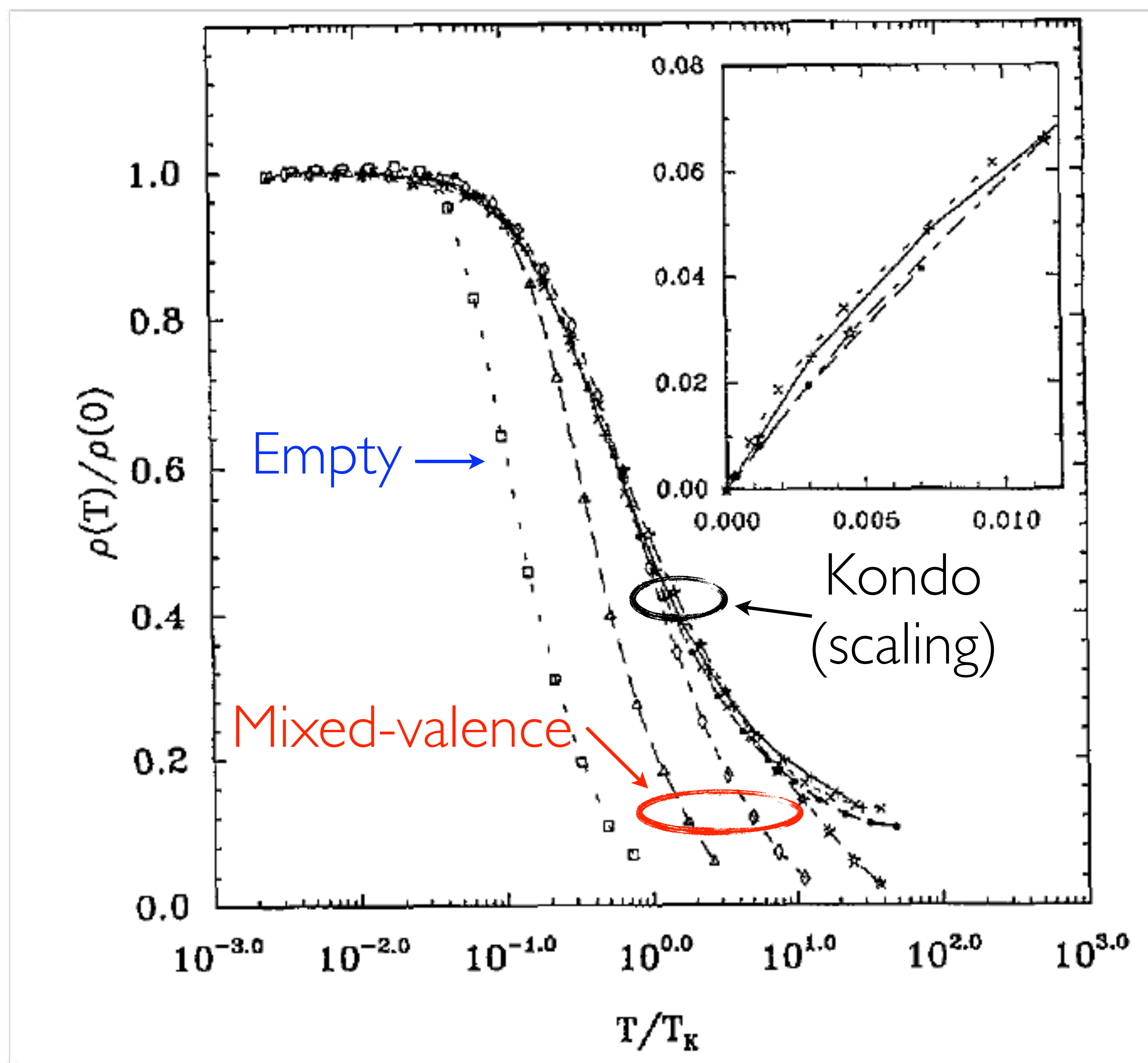
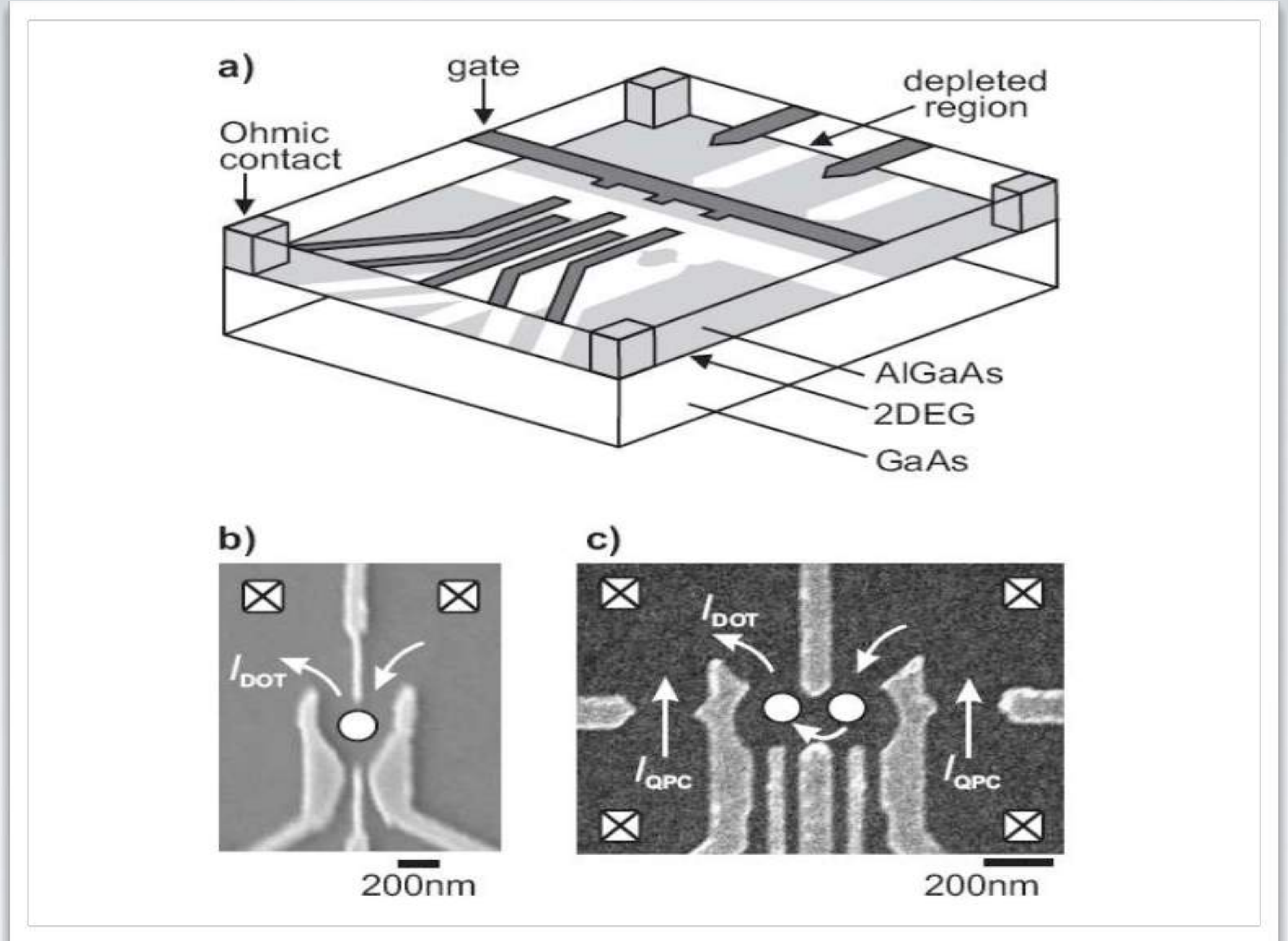

ρ(T)/ρ(0)
T/T_K
Empty
Kondo
(scaling)
Mixed-valence
1.0
0.8
0.6
0.4
0.2
0.0
10^-3.0
10^-2.0
10^-1.0
10^0.0
10^1.0
10^2.0
10^3.0
0.08
0.06
0.04
0.02
0.00
0.000
0.005
0.010

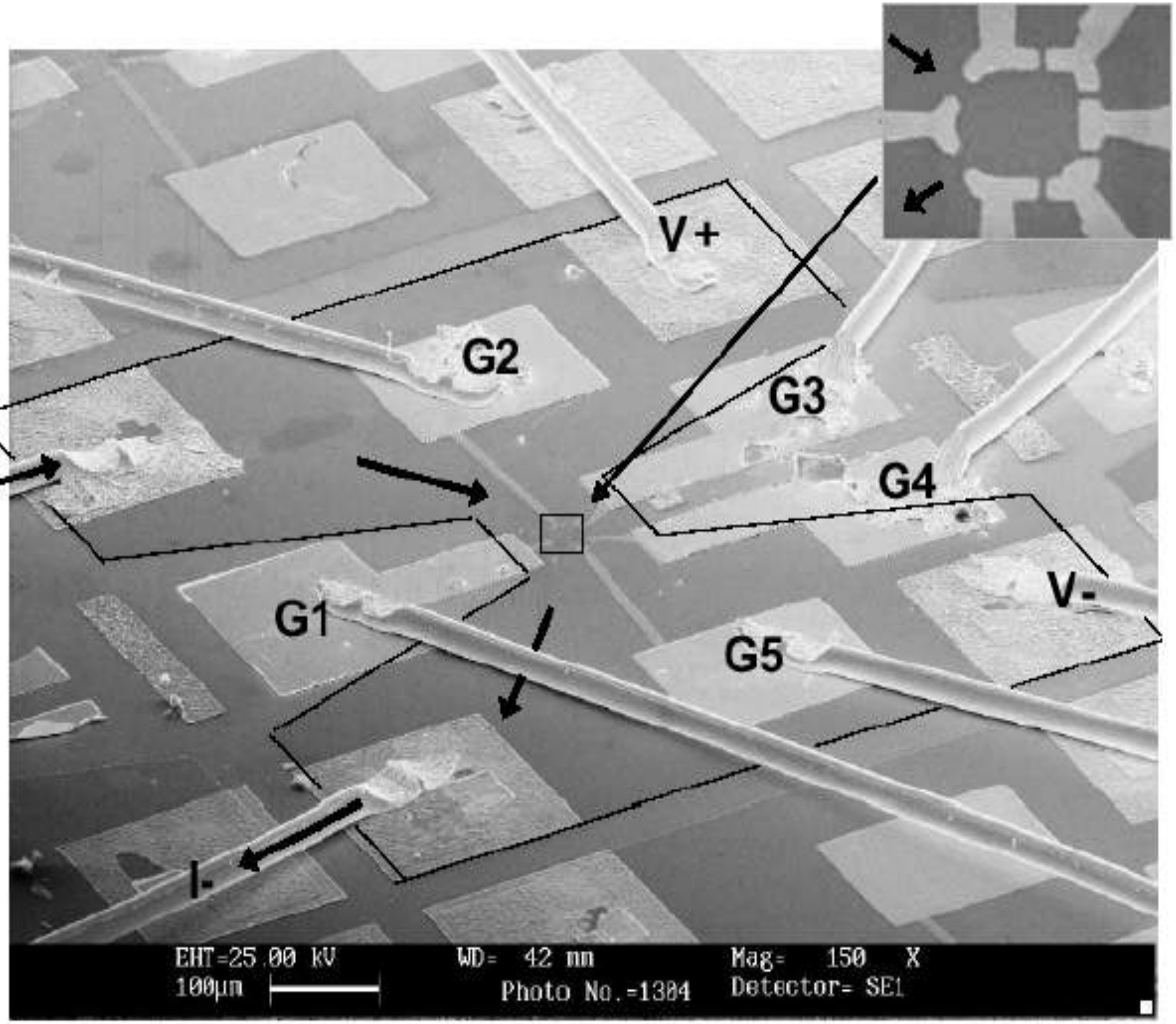
This lecture

- Kondo effect in mesoscopic systems: quantum dots and nanotubes, non-equilibrium effects, etc.

- New developments: hybrid systems (competition between superconductivity and Kondo), Shiba states, etc.



Quantum Dots



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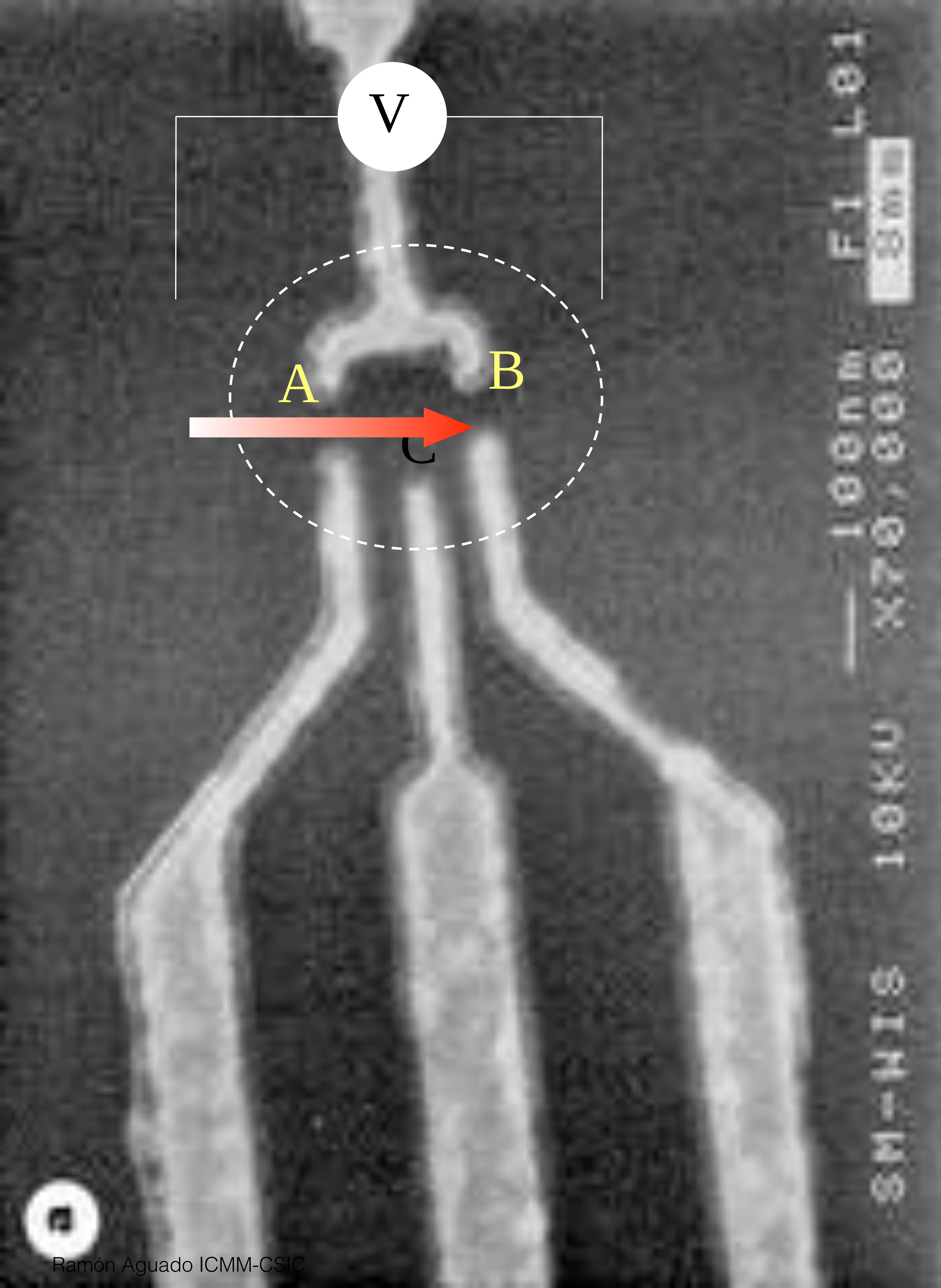
V
A
B
C
100nm
X70,000
10KU
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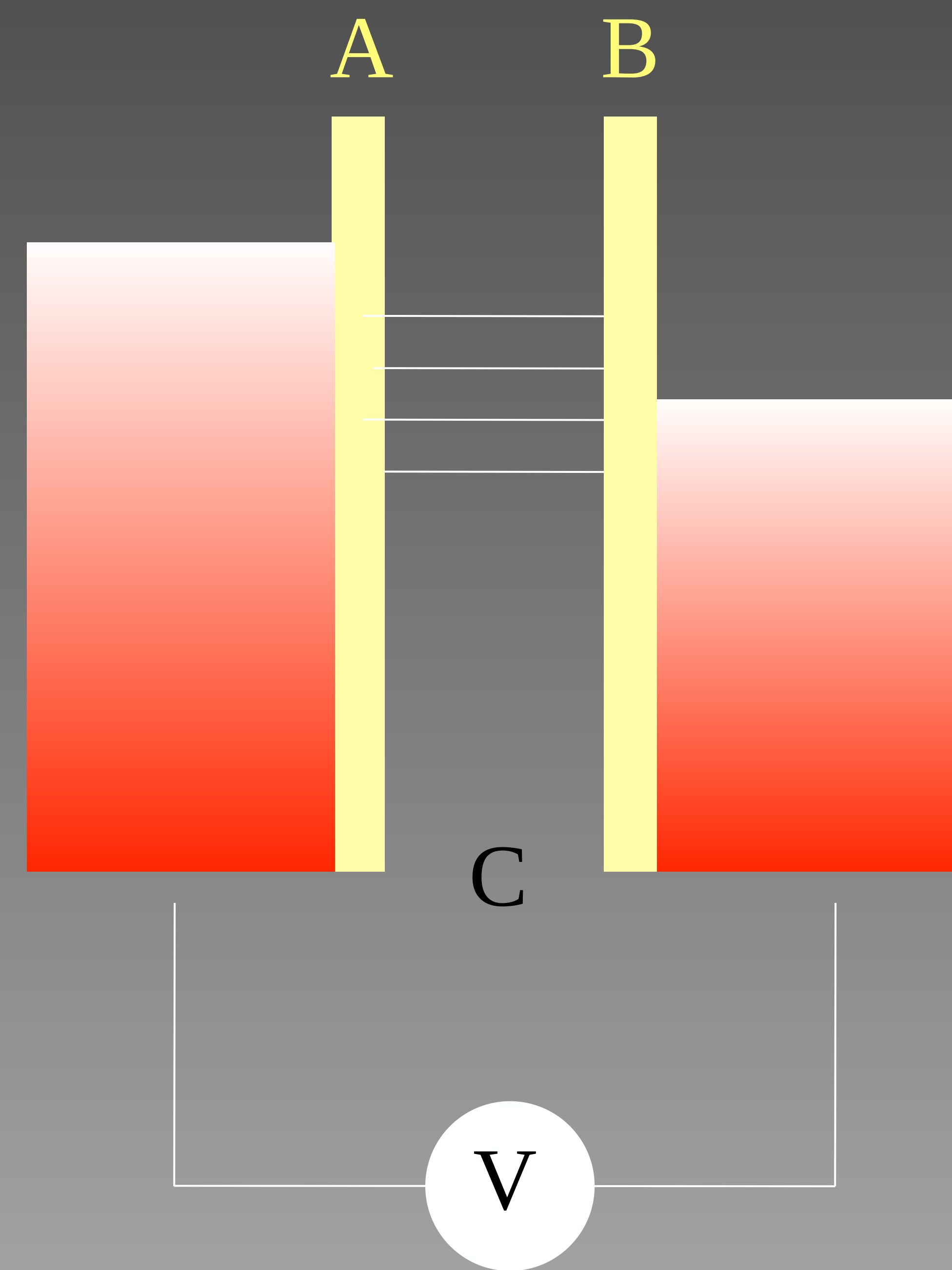
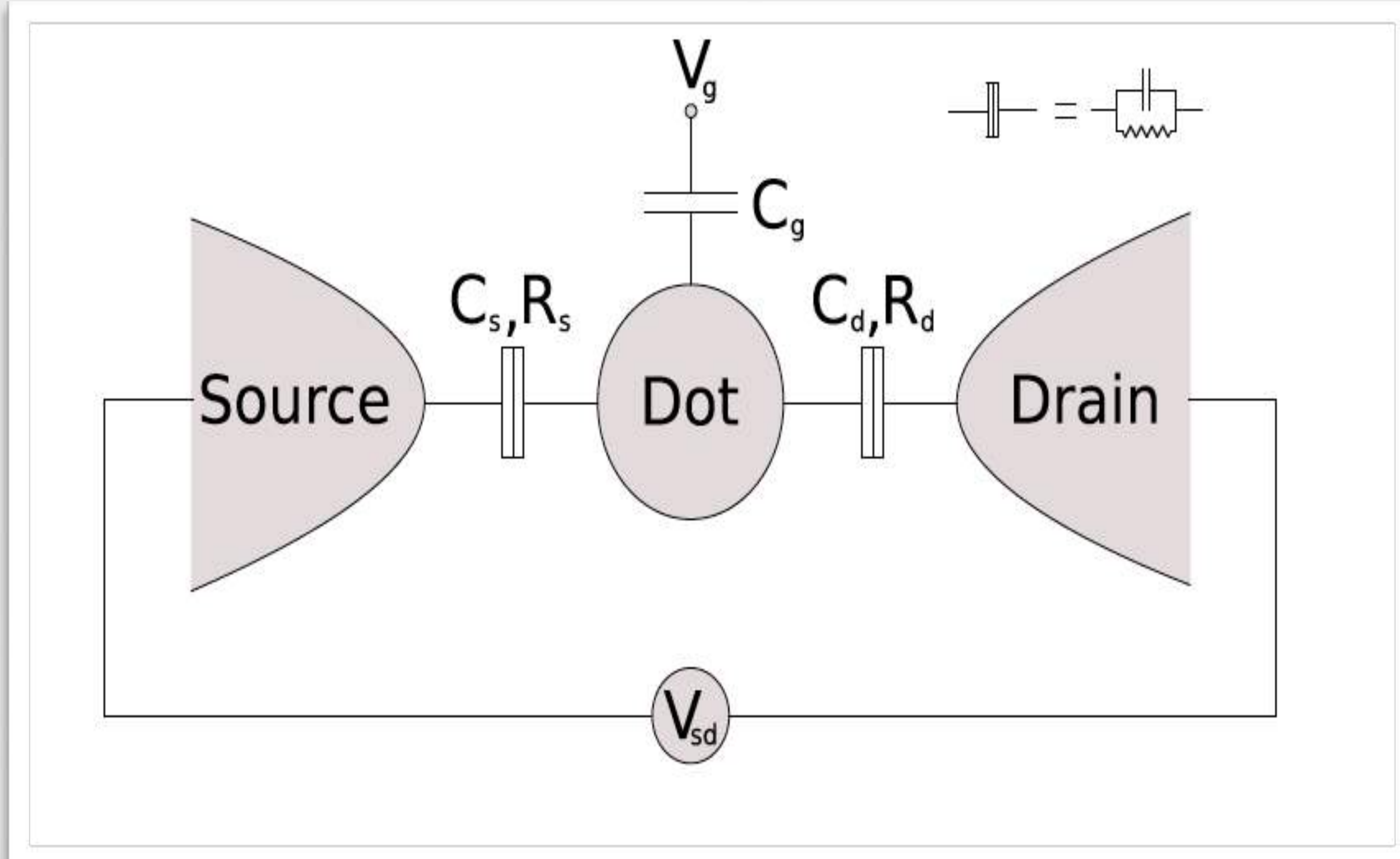

A
B
C
V

SINGLE ELECTRON TRANSISTOR



CHARGING ENERGY

Isolated island: Charge is quantized

$$Q = Ne$$

Energy

$$E = \frac{Q^2}{2C} = \frac{e^2}{2C}N^2$$

Island close to a gate: the gate shifts the potential and induces charge

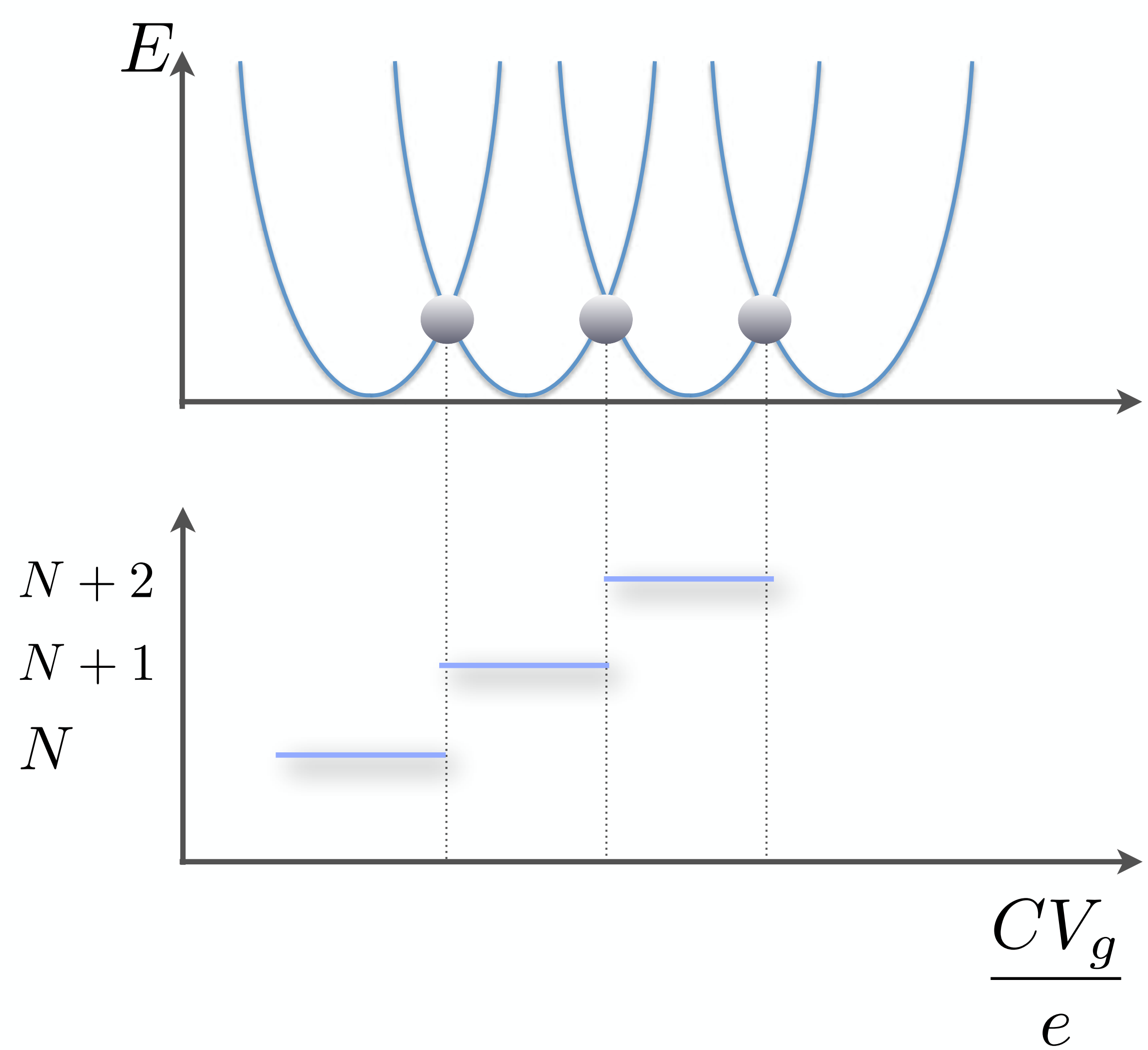
$$E = \frac{e^2}{2C}(N - \frac{C_g V_g}{e})^2$$

CHARGING ENERGY

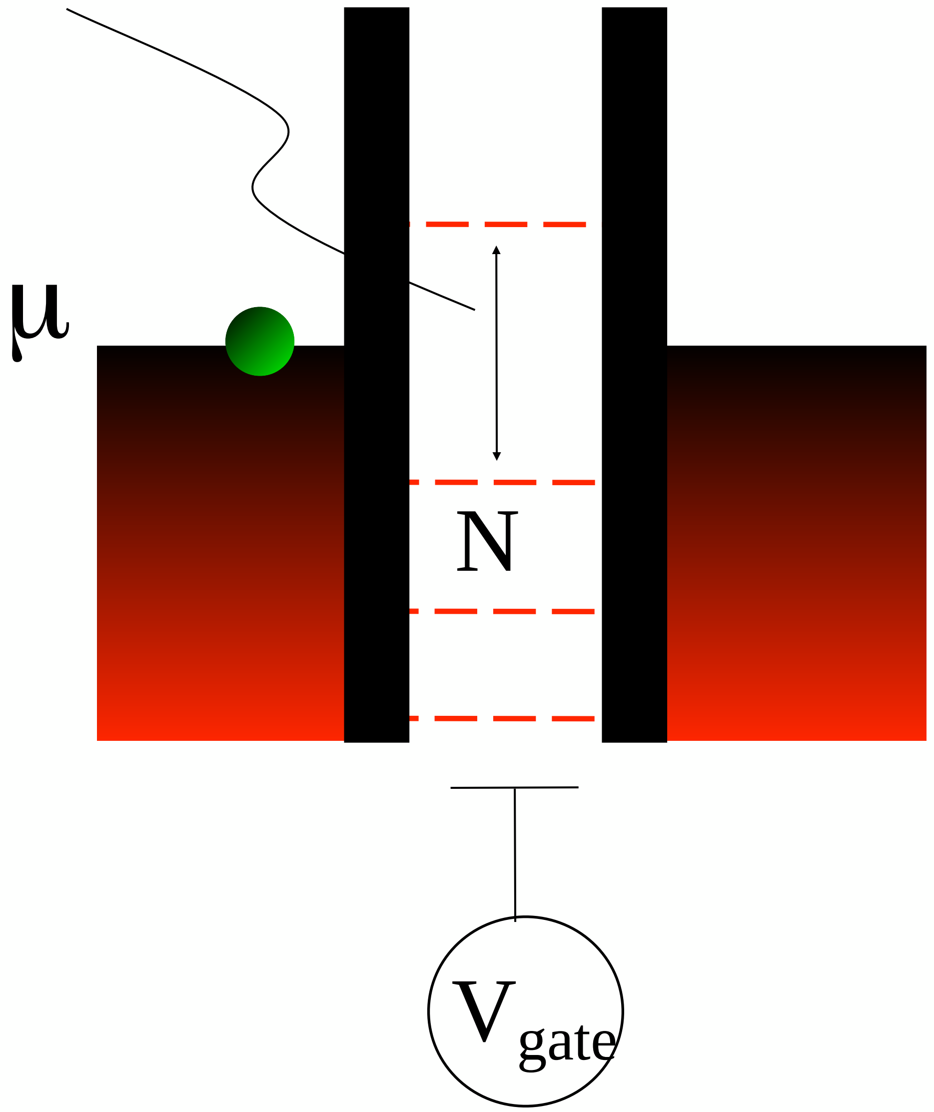
$$E = \frac{e^2}{2C}(N - \frac{C_g V_g}{e})^2$$

Electrons can enter the island when:

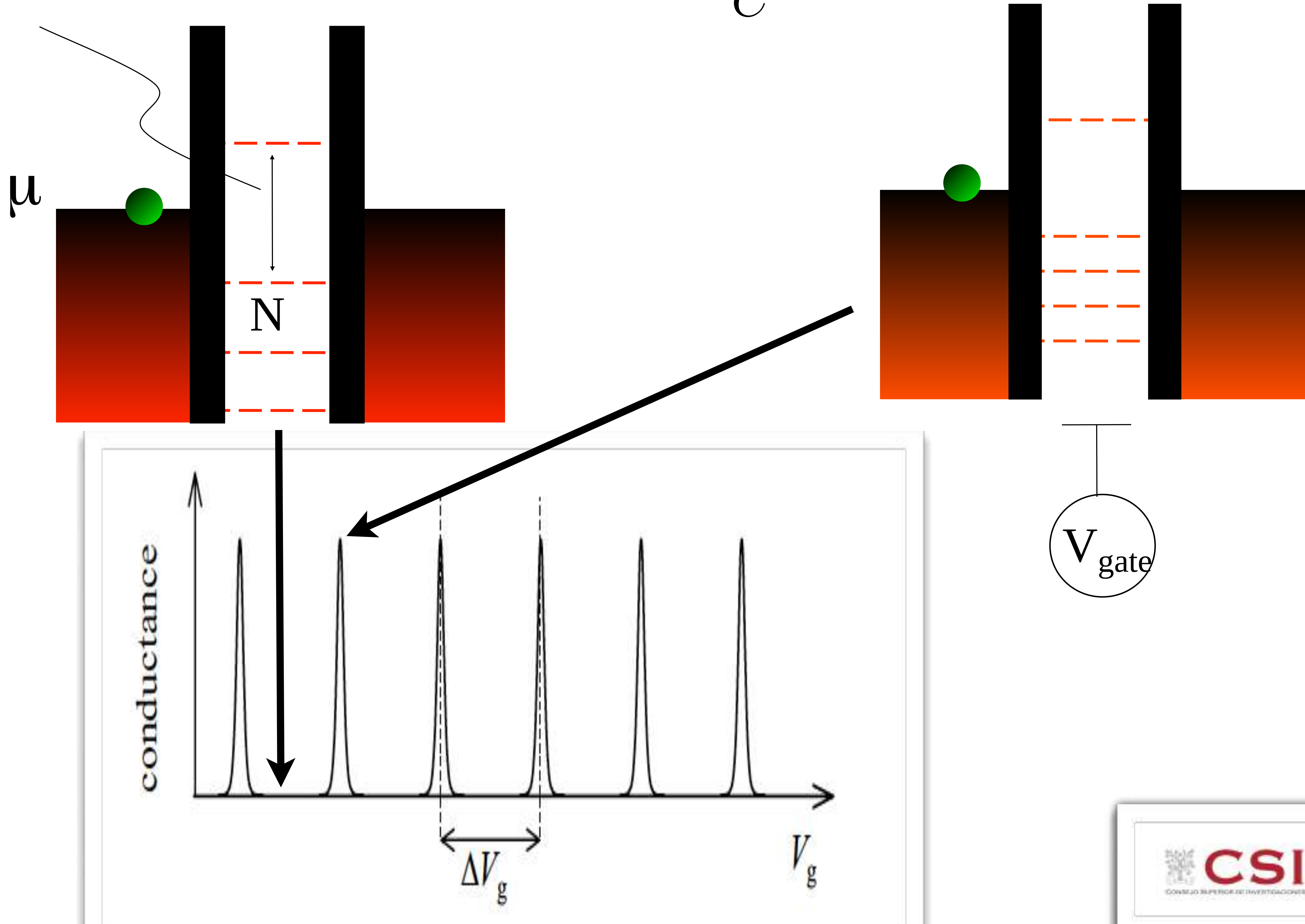
$$E(N+1, V_g) = E(N, V_g) = \frac{e^2}{C}$$



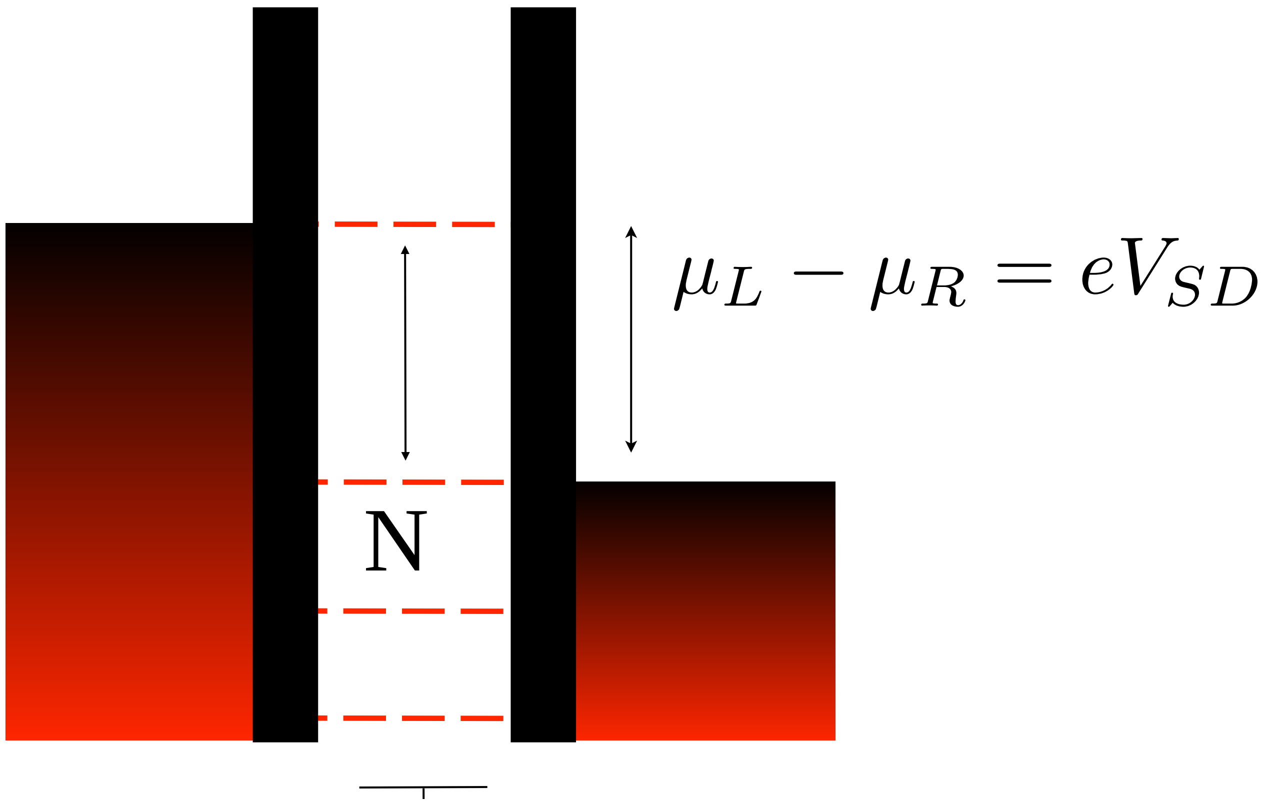
$$\mu_{dot}(N+1, V_g) - \mu_{dot}(N, V_g) = \frac{e^2}{C} + \Delta\varepsilon$$

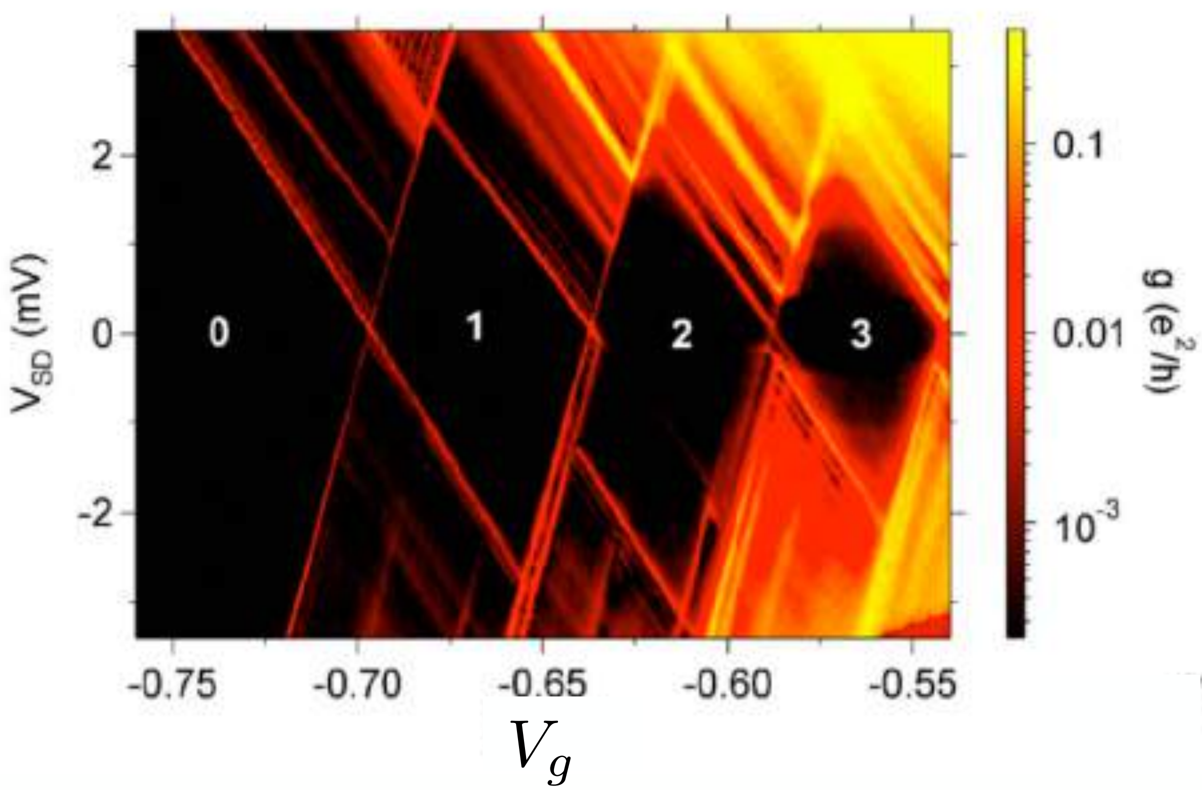


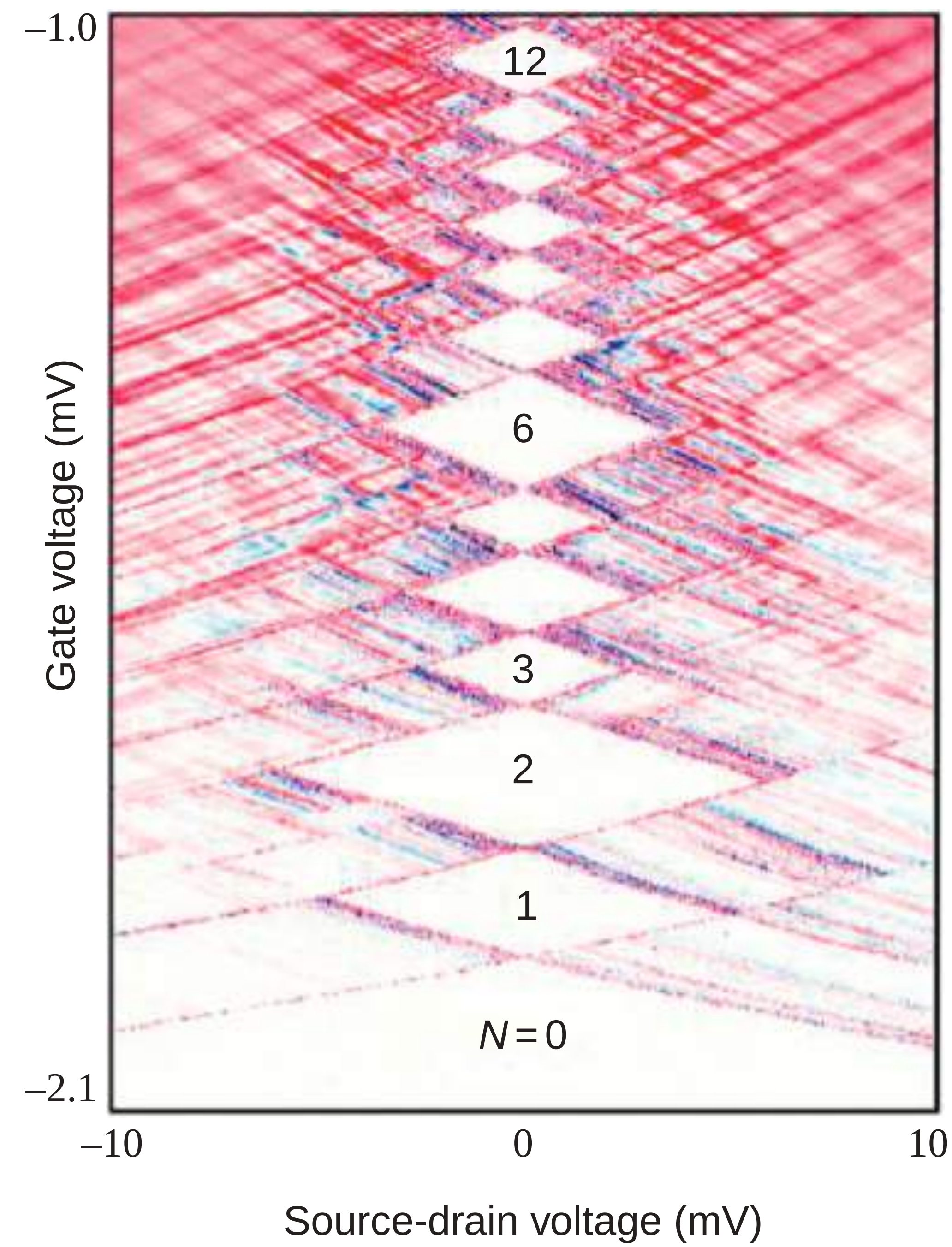
$$\mu_{dot}(N+1, V_g) - \mu_{dot}(N, V_g) = \frac{e^2}{C} + \Delta\varepsilon$$



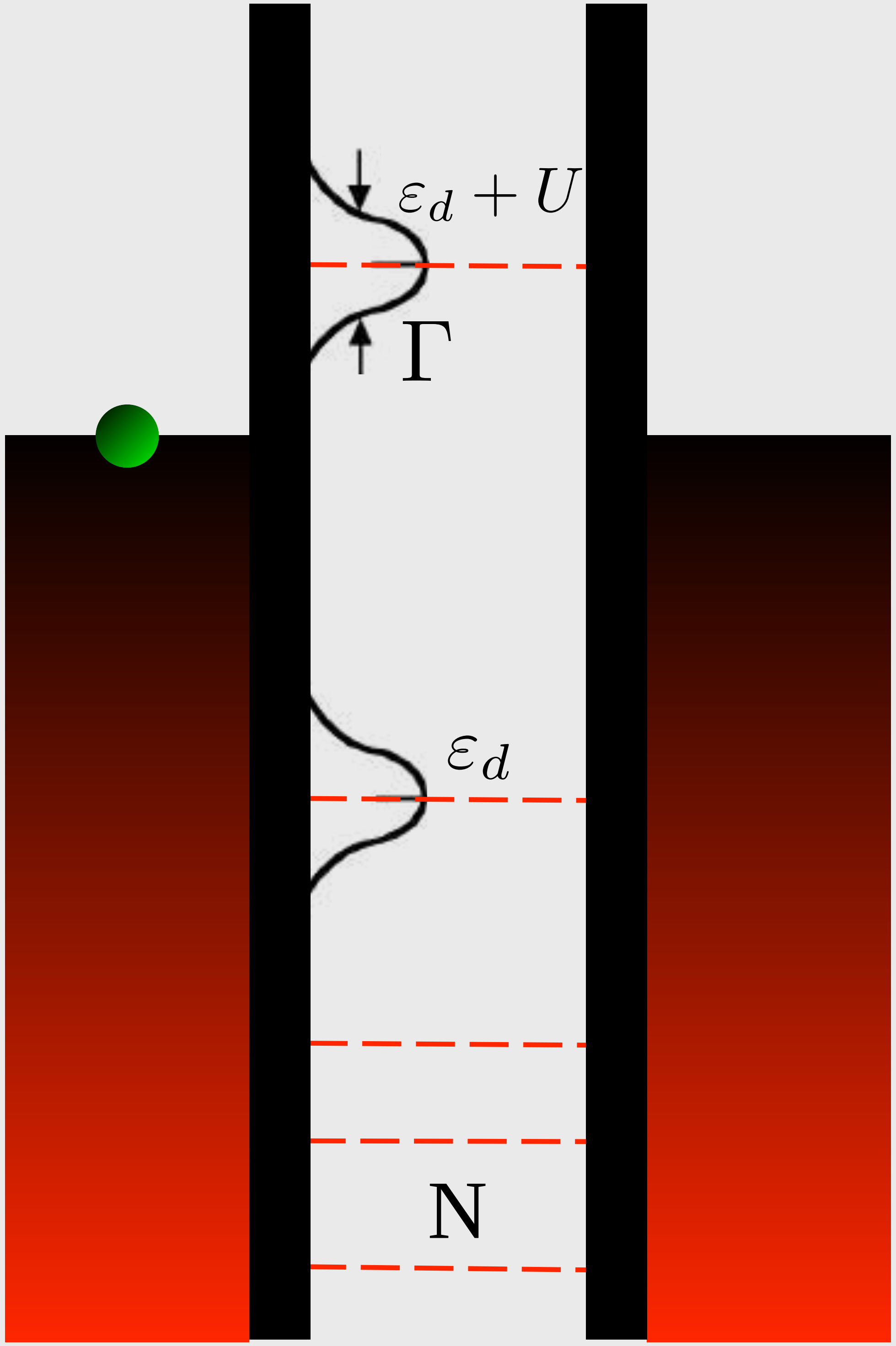
$$\mu_{dot}(N+1, V_g) - \mu_{dot}(N, V_g) = \frac{e^2}{C} + \Delta\varepsilon$$ Coulomb Diamonds







Small quantum dots are artificial realizations of the Anderson model!!



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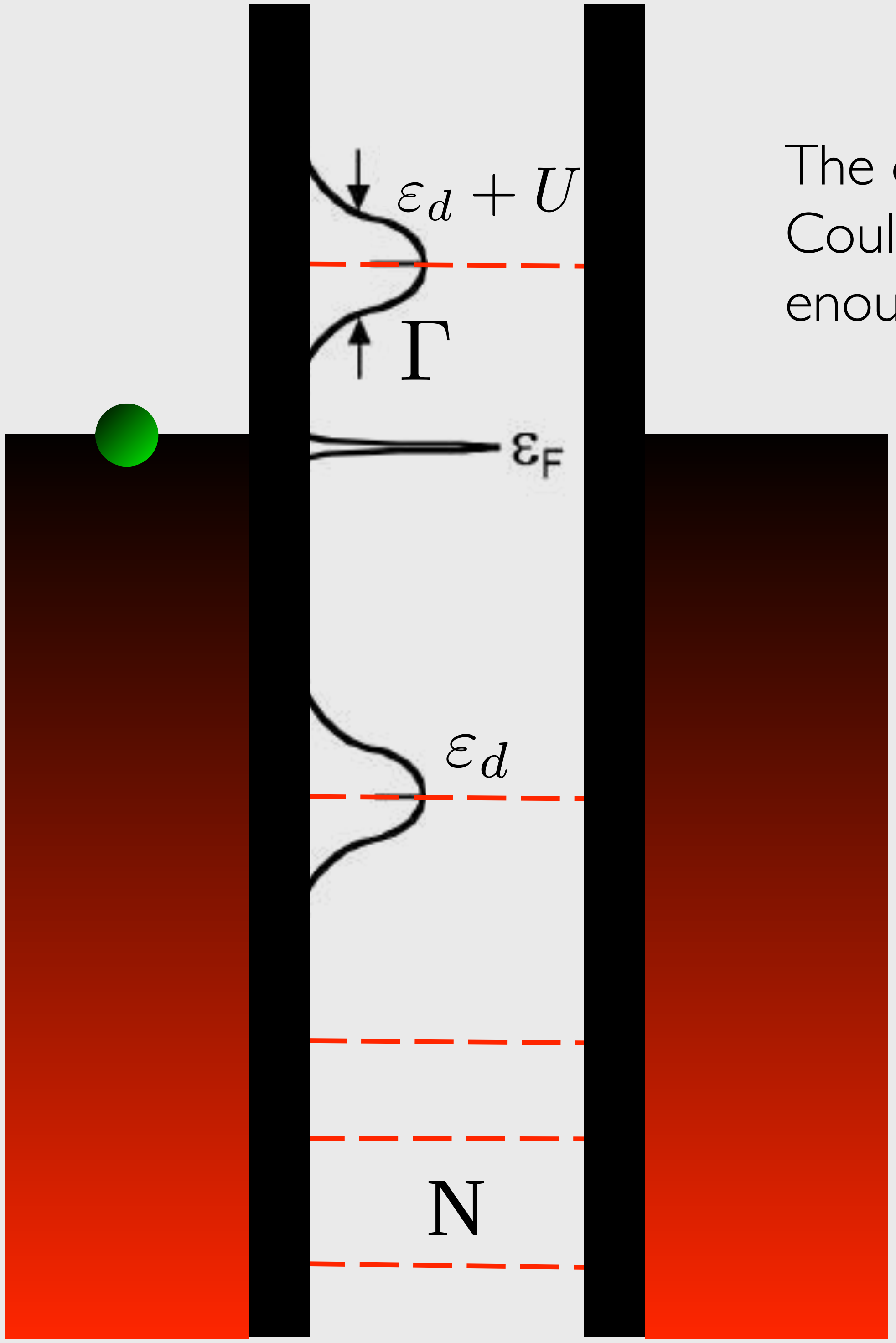
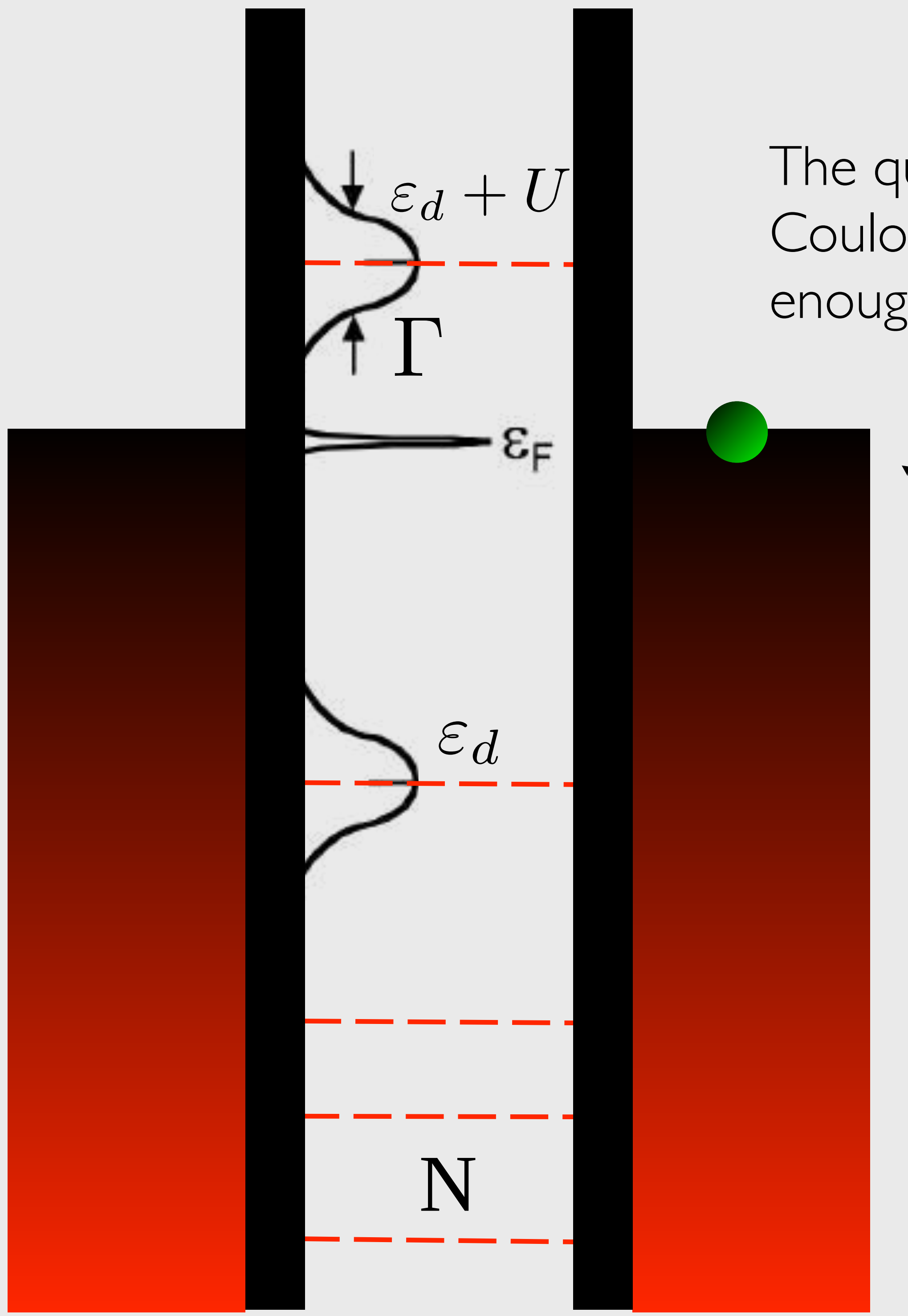


The question is can we go from Coulomb Blockade to Kondo at low enough temperatures?

Small quantum dots are artificial realizations of the Anderson model!!



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Enhancement of conductance as a consequence of the Kondo resonance: note that current flows at fixed occupancy (spin flips)

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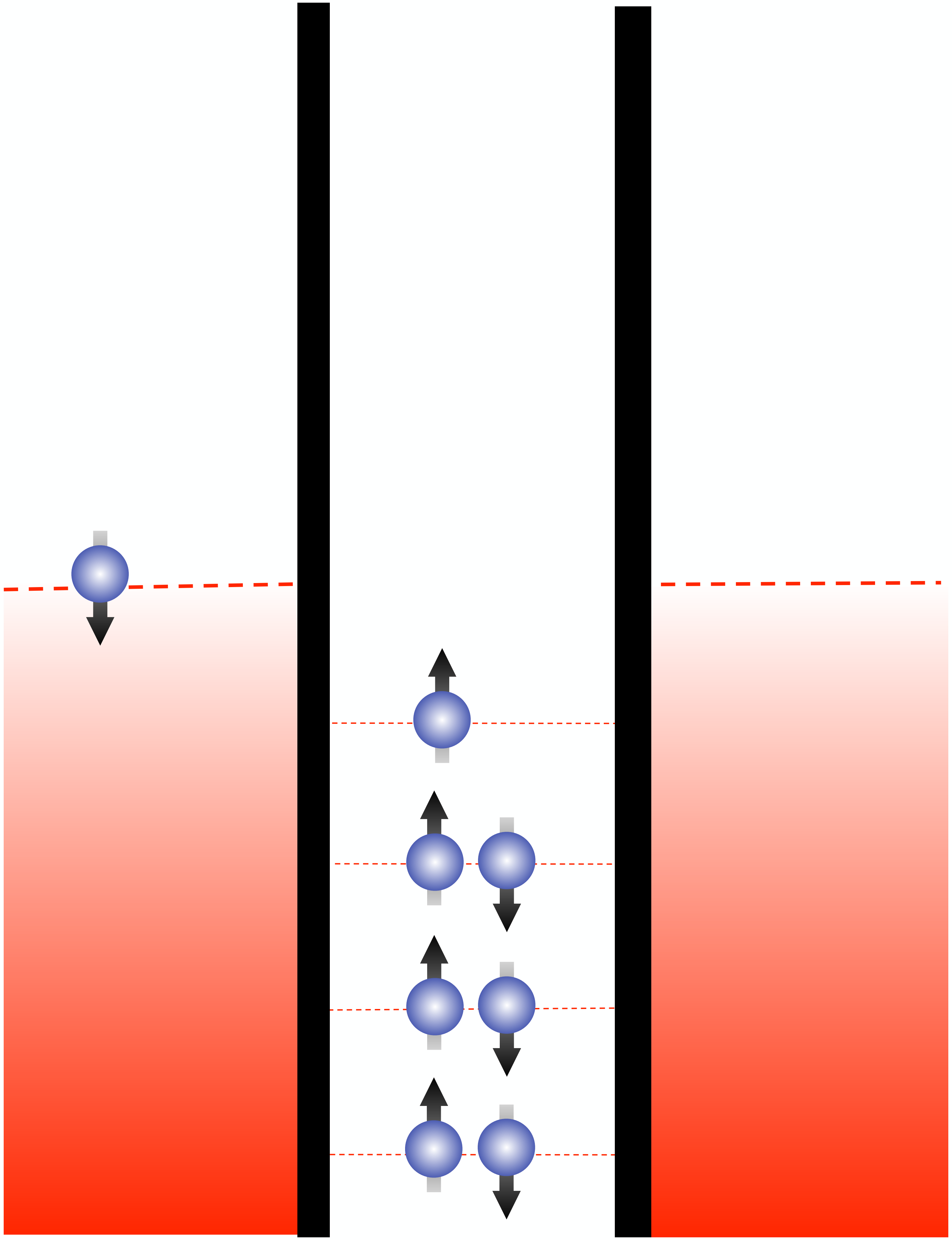
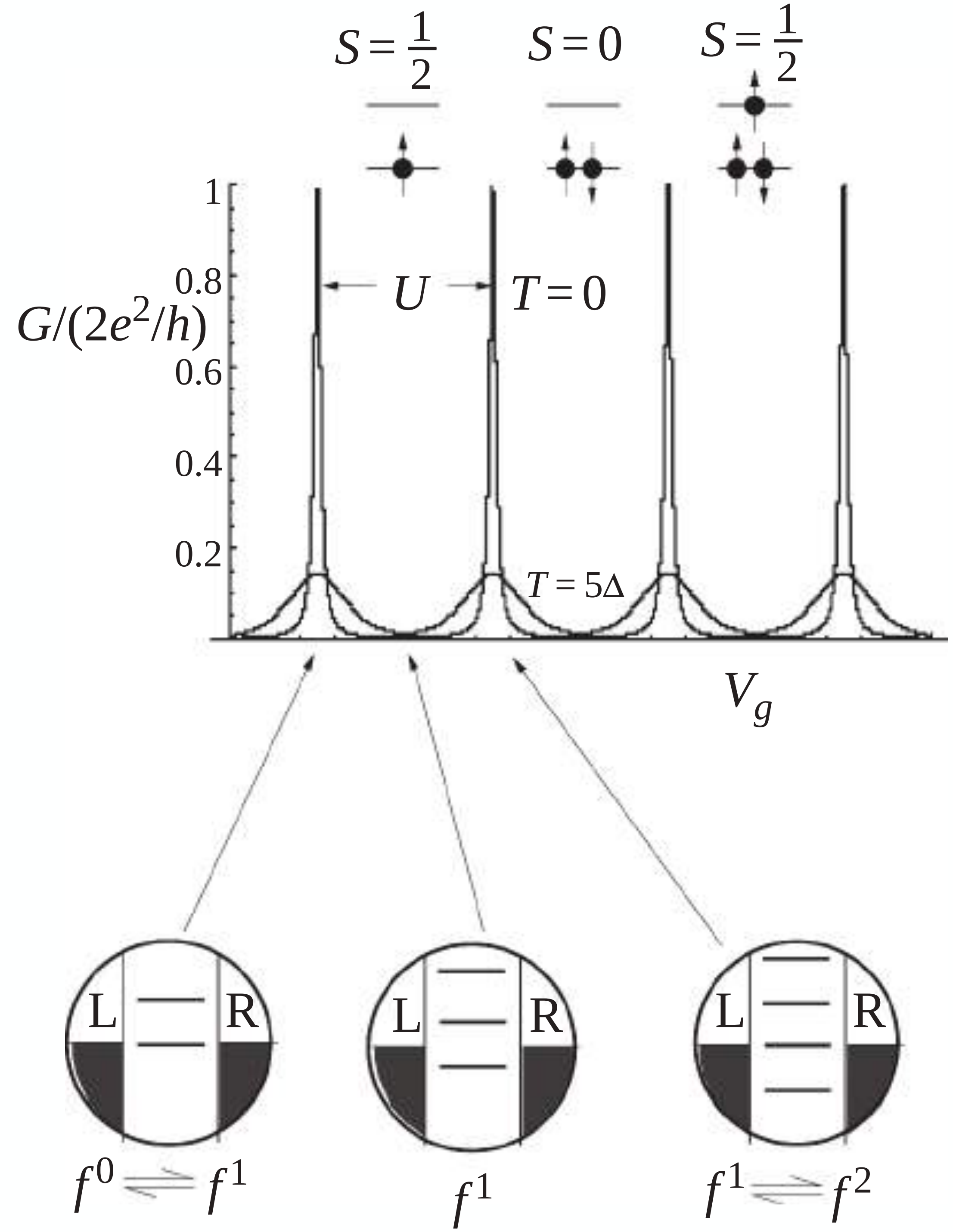
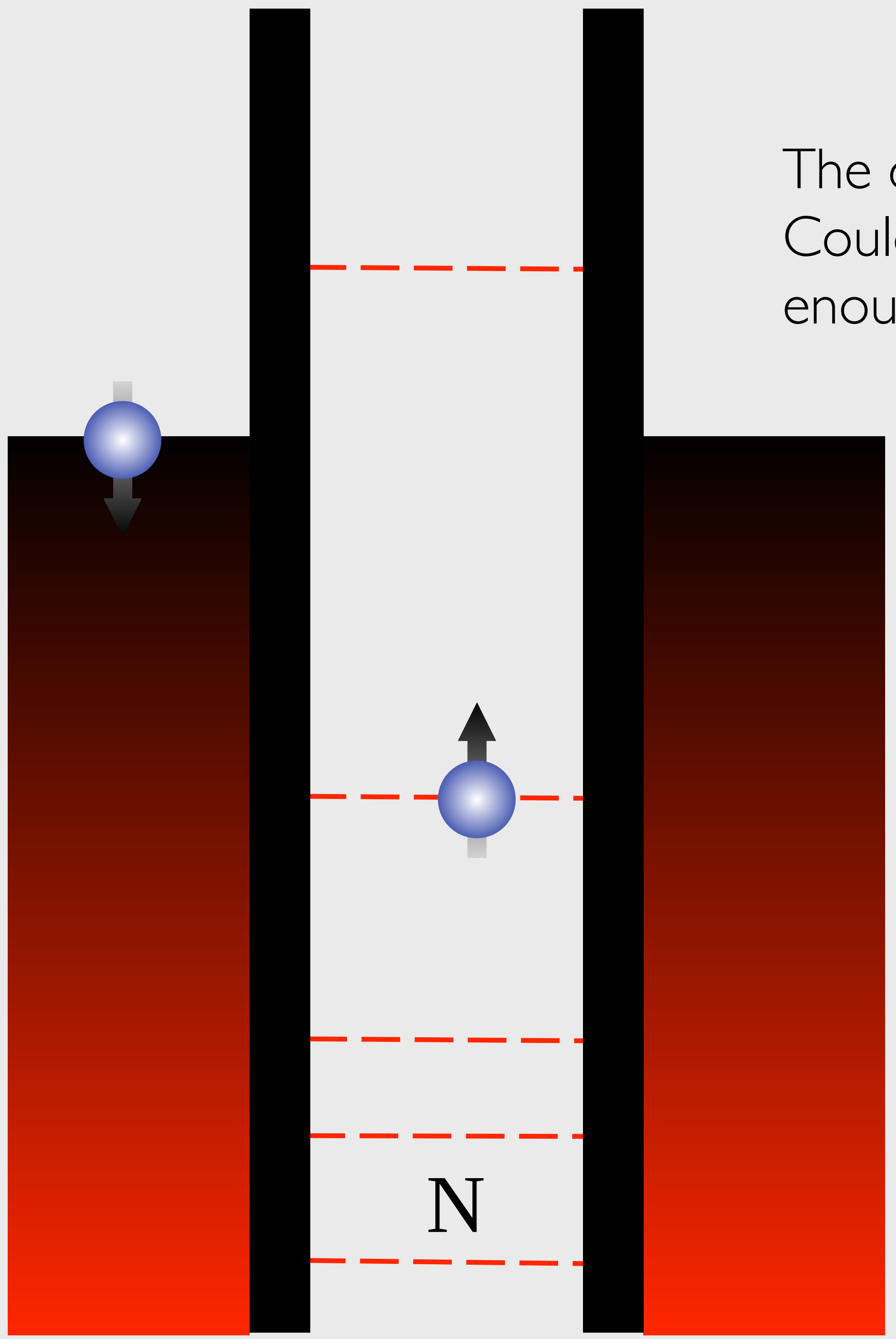




$S=\frac{1}{2}$
$S=0$
$S=\frac{1}{2}$
$G/(2e^2/h)$
1
0.8
0.6
0.4
0.2
U
$T=0$
$T=5\Delta$
V_g
L
R
$f^0 \rightleftharpoons f^1$
f^1
$f^1 \rightleftharpoons f^2$

Small quantum dots are artificial realizations of the Anderson model!!



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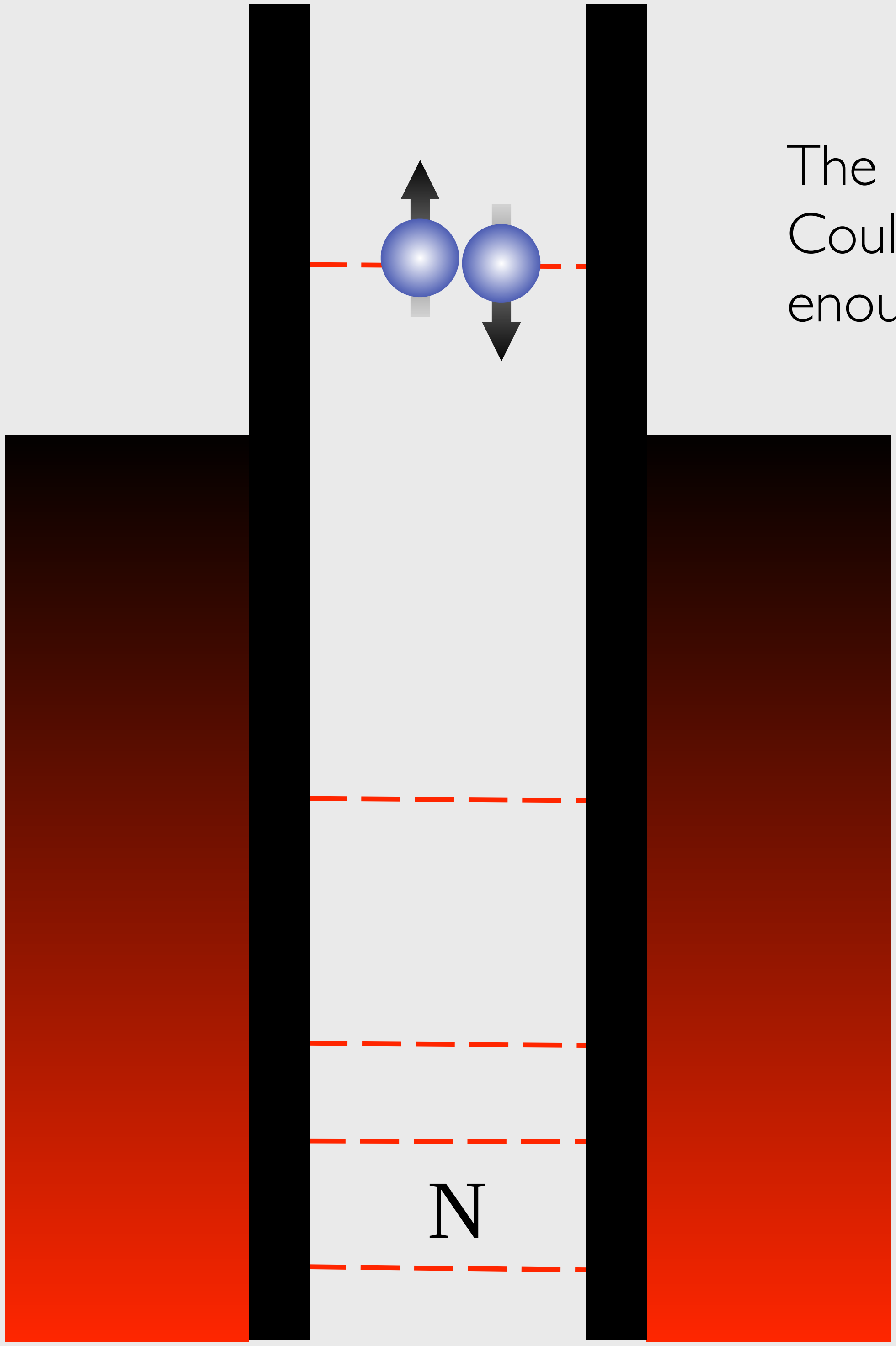

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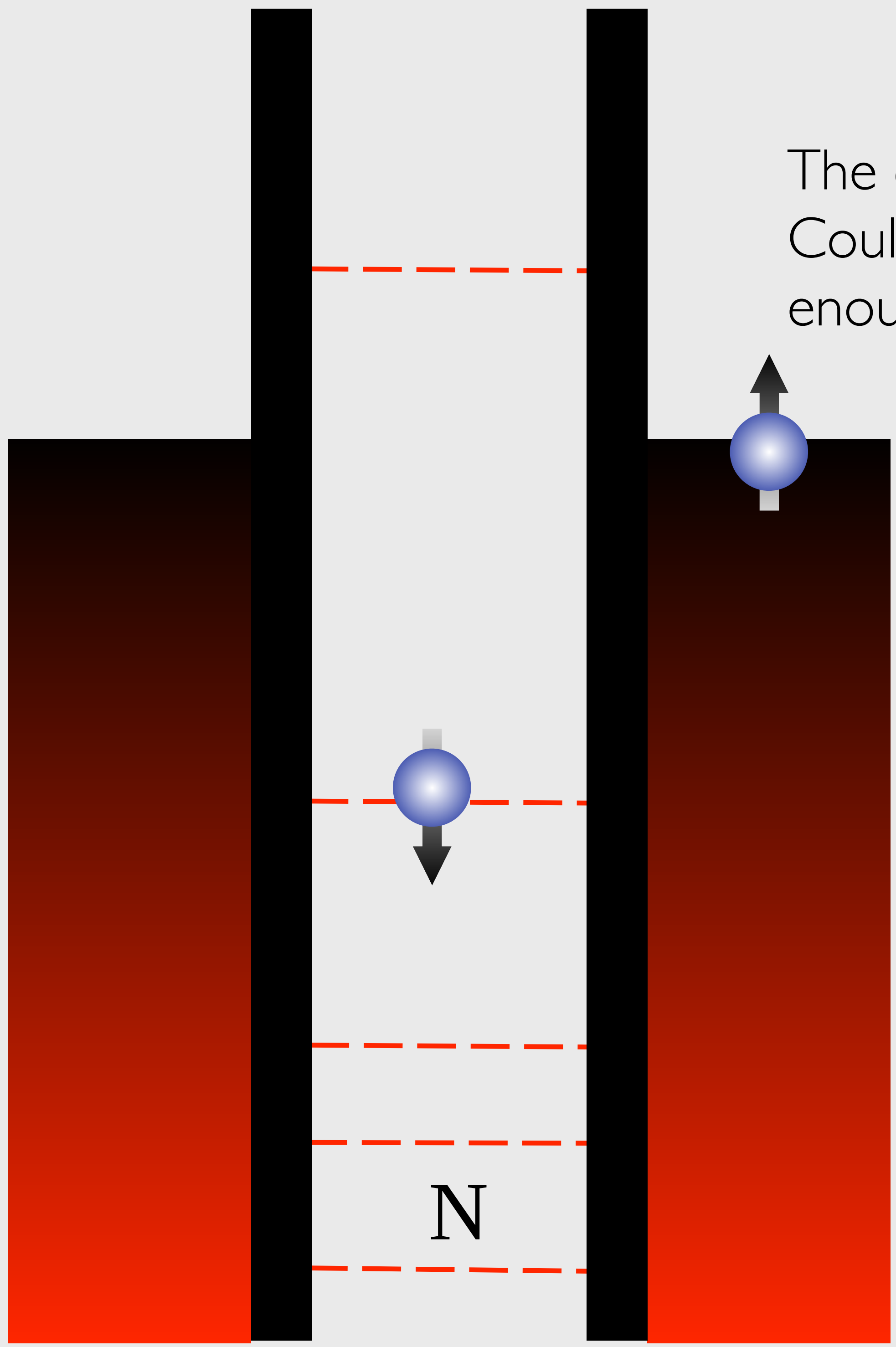

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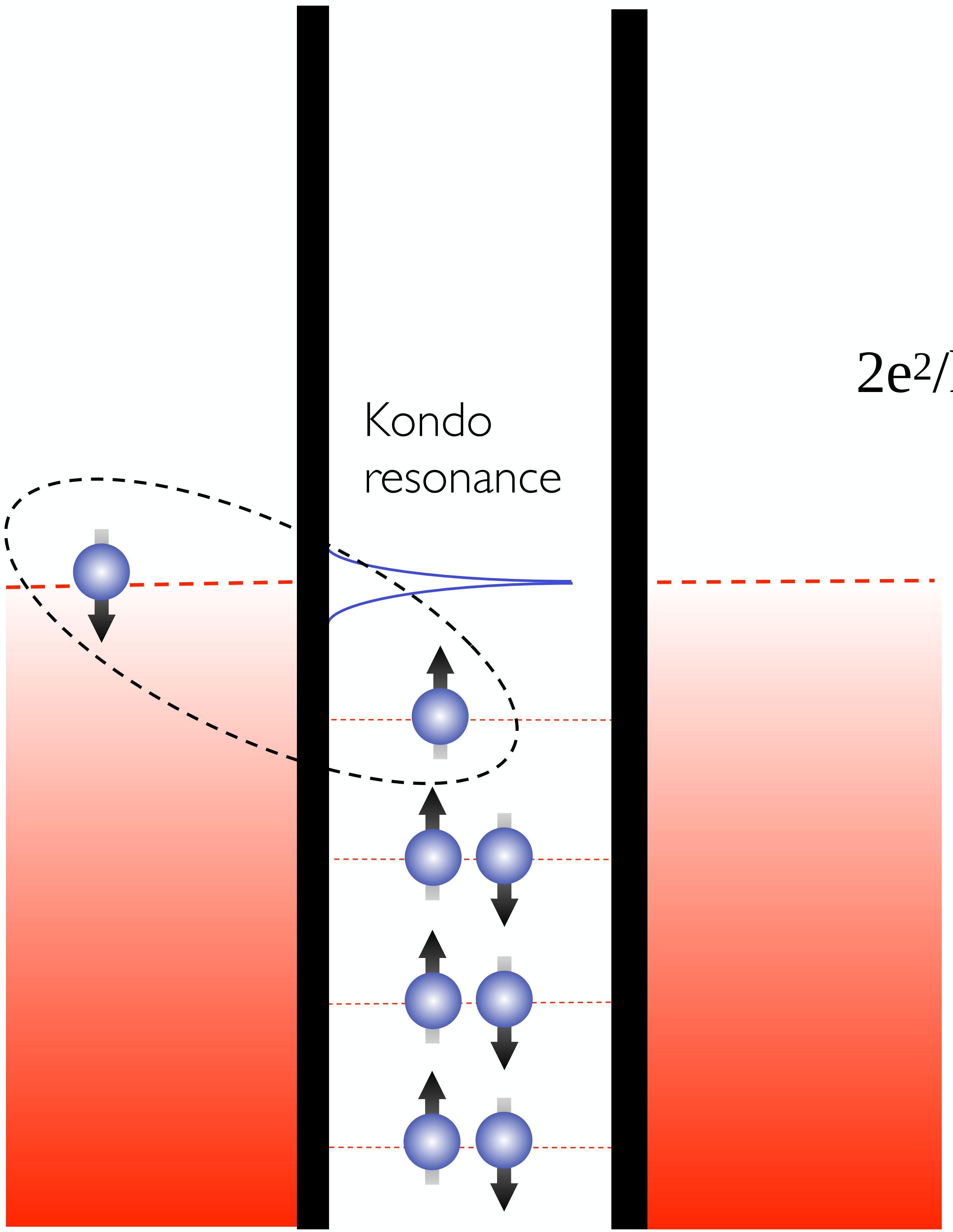
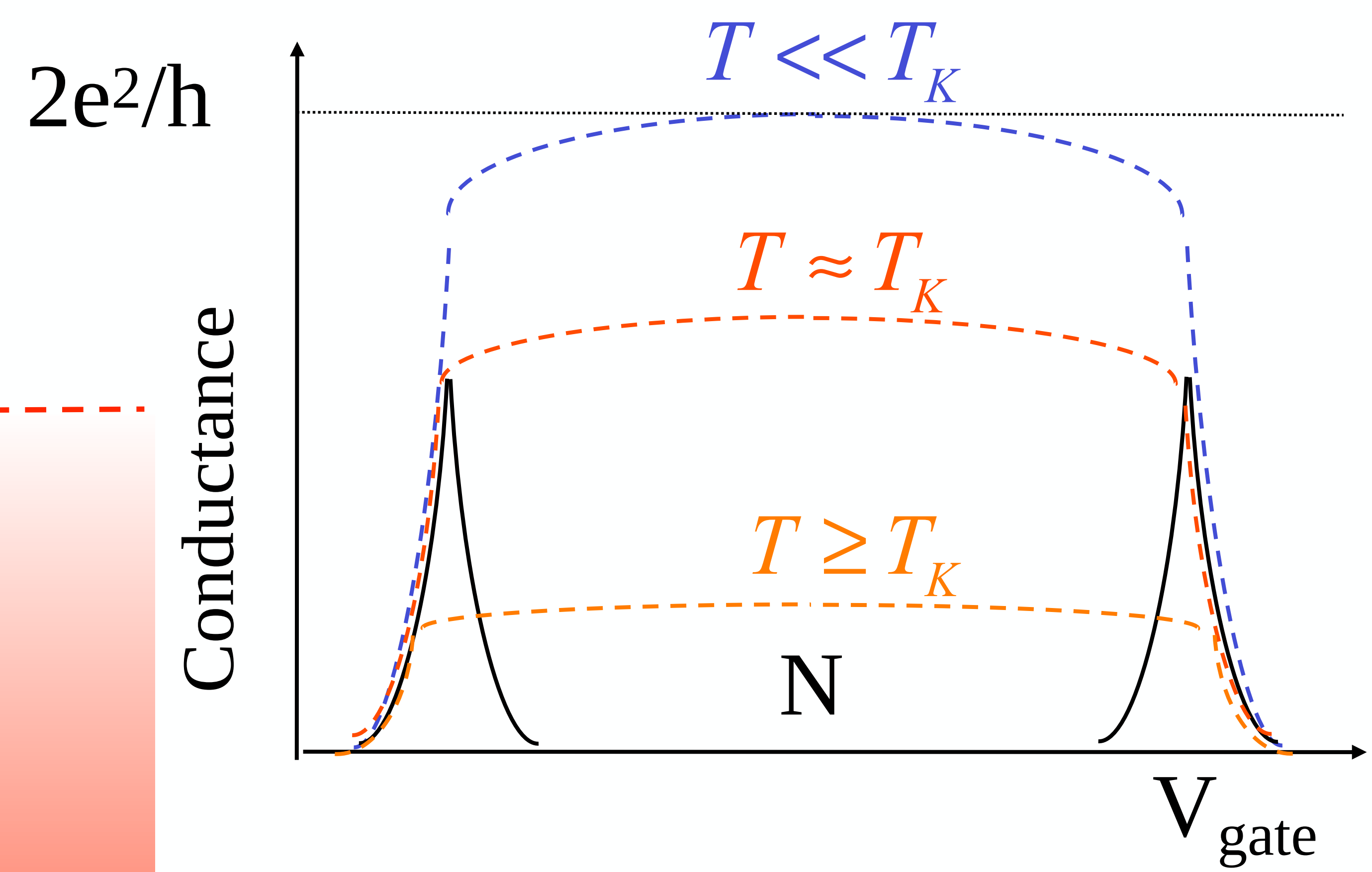
Enhancement of conductance as a consequence of the Kondo resonance: **note that current flows at fixed occupancy** (spin flips)


Kondo
resonance

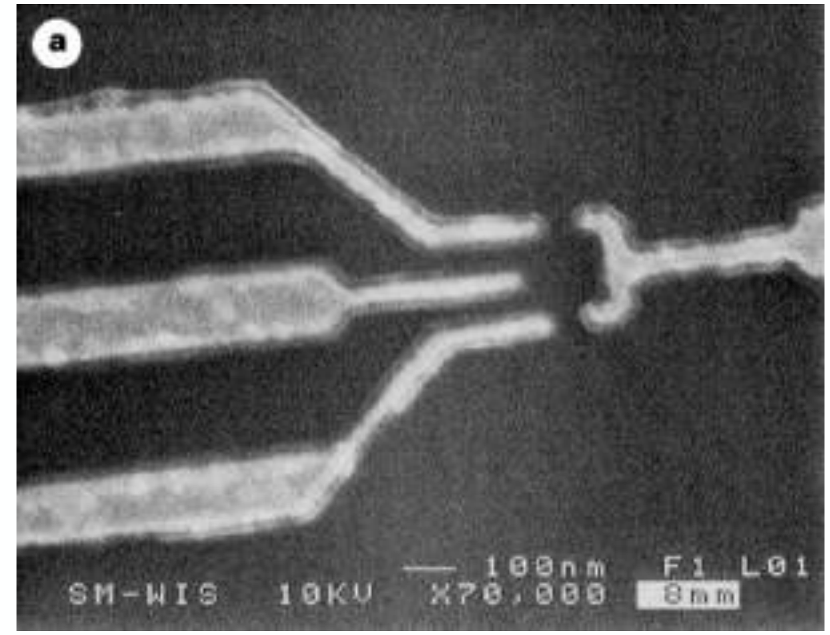

$2e^2/h$
$T << T_K$
$T \approx T_K$
$T \geq T_K$
Conductance
N
V_{gate}

Kondo effect in a single-electron transistor

D. Goldhaber-Gordon*†, Hadas Shtrikman†, D. Mahalu†, David Abusch-Magder*, U. Meirav† & M. A. Kastner*

* *Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachussetts 02139, USA*

† *Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel*



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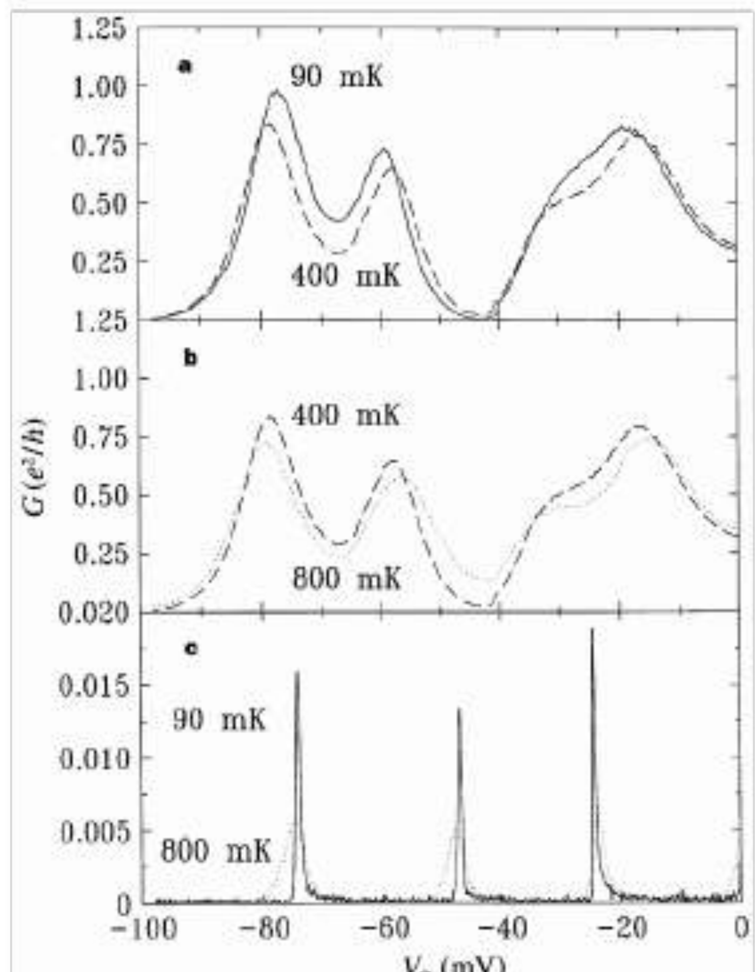

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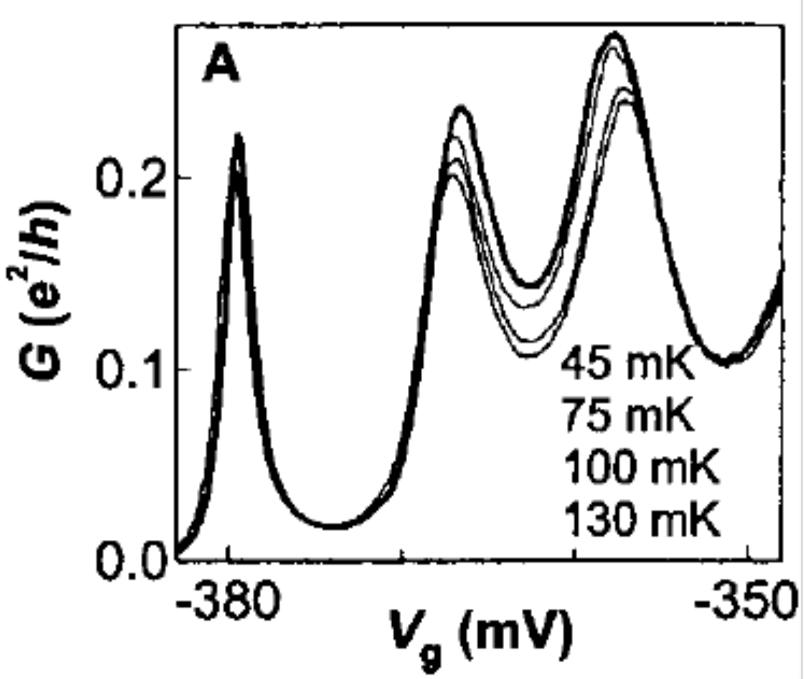
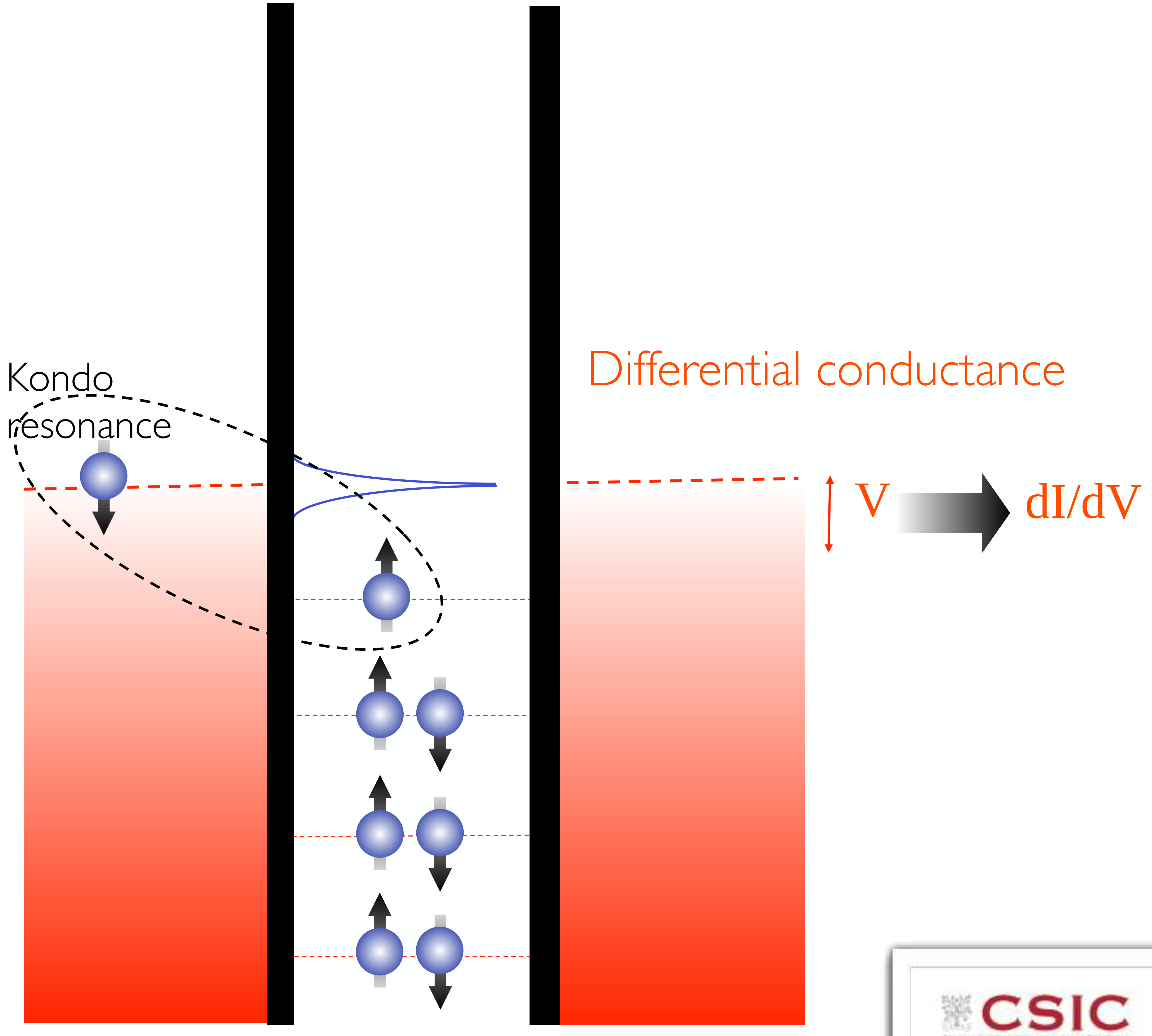
A Tunable Kondo Effect in Quantum Dots

Sara M. Cronenwett, Tjerk H. Oosterkamp, Leo P. Kouwenhoven

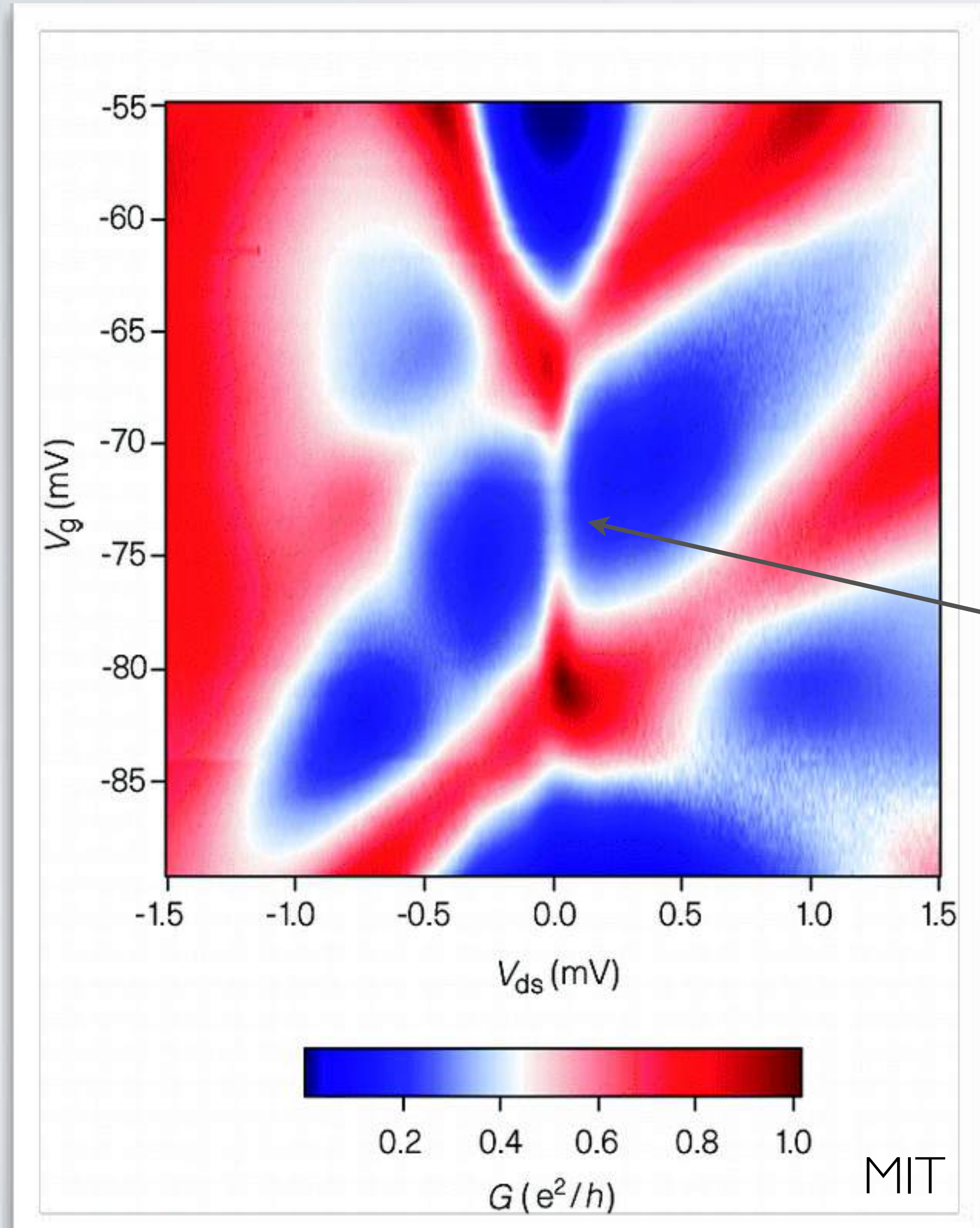
A tunable Kondo effect has been realized in small quantum dots. A dot can be switched from a Kondo system to a non-Kondo system as the number of electrons on the dot is changed from odd to even. The Kondo temperature can be tuned by means of a gate voltage as a single-particle energy state nears the Fermi energy. Measurements of the temperature and magnetic field dependence of a Coulomb-blockaded dot show good agreement with predictions of both equilibrium and nonequilibrium Kondo effects.

540 24 JULY 1998 VOL 281 SCIENCE www.sciencemag.org

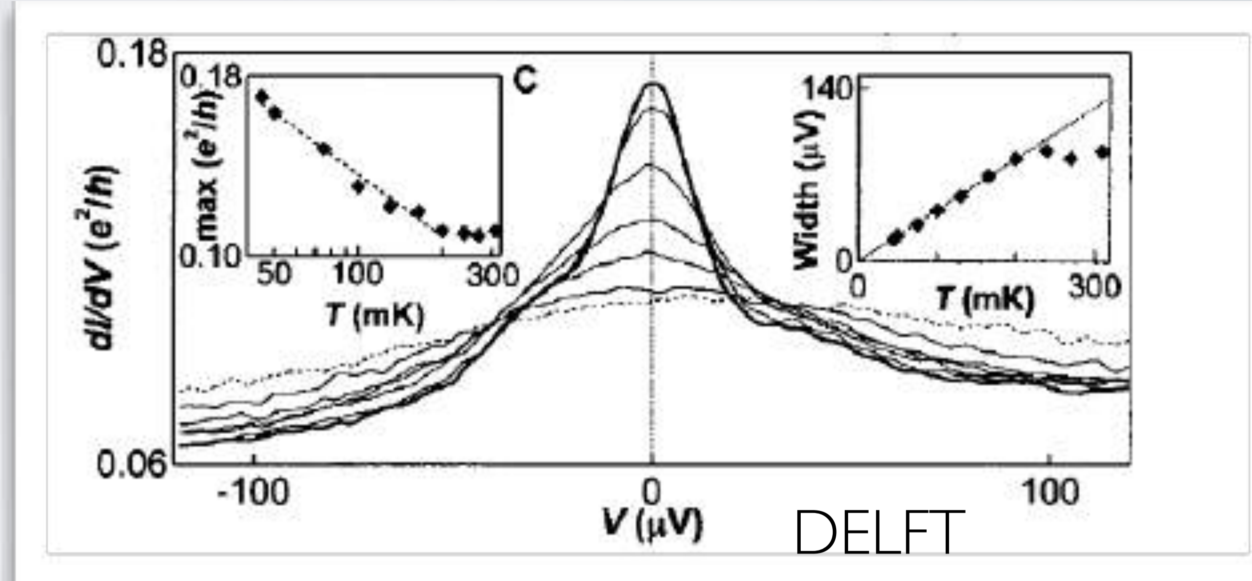



Kondo
resonance
Differential conductance
V
dI/dV

Direct measurement of the Kondo resonance!!



MIT

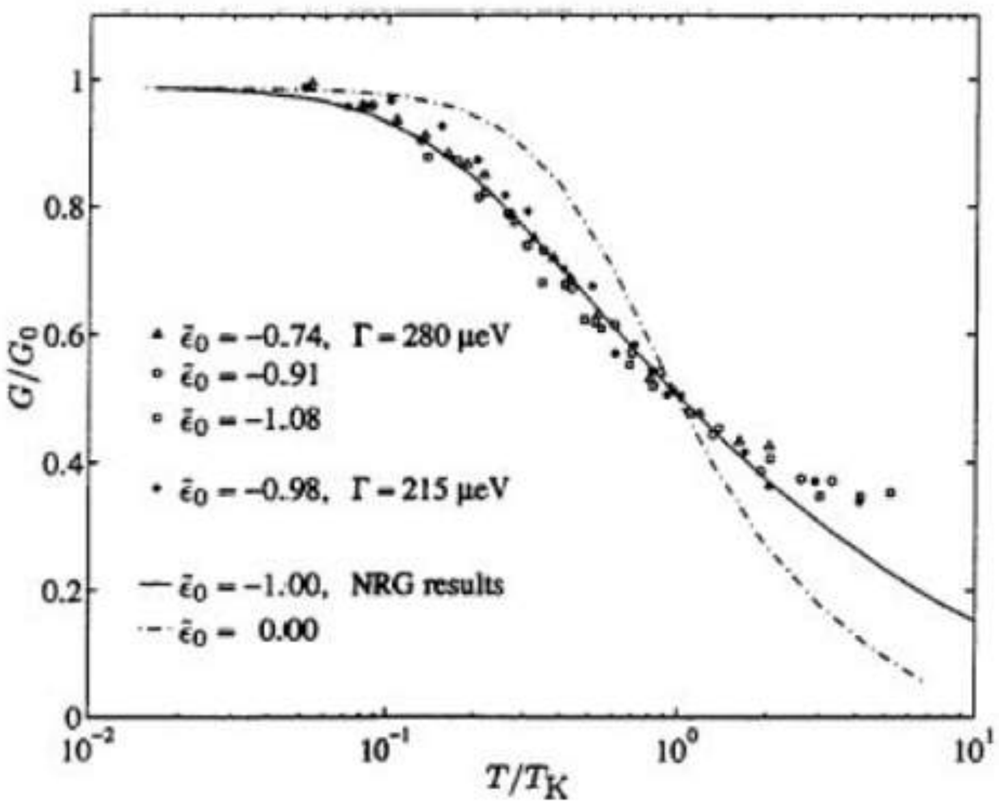


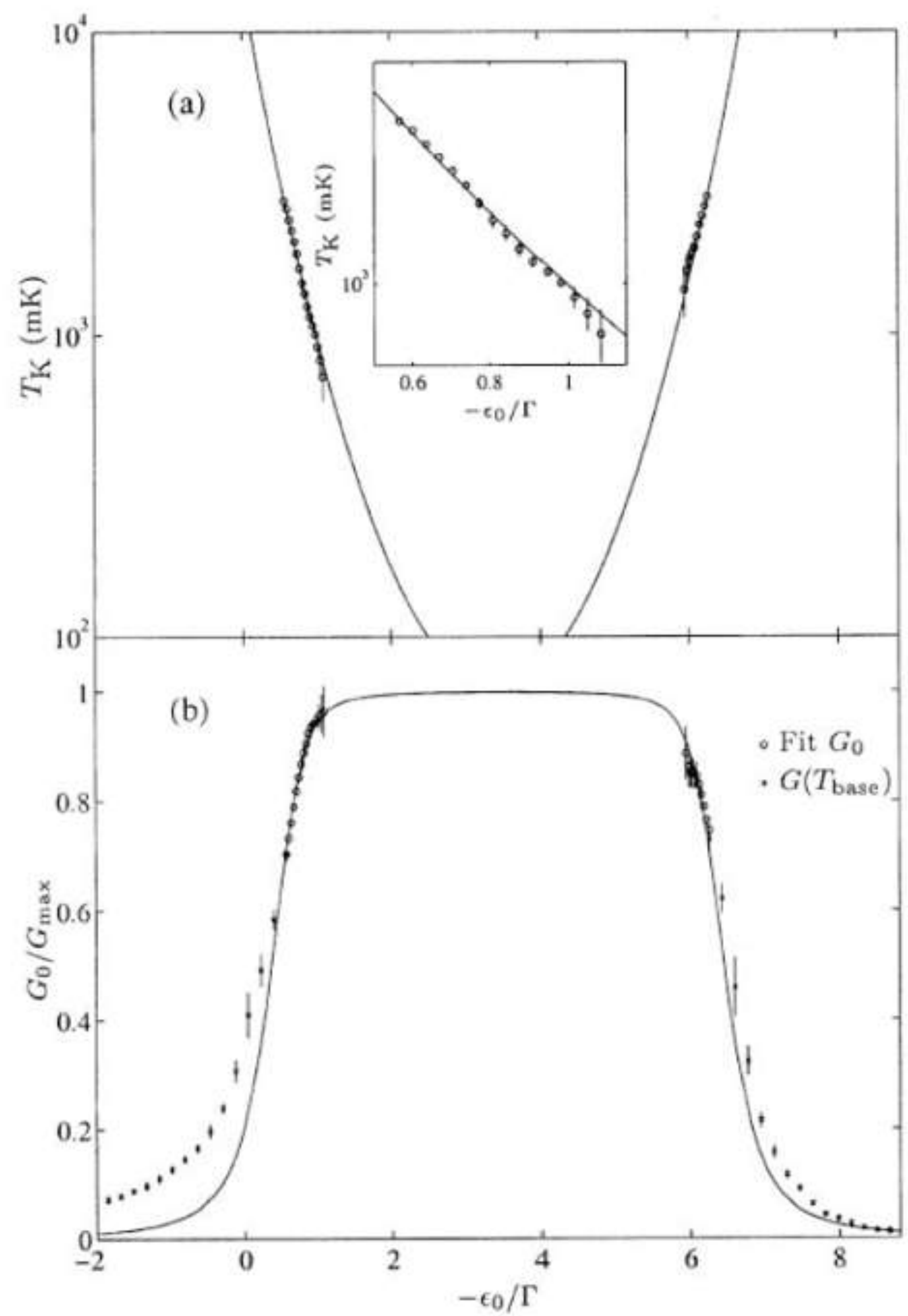
DELFT

Resonance in the middle of the Coulomb Blockade valley: very sharp indicating long lifetime (many-body Kondo) as opposed to single particle

Quantitative Tests of Kondo Theory

$$T_K = \frac{\sqrt{\Gamma U}}{2} \exp\left(\frac{\pi \varepsilon_0 (\varepsilon_0 + U)}{\Gamma U}\right)$$

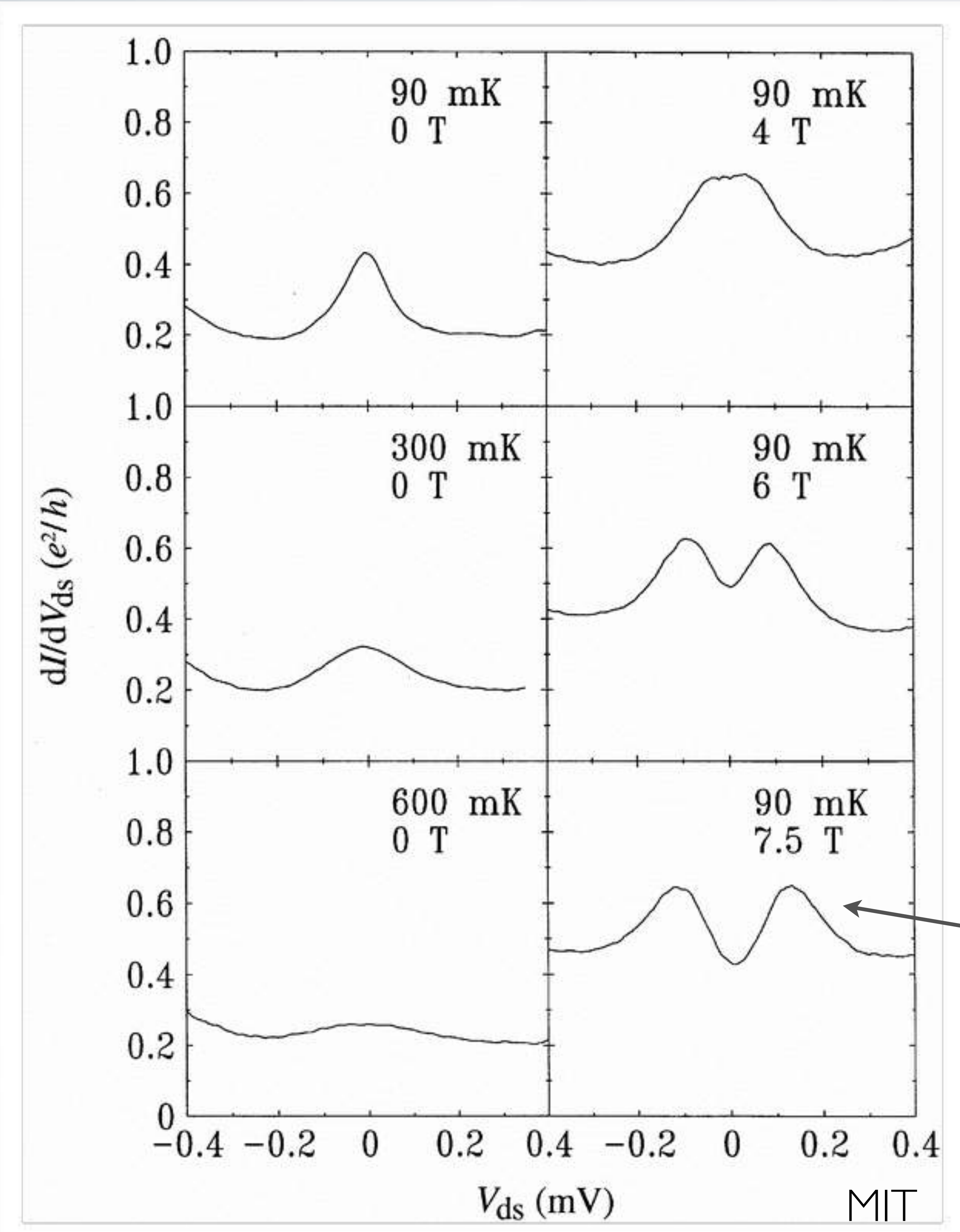


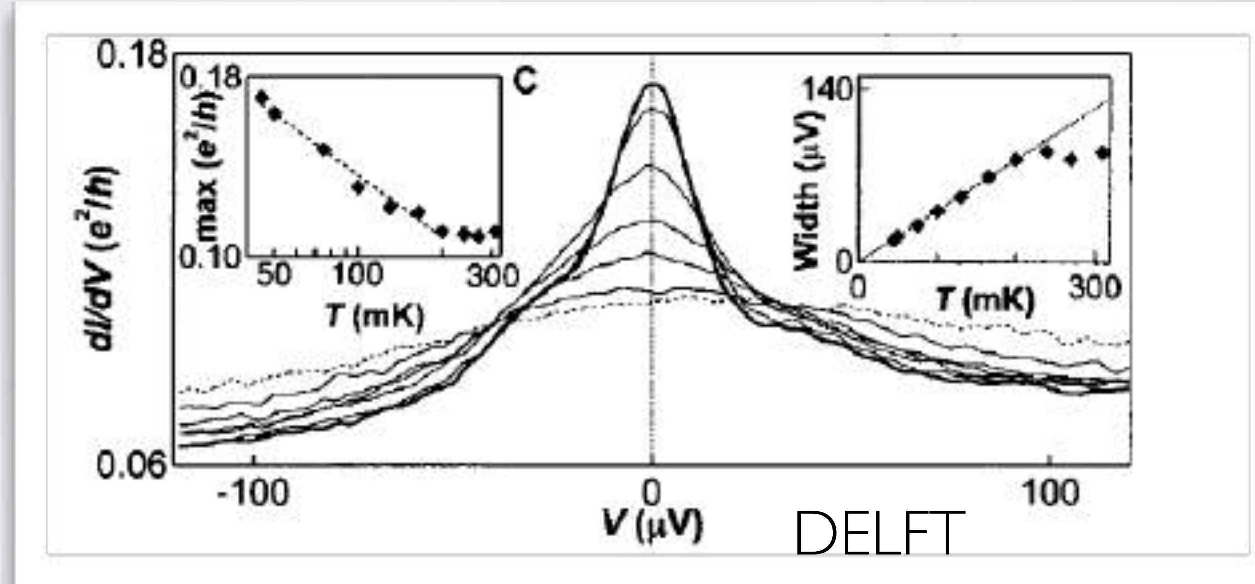


G-G, Kastner et al., PRL **81**, 5225 (1998)

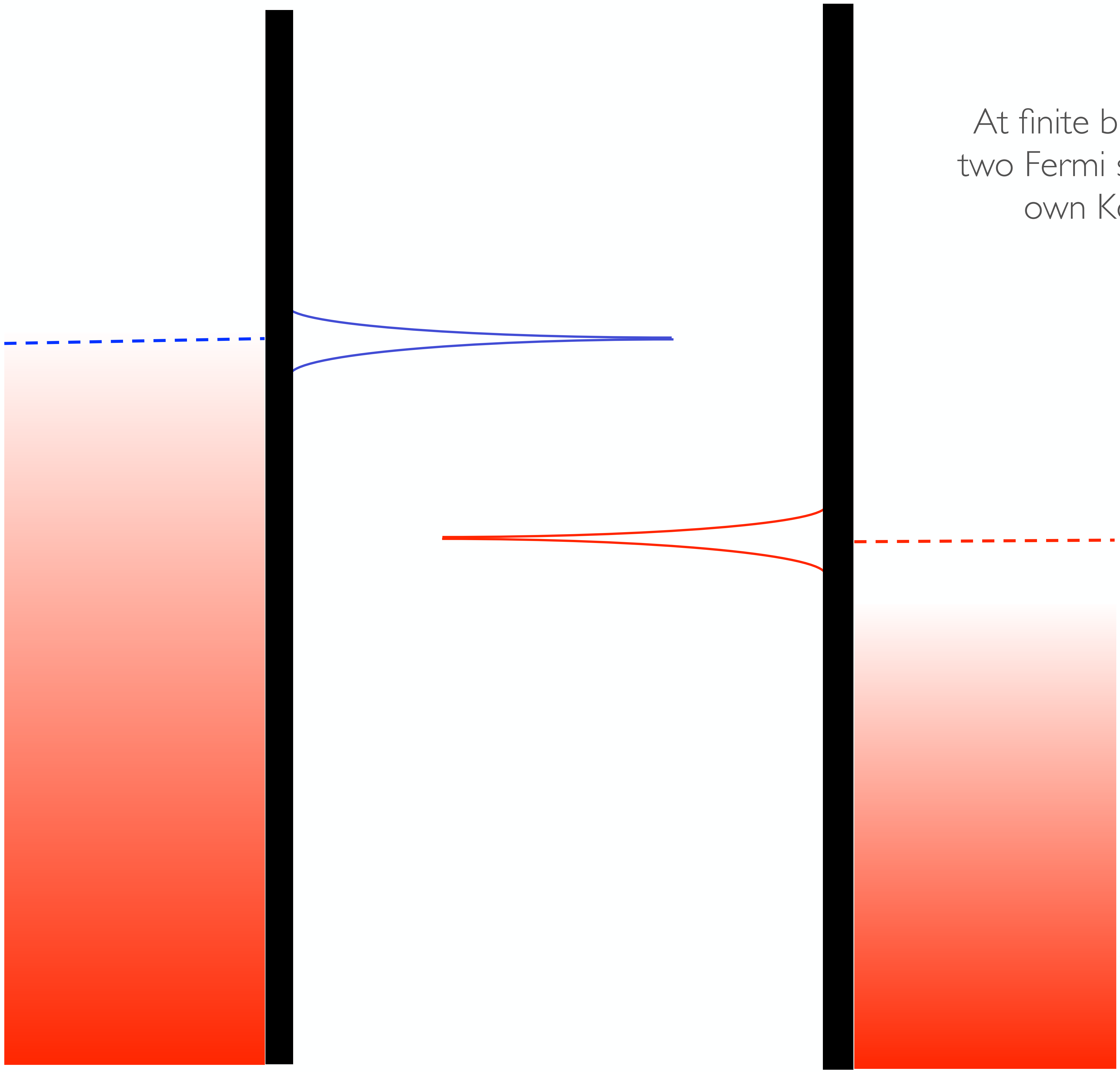
CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Direct measurement of the Kondo resonance!!





Kondo resonance splits with magnetic field with twice the Zeeman splitting. Why???



At finite bias voltage there are two Fermi surfaces, each with its own Kondo resonance.

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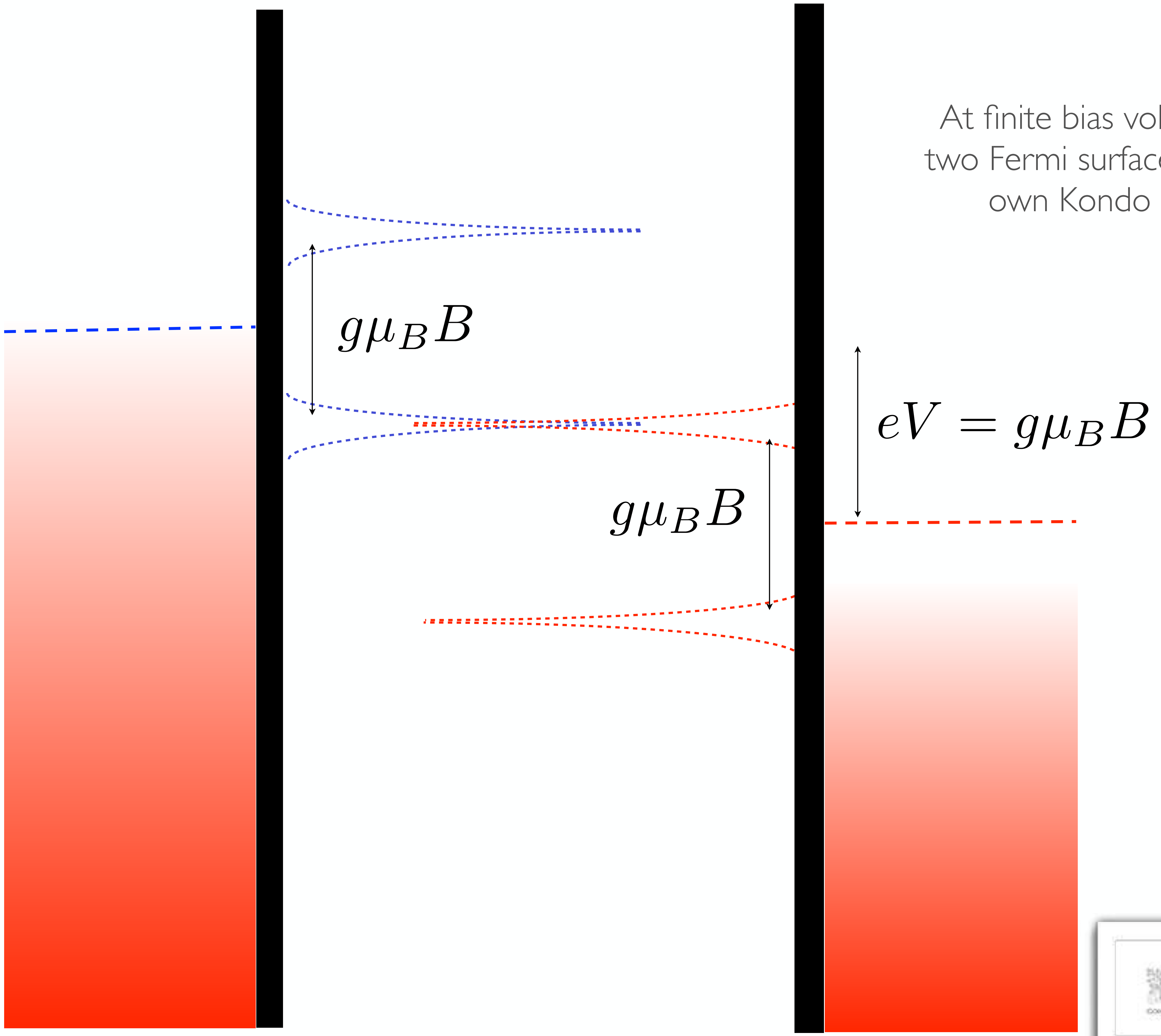


Theoretical prediction

VOLUME 70, NUMBER 17 PHYSICAL REVIEW LETTERS 26 APRIL 1993

Low-Temperature Transport Through a Quantum Dot: The Anderson Model Out of Equilibrium

Yigal Meir
Department of Physics, University of California, Santa Barbara, California 93106

Ned S. Wingreen
NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540

Patrick A. Lee
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 7 December 1992)

The infinite-U Anderson model is applied to nonequilibrium transport through a quantum dot containing two spin levels weakly coupled to two leads. At low temperatures, the Kondo peak in the equilibrium density of states is split upon the application of a voltage bias. The split peaks, one at the chemical potential of each lead, are suppressed by nonequilibrium dissipation. In a magnetic field, the Kondo peaks shift away from the chemical potentials by the Zeeman energy, leading to an observable peak in the differential conductance when the nonequilibrium bias equals the Zeeman energy.

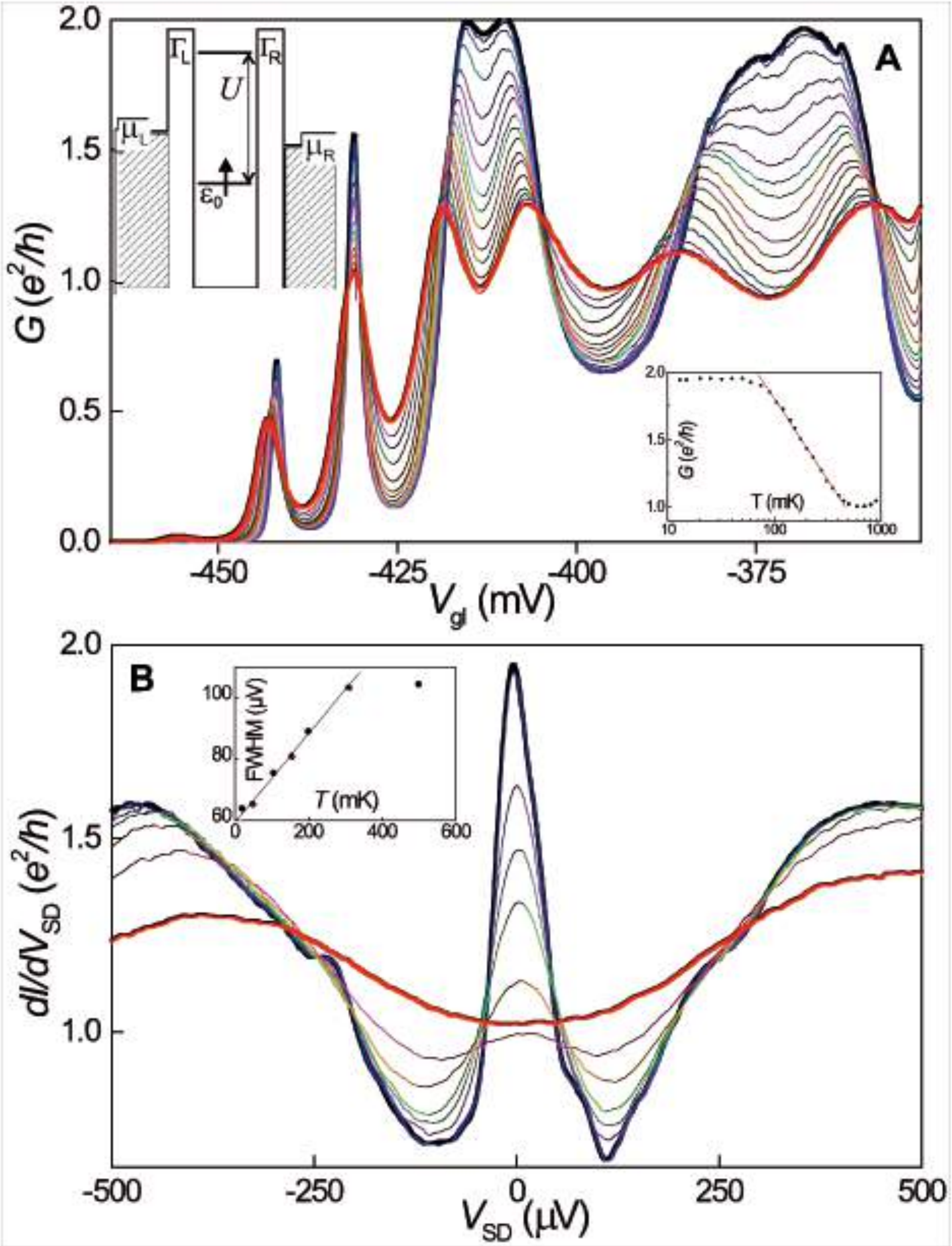
Slave boson 1/N+Keldysh

Disclaimer

- Beyond the superficial description of "splitting by voltage and magnetic field" there are a lot of theoretical developments (not discussed here).

- With relevant questions such as: how voltage=non-equilibrium effects induces dephasing?, how the logarithmic divergences are cut-off? etc.

See, e. g. Non-Equilibrium Transport through a Kondo-Dot in a Magnetic Field: Perturbation Theory and Poor Man's Scaling, A. Rosch, J. Paaske, J. Kroha, P. Wölfle Phys. Rev. Lett. 90, 076804 (2003)



- Symmetric unitary limit of $2e^2/h$ conductance has been achieved in a SET

- Conductance saturates at low T

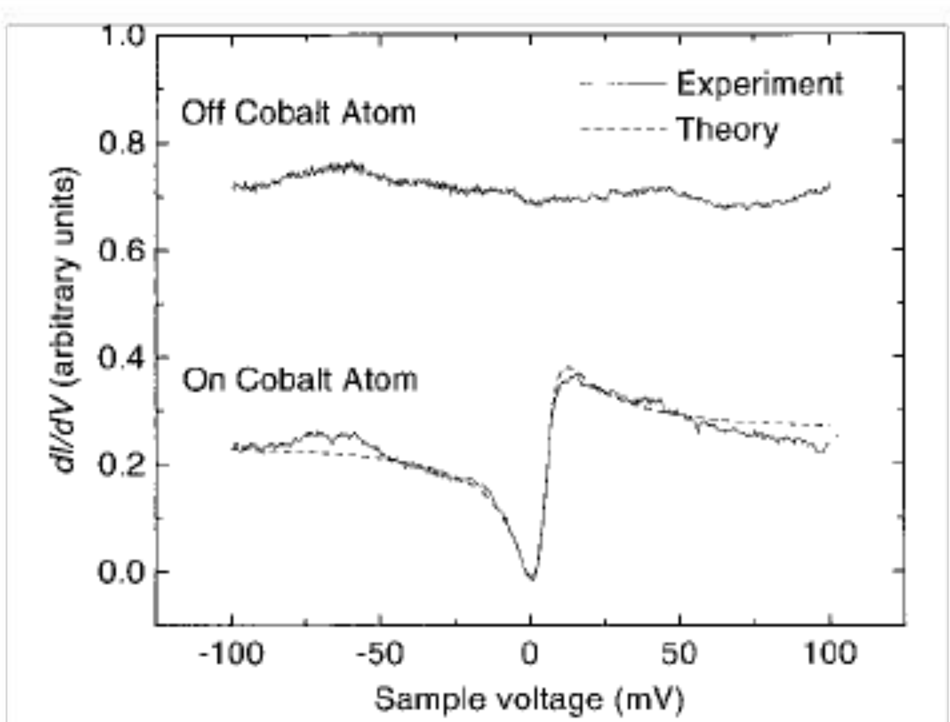
van der Wiel, Kouwenhoven et al. Science **289**, 2105 (2000).

Tunneling into a Single Magnetic Atom: Spectroscopic Evidence of the Kondo Resonance

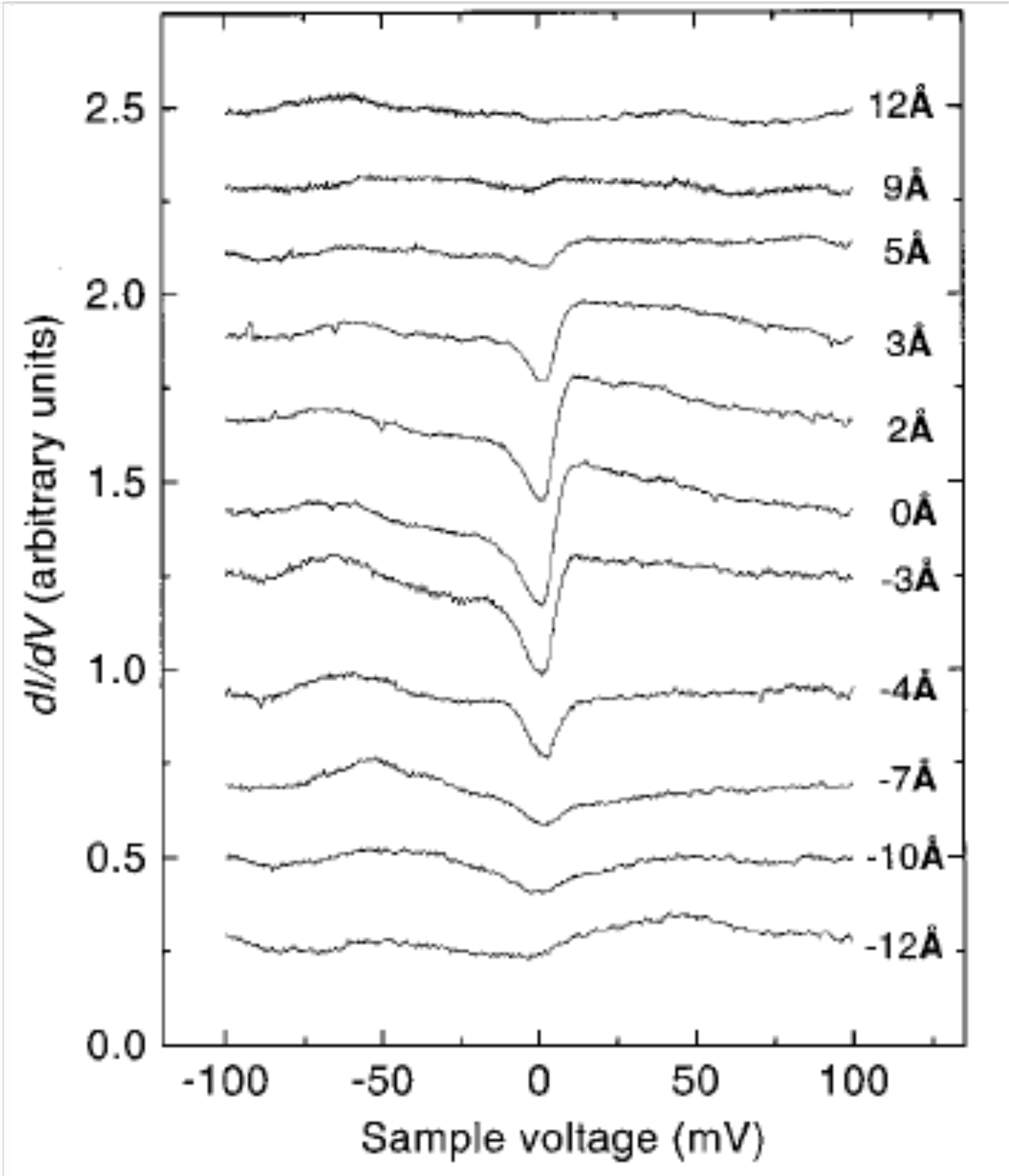
V. Madhavan, W. Chen, T. Jamneala, M. F. Crommie, N. S. Wingreen

The Kondo effect arises from the quantum mechanical interplay between the electrons of a host metal and a magnetic impurity and is predicted to result in local charge and spin variations around the magnetic impurity. A cryogenic scanning tunneling microscope was used to spatially resolve the electronic properties of individual magnetic atoms displaying the Kondo effect. Spectroscopic measurements performed on individual cobalt atoms on the surface of gold show an energetically narrow feature that is identified as the Kondo resonance—the predicted response of a Kondo impurity. Unexpected structure in the Kondo resonance is shown to arise from quantum mechanical interference between the *d* orbital and conduction electron channels for an electron tunneling into a magnetic atom in a metallic host.

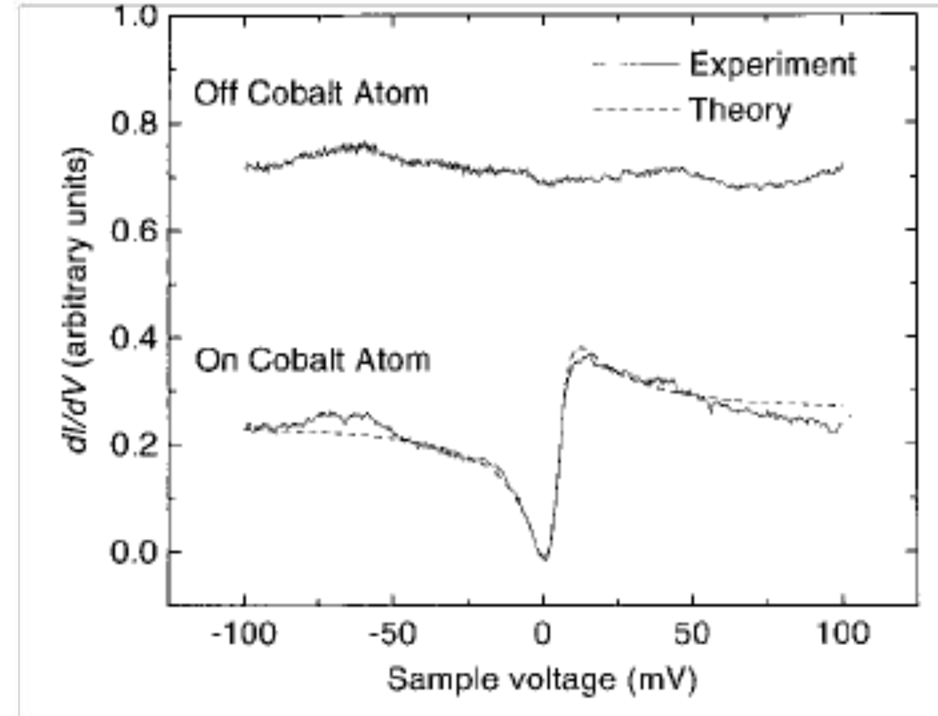
www.sciencemag.org • SCIENCE • VOL. 280 • 24 APRIL 1998 567



SAME YEAR OF THE QUANTUM DOT EXPERIMENTS!!

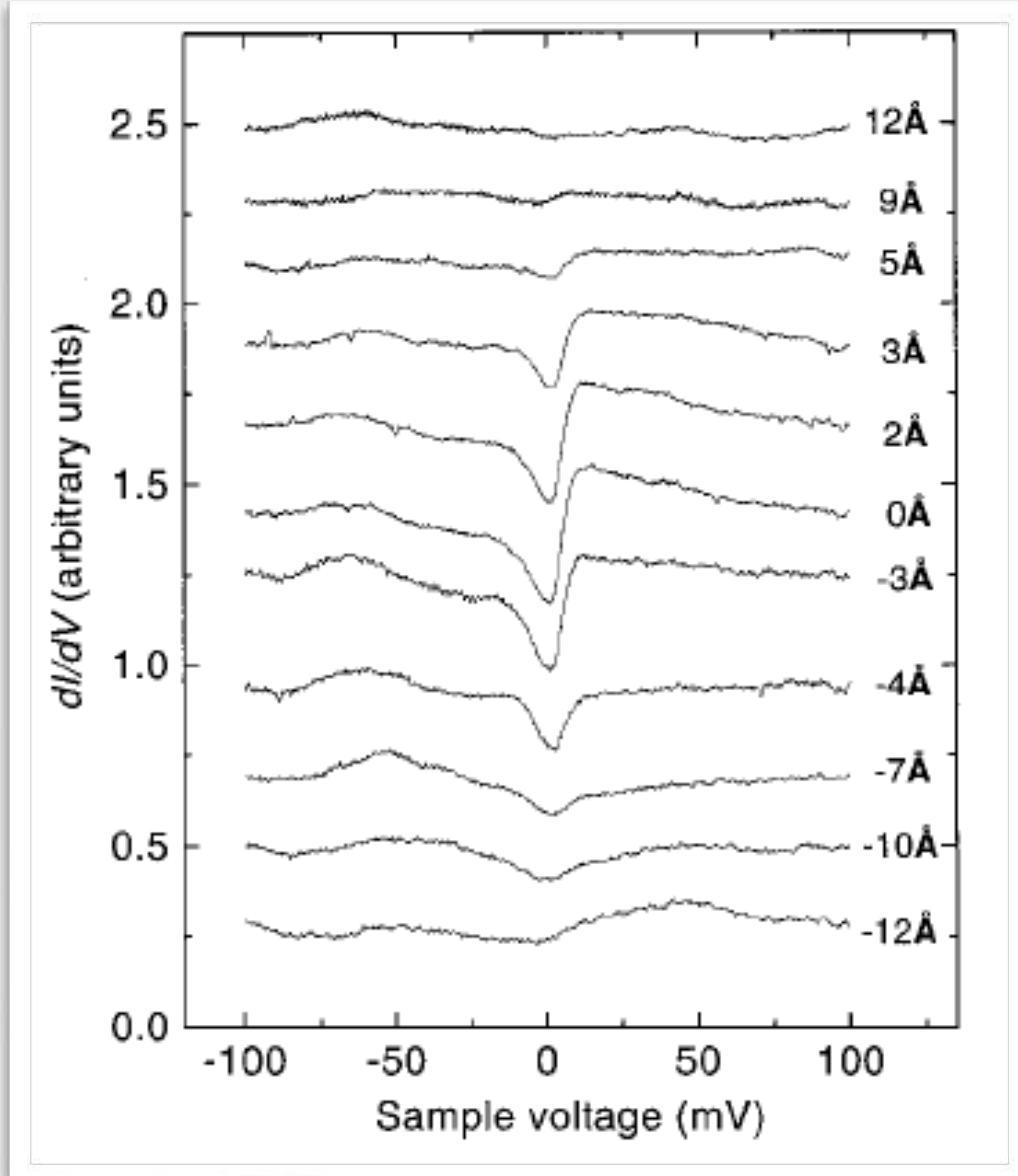


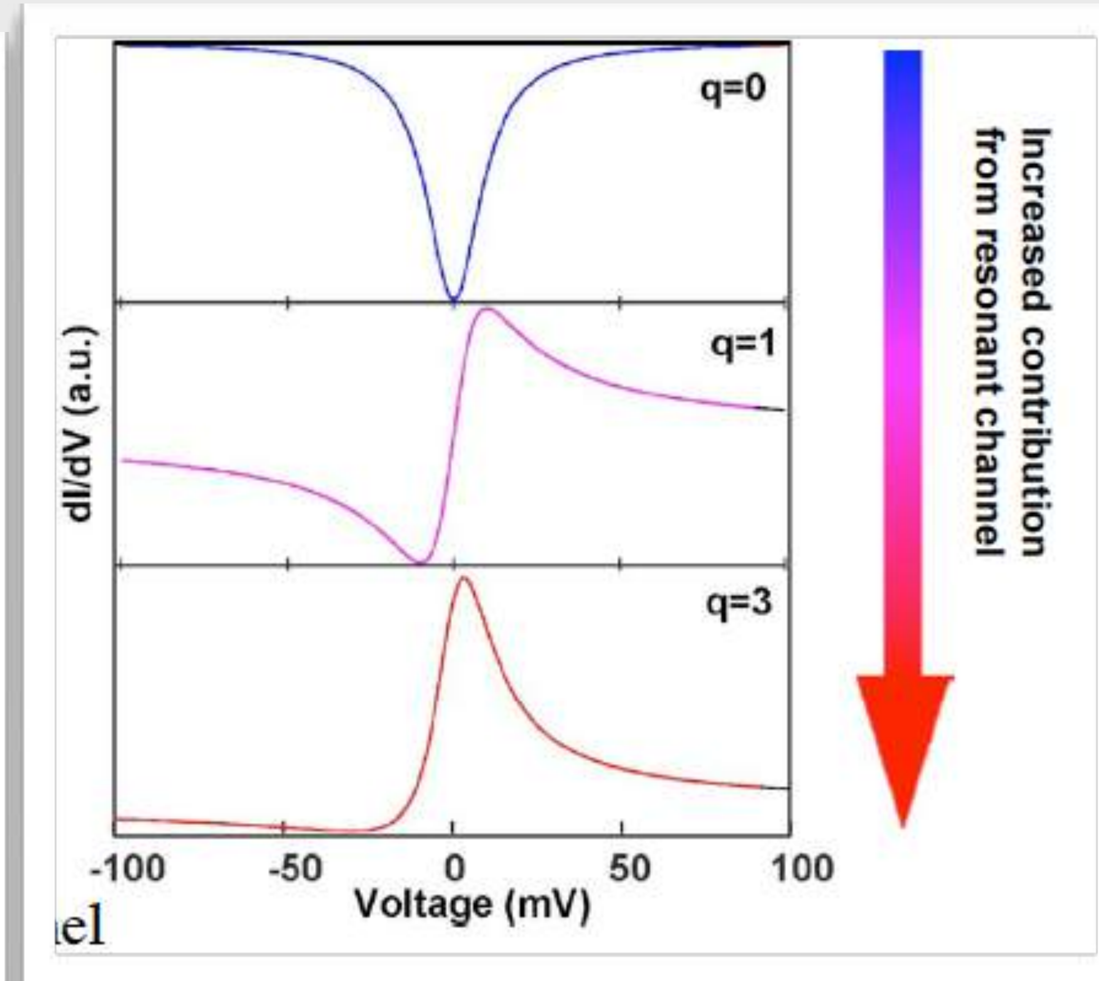
dI/dV (arbitrary units)
Sample voltage (mV)
12Å
9Å
5Å
3Å
2Å
0Å
-3Å
-4Å
-7Å
-10Å
-12Å



Experiment
Theory
Off Cobalt Atom
On Cobalt Atom
dI/dV (arbitrary units)
Sample voltage (mV)

FANO LINESHAPES: interference of two paths

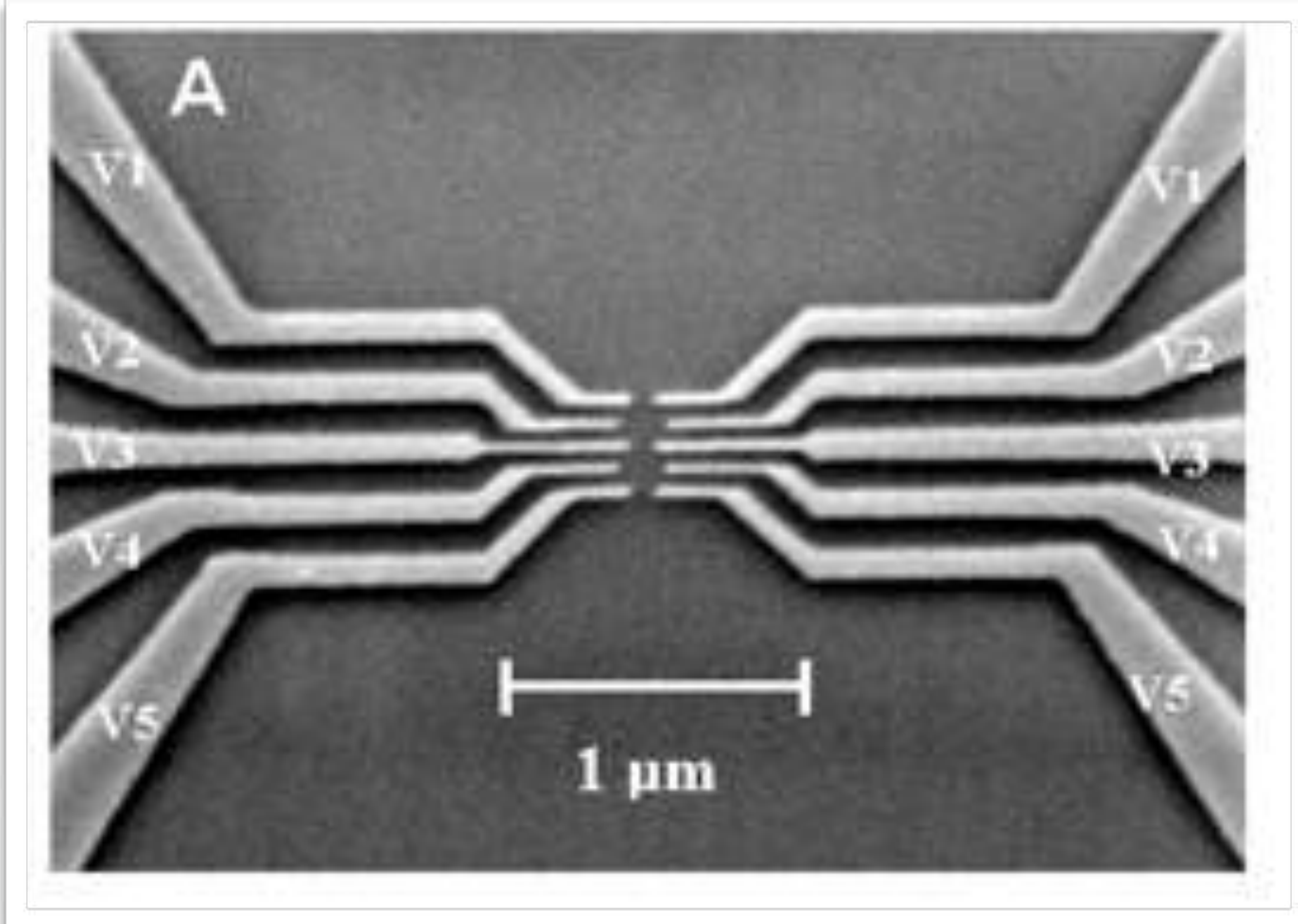




$$\sim \frac{(\tilde{\varepsilon}+q)^2}{\tilde{\varepsilon}^2+1}$$

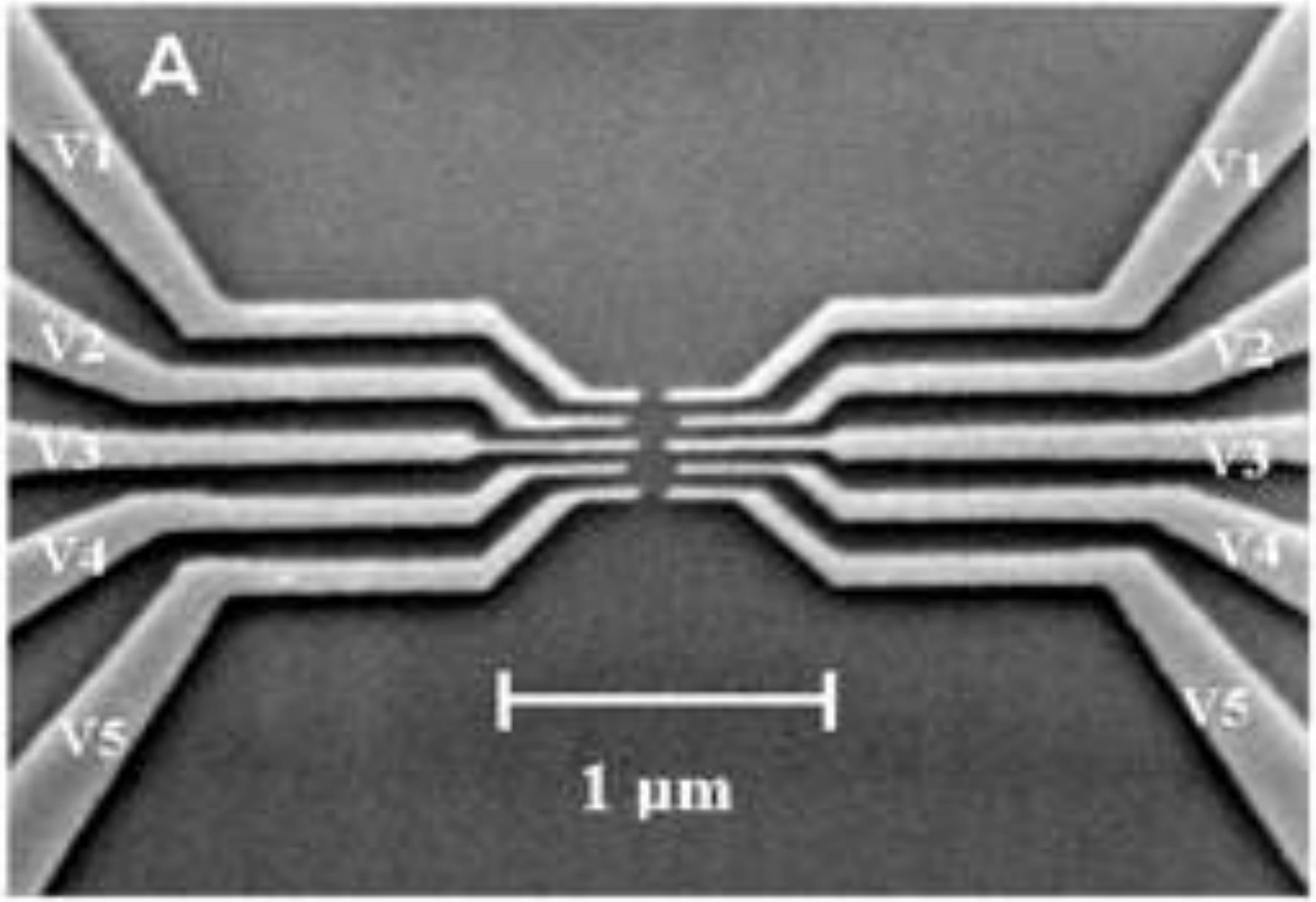
$$\tilde{\varepsilon} = \frac{\varepsilon - \varepsilon_0}{\Gamma}$$

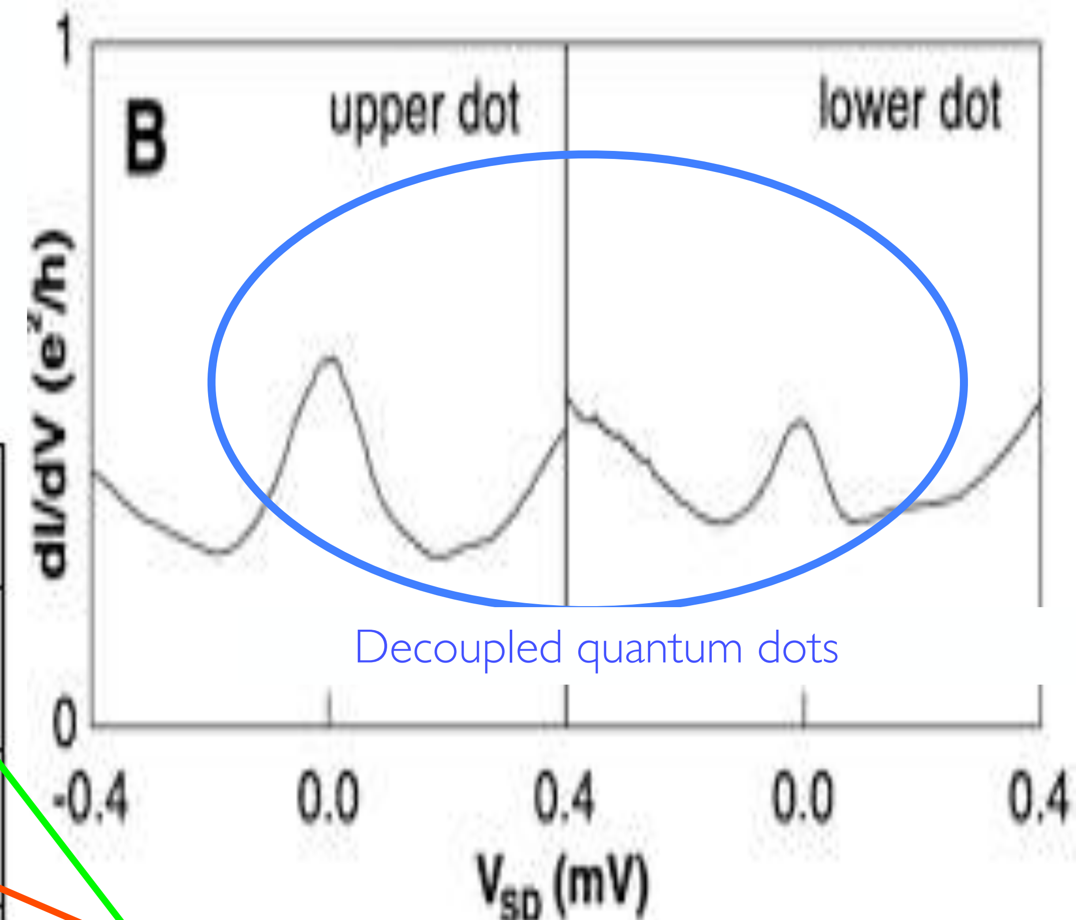
Double dots: two-impurity

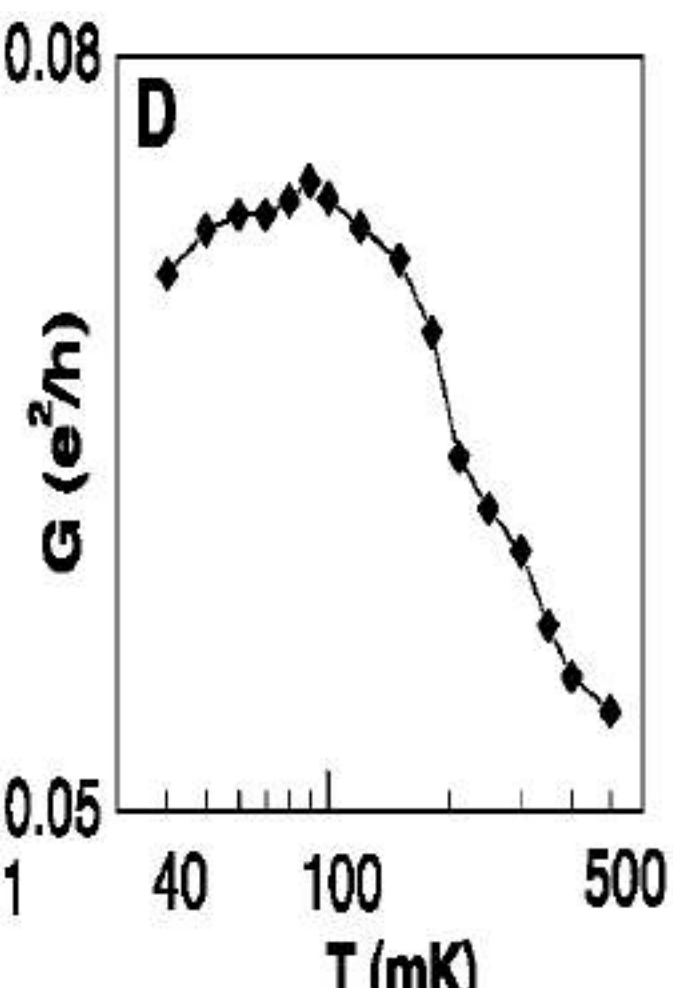


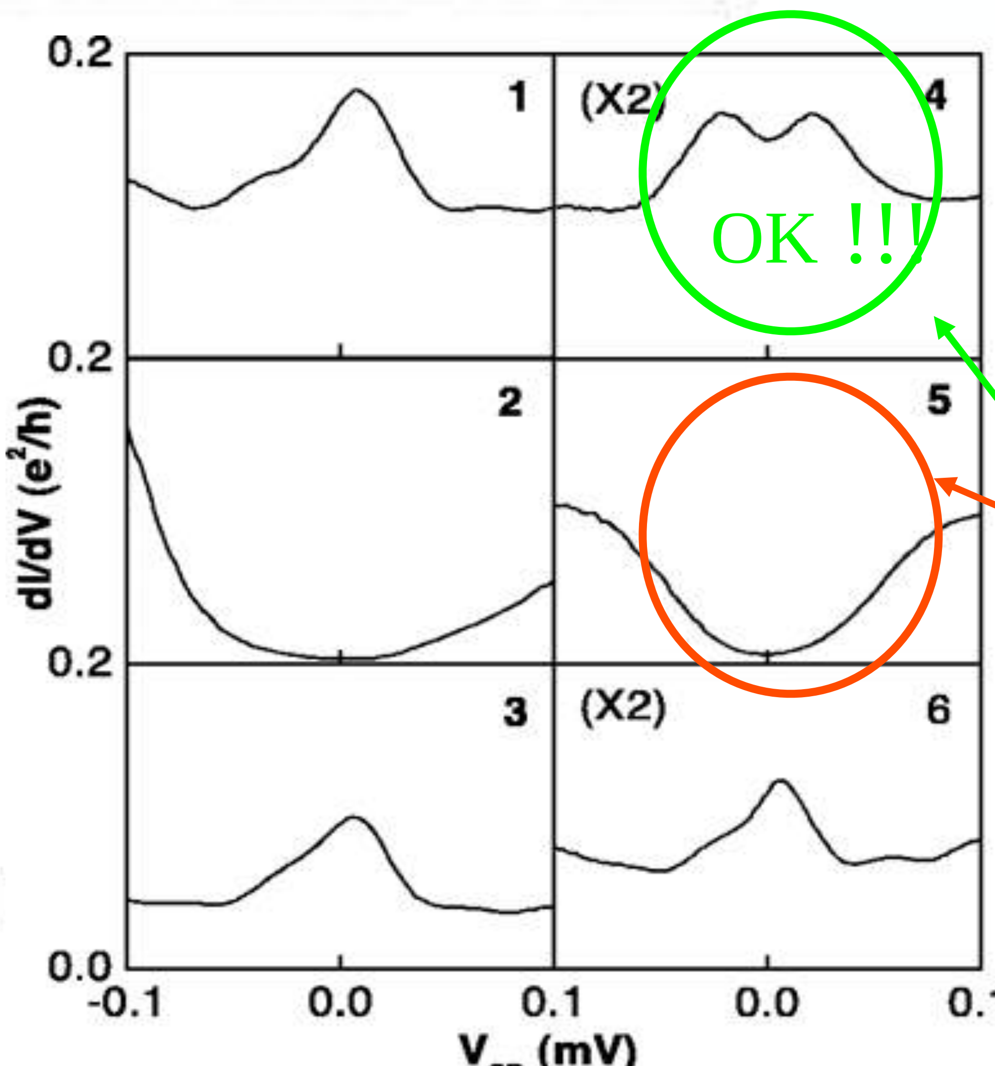
The Kondo effect in an artificial molecule, H. Jeong, A. M. Chang and M. R. Melloch, Science, 293, 2221 (2001)

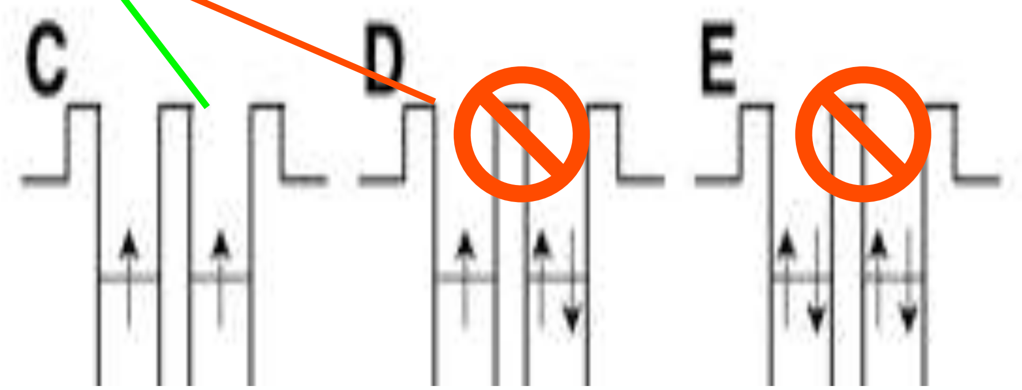
The Kondo effect in an artificial molecule, H. Jeong, A. M. Chang and M. R. Melloch, Science, 293, 2221 (2001)

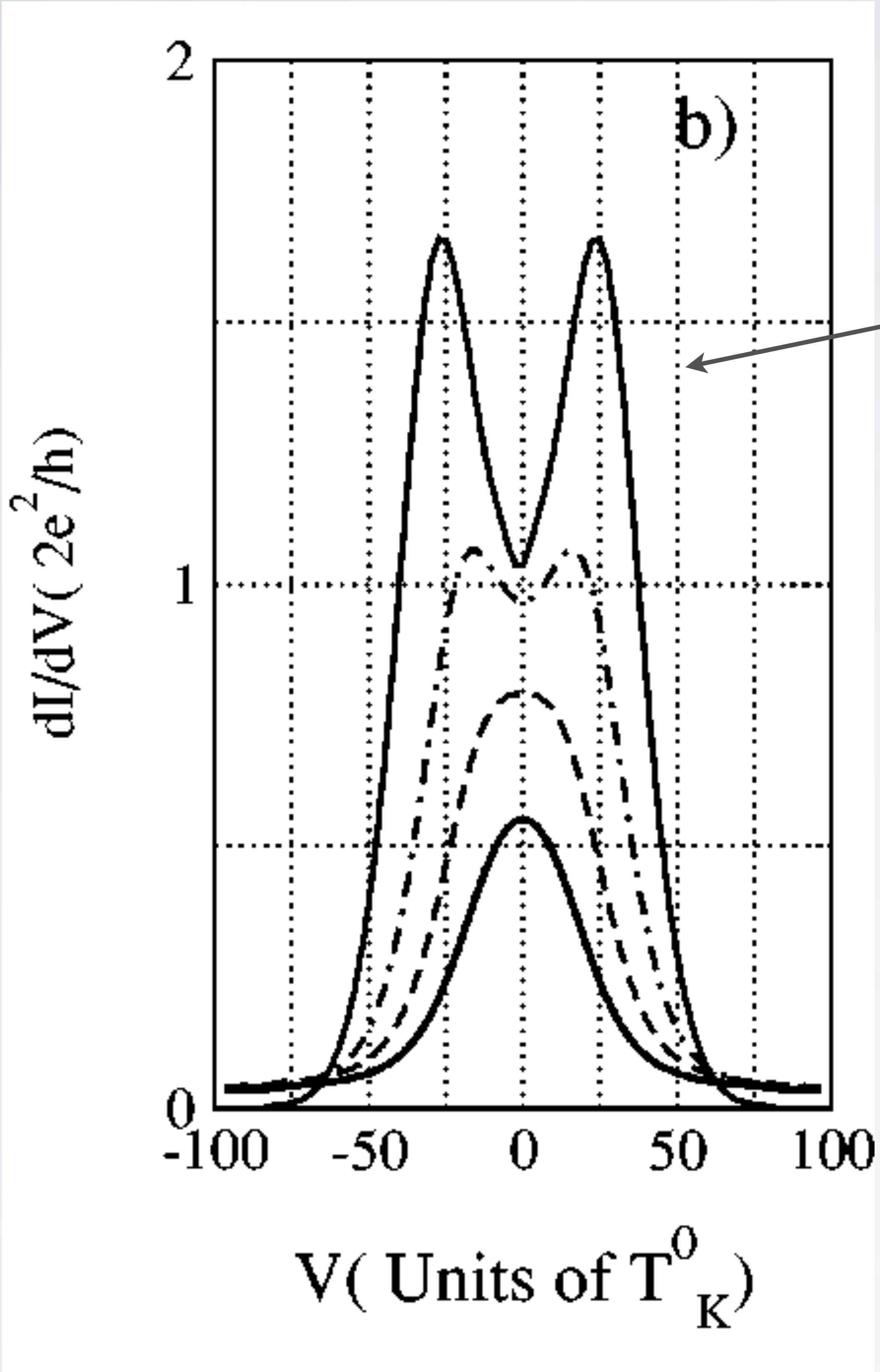










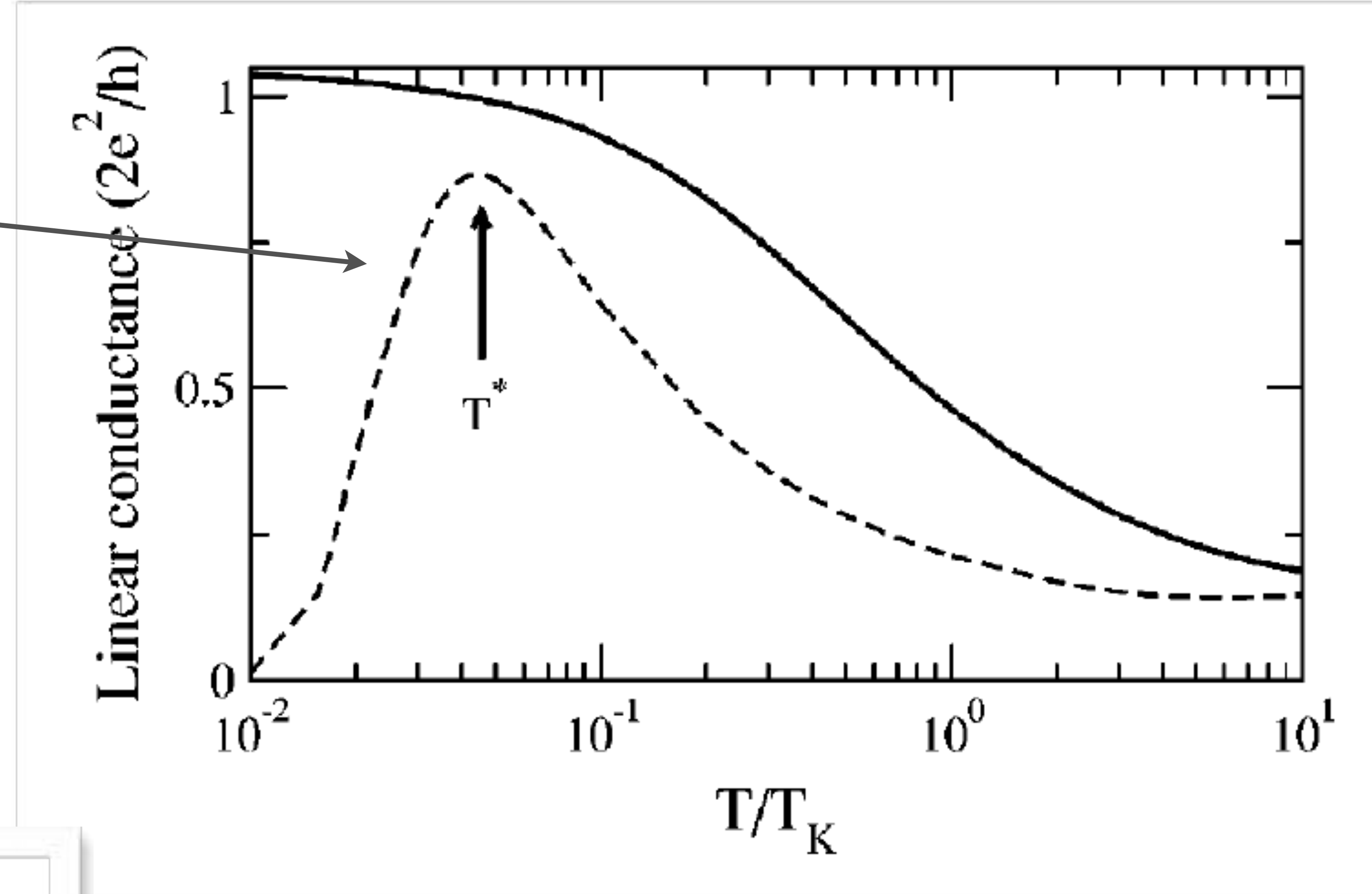


- Splitting of Kondo peak as coupling between dots increases.

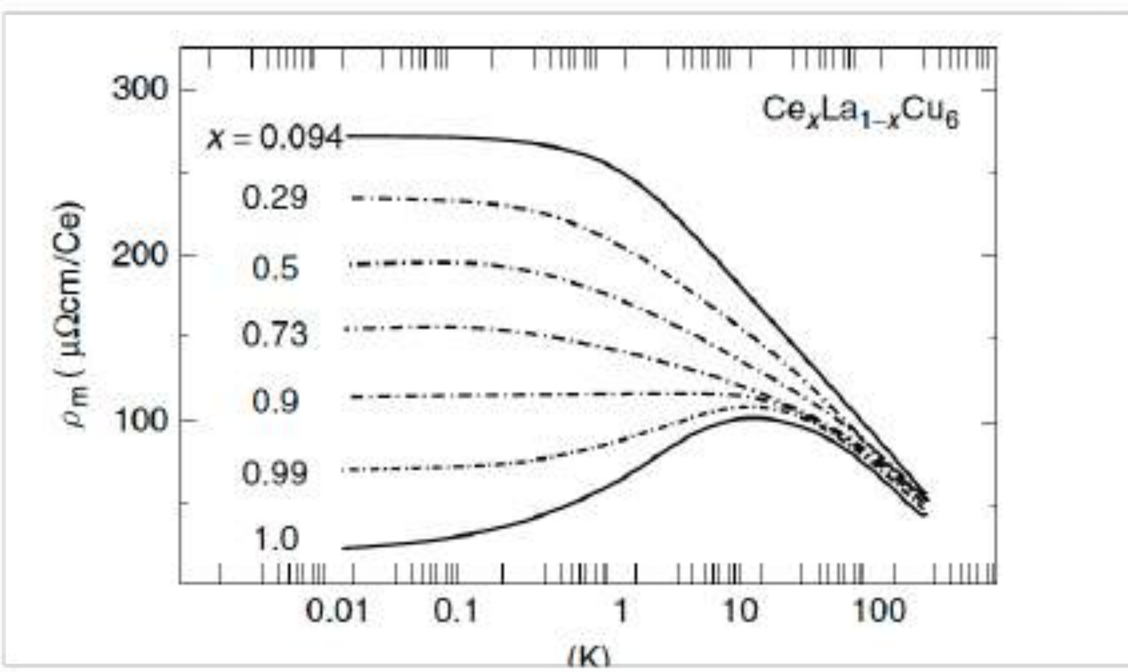
Slave boson 1/N+Keldysh

Aguado and Langreth, Phys. Rev. Lett. 85, 1946 (2000)
Aguado and Langreth, Phys. Rev. B 67, 245307 (2003)

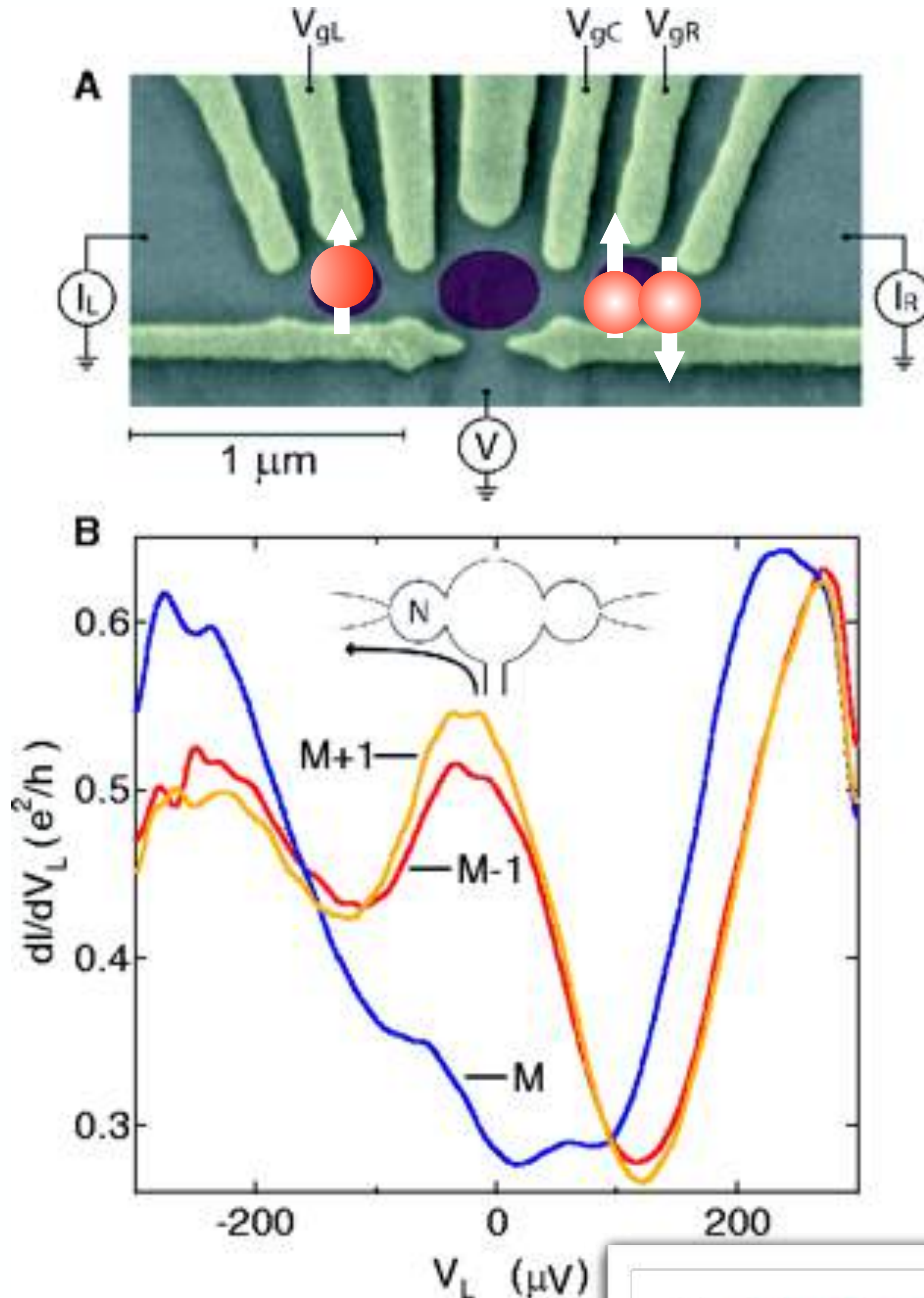
- Non-monotonic conductance: development of coherence
- Similar to heavy fermion physics in mesoscopics!!



Aguado and Langreth, Phys. Rev. B 67, 245307 (2003)



When the right dot contains an even number of electrons the left dot shows Kondo effect.



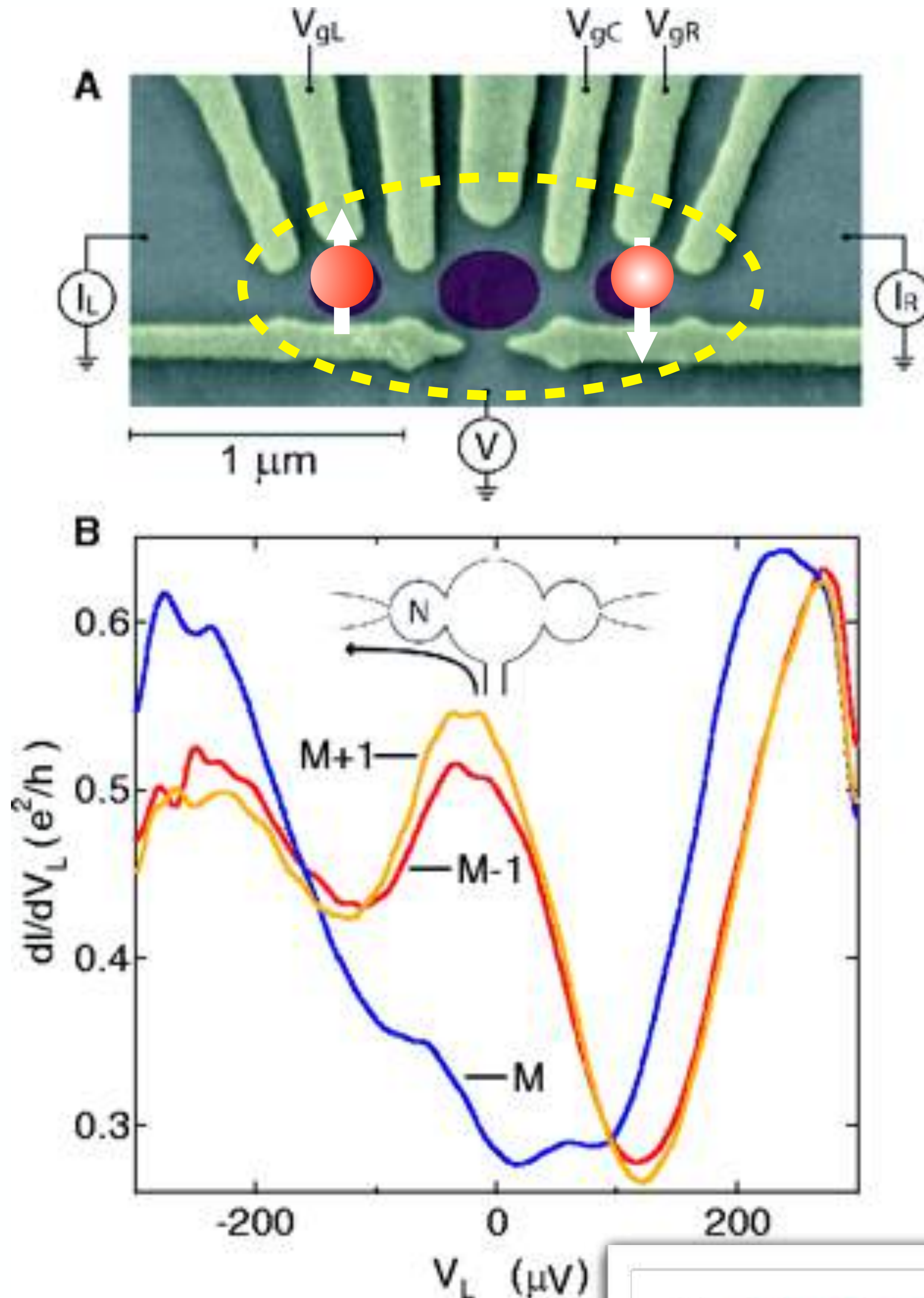
Tunable Nonlocal Spin Control in a Coupled–Quantum Dot System

N. J. Craig,[1] J. M. Taylor,[1] E. A. Lester,[1] C. M. Marcus,[1]* M. P. Hanson,[2] A. C. Gossard[2]

The effective interaction between magnetic impurities in metals that can lead to various magnetic ground states often competes with a tendency for electrons near impurities to screen the local moment (known as the Kondo effect). The simplest system exhibiting the richness of this competition, the two-impurity Kondo system, was realized experimentally in the form of two quantum dots coupled through an open conducting region. We demonstrate nonlocal spin control by suppressing and splitting Kondo resonances in one quantum dot by changing the electron number and coupling of the other dot. The results suggest an approach to nonlocal spin control that may be relevant to quantum information processing.

www.sciencemag.org SCIENCE VOL 304 23 APRIL 2004 565

When the right dot contains an odd number of electrons Kondo effect is suppressed.



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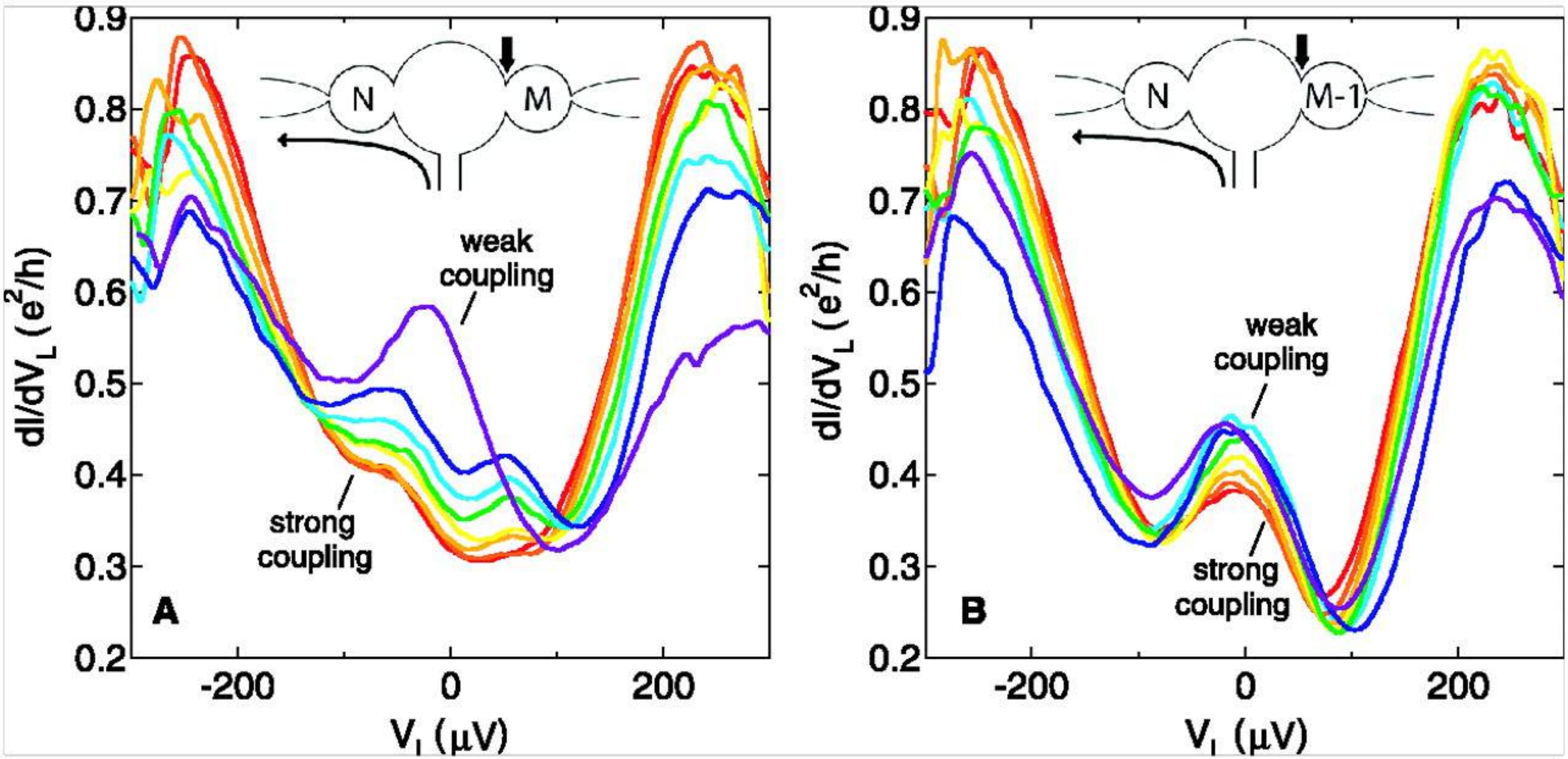
Tunable coupling



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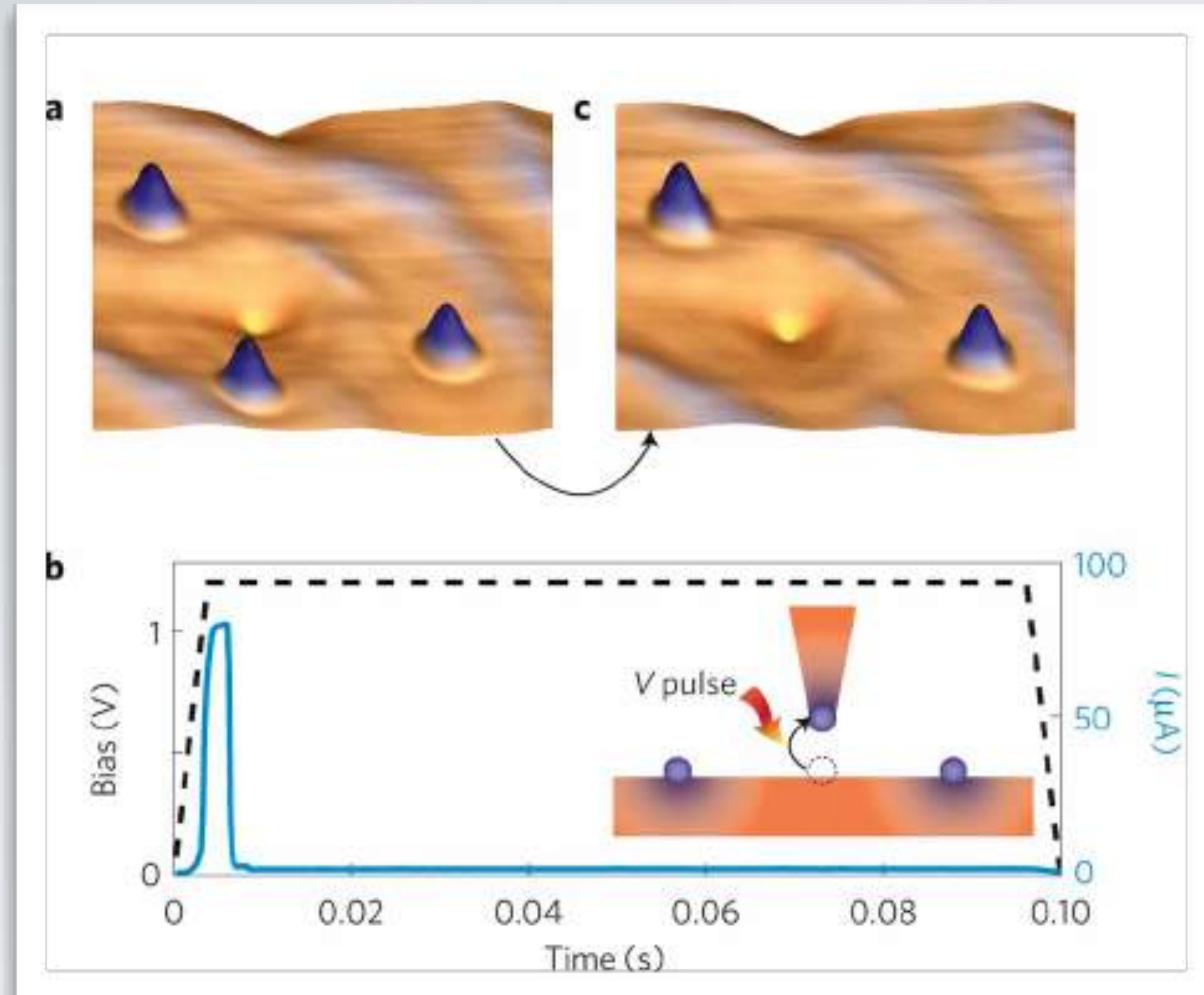
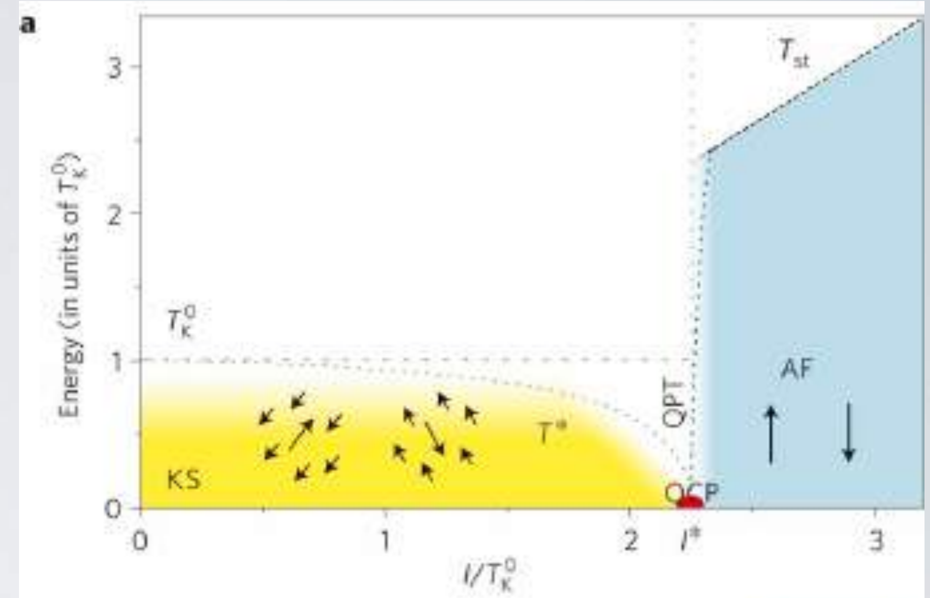
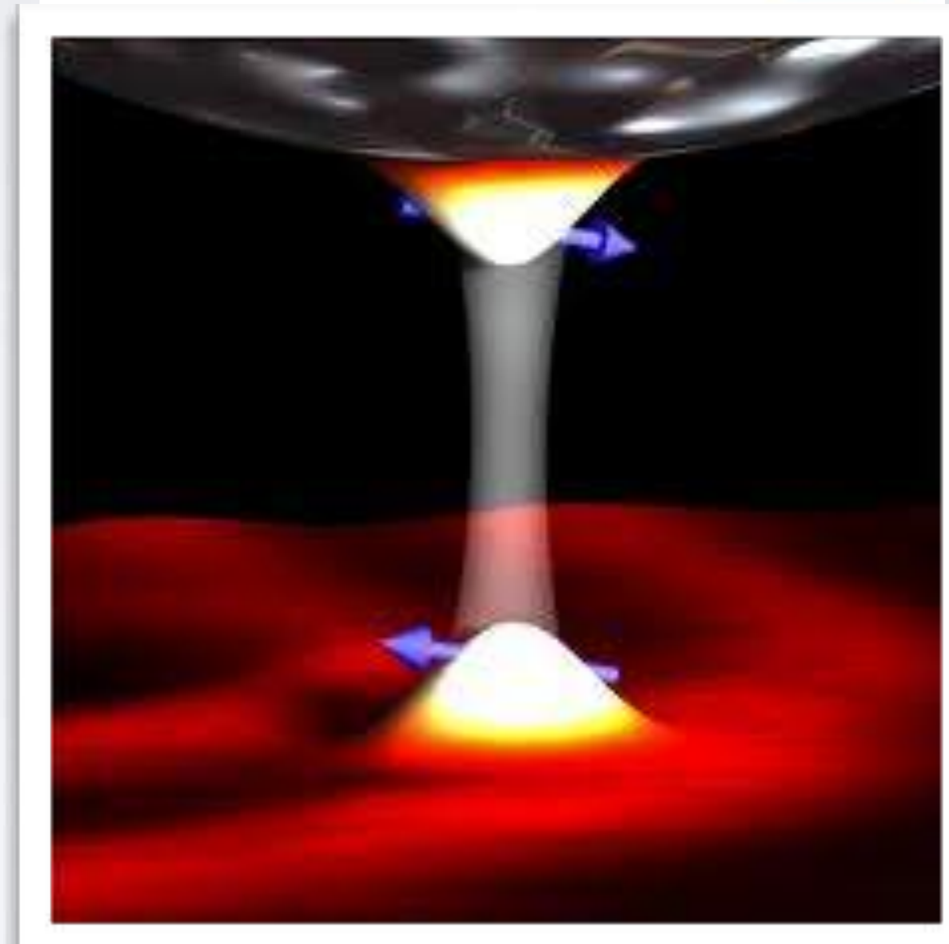
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

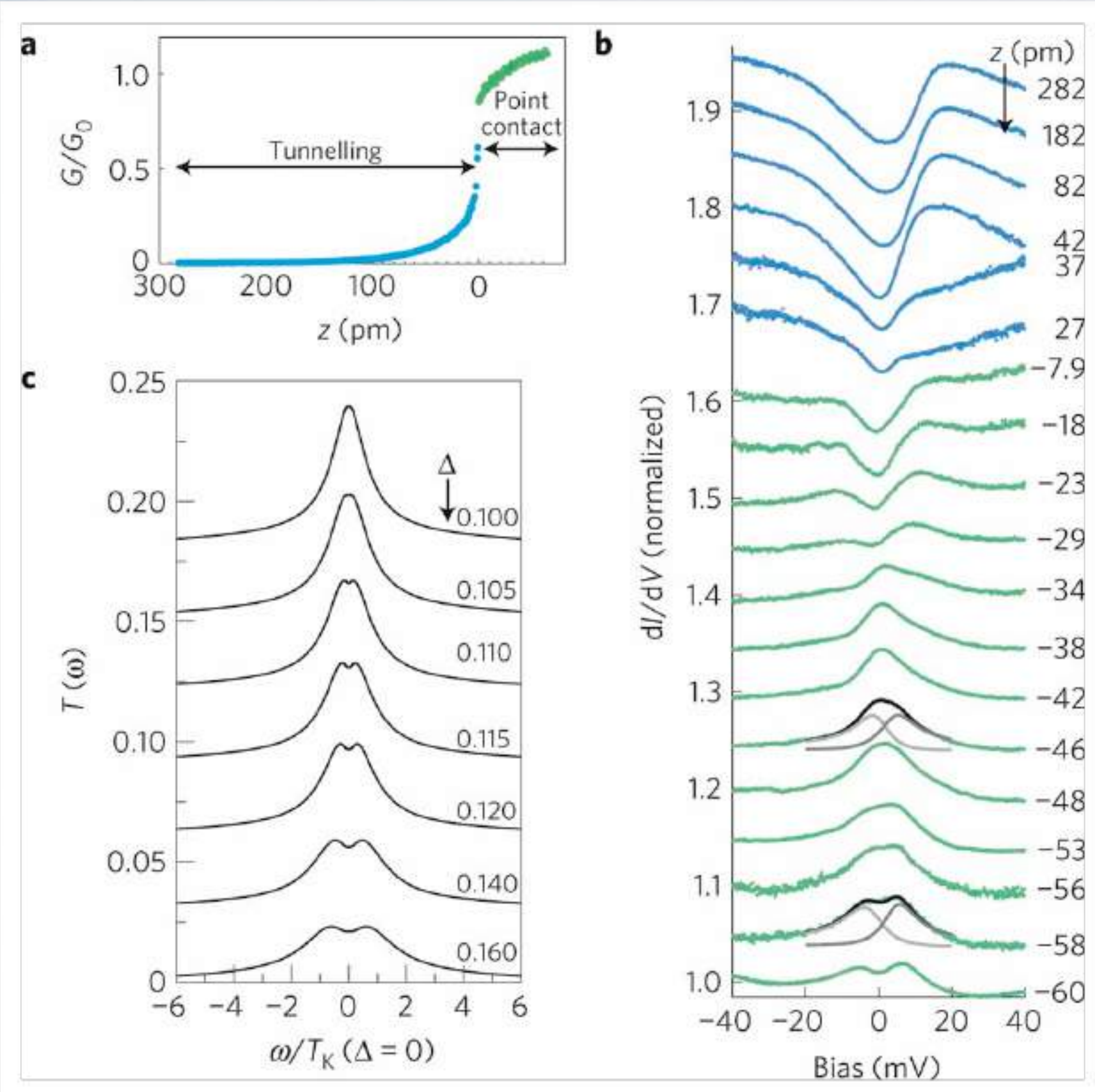




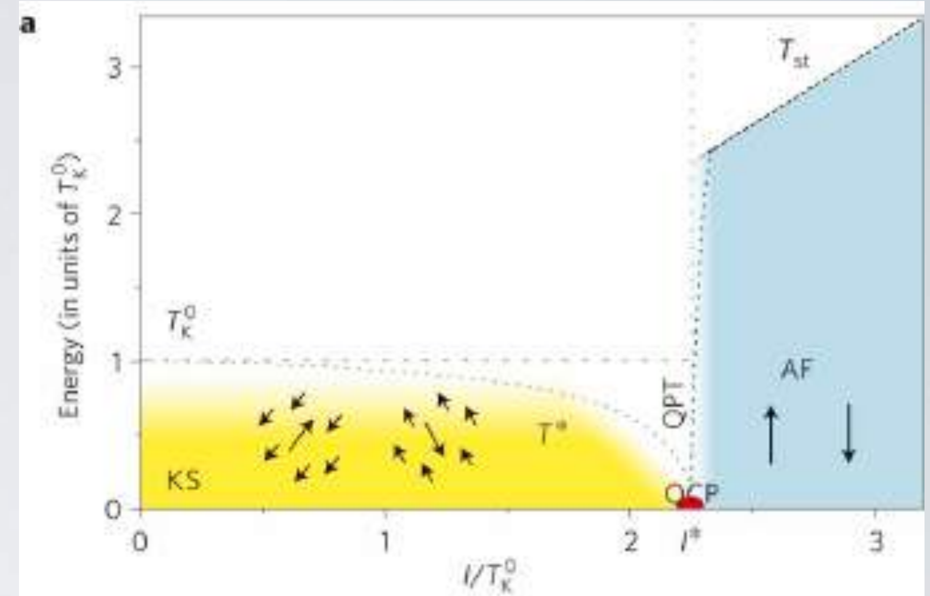


- Two magnetic atoms, one attached to the tip of a STM and one adsorbed on a metal surface, each constituting a Kondo system.
- Cobalt dimers clamped between an STM tip and a gold surface.
- Control of the tip-sample distance with sub-picometer resolution allows to tune the interaction between the two cobalt atoms with unprecedented precision.

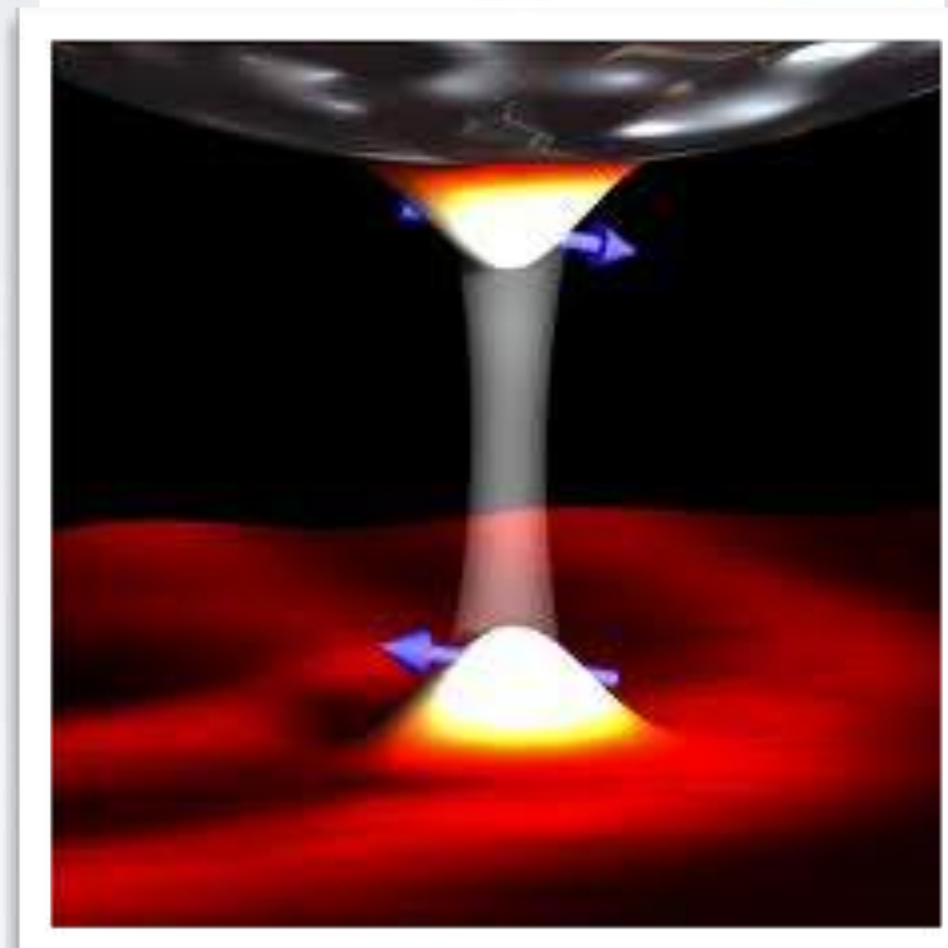
Bork et al, Nat. Phys. 7, 901 (2011)



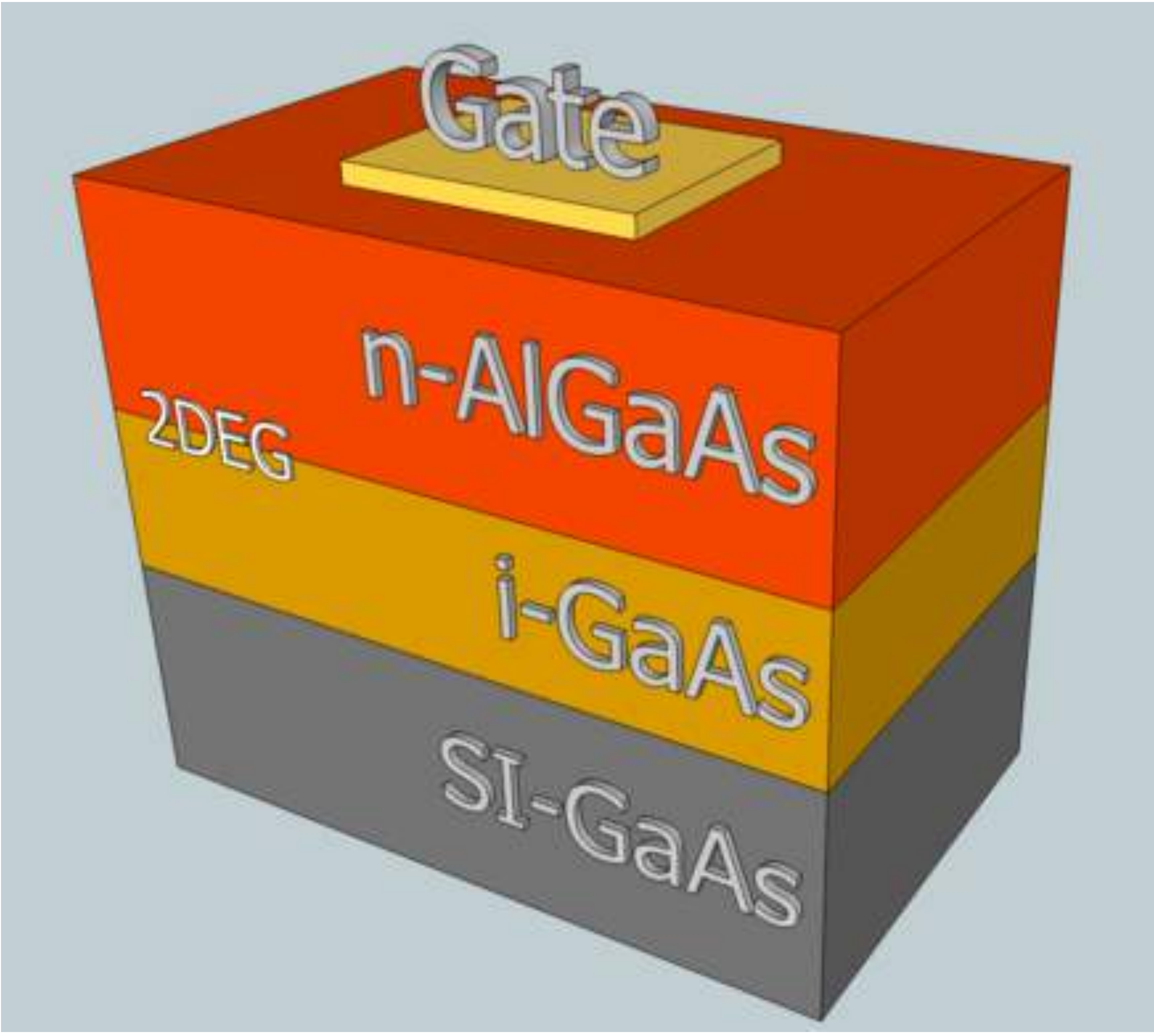
a
G/G$_0$
1.0
0.5
0
300
200
100
0
z (pm)
Tunnelling
Point contact
b
z (pm)
282
182
82
42
37
27
−7.9
−18
−23
−29
−34
−38
−42
−46
−48
−53
−56
−58
−60
dI/dV (normalized)
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
−40
−20
0
20
40
Bias (mV)
c
T (ω)
0.25
0.20
0.15
0.10
0.05
0
Δ
0.100
0.105
0.110
0.115
0.120
0.140
0.160
−6
−4
−2
0
2
4
6
ω/T$_K$ (Δ = 0)

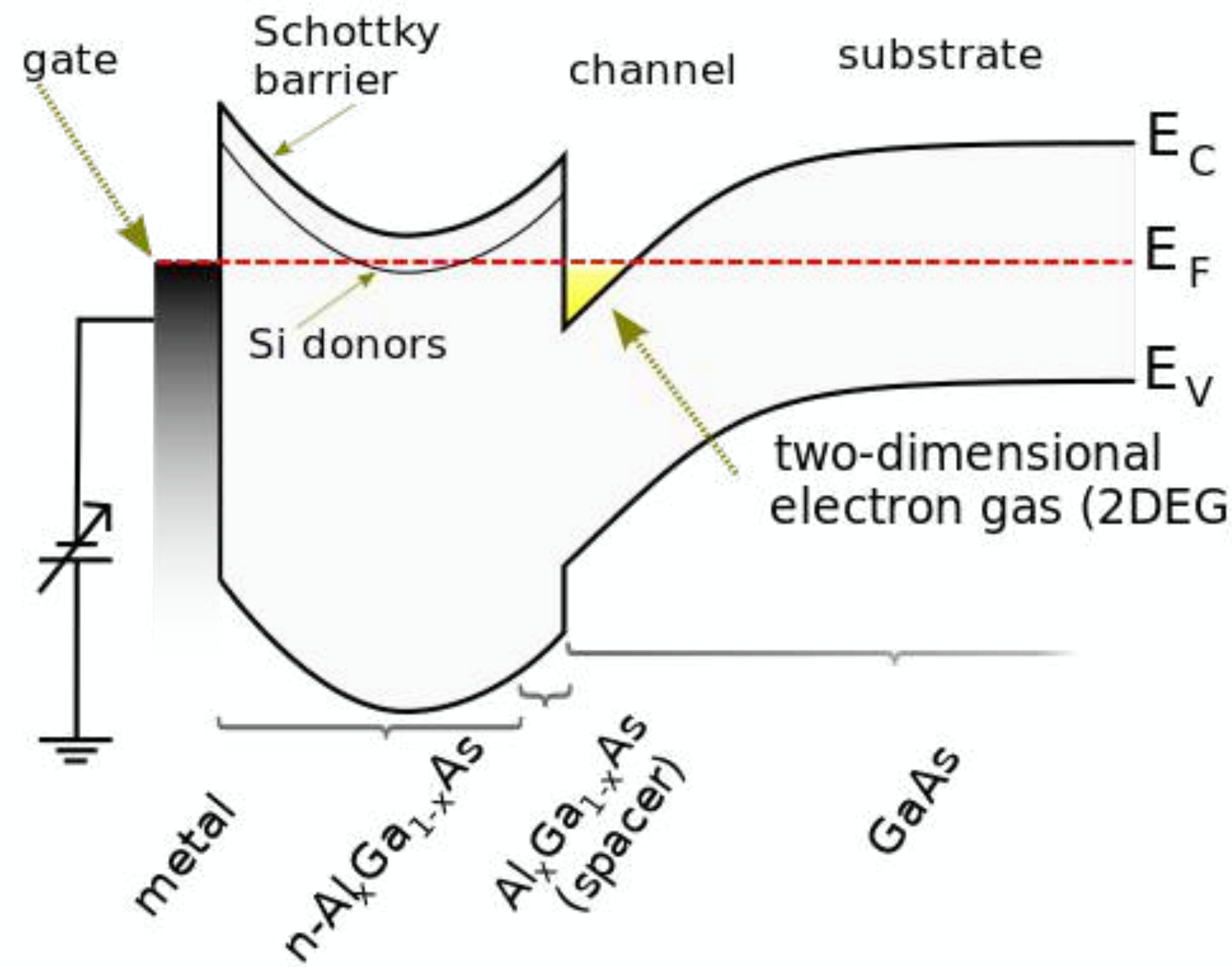


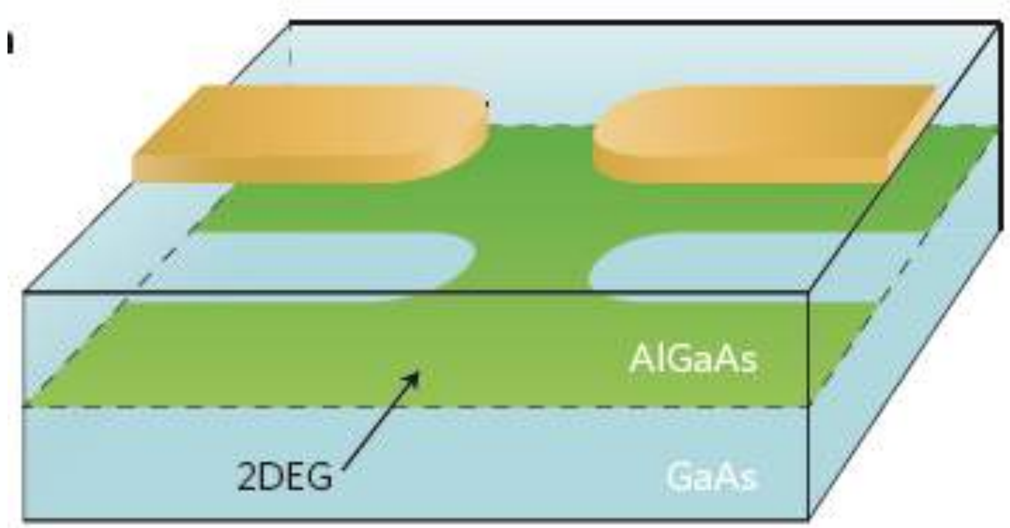
a
Energy (in units of T_K^0)
T_{st}
T_K^0
T^*
KS
AF
QPT
QCP
I^*
I/T_K^0
0
1
2
3

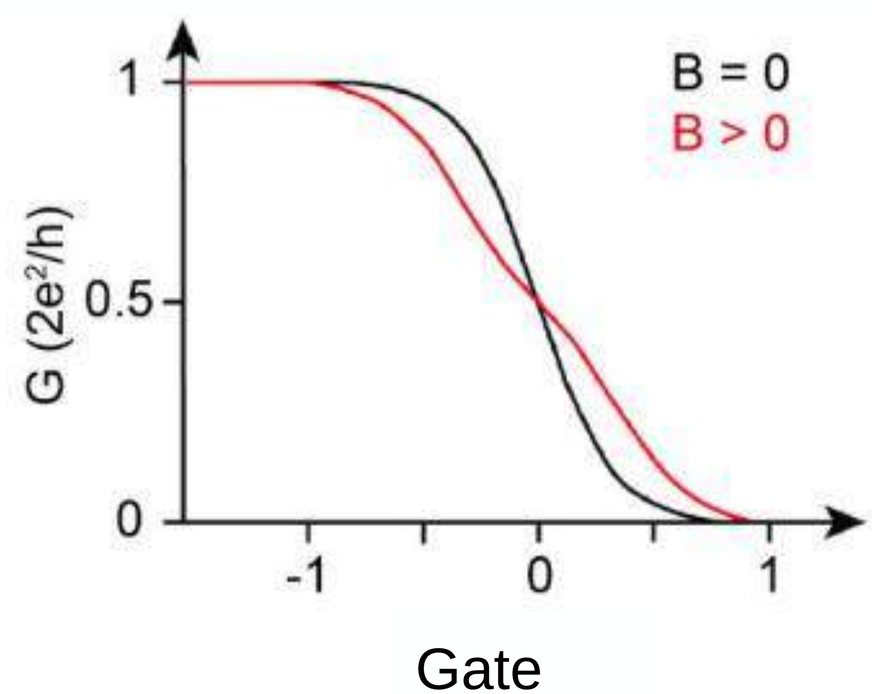


Quantum point contacts

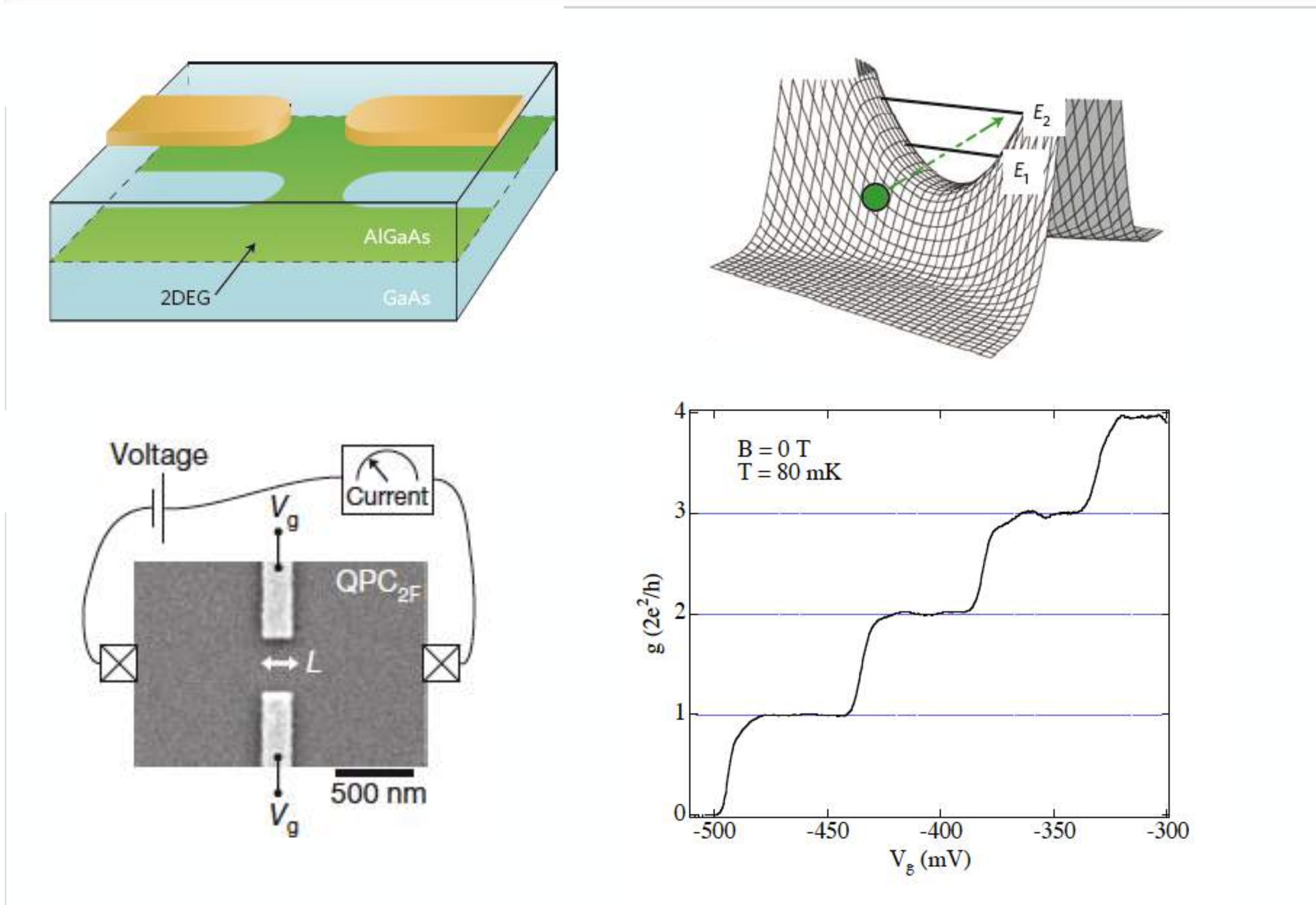

Gate
n-AlGaAs
2DEG
i-GaAs
SI-GaAs


gate
Schottky barrier
channel
substrate
E_C
E_F
E_V
Si donors
two-dimensional electron gas (2DEG)
metal
n-$Al_xGa_{1-x}As$
$Al_xGa_{1-x}As$ (spacer)
GaAs

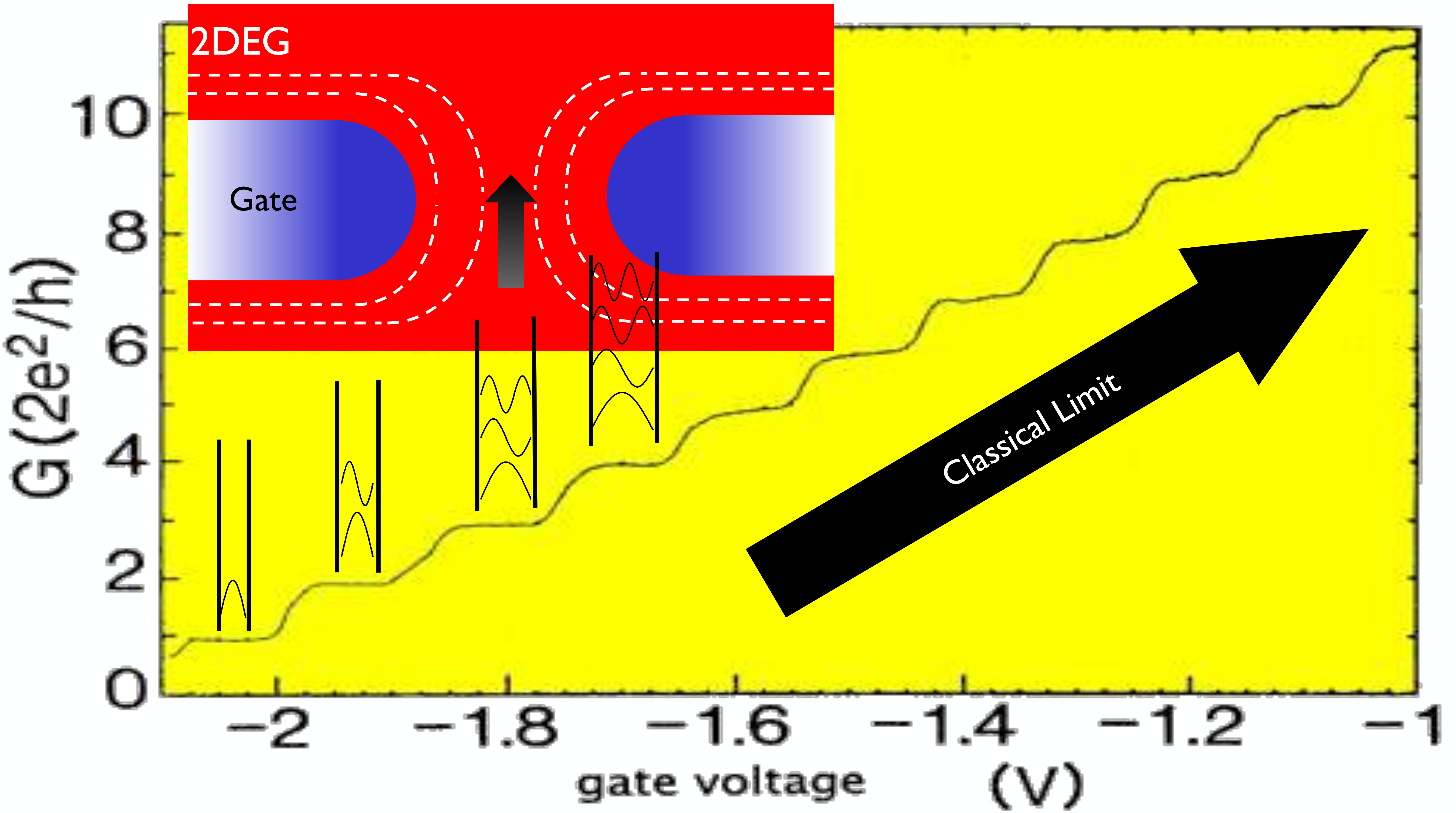

AlGaAs
2DEG
GaAs


B = 0
B > 0
G ($2e^2/h$)
1
0.5
0
-1
0
1
Gate

Quantum point contacts



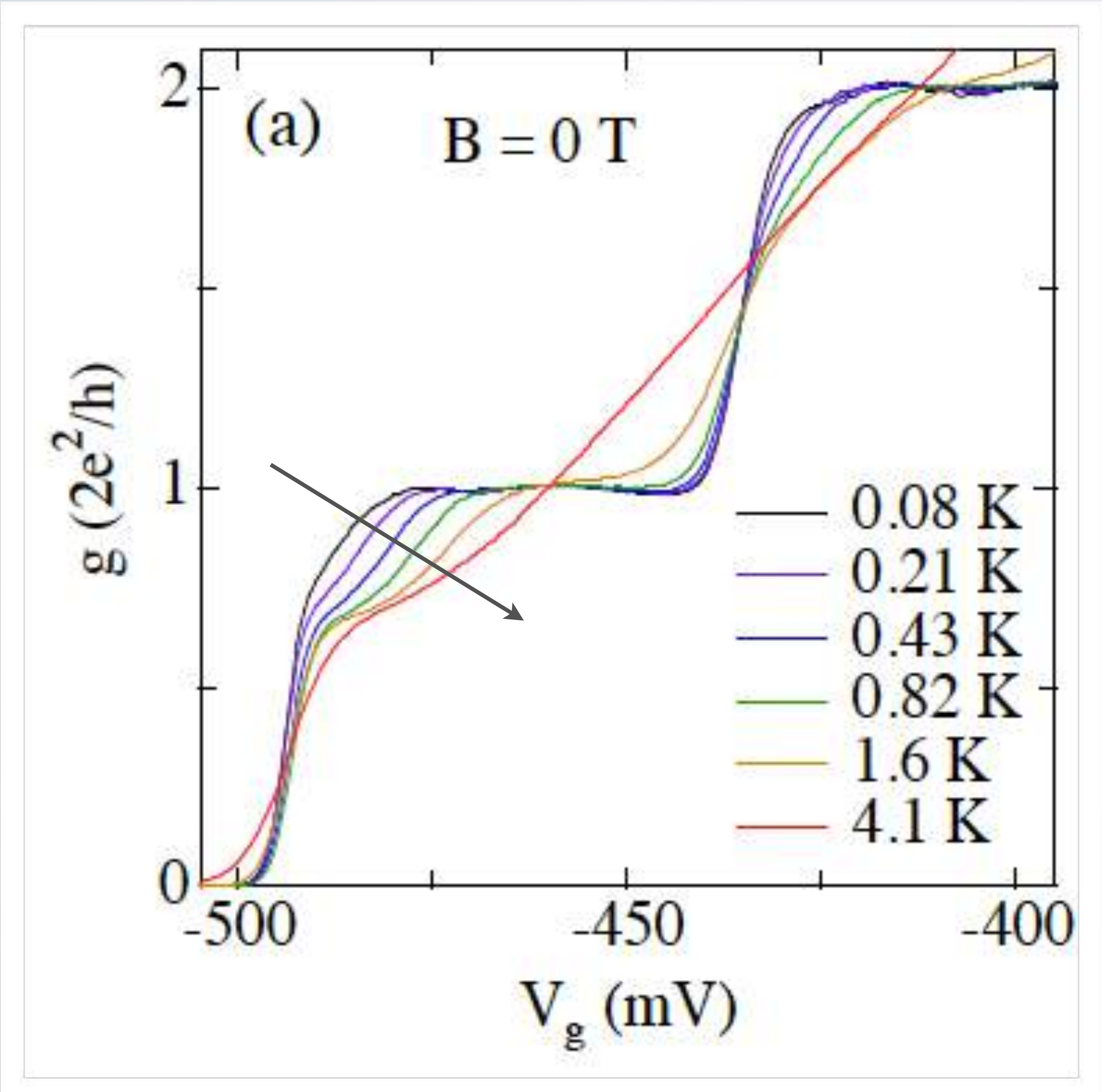
Conductance quantisation



- "Quantized conductance of point contacts in a two-dimensional electron gas", B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson , L. P. Kouwenhoven, D. van der Marel, C. T. Foxon, Phys. Rev. Lett, 60, 848 (1988).

- "One-dimensional transport and the quantisation of the ballistic resistance", D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J.E.F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie and G. A. C Jones, J. Phys. C: Solid State Phys. 21 L209 (1988).

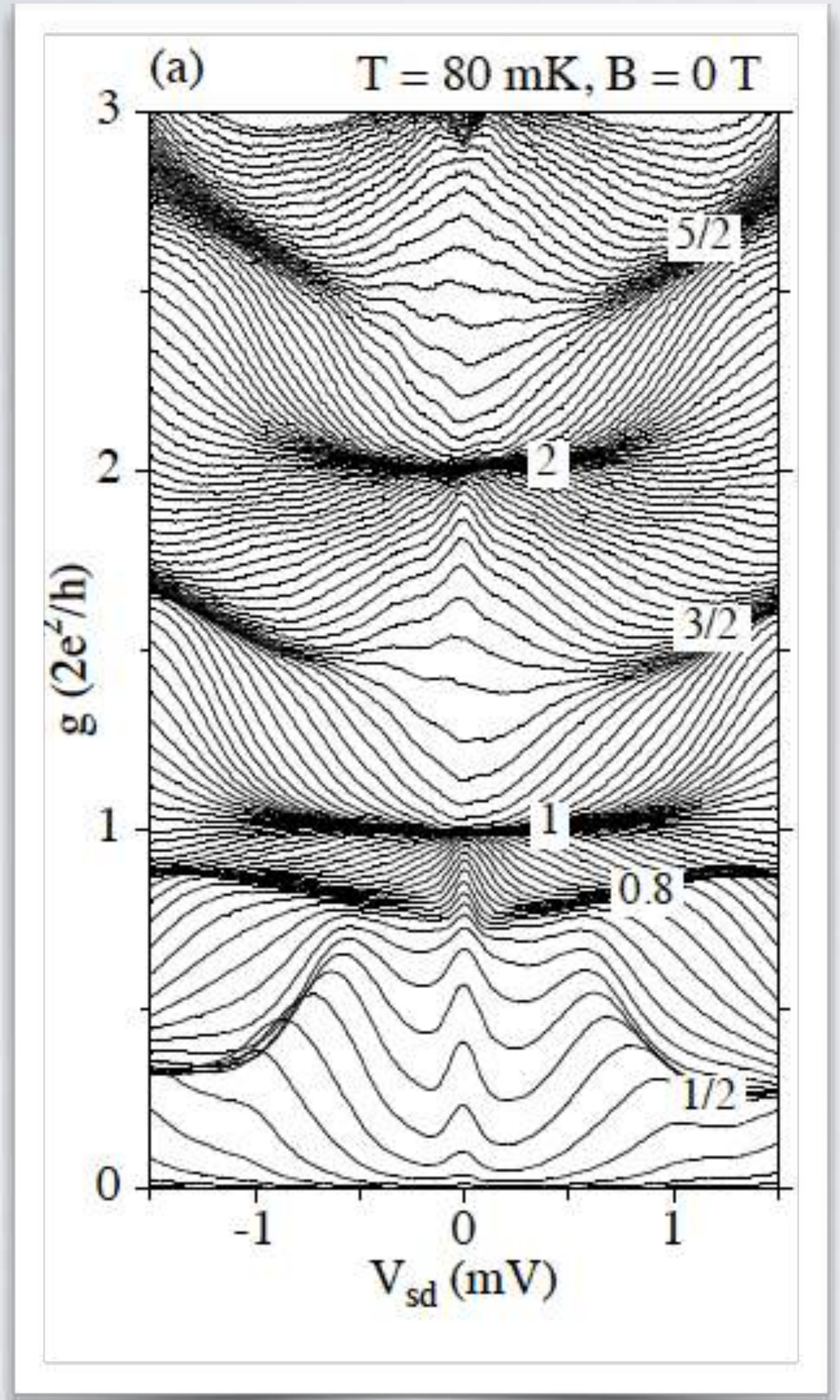
0.7 anomaly



Clear shoulder as one increases temperature!!

Sara Cronenwett's thesis, Marcus Lab, Harvard 2002

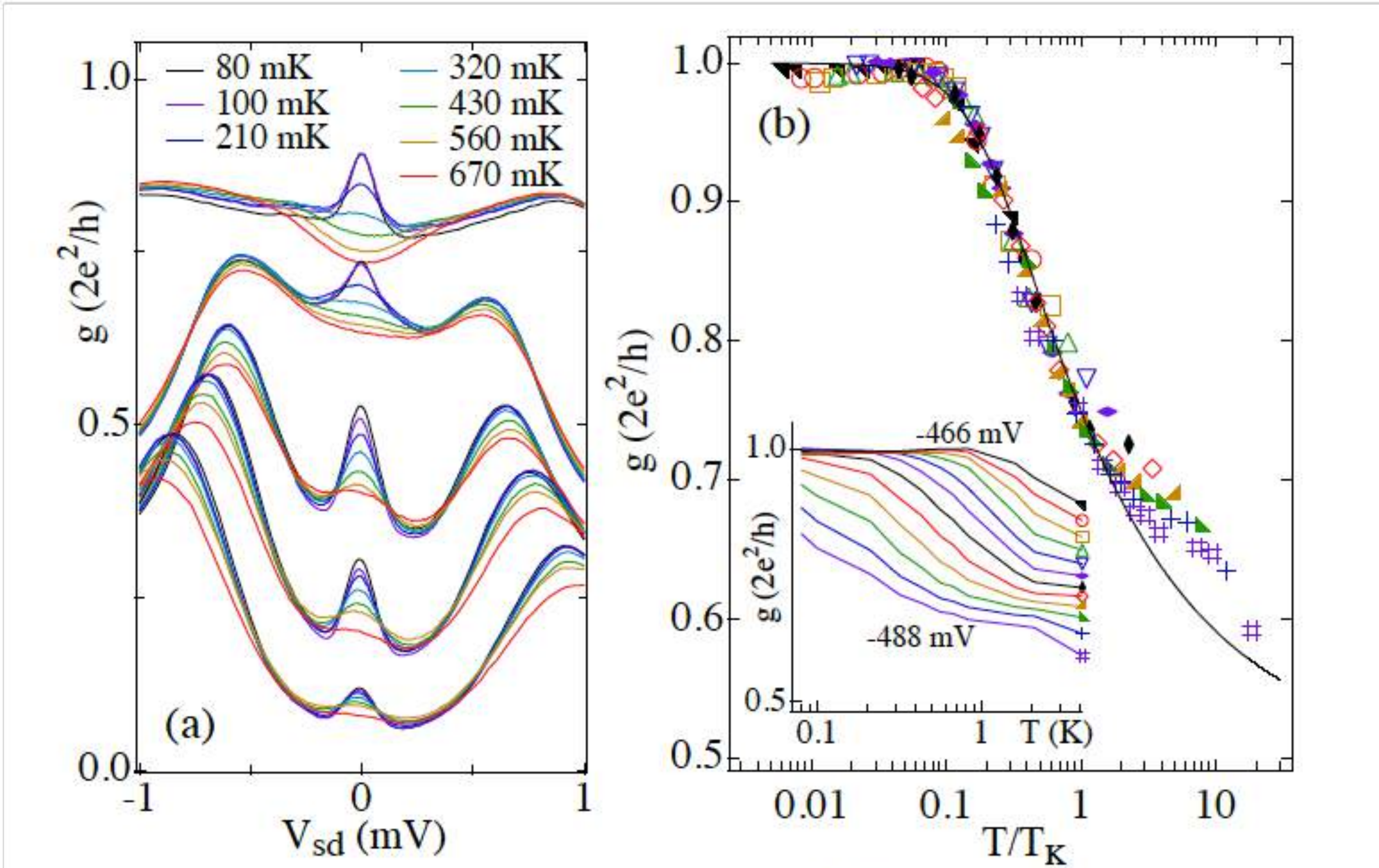
0.7 anomaly



The differential conductance at finite bias shows clear zero-bias anomalies at low temperatures. Kondo physics in an open system?

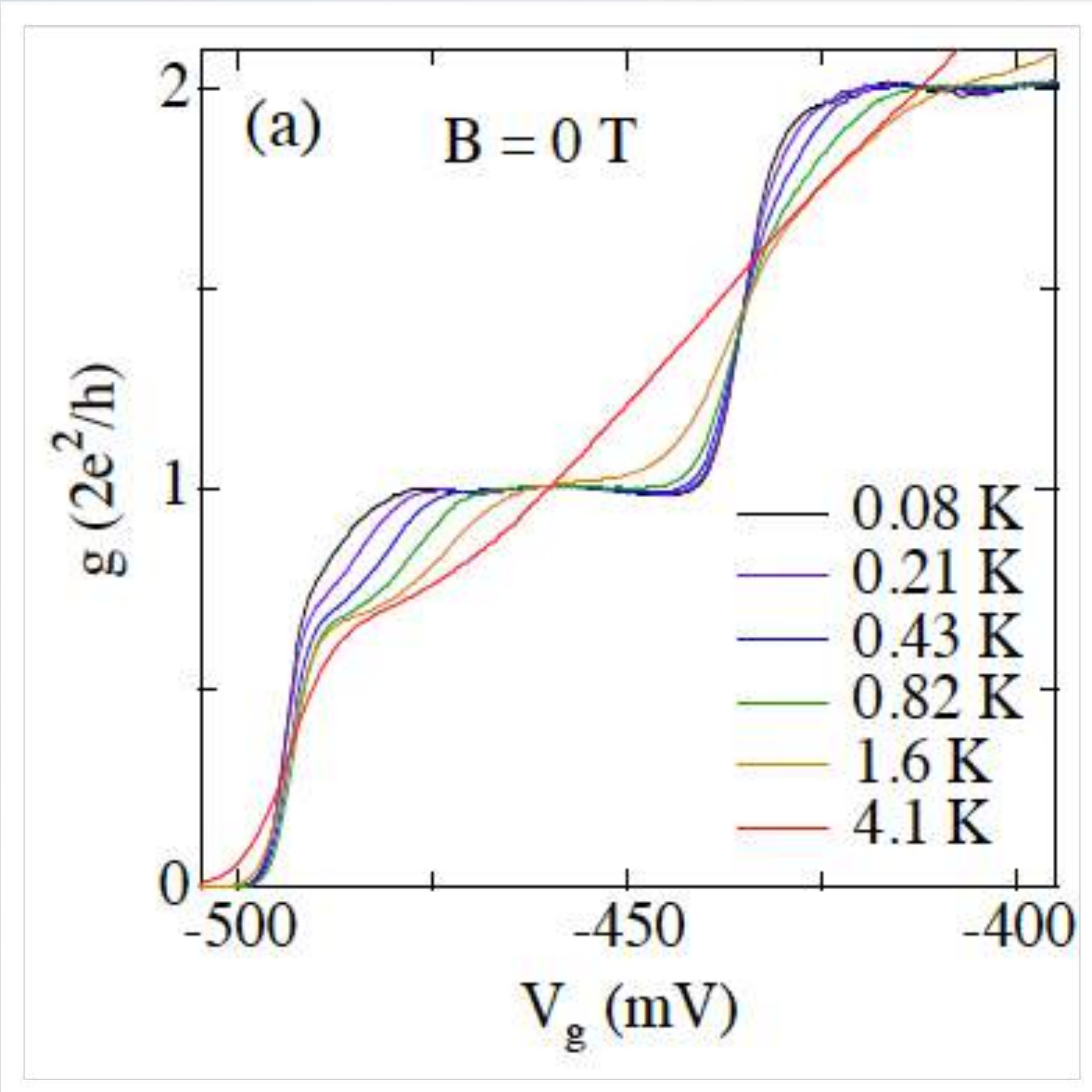
Sara Cronenwett's thesis, Marcus Lab, Harvard 2002

Kondo features



Cronenwett et al, Phys. Rev. Lett. 88, 226805 (2002).

0.7 anomaly



The anomaly at "high temperatures" is originated from a magnetic moment (similar to Coulomb blockade, hence the conductance reduction) which is screened at low temp. by Kondo

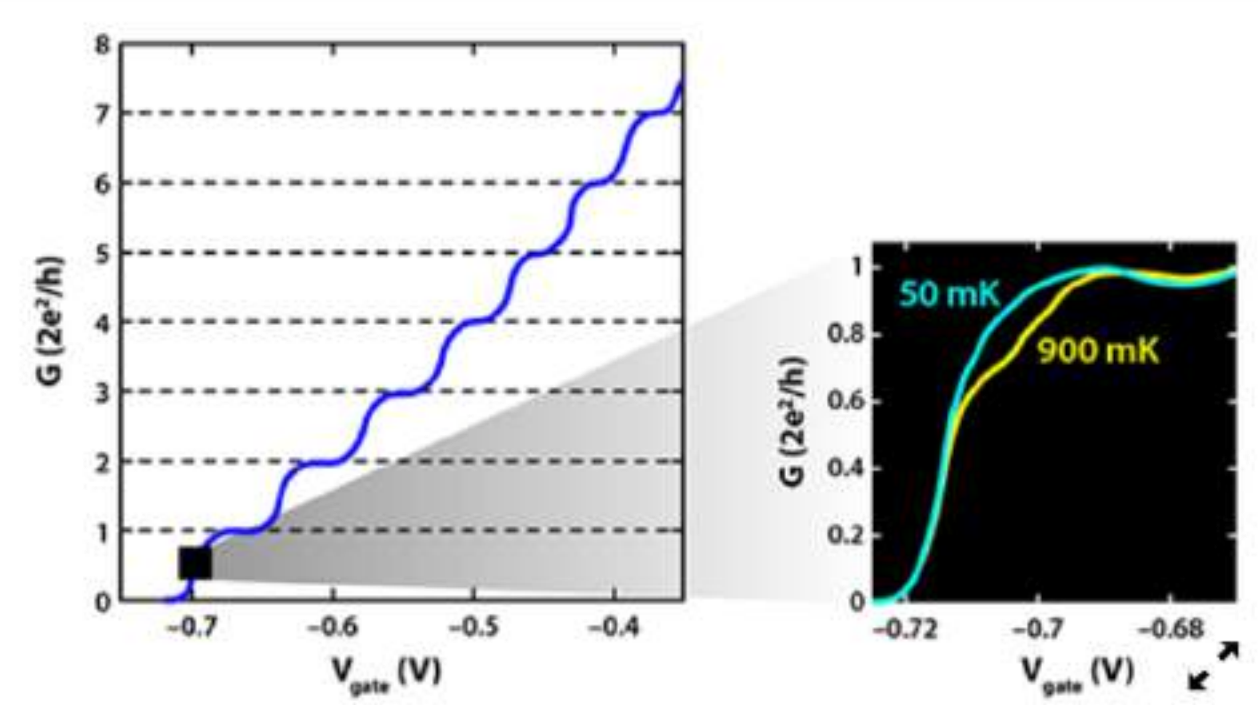
VIEWPOINT

Kondo Physics in a Quantum Channel

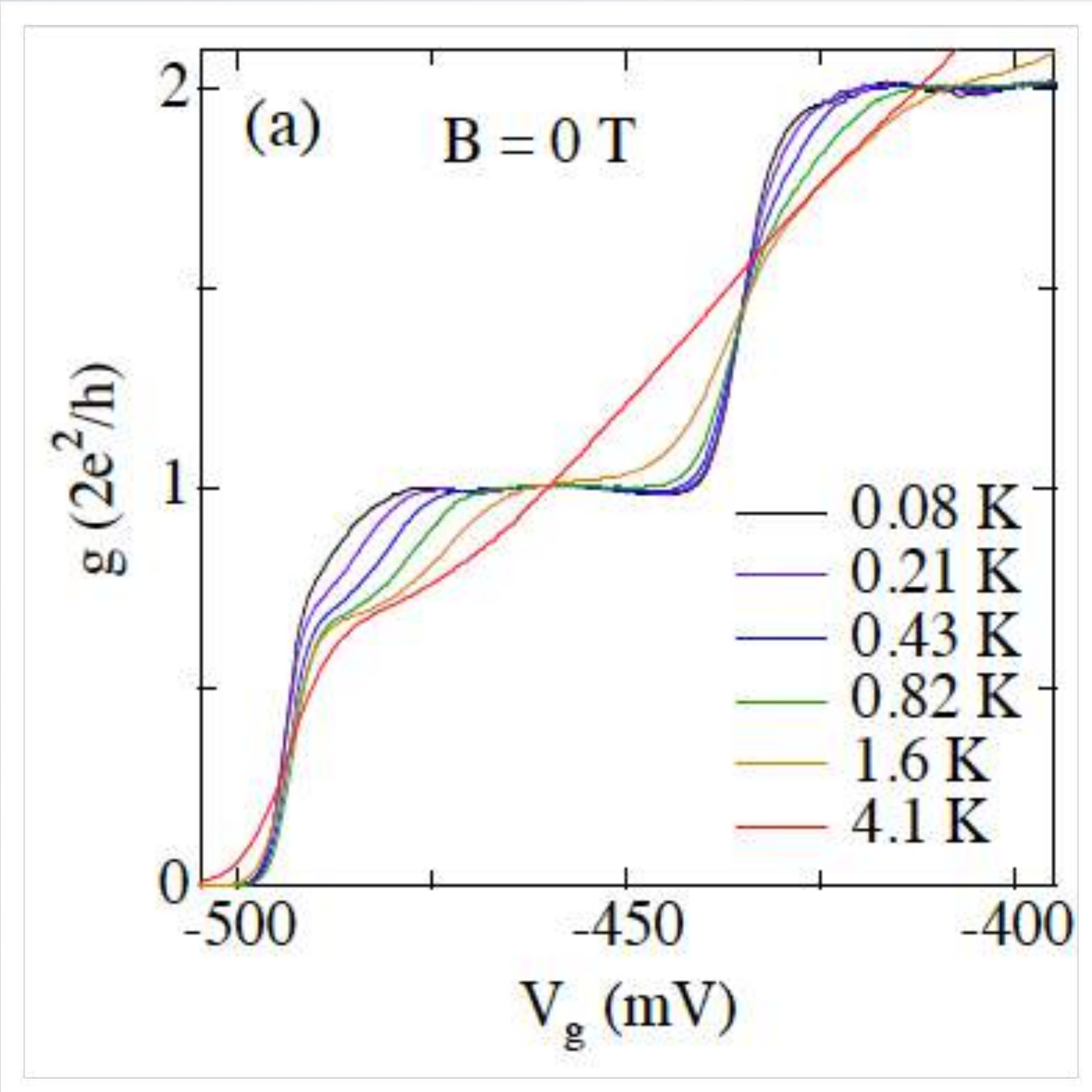
Ramón Aguado
Theory and Simulation of Materials Department, Instituto de Ciencia de Materiales de Madrid (ICMM), Consejo Superior de Investigaciones Cientificas (CSIC), Cantoblanco, 28049 Madrid, Spain

March 28, 2016 • *Physics* 9, 32

Using a scanning gate microscope, researchers have shown that electron waves scattered from a quantum point contact carry the imprint of interactions with localized electron spins.



0.7 anomaly



- Theory predicts that such moments can form near pinch-off (Rejec and Meir, Nature 442, 900-903, 2006) but not consensus.
- Importantly, theory also predict that long contacts can host <u>**two impurities**</u>

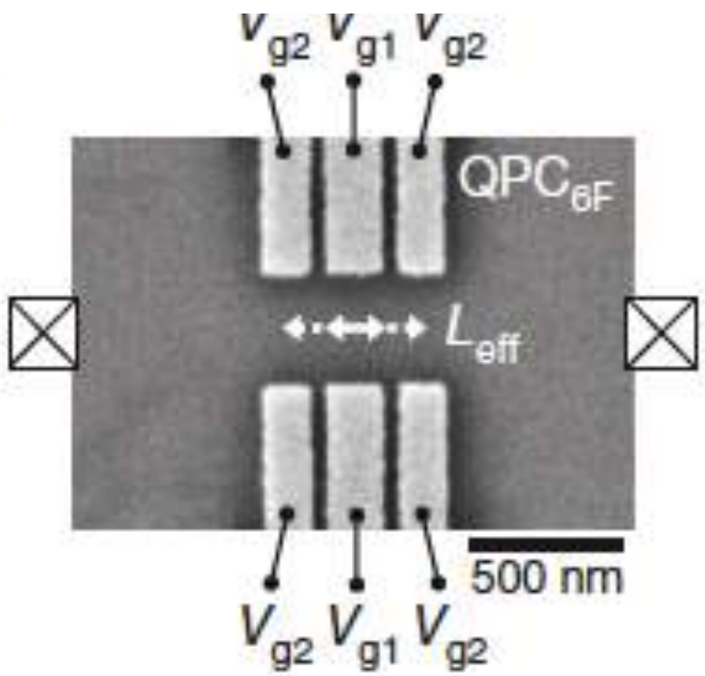
Two-impurity Kondo physics in length-tunable QPCs

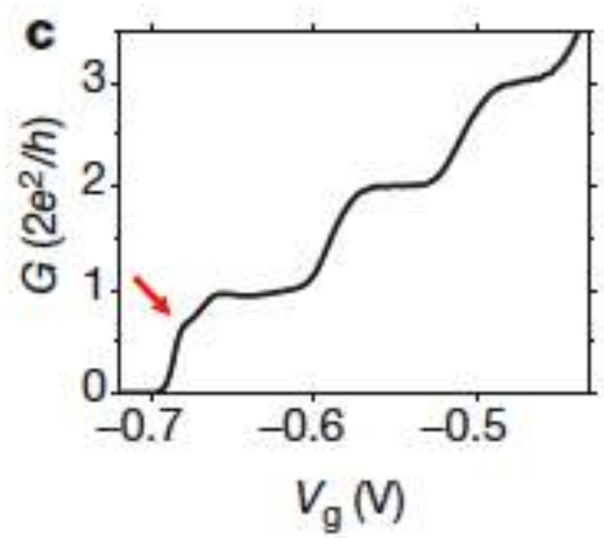
Nature 501, 79–83 (2013)

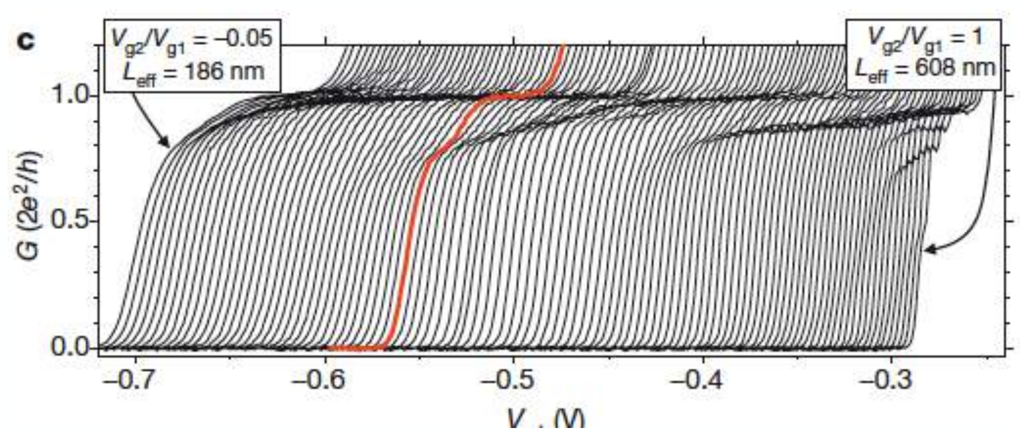
doi:10.1038/nature12491

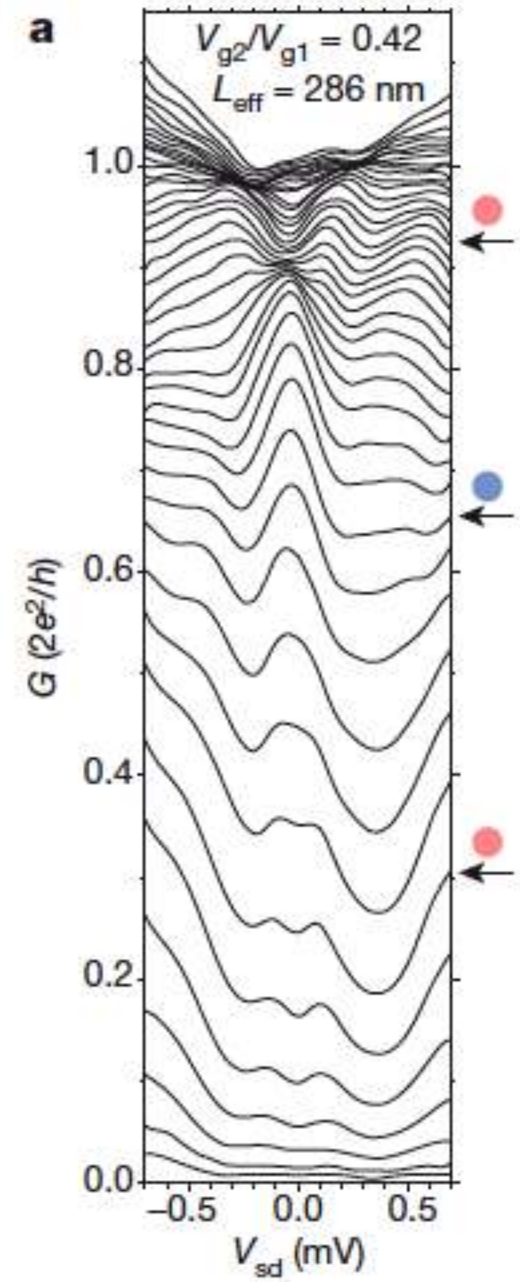
Odd and even Kondo effects from emergent localization in quantum point contacts

M. J. Iqbal[1], Roi Levy[2], E. J. Koop[1], J. B. Dekker[1], J. P. de Jong[1], J. H. M. van der Velde[1], D. Reuter[3], A. D. Wieck[3], Ramón Aguado[4], Yigal Meir[2,5] & C. H. van der Wal[1]









Two-impurity Kondo physics in length-tunable QPCs recently demonstrated

Nature 501, 79–83 (2013)

doi:10.1038/nature12491

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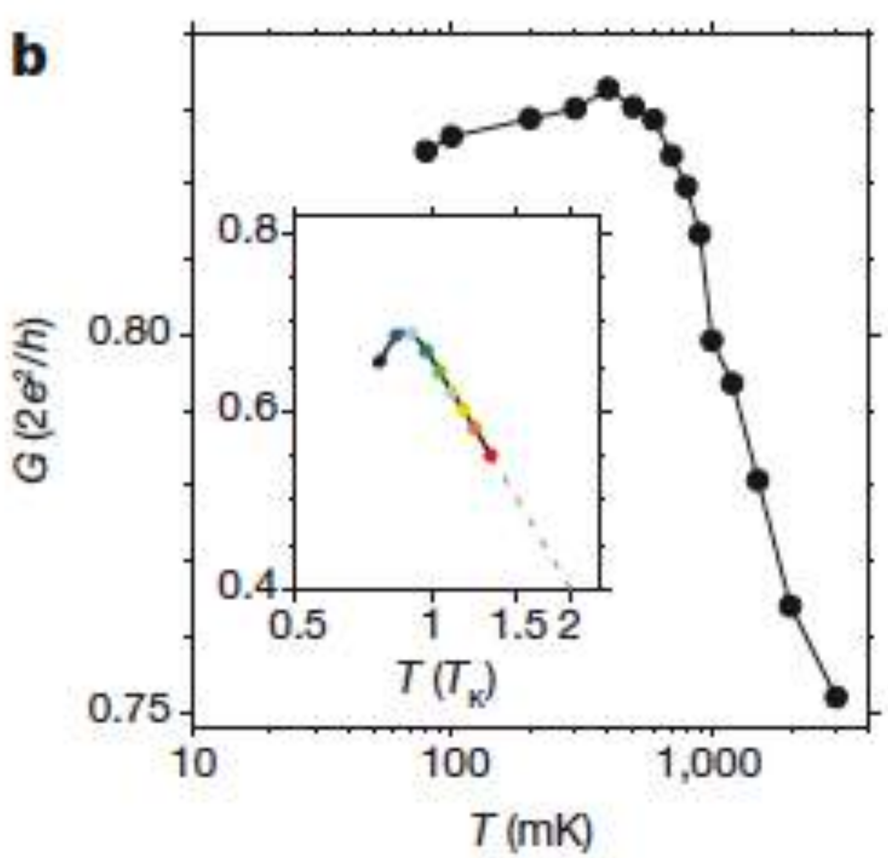
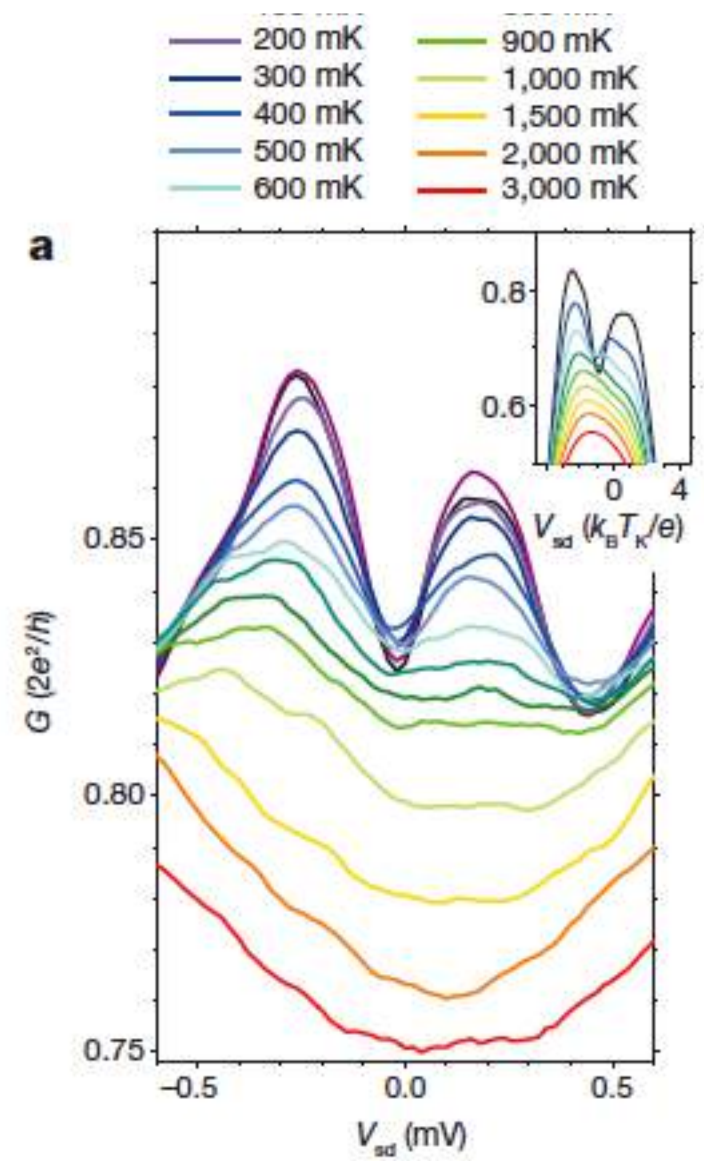
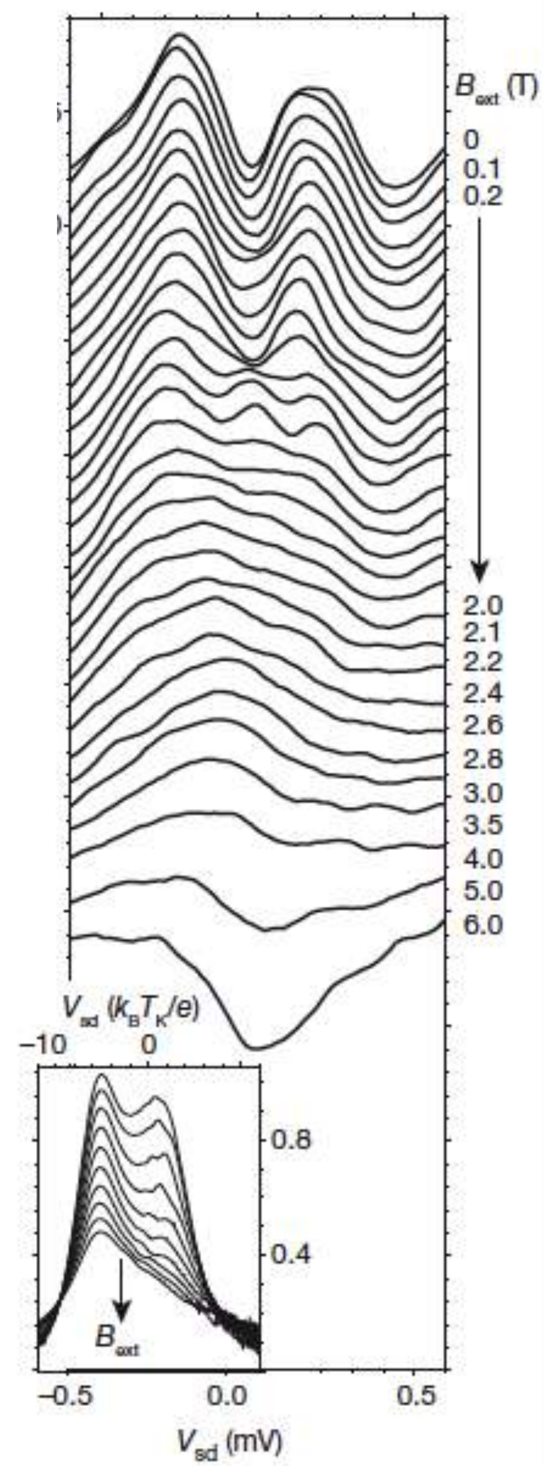
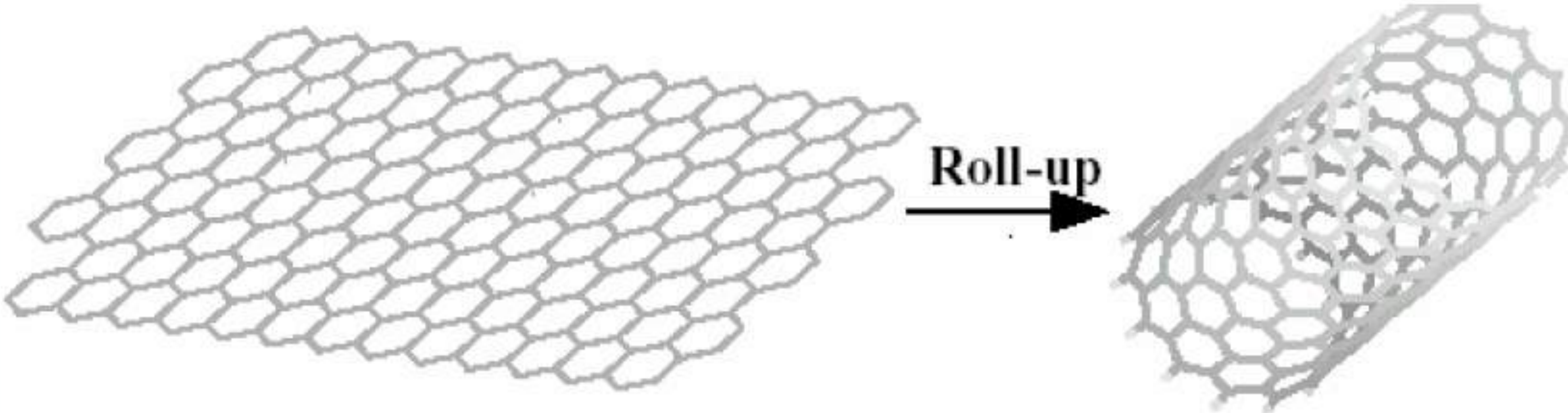






Splitting and Non-monotonic conductance: development of coherence!!

Nanotubes


Roll-up

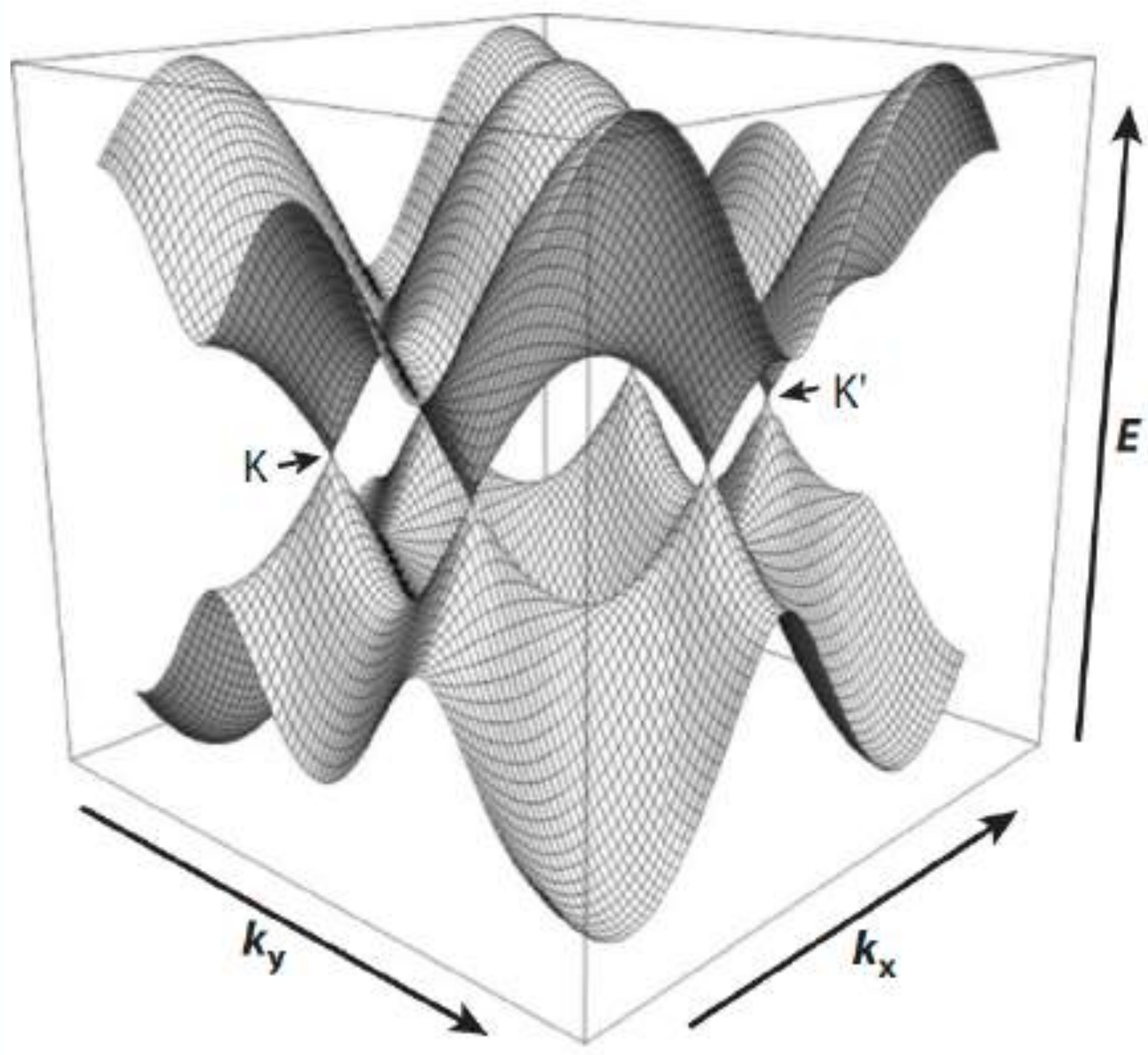
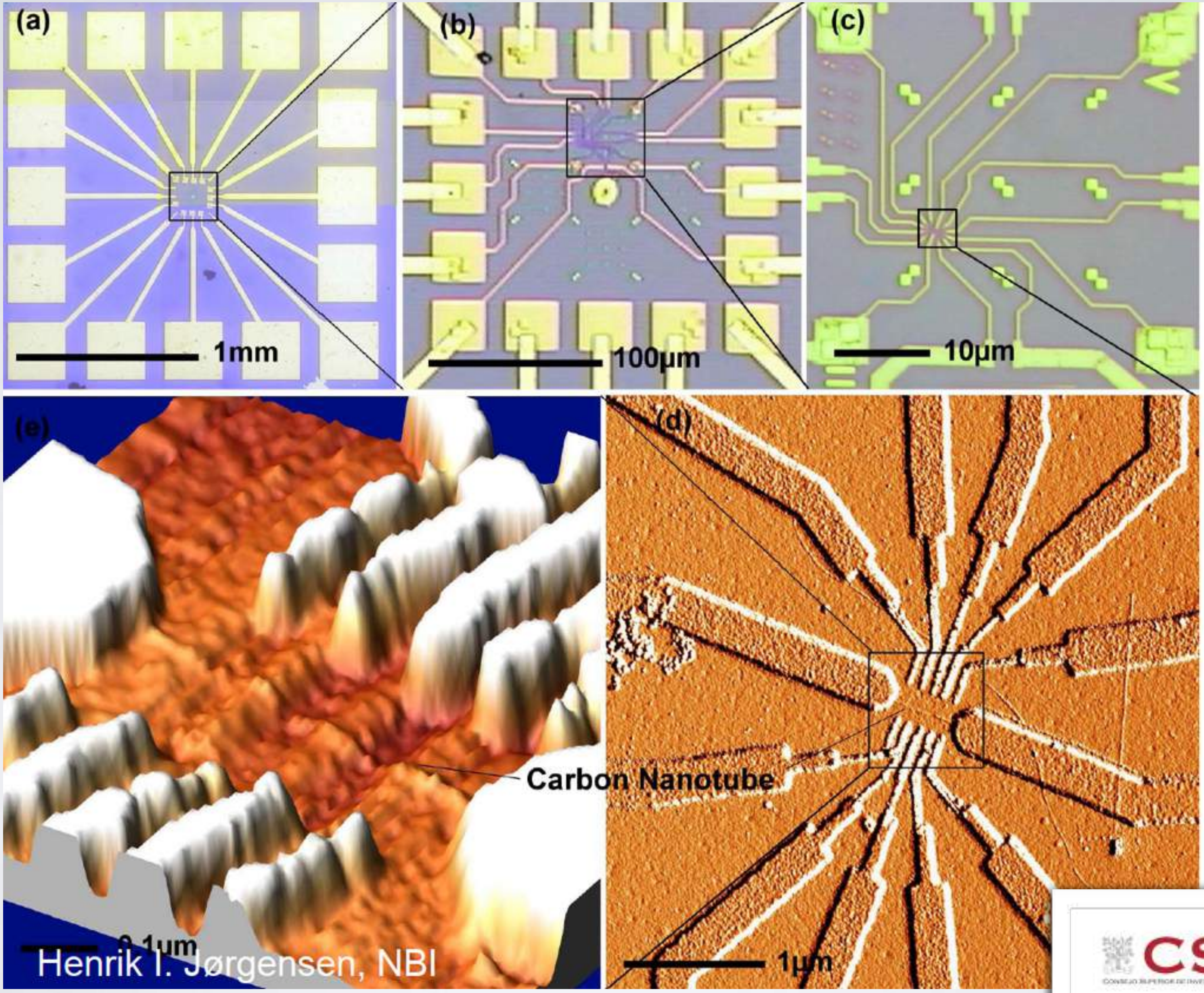
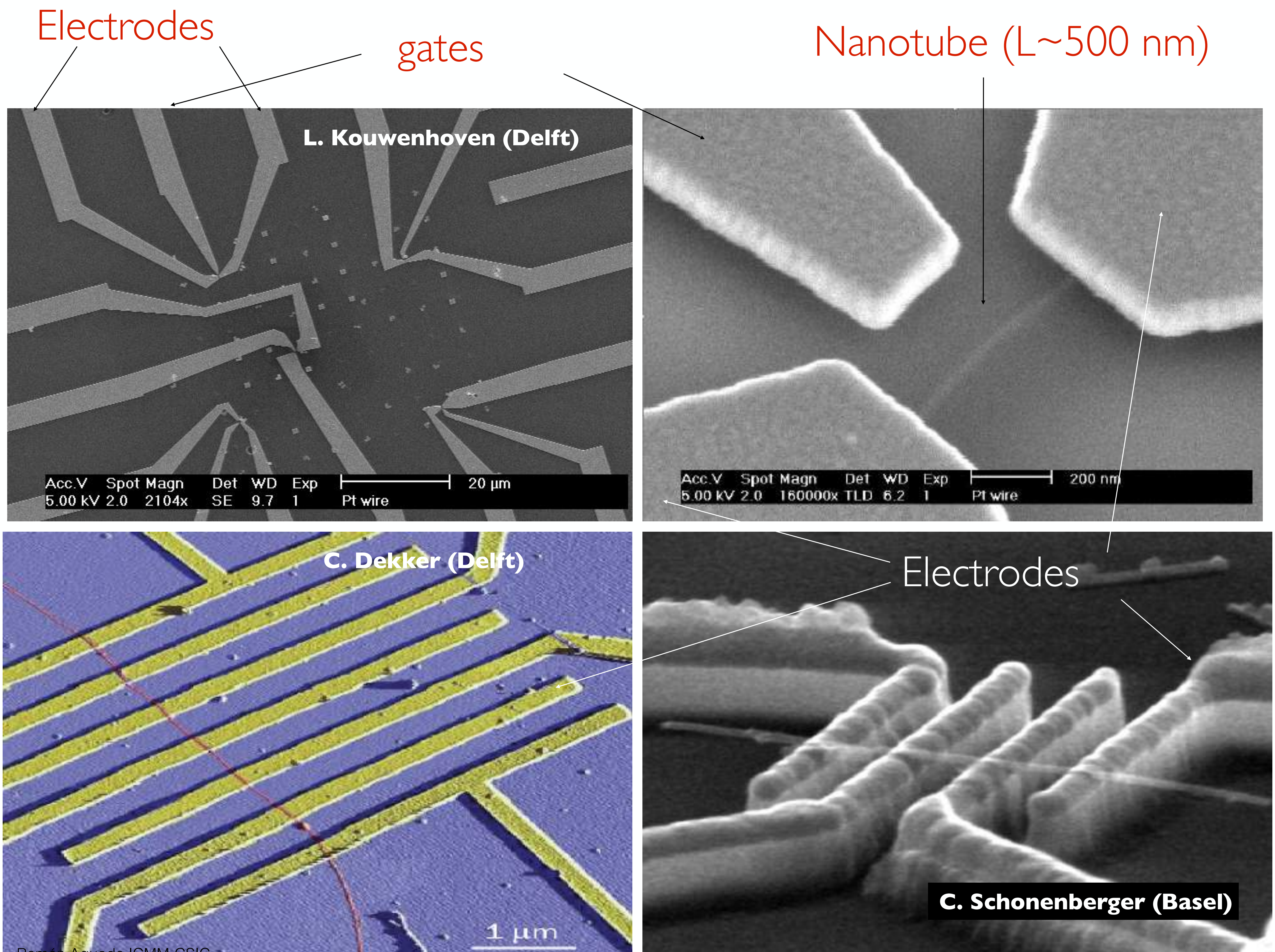

K'
K
E
k_y
k_x

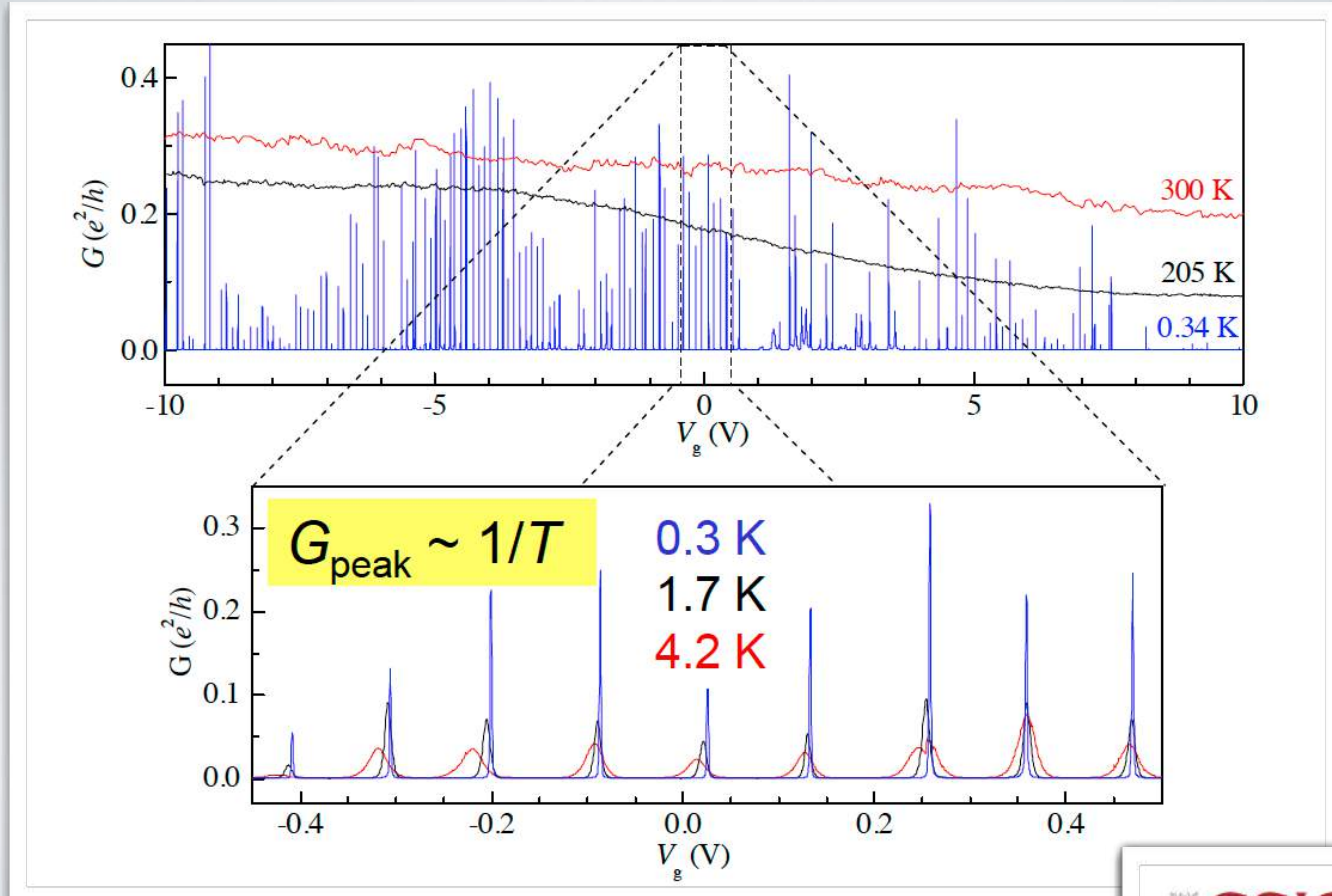
Image: Jesper Nygard, Niels Bohr, Copenhagen



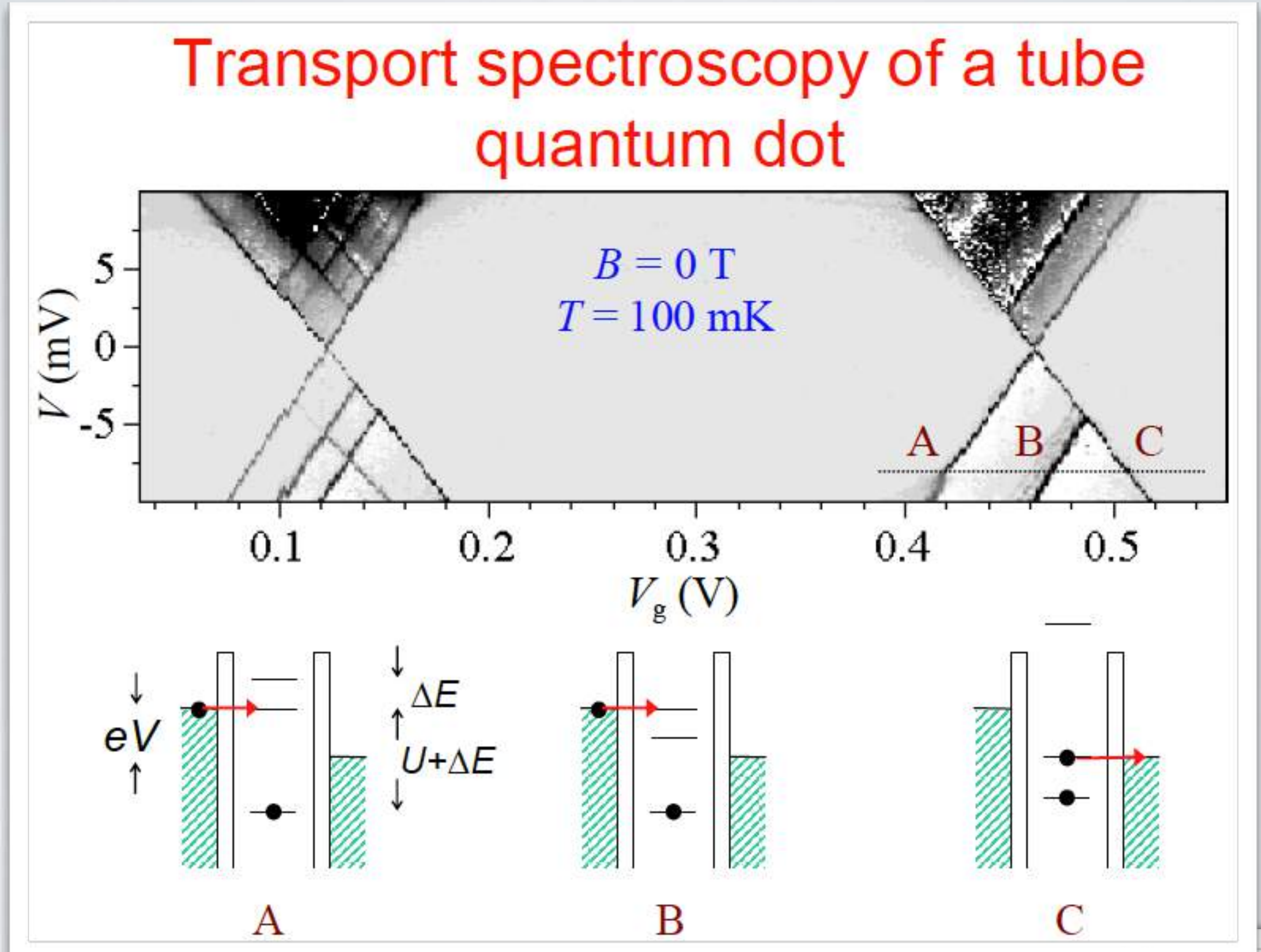
CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

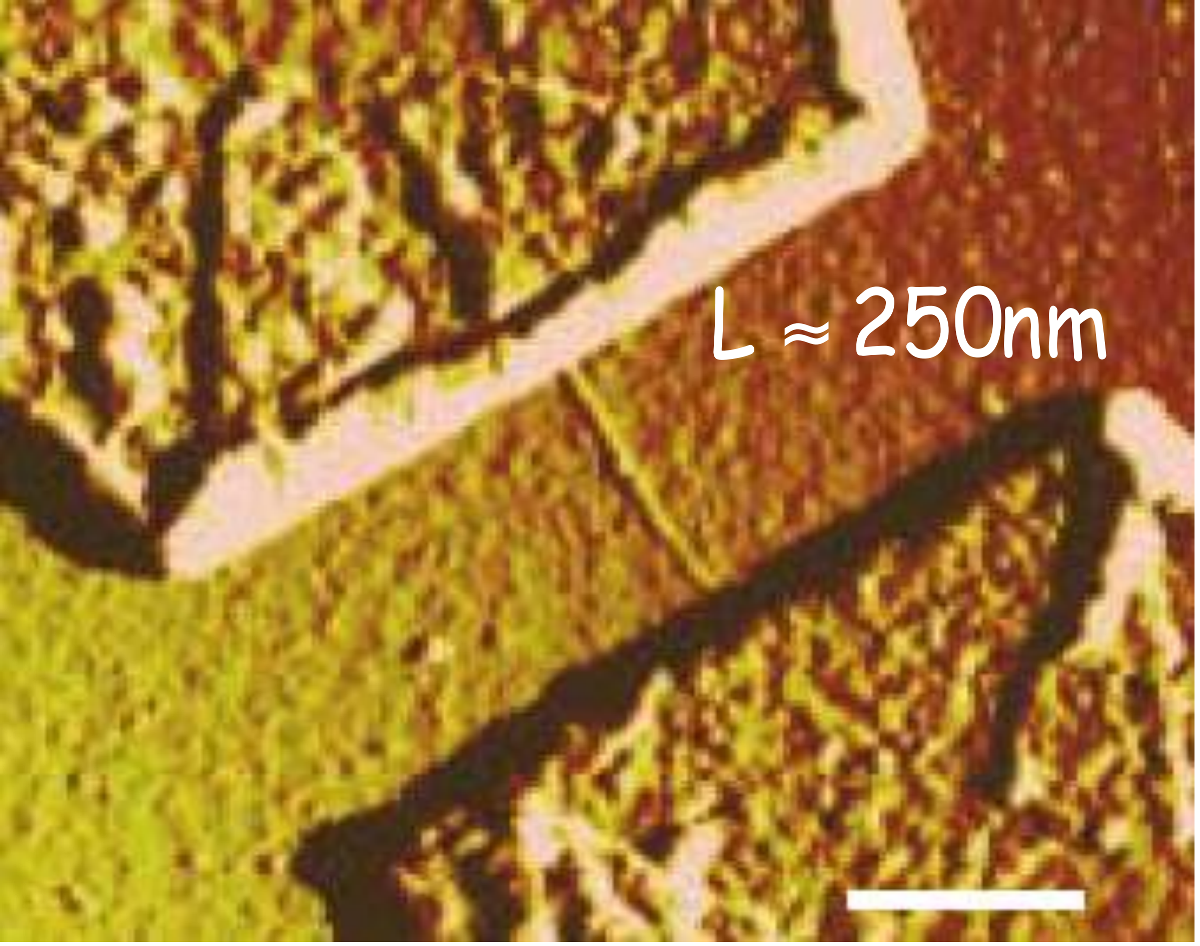

Electrodes
gates
Nanotube (L~500 nm)
L. Kouwenhoven (Delft)
Acc.V Spot Magn Det WD Exp 20 µm
5.00 kV 2.0 2104x SE 9.7 1 Pt wire
Acc.V Spot Magn Det WD Exp 200 nm
5.00 kV 2.0 160000x TLD 6.2 1 Pt wire
C. Dekker (Delft)
1 µm
Electrodes
C. Schonenberger (Basel)

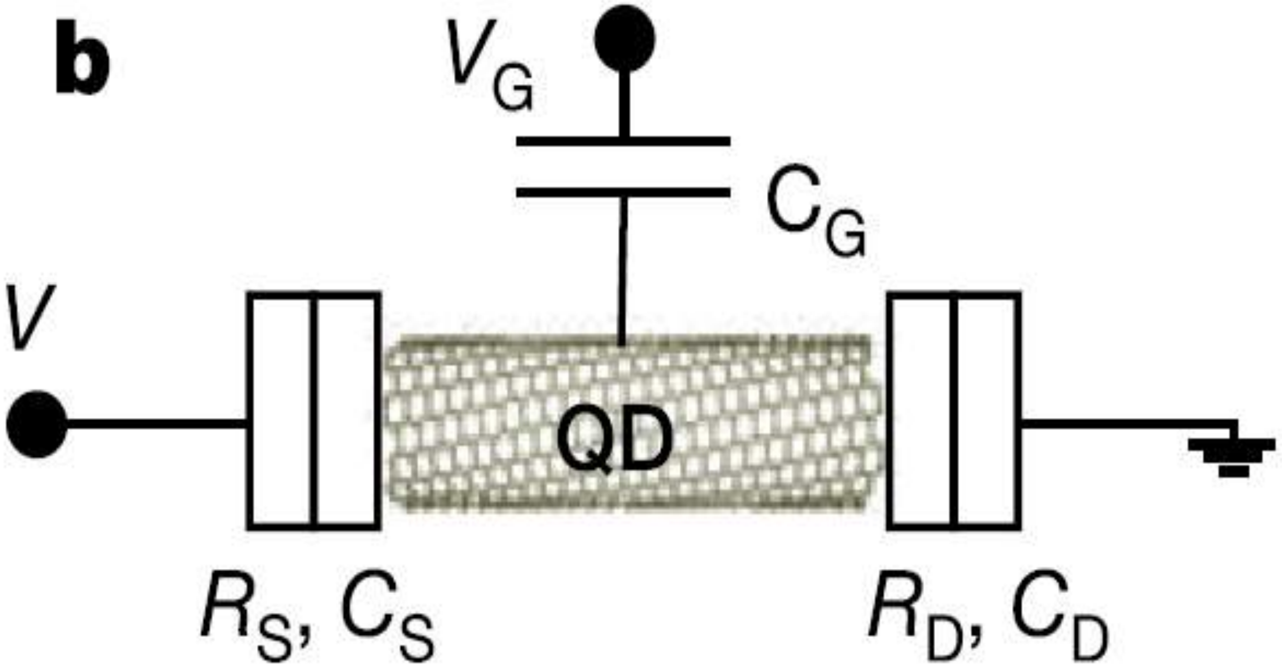
Quantum dot behaviour



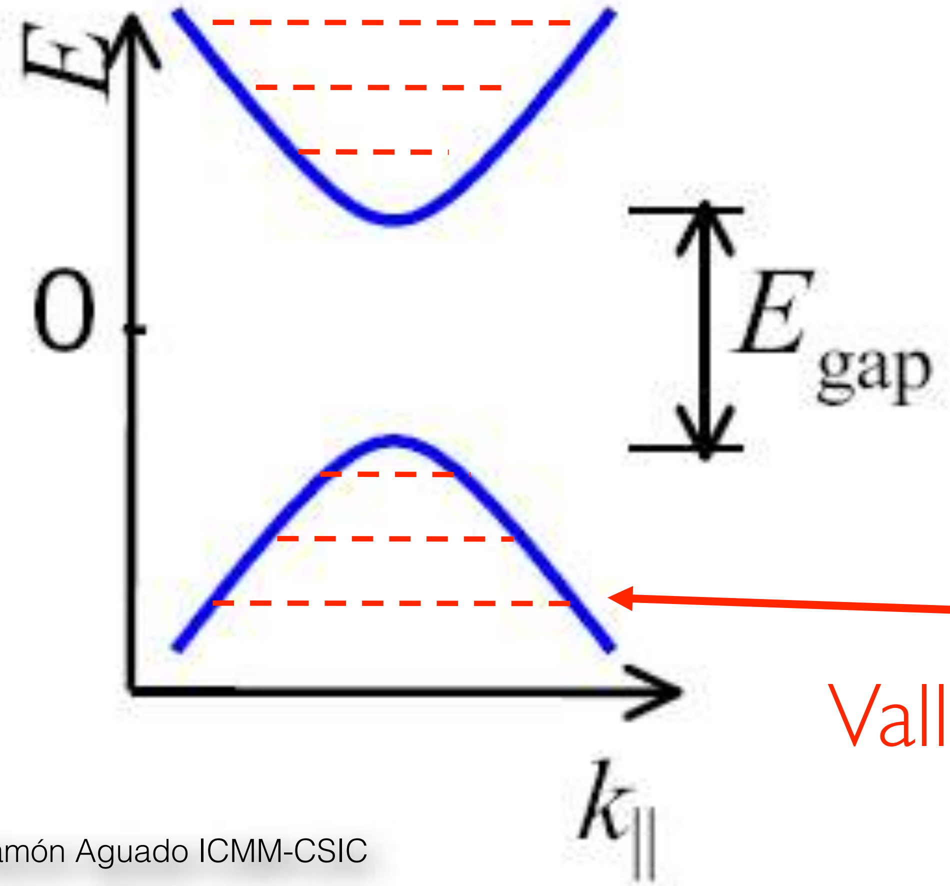
Quantum dot behaviour

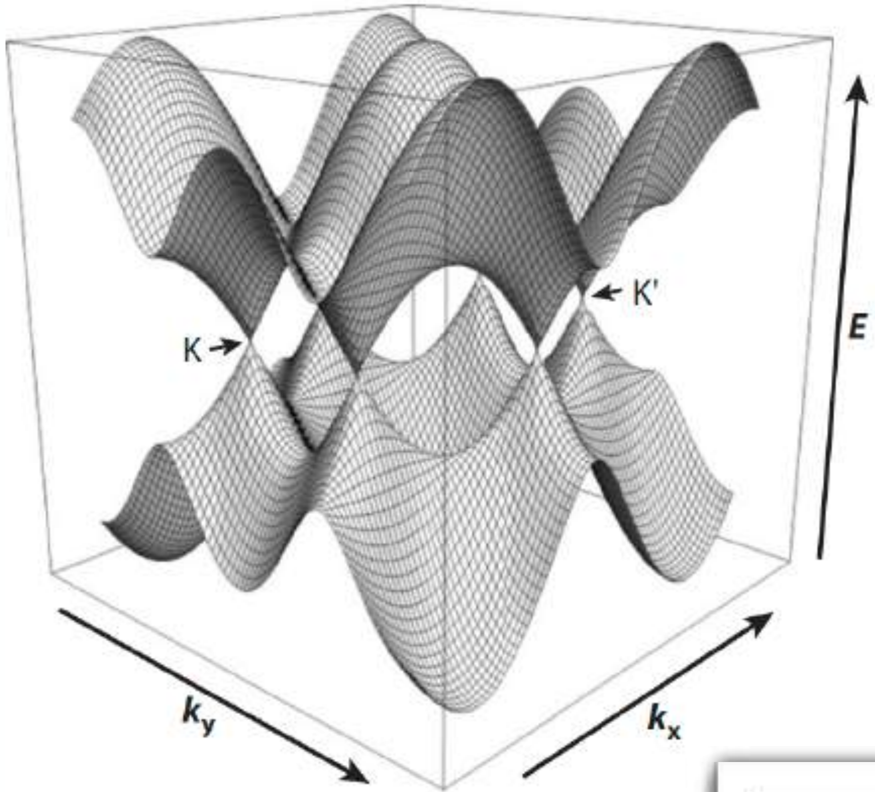






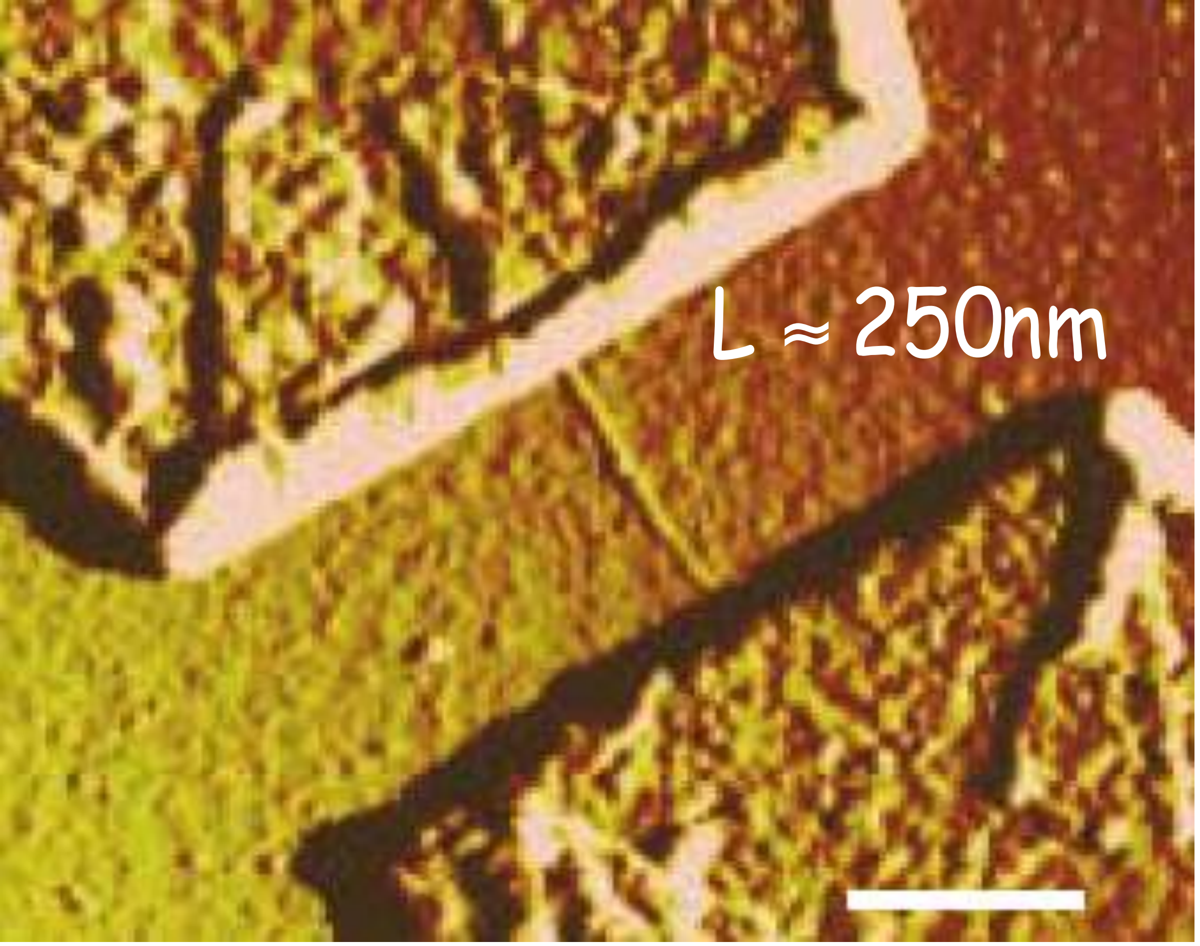
Short nanotubes, quantized levels



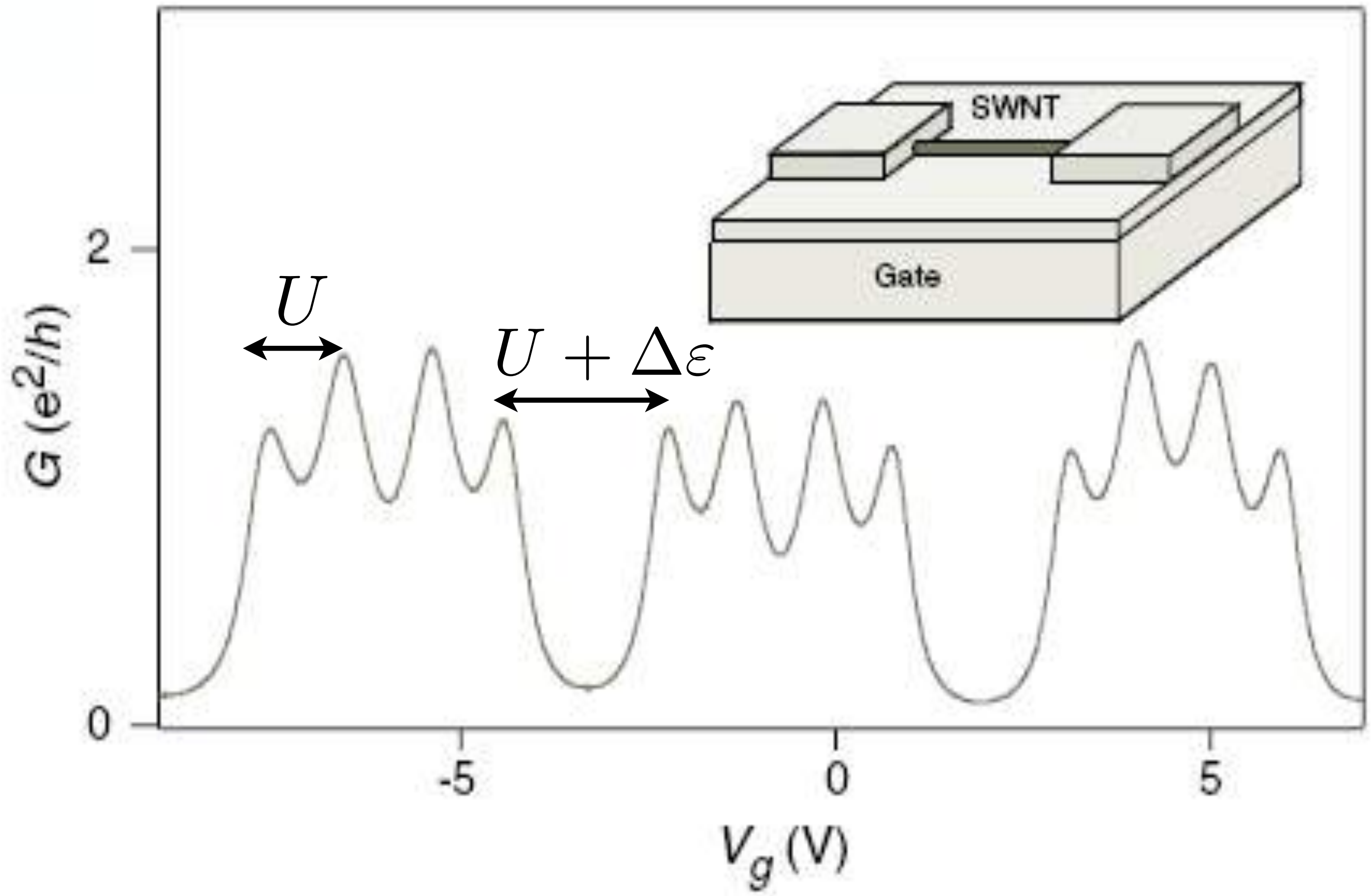


Valley degeneracy?

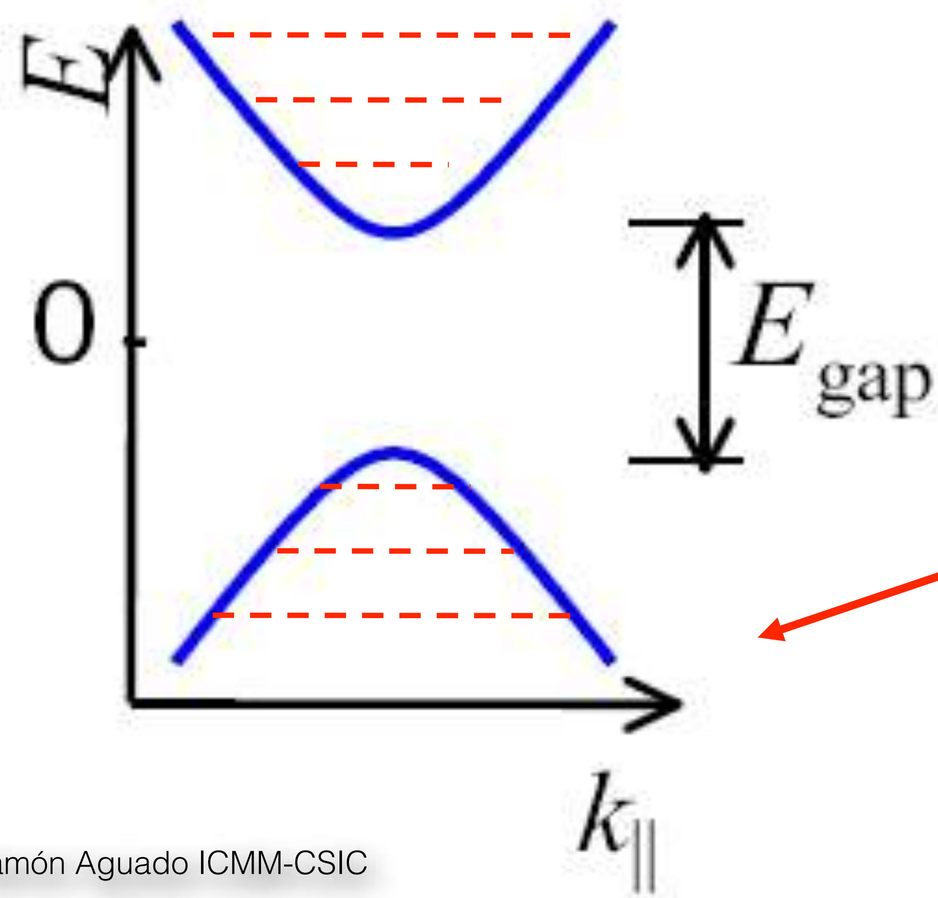
CSIC



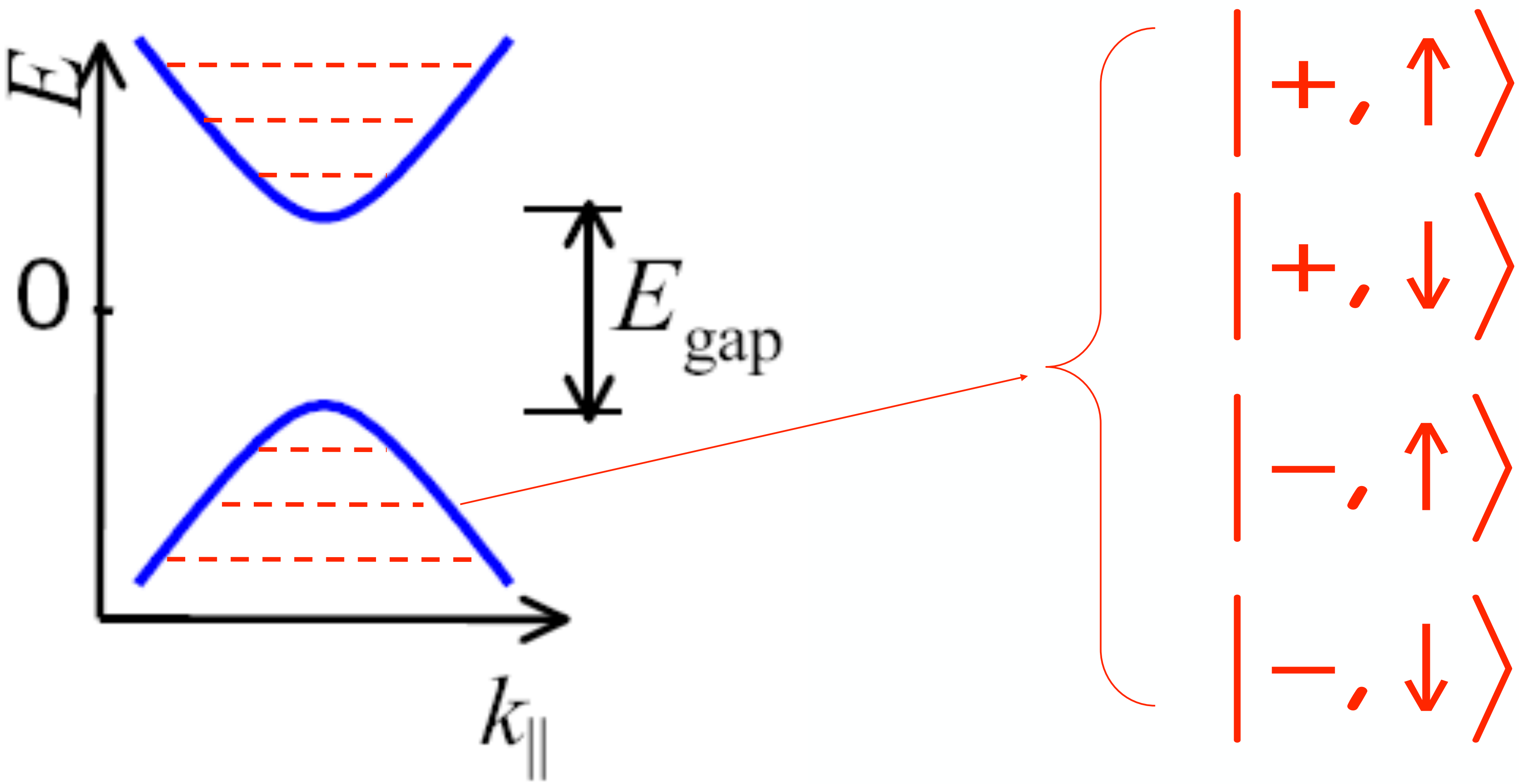
Wenjie Liang et al, PRL, 88, 126801 (2001)



Short nanotubes, quantized levels

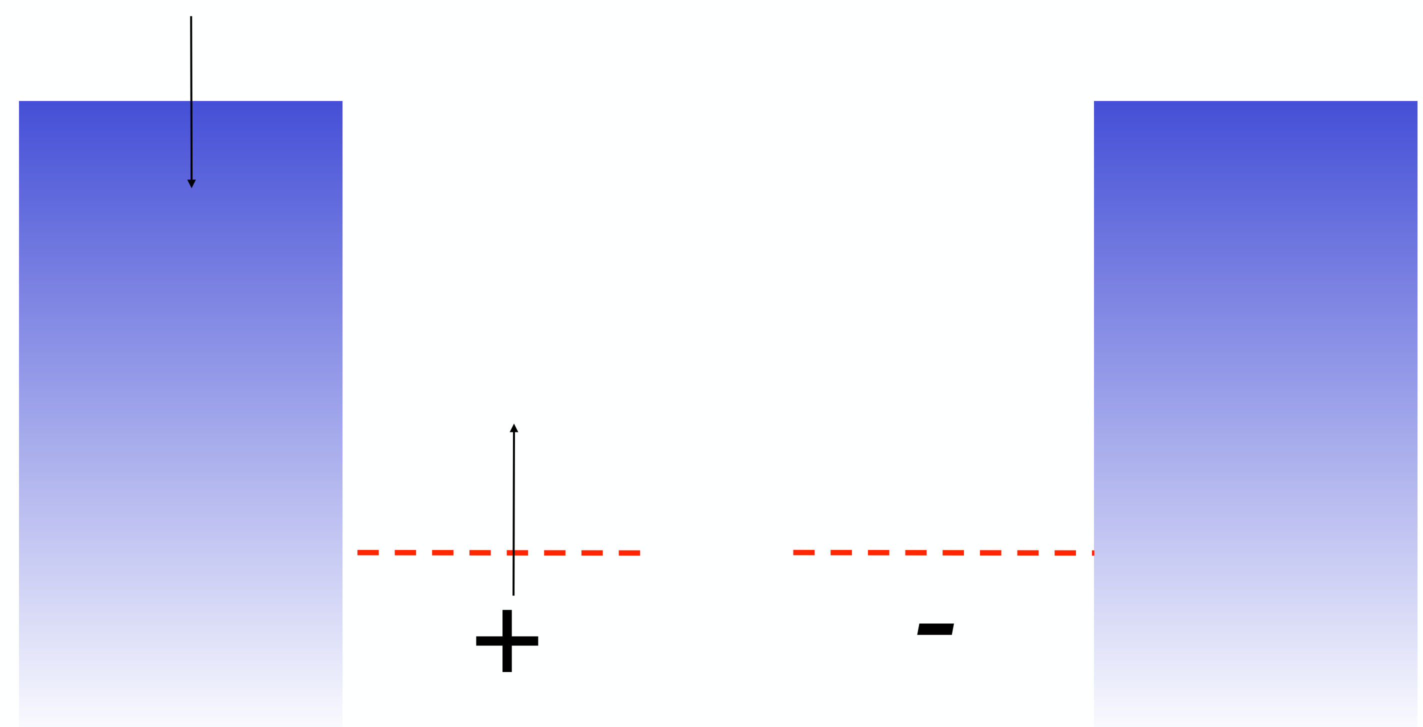


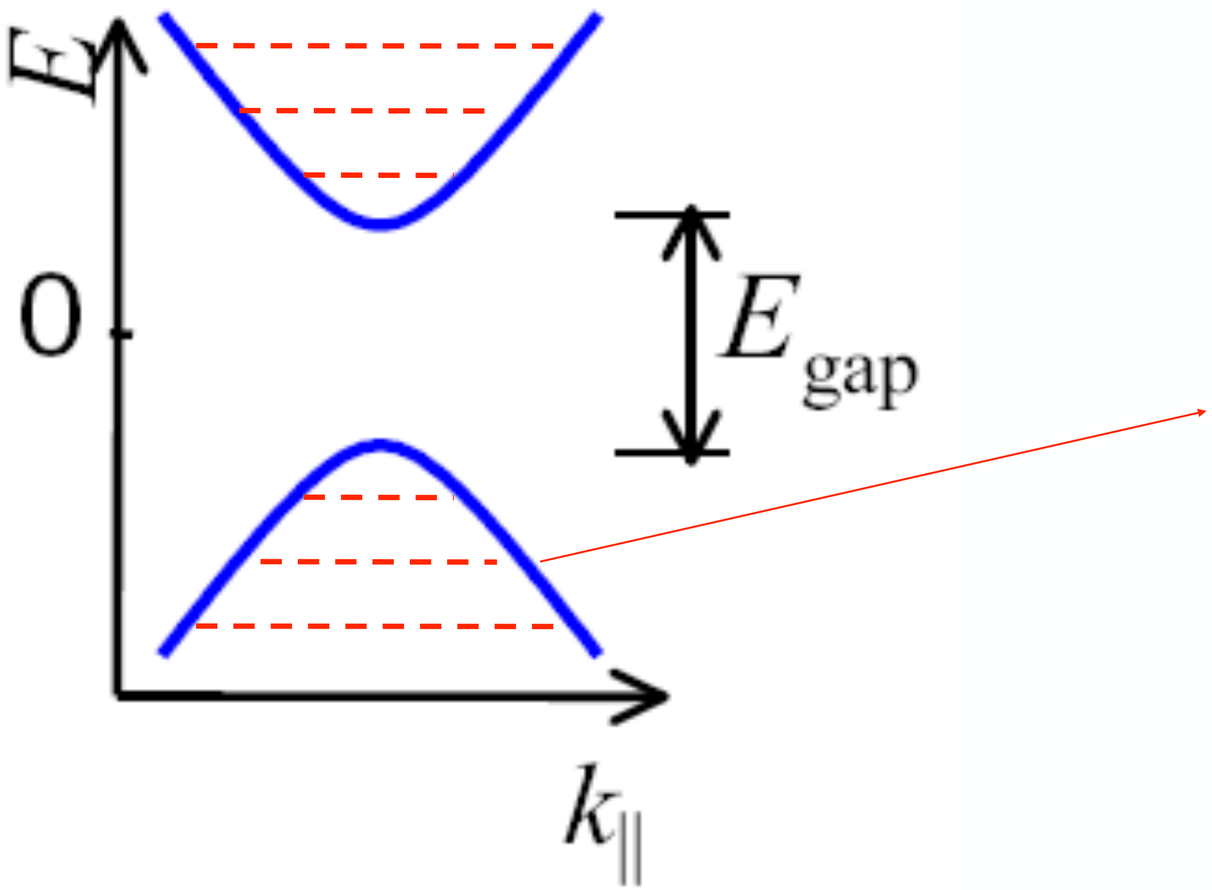
Yes: Each level is fourfold degenerate (spin+valley). Visible in Coulomb Blockade!!!



At low temperatures both spin and valley quantum fluctuations

$$|+,\uparrow\rangle \rightarrow |+,\downarrow\rangle$$

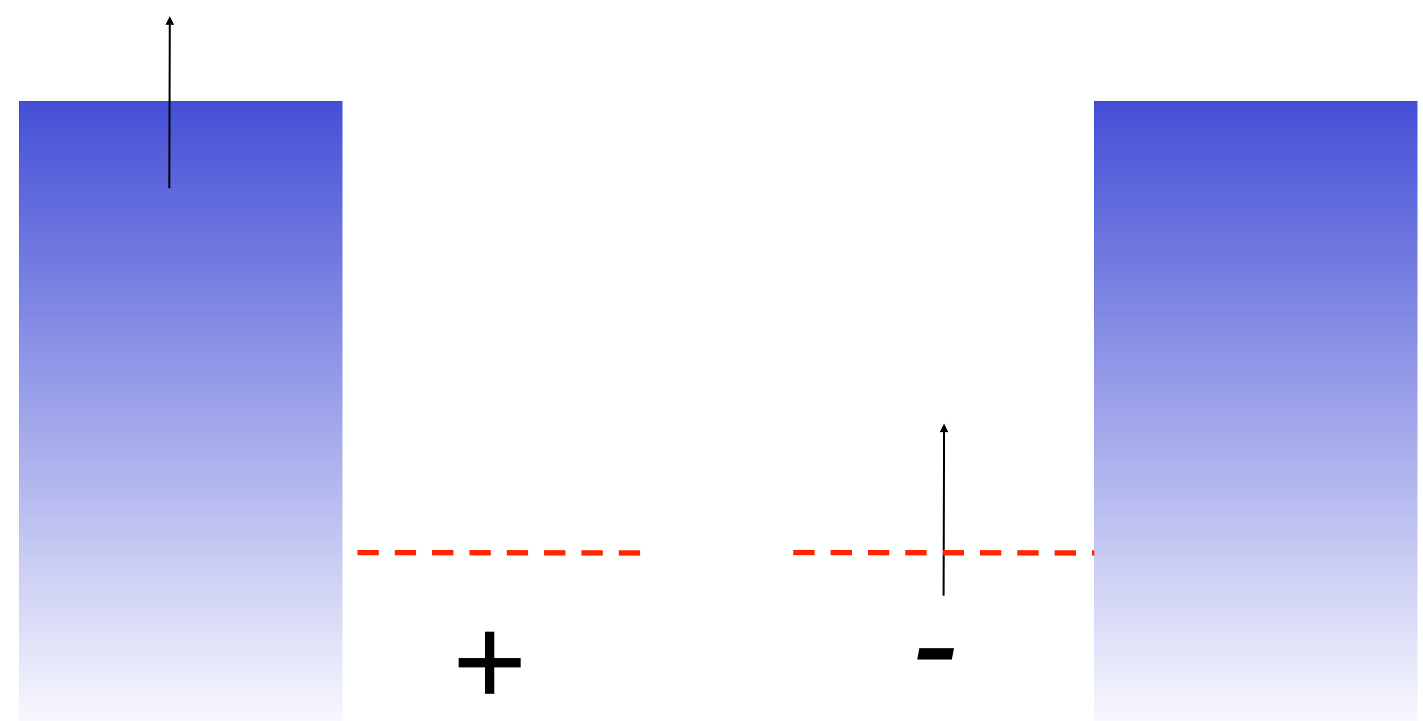


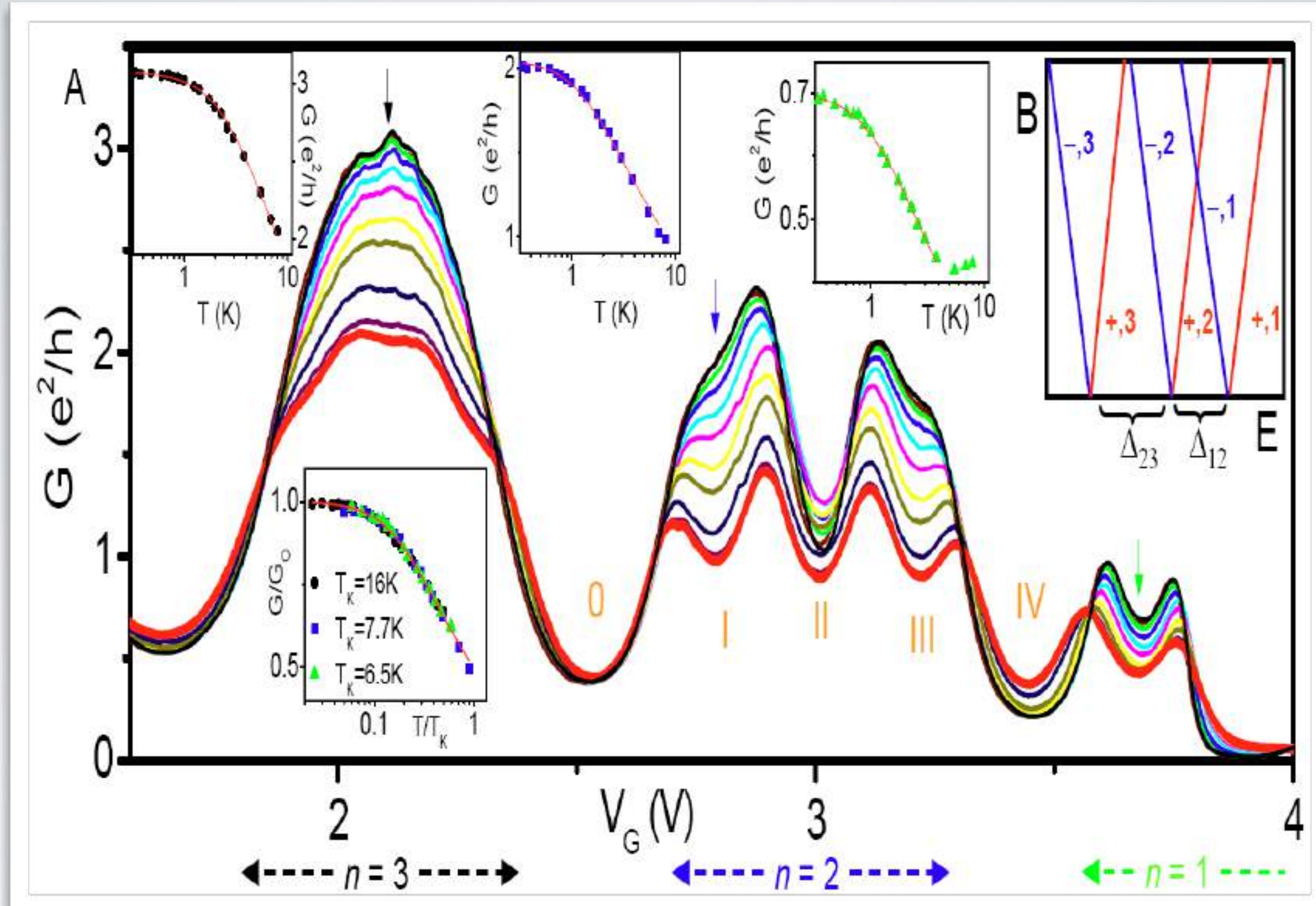


$$\left.\begin{array}{l}|+,\uparrow\rangle \\ |+,\downarrow\rangle \\ |-,\uparrow\rangle \\ |-,\downarrow\rangle\end{array}\right.$$

At low temperatures both spin and valley quantum fluctuations

$$|-,\uparrow\rangle \rightarrow |+,\uparrow\rangle$$





Experiments from Delft (Pablo Jarillo-Herrero's PhD)

SU(4) Kondo effect?

Difficult to tell at this level linear conductance is the same for SU(2) and SU(4)!!

Remember Langreth's sum rule for the phase shift

$$G = N\frac{e^2}{h}sin^2\delta = N\frac{e^2}{h}sin^2(\pi\langle n\rangle)$$

SU(N=2)

$$\langle n\rangle = \frac{1}{2}$$

$$G = 2\frac{e^2}{h}sin^2(\frac{\pi}{2}) = \frac{2e^2}{h}$$

SU(N=4)

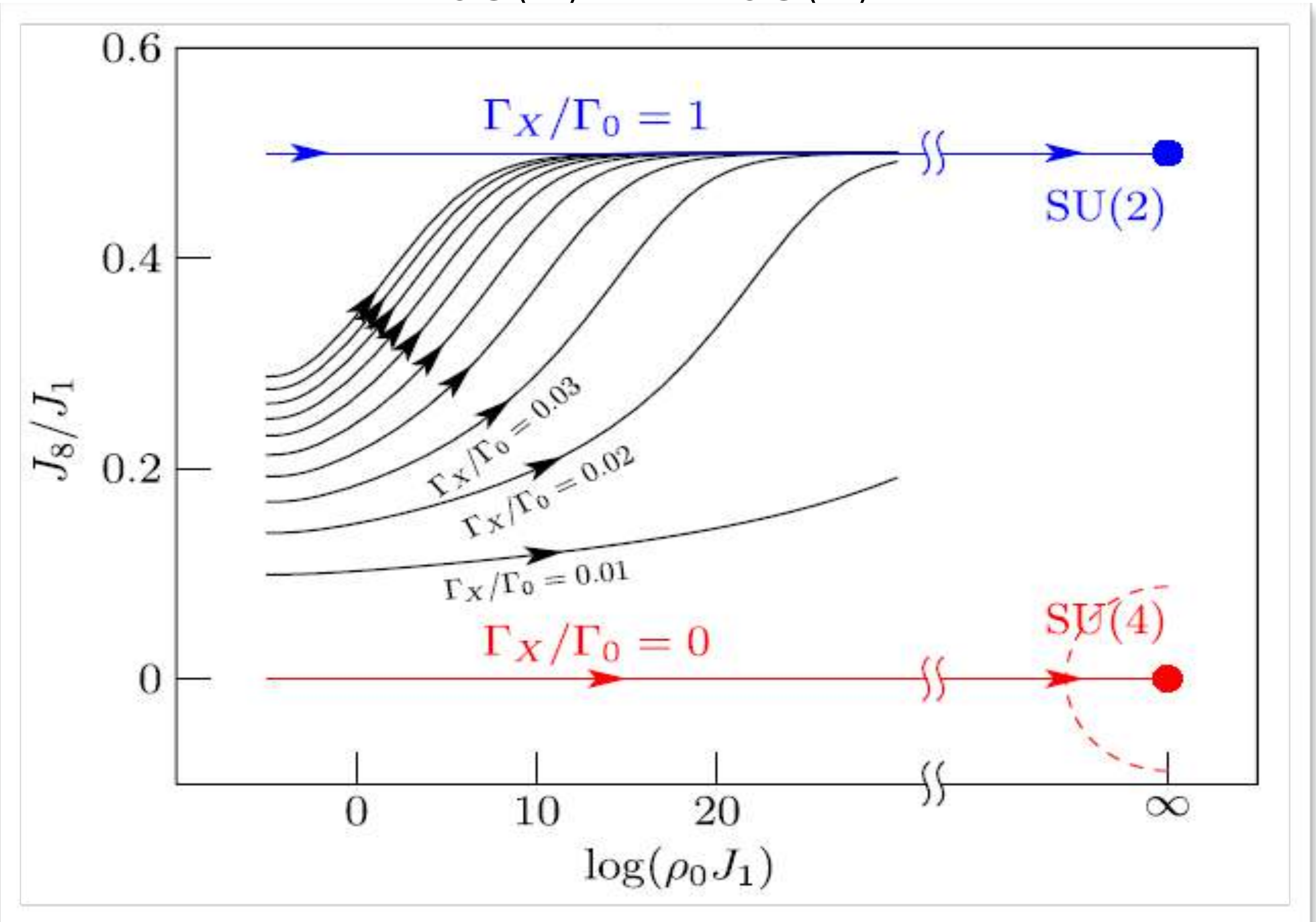
$$\langle n\rangle = \frac{1}{4}$$

$$G = 4\frac{e^2}{h}sin^2(\frac{\pi}{4}) = \frac{2e^2}{h}$$

SU(4) Kondo effect?

Moreover, it is not obvious whether the valley is conserved during tunneling (e. g. bad contacts). Valley mixing can lead to SU(2) spin Kondo.

SCALING EQUATIONS HAVE TWO FIXED POINTS: SU(2) AND SU(4)

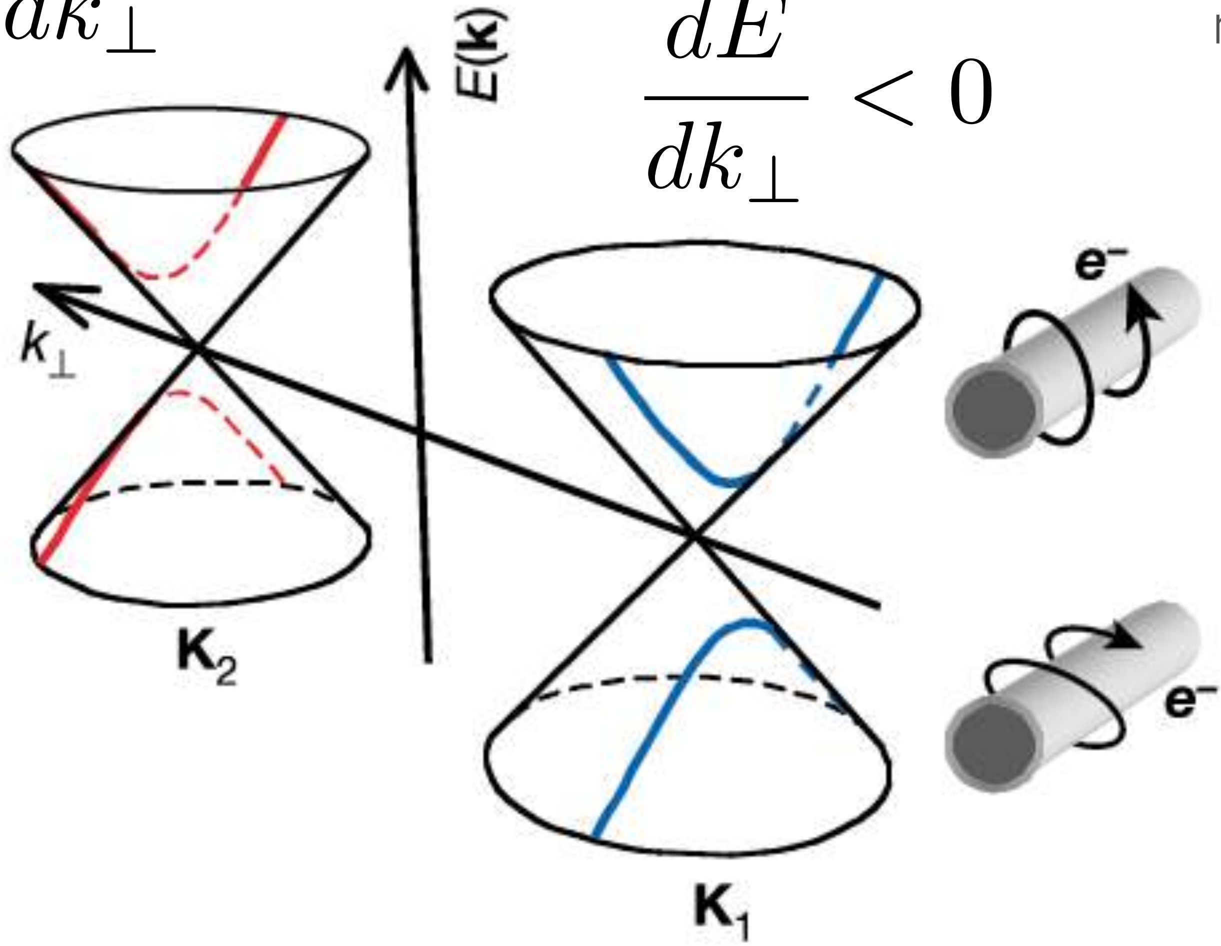


"SU(4) Kondo effect in carbon nanotubes", Mahn-Soo Choi, Rosa López and Ramón Aguado, Phys. Rev. Lett. 95, 067204 (2005)

How to detect SU(4) Kondo effect?

$$\frac{dE}{dk_\perp} > 0$$

$$\frac{dE}{dk_\perp} < 0$$



Semiclasically, the two orbital states correspond to electrons circling the tube either clockwise or anticlockwise. Such motion has a corresponding orbital moment

$$\mu_{orb} = \frac{eDv_F}{4}$$

Determination of electron orbital magnetic moments in carbon nanotubes

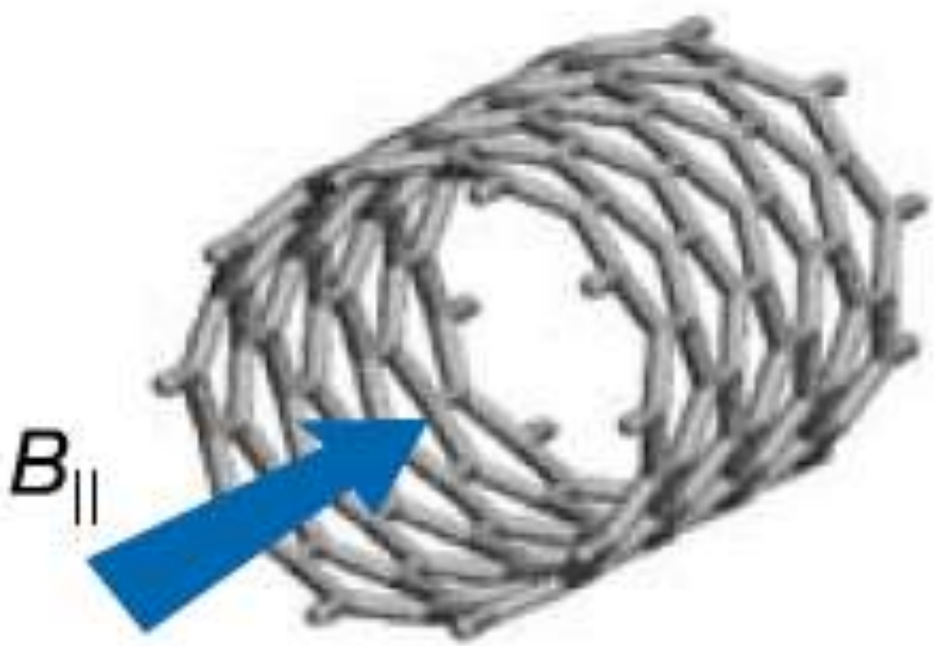
E. D. Minot*, Yuval Yaish*, Vera Sazonova & Paul L. McEuen

Laboratory of Atomic and Solid-State Physics, Cornell University, Ithaca, New York 14853, USA

NATURE | VOL 428 | 1 APRIL 2004 | www.nature.com/nature

How to detect SU(4) Kondo effect?

An axial magnetic field will couple to this orbital moment!



$$\Delta E = \pm \mu_{orb} B_{||} = \frac{e D v_F B_{||}}{4}$$

Determination of electron orbital magnetic moments in carbon nanotubes

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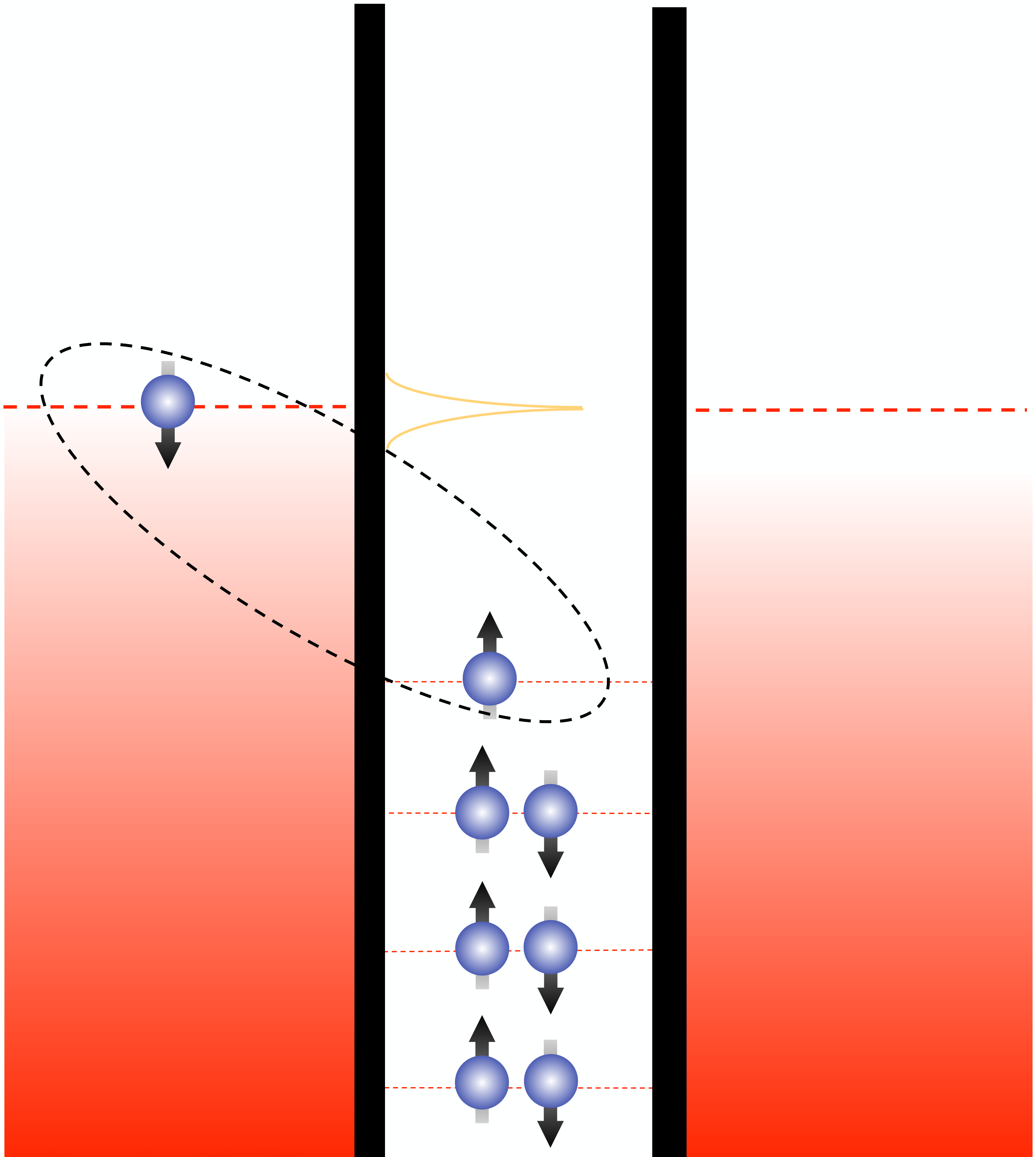
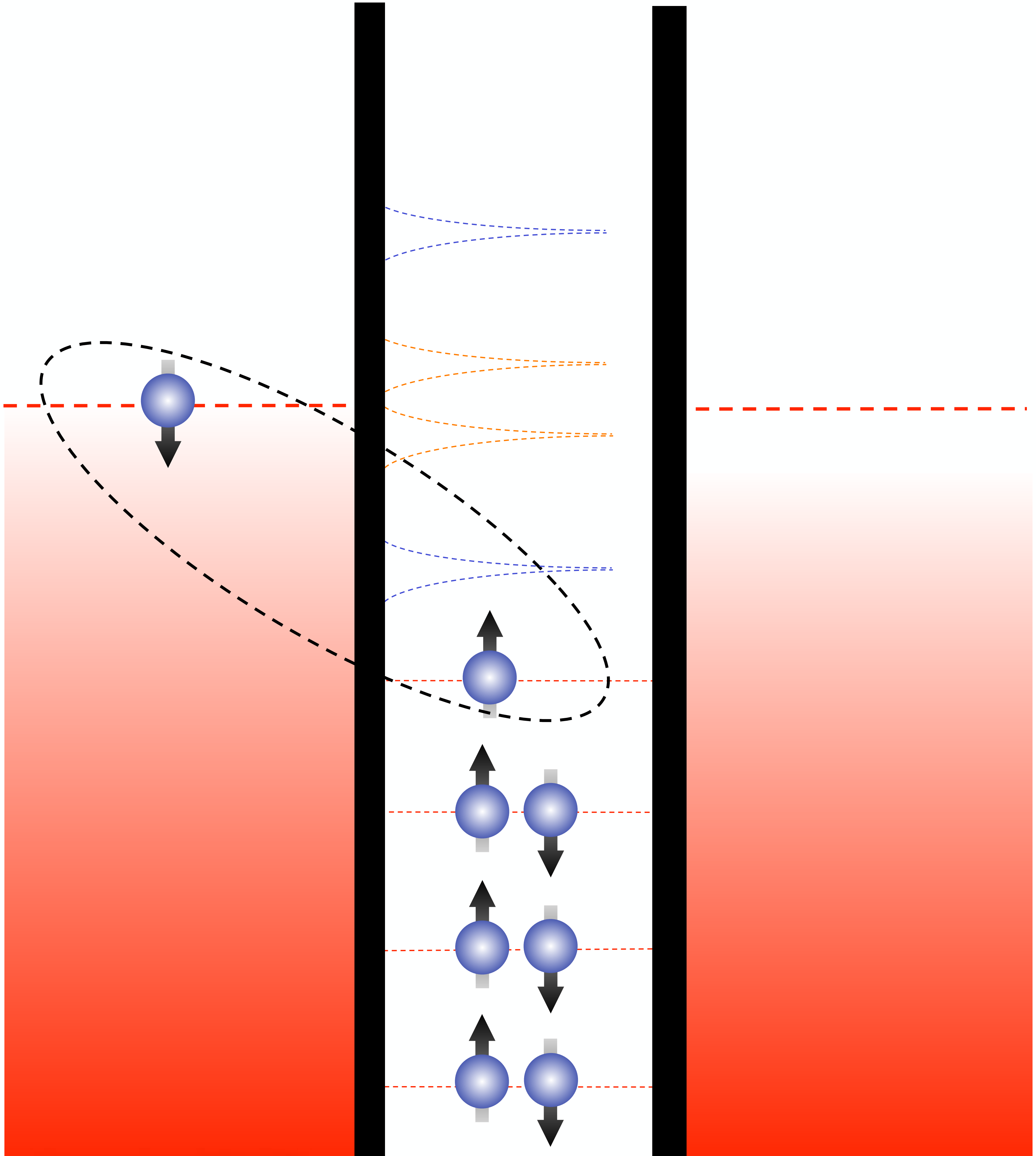


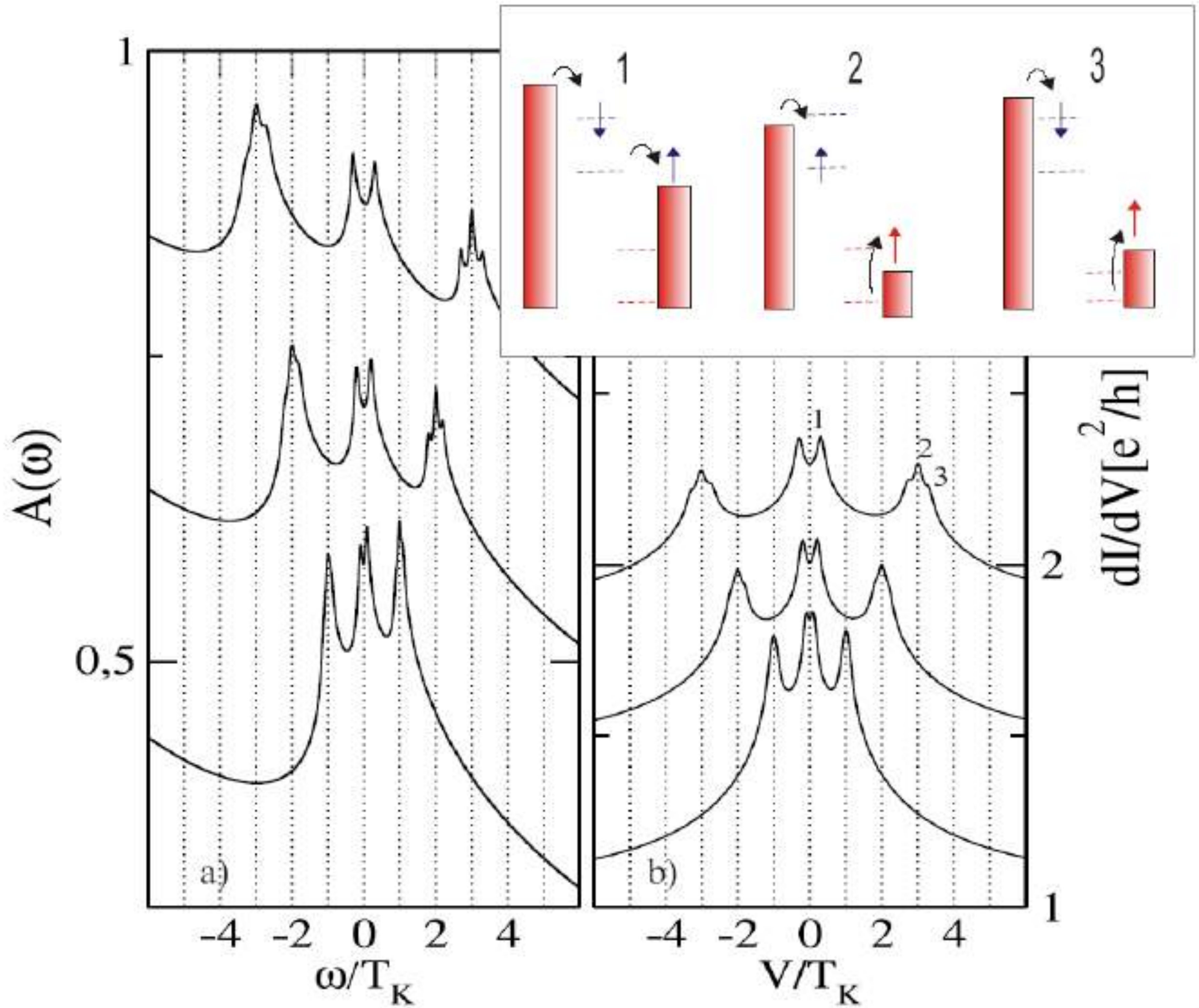
Ramón Aguado ICMM-CSIC

$B_{||} \neq 0$

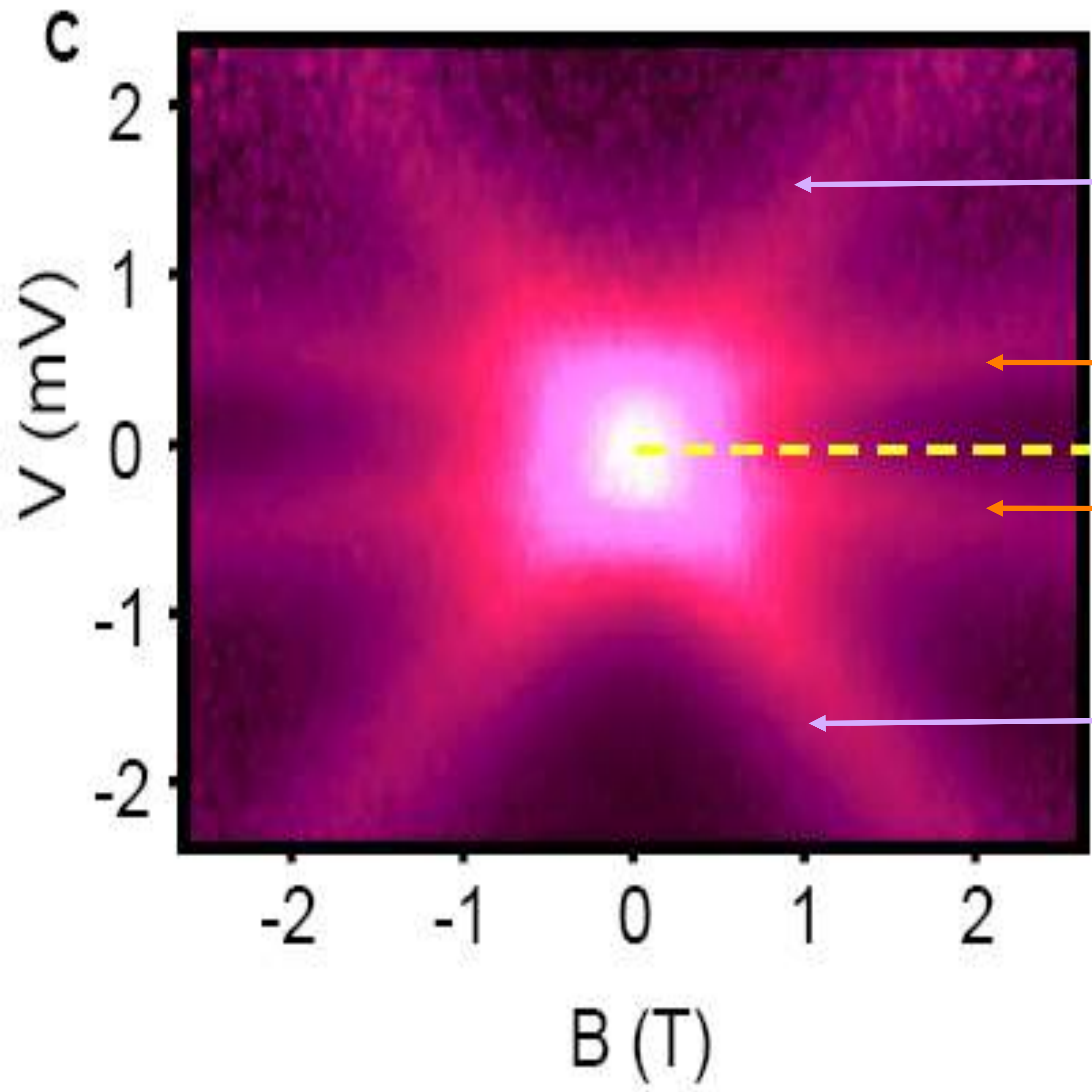


- Spin Kondo splits as Δ_{Zeeman}.
- Orbital Kondo splits as $2\Delta_{orbital}$.

Slave boson 1/N+Keldysh



"SU(4) Kondo effect in carbon nanotubes", Mahn-Soo Choi, Rosa López and Ramón Aguado, Phys. Rev. Lett. 95, 067204 (2005)

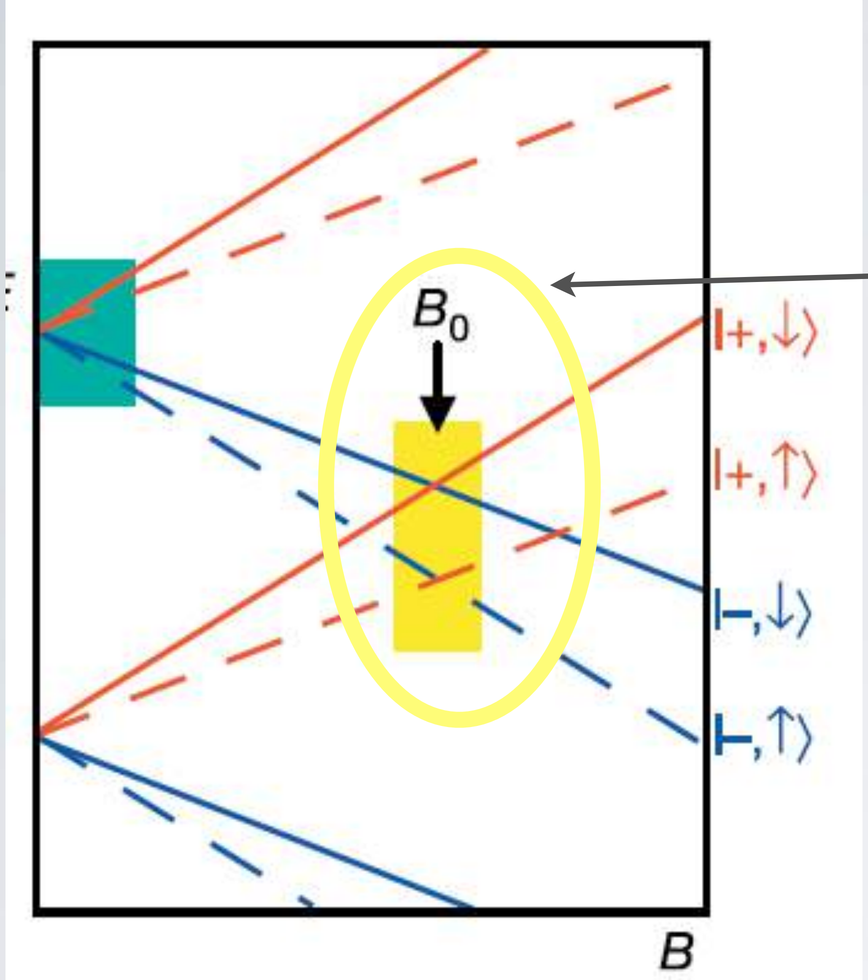


Full agreement theory and experiment

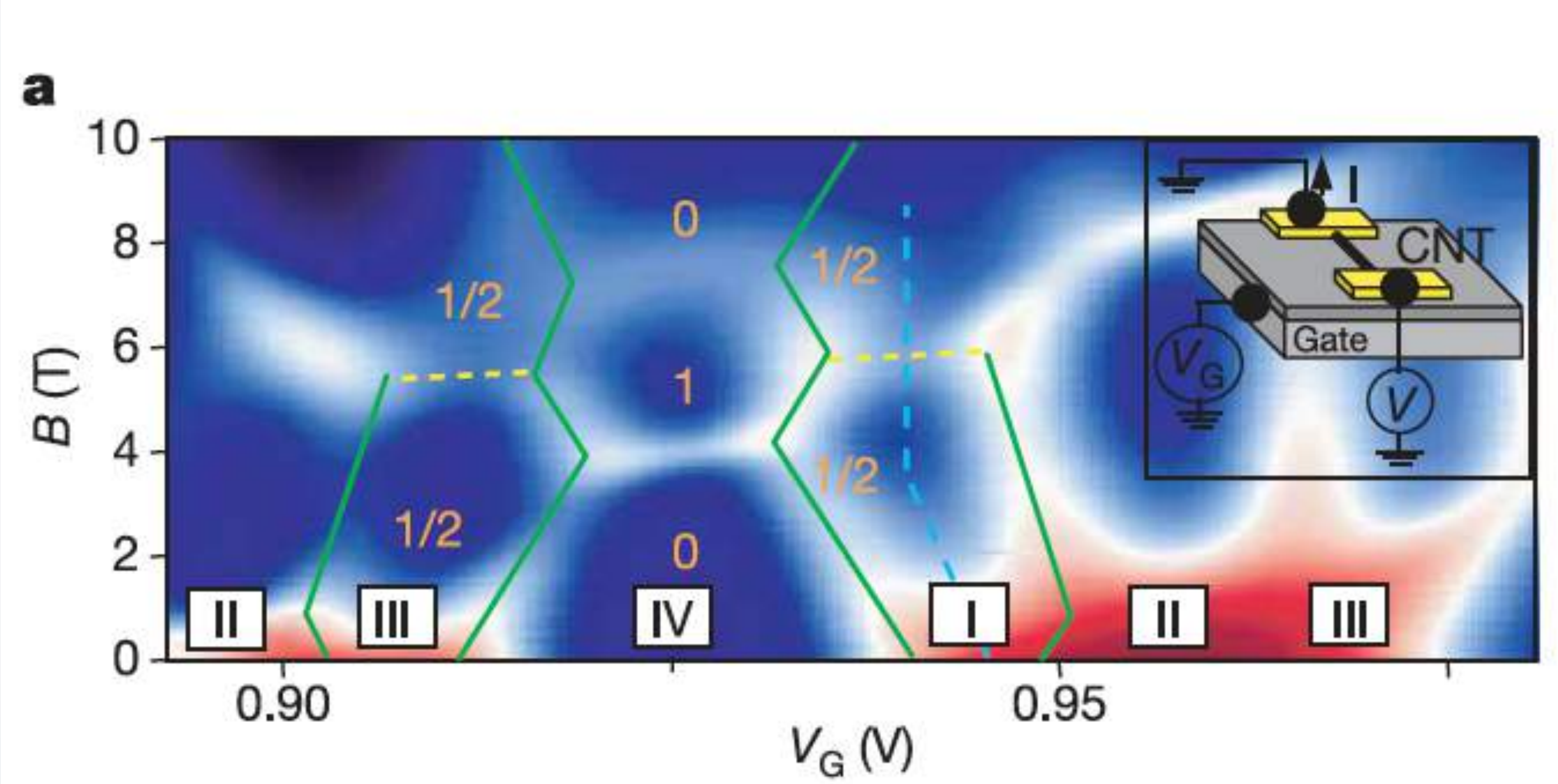
Zeeman splitting

Orbital splitting

Pablo Jarillo-Herrero, Jing Kong, Herre S.J. van der Zant, Cees Dekker, Leo P. Kouwenhoven, Silvano De Franceschi, Nature 434, 484-488 (2005)



Also **pure orbital** Kondo effect is possible at high magnetic fields (spins are fully polarized)



This physics has also been observed in Silicon transistors with single dopants (two-valley degeneracy of Silicon gives exotic Kondo)

PRL **108,** 046803 (2012) PHYSICAL REVIEW LETTERS week ending 27 JANUARY 2012

Magnetic-Field Probing of an SU(4) Kondo Resonance in a Single-Atom Transistor

G. C. Tettamanzi,[1,2,*] J. Verduijn,[1,2] G. P. Lansbergen,[1] M. Blaauboer,[1] M. J. Calderón,[3] R. Aguado,[3] and S. Rogge[1,2]

[1]*Delft University of Technology, Kavli Institute of Nanoscience, Department of Quantum Nanoscience, Lorentzweg 1, 2628 CJ Delft, The Netherlands*

[2]*Centre for Quantum Computation and Communication Technology, School of Physics, University of New South Wales, Sydney, New South Wales 2052, Australia*

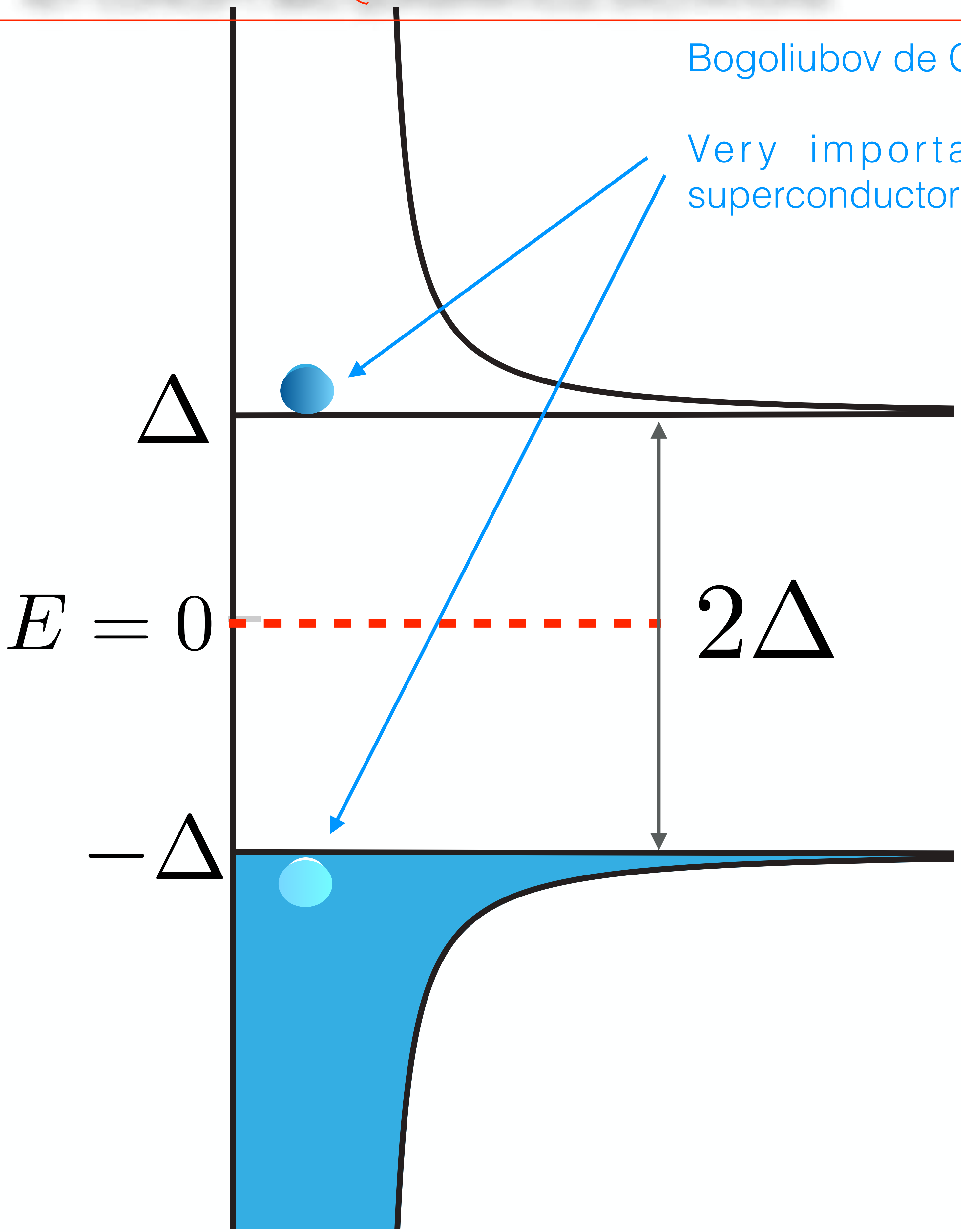
[3]*Instituto de Ciencia de Materiales de Madrid, ICMM-CSIC Cantoblanco, E-28049 Madrid, Spain*

(Received 15 February 2011; published 25 January 2012)

Semiconductor devices have been scaled to the point that transport can be dominated by only a single dopant atom. As a result, in a Si fin-type field effect transistor Kondo physics can govern transport when one electron is bound to the single dopant. Orbital (valley) degrees of freedom, apart from the standard spin, strongly modify the Kondo effect in such systems. Owing to the small size and the s-like orbital symmetry of the ground state of the dopant, these orbital degrees of freedom do not couple to external magnetic fields which allows us to tune the symmetry of the Kondo effect. Here we study this tunable Kondo effect and demonstrate experimentally a symmetry crossover from an SU(4) ground state to a pure orbital SU(2) ground state as a function of magnetic field. Our claim is supported by theoretical calculations that unambiguously show that the SU(2) symmetric case corresponds to a pure valley Kondo effect of fully polarized electrons.

DOI: 10.1103/PhysRevLett.108.046803 PACS numbers: 73.23.Hk, 72.15.Qm, 71.27.+a, 71.30.+h

Let's add superconductivity!



Bogoliubov de Gennes (BdG)

Very important: quasiparticle excitations in a superconductor are electron-hole quantum superpositions.

$$H_{BCS} = \sum_{\mathbf{k}\sigma} E_{\mathbf{k}} \gamma^{\dagger}_{\mathbf{k}\sigma} \gamma_{\mathbf{k}\sigma}$$

$$\begin{pmatrix} \gamma_{\mathbf{k}\uparrow} \\ \gamma^{\dagger}_{-\mathbf{k}\downarrow} \end{pmatrix} = \begin{bmatrix} u^*_{\mathbf{k}} & v_{\mathbf{k}} \\ -v^*_{\mathbf{k}} & u_{\mathbf{k}} \end{bmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^{\dagger}_{-\mathbf{k}\downarrow} \end{pmatrix}$$

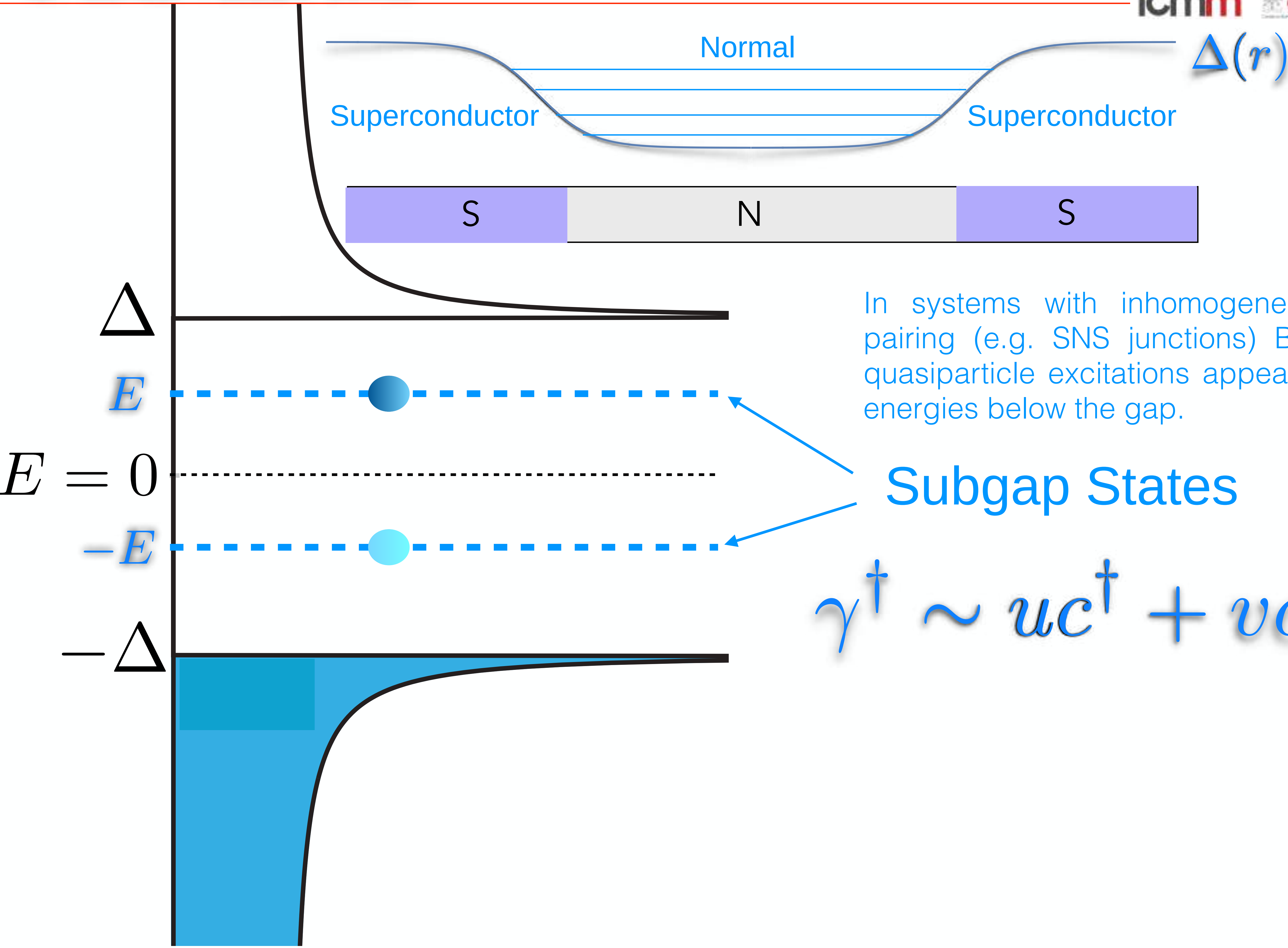
coherent combination of an electron-like and hole-like excitations with opposite spins.

$$|u_{\mathbf{k}}|^2 = \frac{1}{2}(1 + \frac{\epsilon_{\mathbf{k}}}{E_{\mathbf{k}}})$$

$$|v_{\mathbf{k}}|^2 = \frac{1}{2}(1 - \frac{\epsilon_{\mathbf{k}}}{E_{\mathbf{k}}})$$

$$u_{\mathbf{k}} v_{\mathbf{k}} = \frac{\Delta}{2E_{\mathbf{k}}}$$

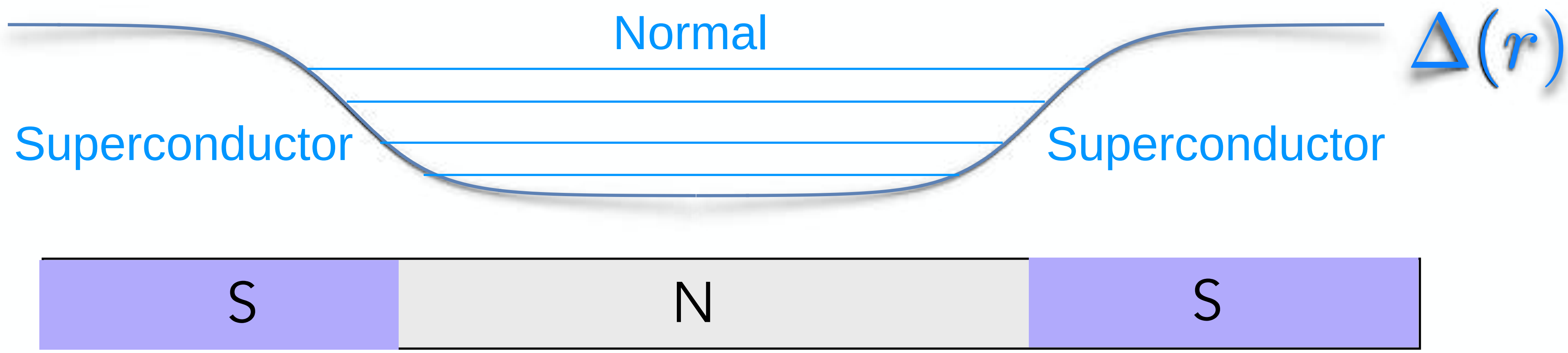
$$E_{\mathbf{k}} = \sqrt{\epsilon^2_{\mathbf{k}} + \Delta^2}$$



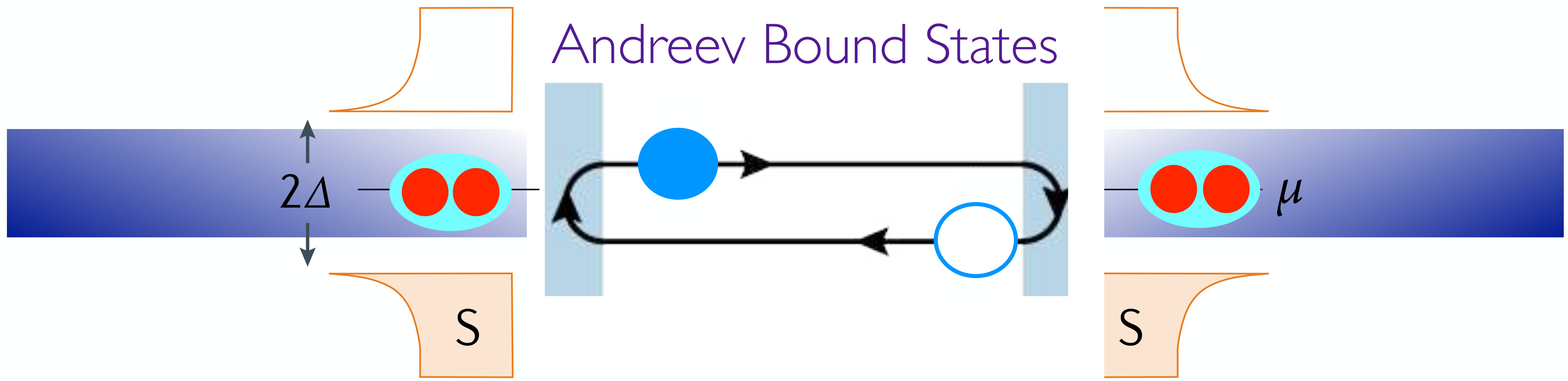
In systems with inhomogeneous pairing (e.g. SNS junctions) BdG quasiparticle excitations appear at energies below the gap.

Subgap States

$$\gamma^\dagger \sim u c^\dagger + v c$$

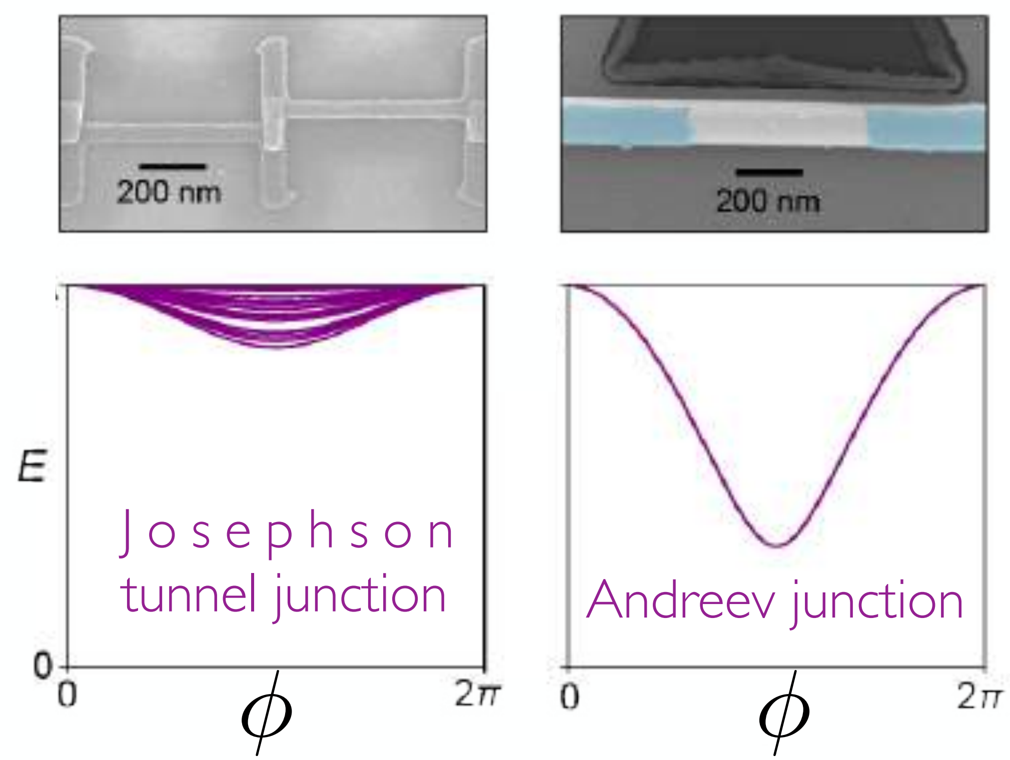


<u>Novel Josephson couplings</u> depending on properties of the normal (semiconducting) region.



Standard tunnel junction

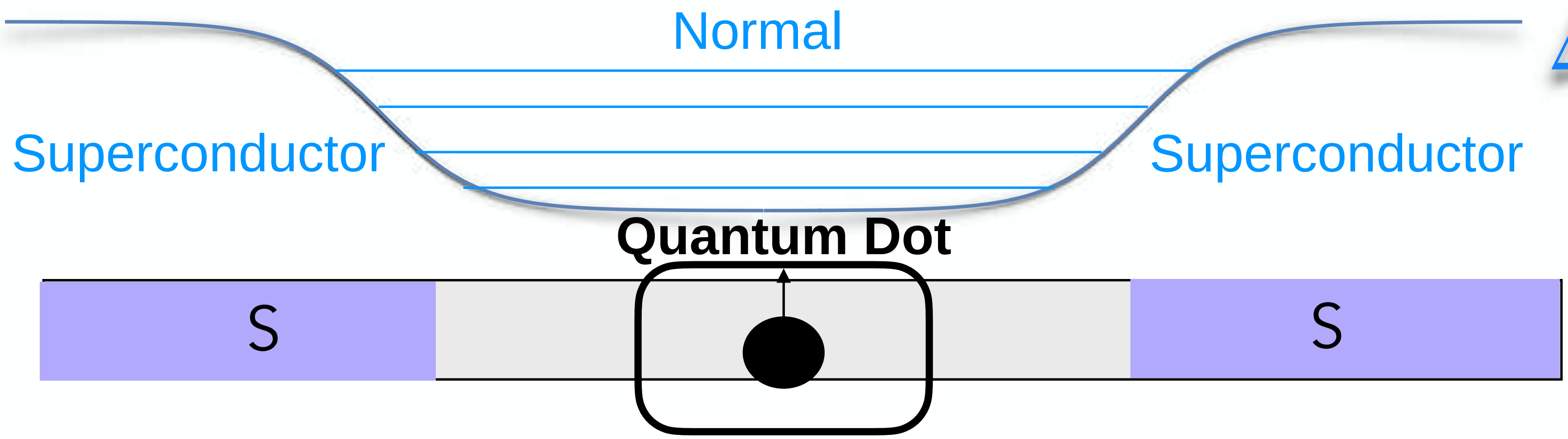
$$I_J(\phi) = I_c sin\phi$$



Andreev SNS junction: single (or few) fermionic-like microscopic BdG quasiparticle degrees of freedom

$$I_J(\phi) \neq I_c sin\phi$$

$\Delta(r)$



Kondo + superconductivity

What happens when a magnetic impurity tries to live inside a superconductor?

It turns out that even a classical spin may lower the energy of the system and create bound states inside the superconducting gap.

Yu-Shiba-Rusinov states (YSR)

Yu (1965)

BOUND STATE IN SUPERCONDUCTORS WITH PARAMAGNETIC IMPURITIES

Yu Luh (Yu Lu)

Received July 10, 1963
Acta Physica Sinica 21, 75-91 (January, 1965)

A generalized c
to investigate
of superconduct
exists around c
gap. An analyt
wave function.
state should ap
verifications c
infrared absorp

Furthermore, th
impurity and th
considered.

Shiba(1968)

Progress of Theoretical Physics, Vol. 40, No. 3, September 1968

Classical Spins in Superconductors

Hiroyuki SHIBA*)

Depar

It is shown that t
with a classical spi
form an "impurity b
observable quantities

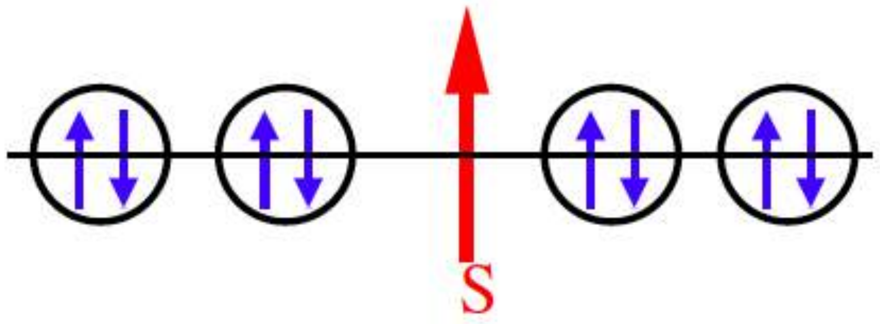
Rusinov(1969)

SUPERCONDUCTIVITY NEAR A PARAMAGNETIC IMPURITY

A. I. Rusinov
P. N. Lebedev Physics Institute, USSR Academy of Sciences
Submitted 28 November 1968
ZhETF Pis. Red. 9, No. 2, 146-149 (20 January 1969)

It is shown in [1] that introduction of a small amount ($\lesssim$1%) of paramagnetic impurities in a superconductor exerts a strong influence on its properties. In particular, the energy gap of the spectrum of such an alloy no longer coincides with the magnitude of the ordering ... in [1] was carried out in the Born approximation with respect ... will be shown below that in the case of a superconductor al- ... e of the scattering of the electrons by the magnetic impurity ... g results.

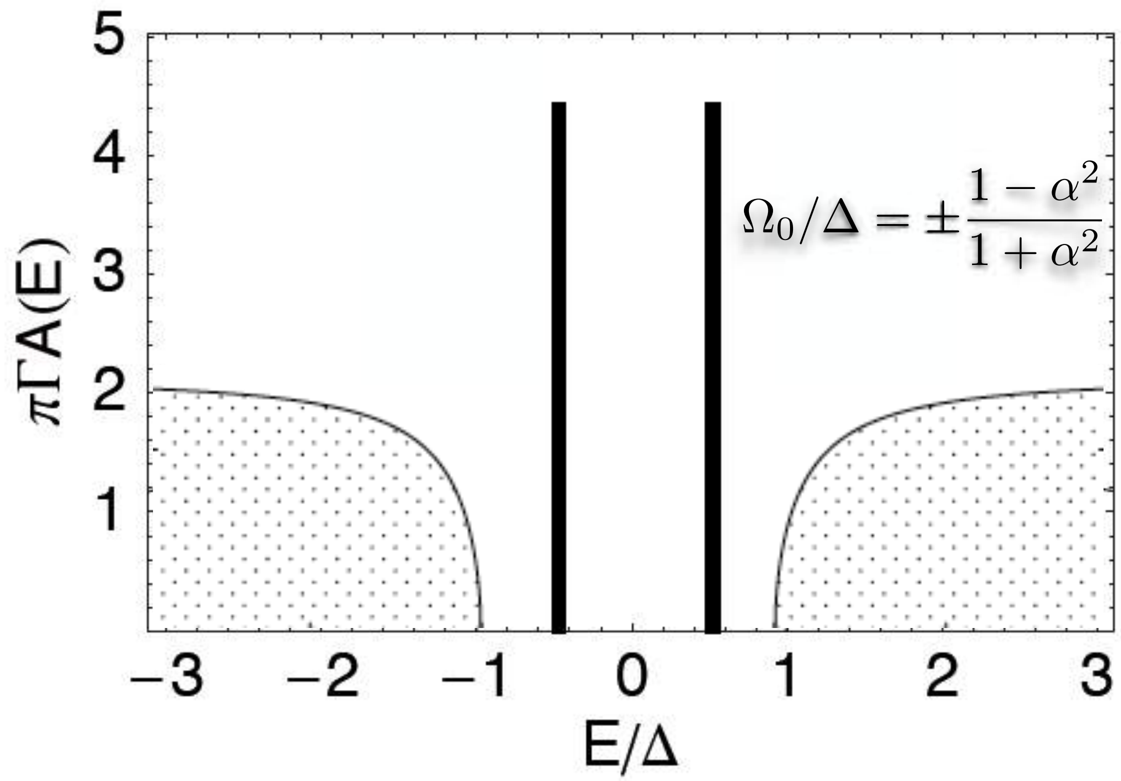
$$\omega = \pm\,\omega_B \equiv \pm\,\Delta_0 \frac{1-((J/2)\,S\pi\rho)^2}{1+((J/2)\,S\pi\rho)^2}$$

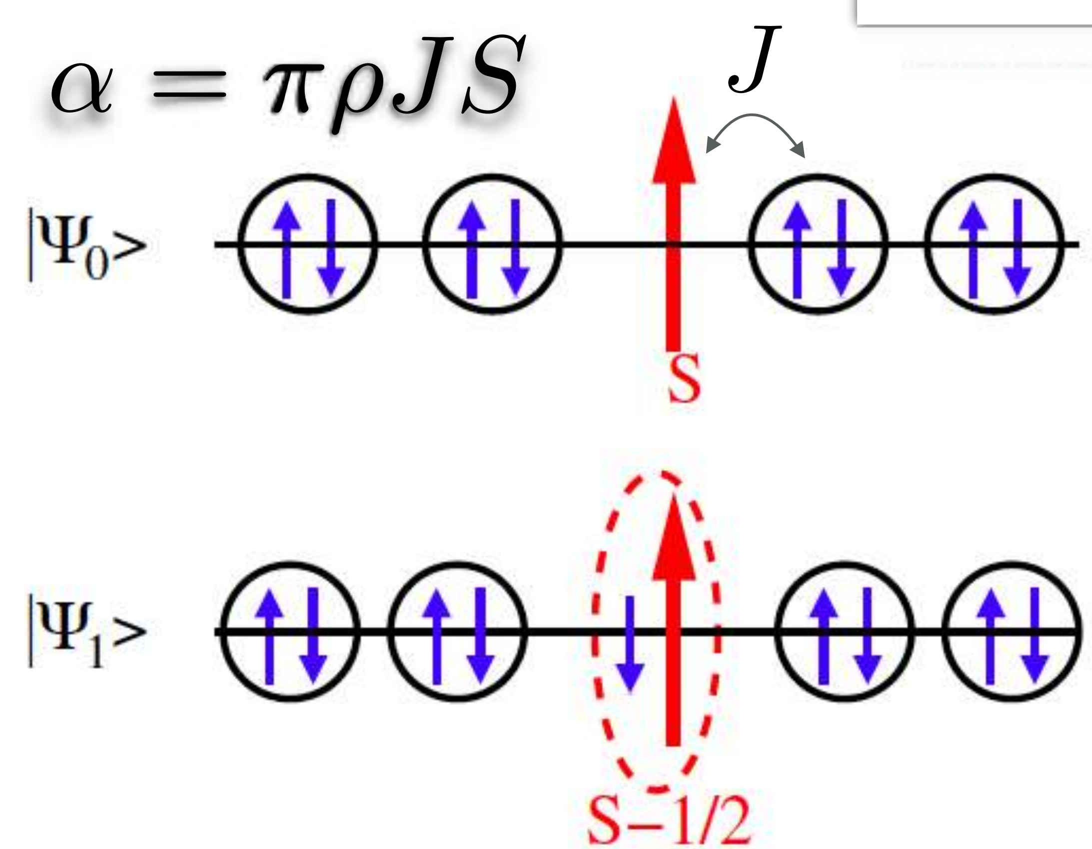


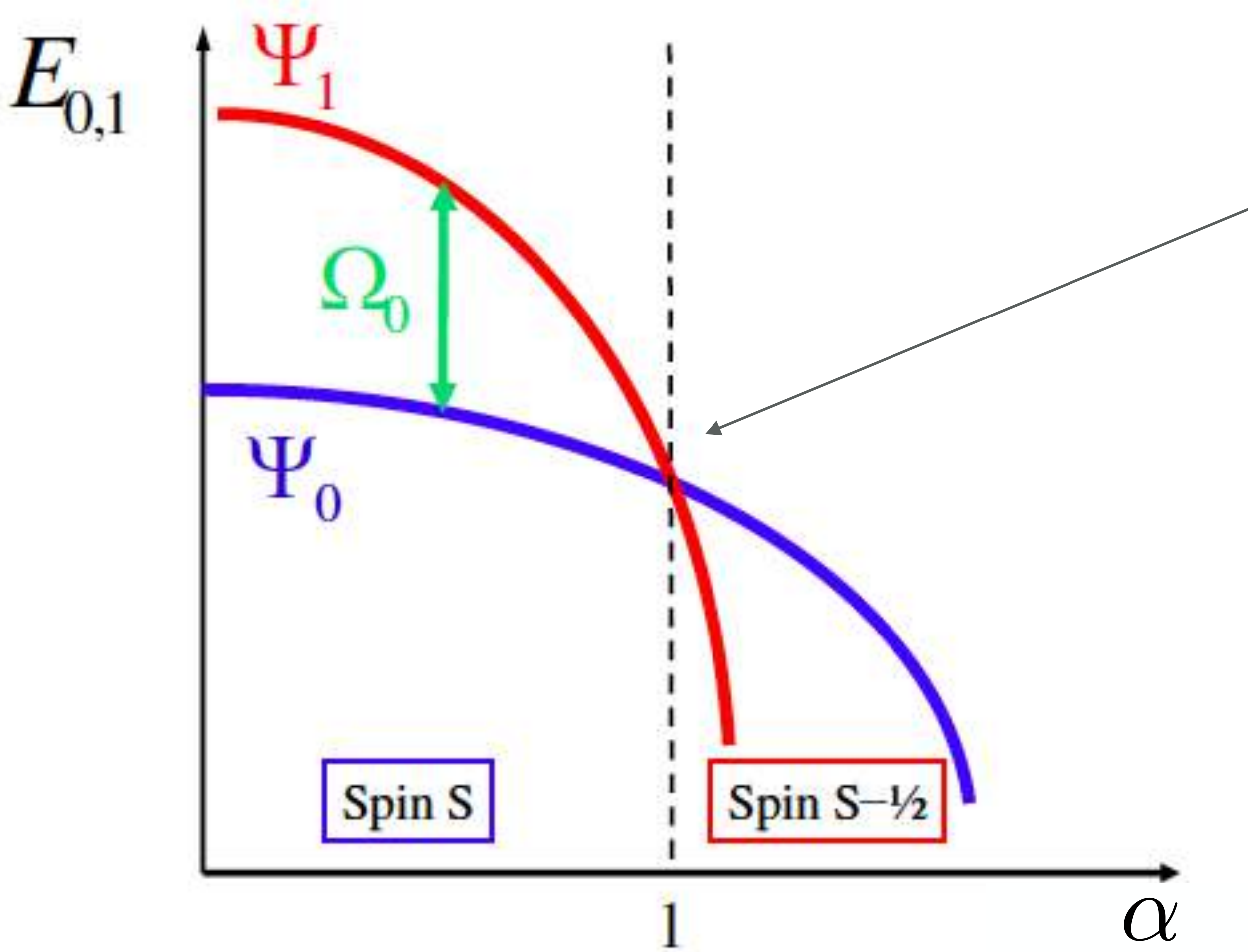
- An impurity spin in a superconducting host strongly affects superconductivity (spin scattering has pair breaking character).

- This is reflected in the appearance of sub-gap states.

- Well known since the 60's (Yu-Shiba-Rusinov): a classical spin creates **bound states** inside the superconducting gap.



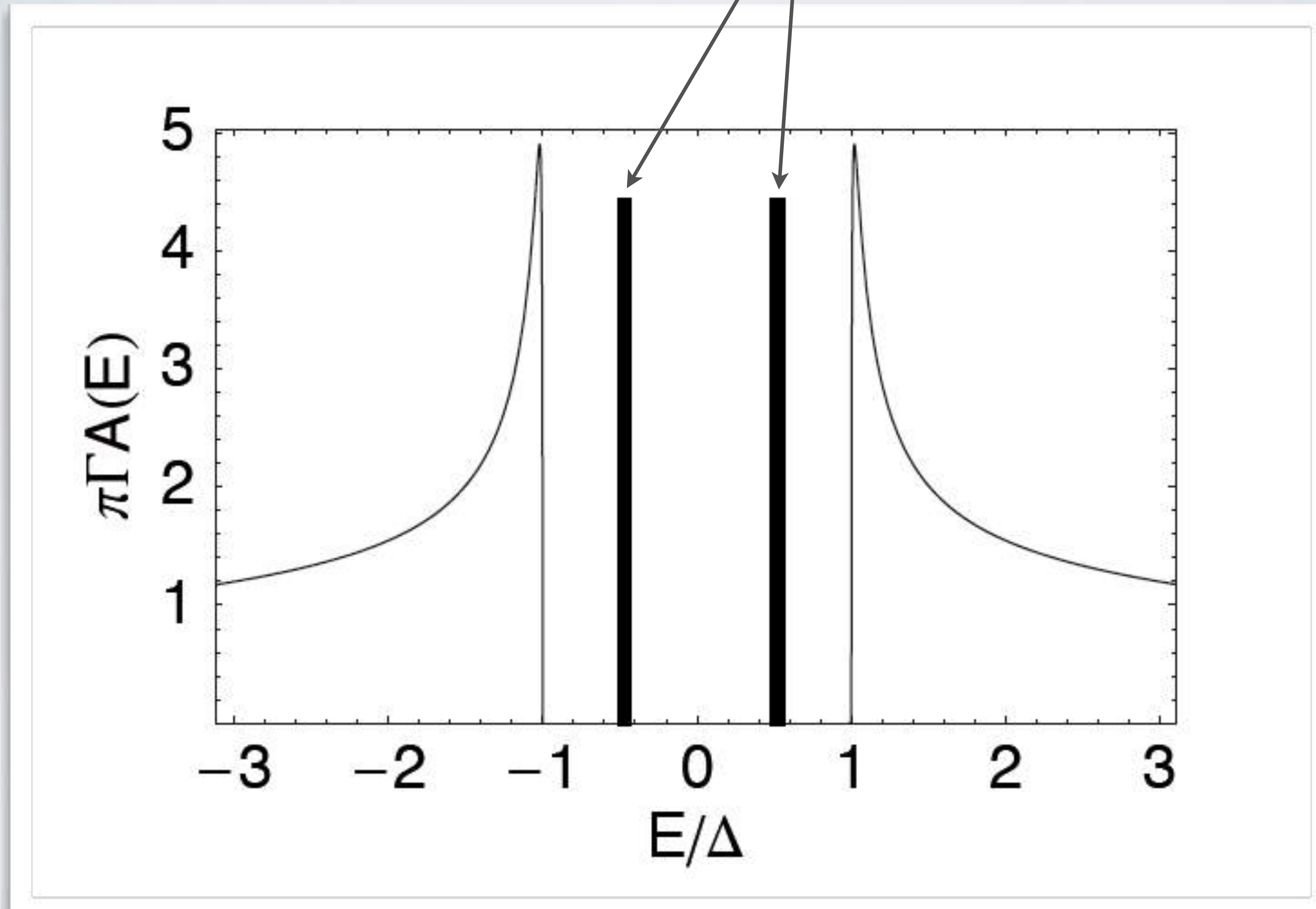




For large exchange coupling the spin of the ground states **changes.** This happens **when the energy of the excitation crosses zero energy.**

For a review see "Impurity-induced states in conventional and unconventional superconductors", Balatsky et al, Reviews of Modern Physics, **78**, 373 (2006)

$$\omega = \pm\,\omega_B = \pm\,\Delta_0 \frac{1 - ((J/2)\,S\pi\rho)^2}{1 + ((J/2)\,S\pi\rho)^2}$$

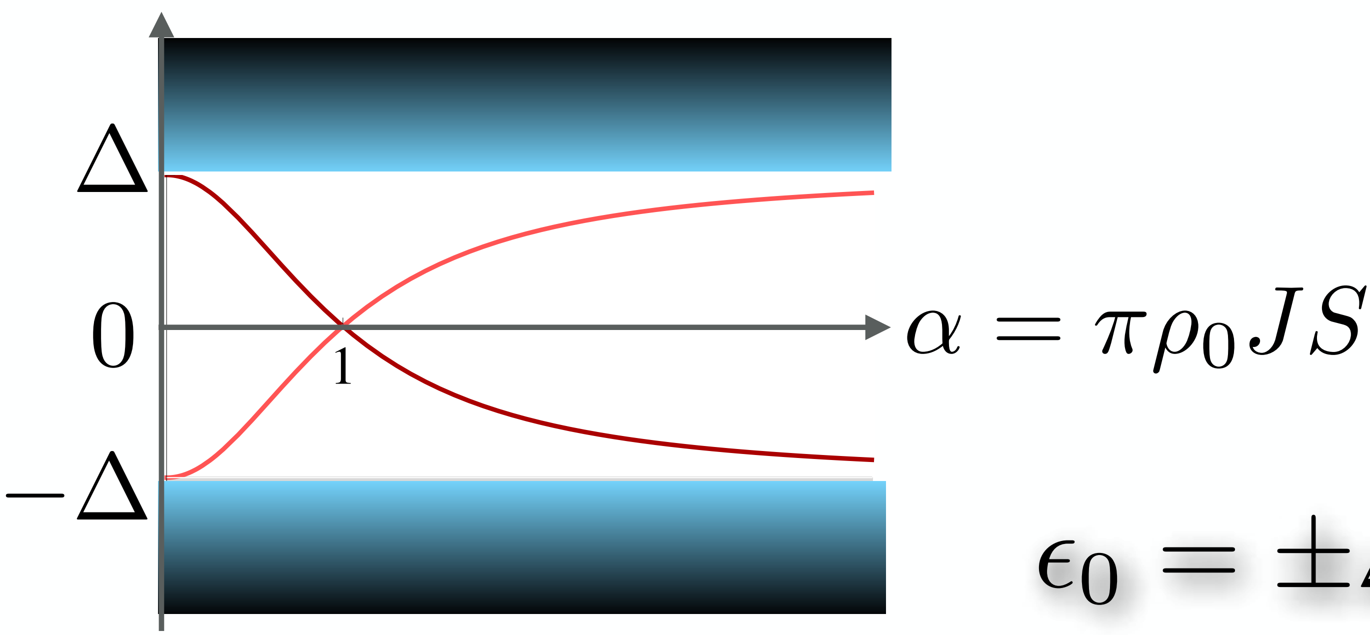


Physical Picture: odd-occupied QDs as spinful impurities

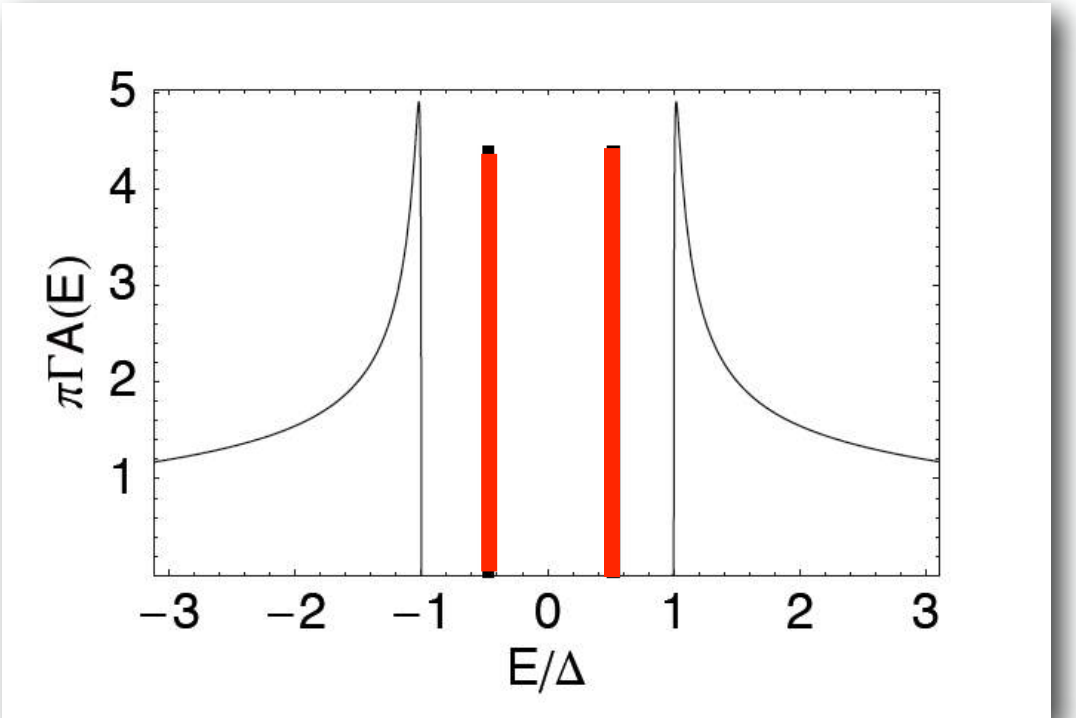
- A spinful impurity in a superconducting host strongly affects superconductivity (spin scattering has pair breaking character).

- Well known since the 60's (Yu-Shiba-Rusinov): a classical spin may lower the energy of the system and create sub-gap bound states.

The ground state parity changes
as sub-gap levels cross zero energy.

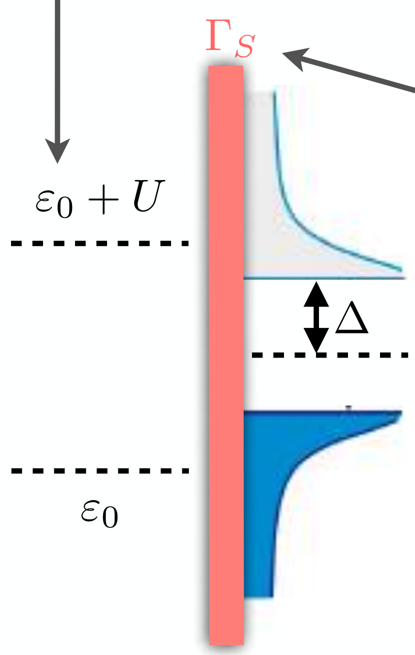


$$\epsilon_0 = \pm\Delta\frac{1-\alpha^2}{1+\alpha^2}$$



THEORETICAL DESCRIPTION:
THE SUPERCONDUCTING ANDERSON MODEL

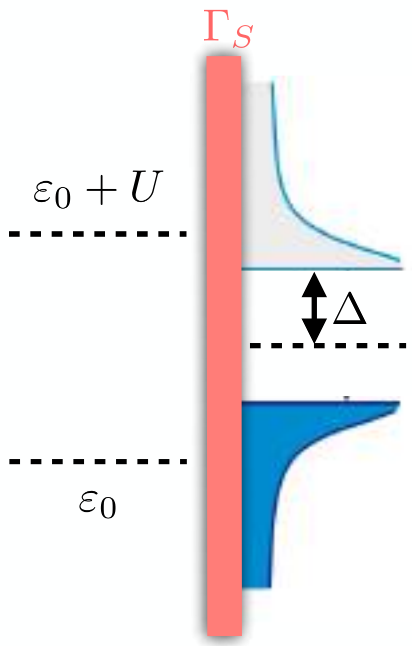
$$\mathcal{H}_D = \sum_{\sigma} \varepsilon_{d\sigma} d_{\sigma}^{\dagger} d_{\sigma} + U n_{d\uparrow} n_{d\downarrow}$$



$$\mathcal{H}_T = \sum_{\alpha,k,\sigma} \left(V_{\alpha k \sigma} c_{\alpha k \sigma}^{\dagger} d_{\sigma} + h.c. \right)$$

$$\mathcal{H}_S = \sum_{k,\sigma} (\varepsilon_{Sk} - \mu_S) c_{Sk\sigma}^{\dagger} c_{Sk\sigma} + \sum_{k} \left(\Delta c_{Sk\uparrow}^{\dagger} c_{S\bar{k}\downarrow}^{\dagger} + h.c \right)$$

THEORETICAL DESCRIPTION: THE SUPERCONDUCTING ANDERSON MODEL



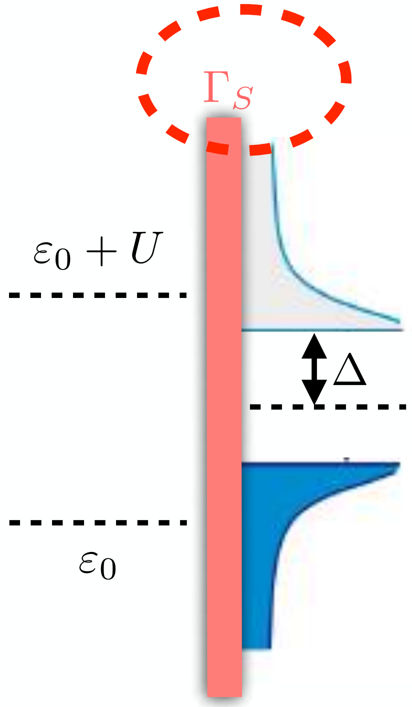
All relevant experimental quantities can be calculated from Green's functions written in the basis of Nambu spinors

$$\Psi_\sigma = \begin{pmatrix} d_\sigma \\ d_{\bar{\sigma}}^\dagger \end{pmatrix}$$

$$\hat{G}_\sigma^r(t,t') \equiv -i\theta(t-t')\langle[\Psi_\sigma(t), \Psi_\sigma^\dagger(t')]_+\rangle$$ etc,...

$$\hat{G}_\sigma^r(\omega)^{-1} = \hat{G}_\sigma^{r(0)}(\omega)^{-1} - \hat{\Sigma}_\sigma(\omega)$$

THEORETICAL DESCRIPTION: THE SUPERCONDUCTING ANDERSON MODEL



Anomalous self-energy coming from the coupling to the superconducting reservoir

$$\hat{\Sigma}(\omega) = \Gamma_S \begin{pmatrix} \frac{\omega}{\sqrt{\Delta^2-\omega^2}} & \frac{\Delta}{\sqrt{\Delta^2-\omega^2}} \\ \frac{\Delta}{\sqrt{\Delta^2-\omega^2}} & \frac{\omega}{\sqrt{\Delta^2-\omega^2}} \end{pmatrix}$$

$\Delta \rightarrow \infty$ Large gap limit: the self-energy is static

$$\hat{\Sigma}(\omega) = \Gamma_S \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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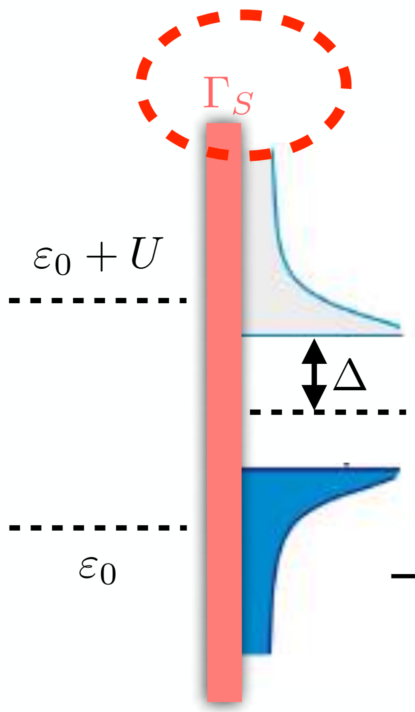
$$\Delta \to \infty$$ Large gap limit: the self-energy is static



$$\hat{\Sigma}(\omega) = \Gamma_S \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The superconducting reservoir can be "integrated out". We can write an impurity Hamiltonian with pairing interaction.

$$\mathcal{H}_D = \sum_\sigma \epsilon_0 d^\dagger_\sigma d_\sigma - \Gamma_S (d^\dagger_\uparrow d^\dagger_\uparrow + d_\uparrow d_\uparrow) + U n_\uparrow n_\downarrow$$

$\Delta \to \infty$

The superconducting reservoir can be "integrated out".
This results in a local superconducting pairing in the QD

$$H_D = \sum_\sigma \varepsilon_\sigma d_\sigma^\dagger d_\sigma - \Gamma_S (d_\uparrow d_\downarrow + d_\uparrow^\dagger d_\downarrow^\dagger) + U n_\uparrow n_\downarrow$$

This problem can be exactly diagonalized: four Bogoliubov De Gennes eigenstates

Doublet sector

$|\uparrow\rangle, |\downarrow\rangle$

BCS Singlet sector

$$|-\rangle = v|0\rangle + u|\uparrow\downarrow\rangle$$

$$|+\rangle = u|0\rangle - v|\uparrow\downarrow\rangle$$

Assuming for the moment spin-degeneracy

$$\epsilon_\uparrow = \epsilon_\downarrow = \epsilon_0 \qquad \xi \equiv \epsilon_0 + \frac{U}{2}$$

$$E_\pm = \frac{U}{2} + \xi \pm \sqrt{\xi^2 + \Gamma_S^2}$$

$$u = \frac{1}{2}\sqrt{1 + \frac{\xi}{\sqrt{\xi^2 + \Gamma_S^2}}} \qquad v = \frac{1}{2}\sqrt{1 - \frac{\xi}{\sqrt{\xi^2 + \Gamma_S^2}}}$$

icmm csic

$$\Delta \rightarrow \infty$$

The superconducting reservoir can be "integrated out".
This results in a local superconducting pairing in the QD

$$H_D = \sum_\sigma \varepsilon_\sigma d^\dagger_\sigma d_\sigma - \Gamma_S (d_\uparrow d_\downarrow + d^\dagger_\uparrow d^\dagger_\downarrow) + U n_\uparrow n_\downarrow$$

This problem can be exactly diagonalized: four Bogoliubov De Gennes eigenstates

Doublet sector

$$|\uparrow\rangle, |\downarrow\rangle$$

BCS Singlet sector

$$|-\rangle = v|0\rangle + u|\uparrow\downarrow\rangle$$

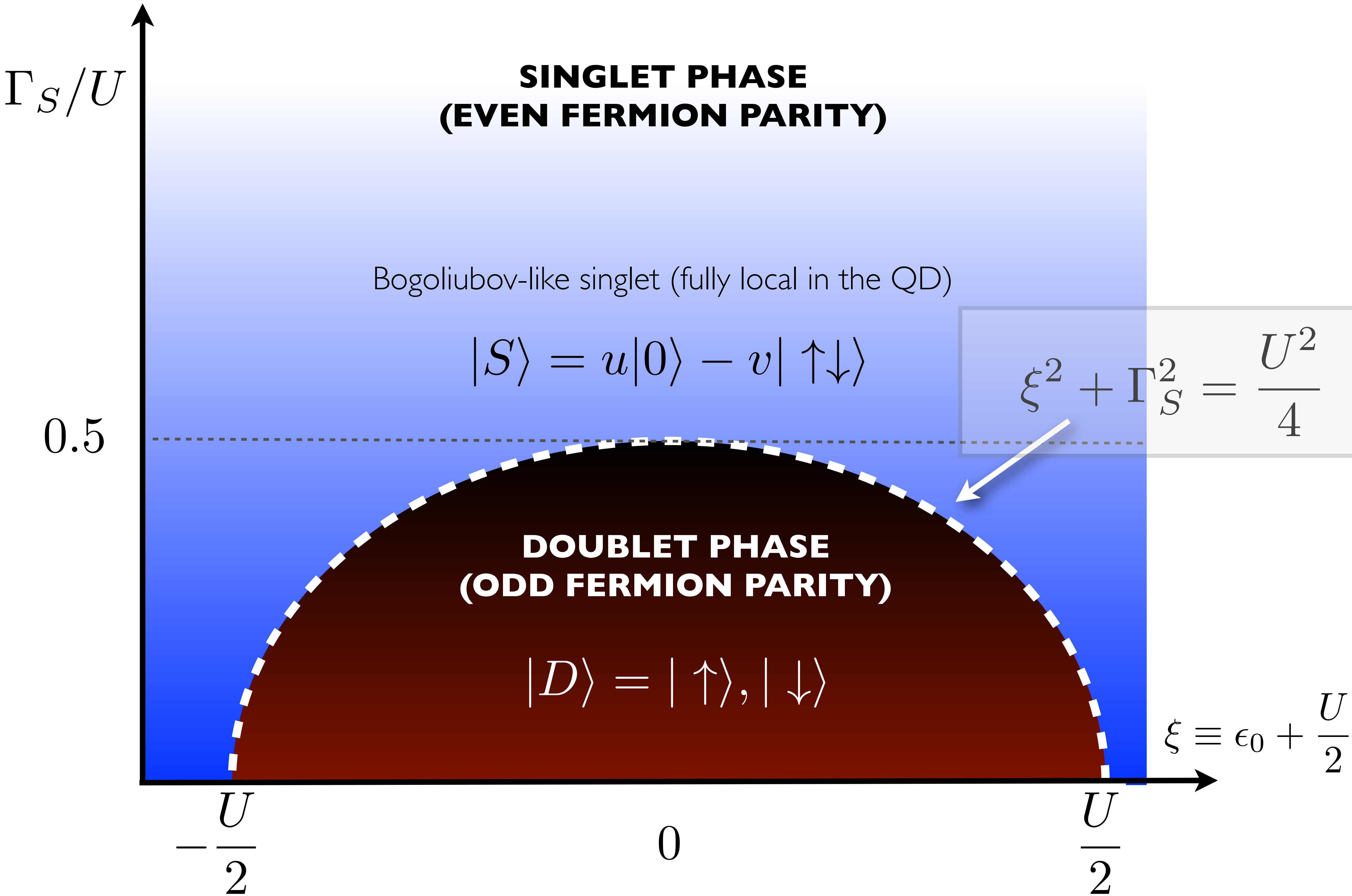
$$|+\rangle = u|0\rangle - v|\uparrow\downarrow\rangle$$

Doublet

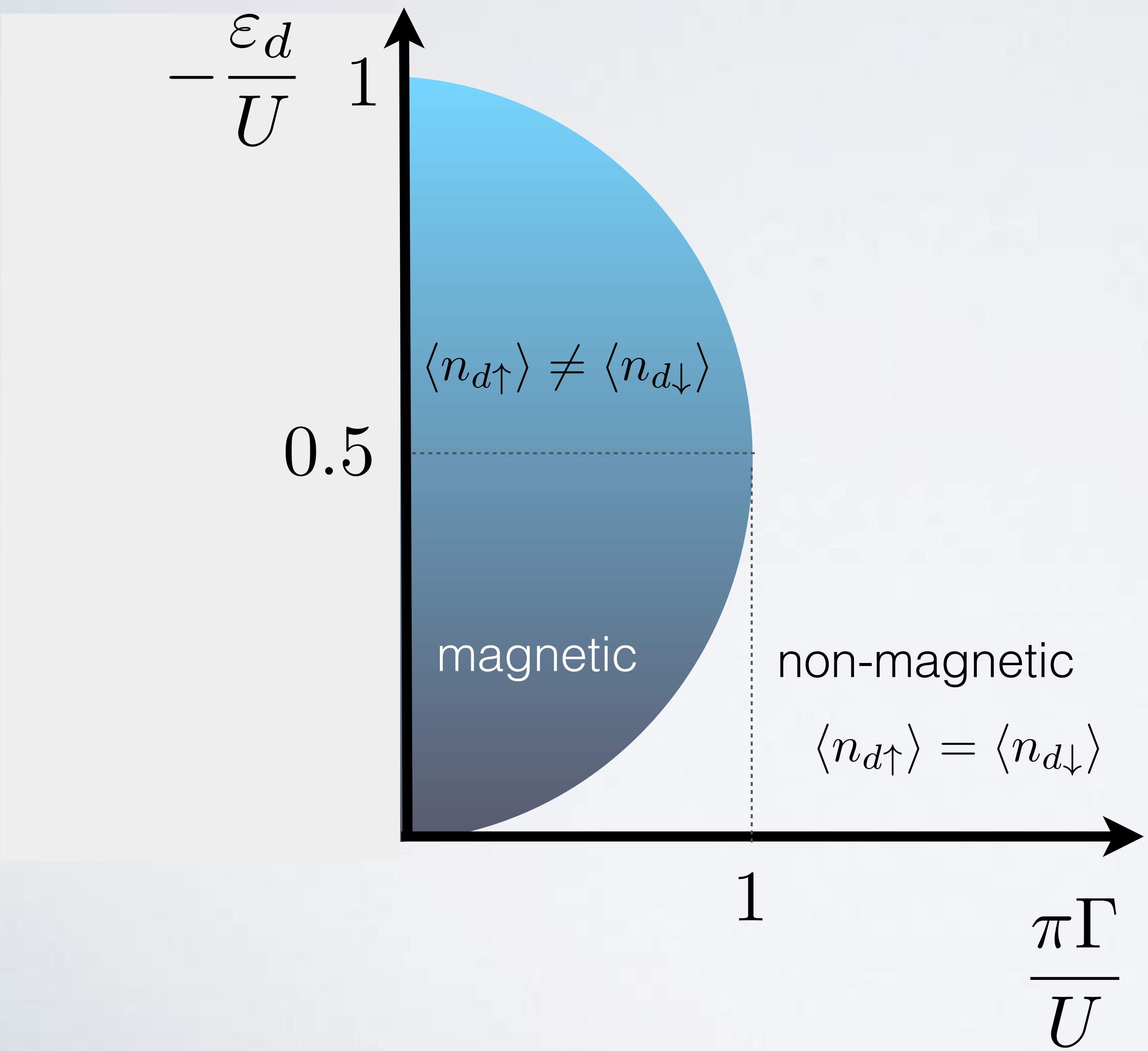
$$H_D = \begin{pmatrix} \epsilon_\uparrow & 0 & 0 & 0 \\ 0 & \epsilon_\downarrow & 0 & 0 \\ 0 & 0 & 0 & \Gamma_S \\ 0 & 0 & \Gamma_S & \epsilon_\uparrow + \epsilon_\downarrow + U \end{pmatrix}$$

Singlet

Exact phase diagram in the "large gap limit": Coulomb blockade and BCS pairing compete in fixing fermion parity of the ground state.



Anderson model with normal reservoir



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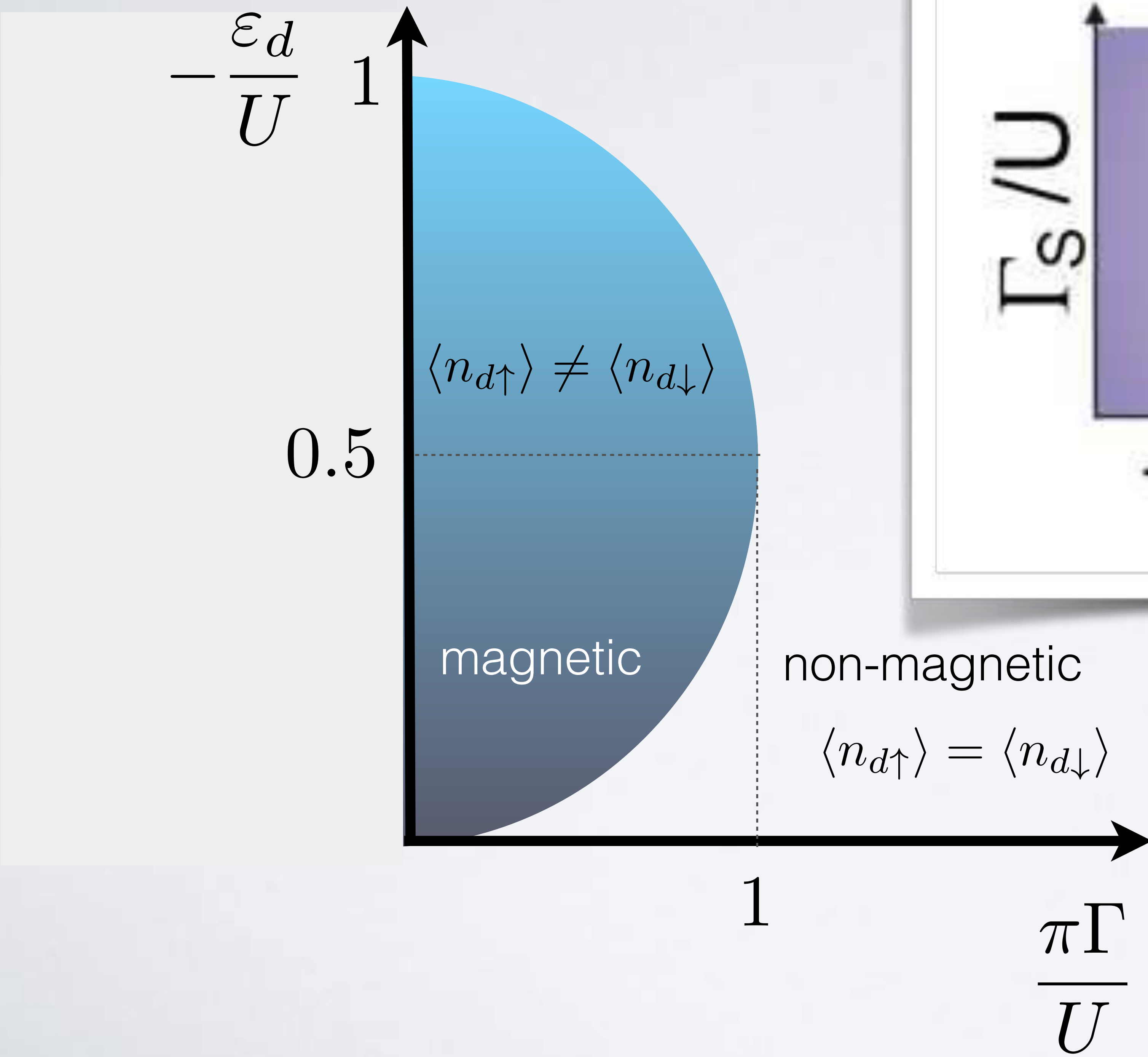
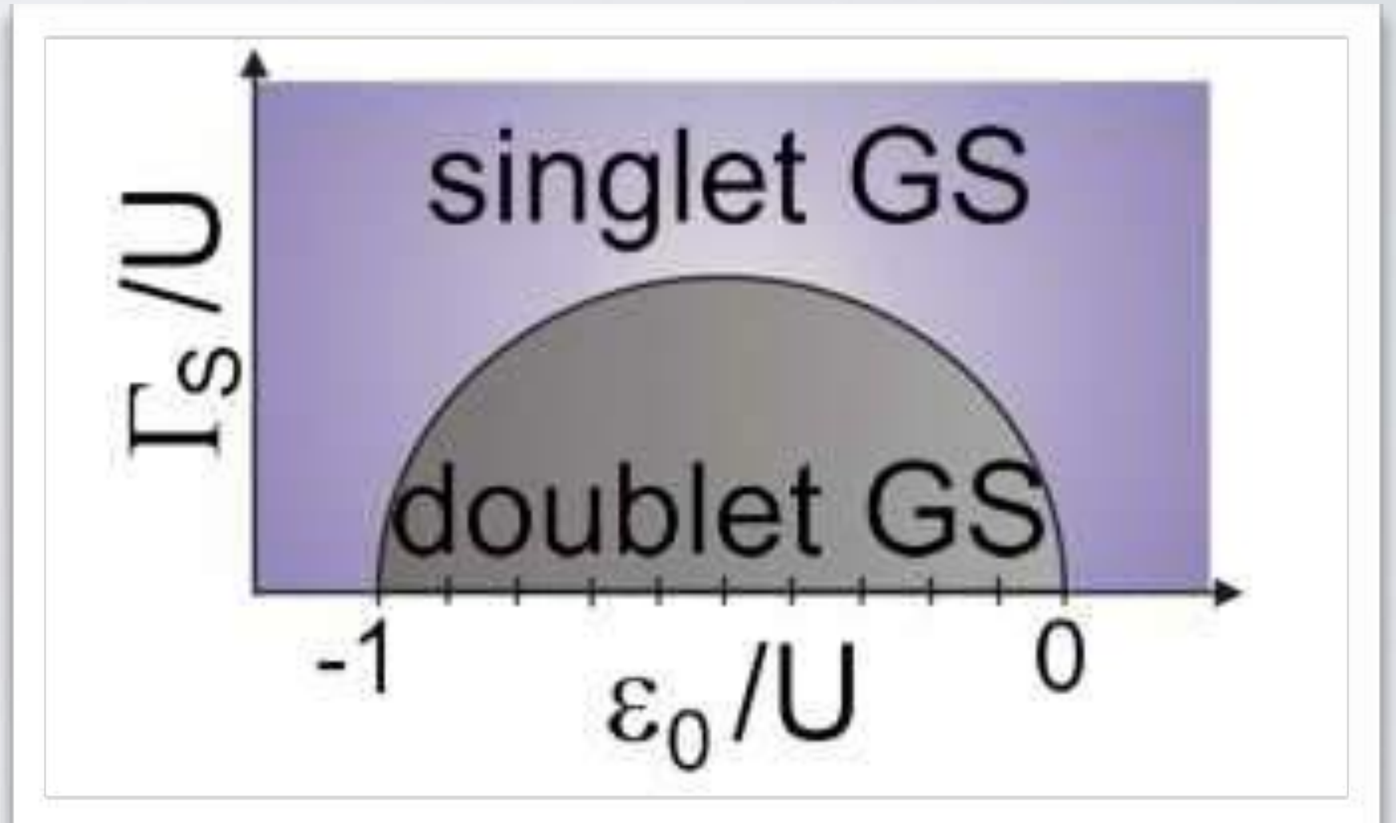
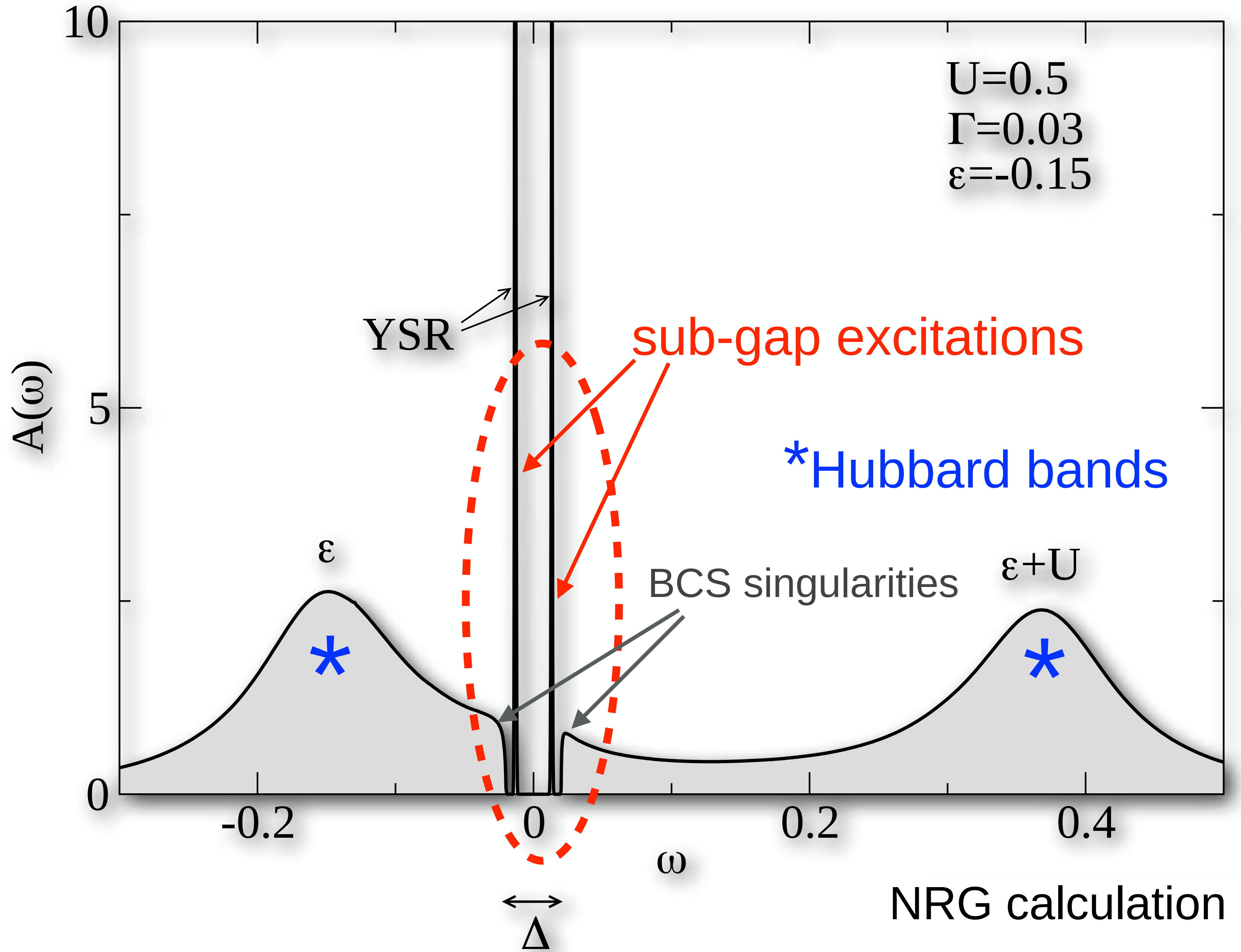
Anderson model with BCS reservoir

Hubbard and BCS pairing compete in fixing fermion parity of the ground state. Strong U wants a doublet singlet state whereas strong coupling to the superconductor wants BCS singlet ground state







- Physical Picture: odd-occupied QD acts as a spinful quantum impurity coupled to the SC.
- The ground state fermion parity changes from even to odd as sub-gap levels cross zero energy (quantum phase transition). Accordingly, the spin state changes from singlet to doublet.
- These levels are the quantum version of Yu-Shiba-Rusinov (YSR) states (subgap states in a superconductor created by pair-breaking from a classical spin impurity).

For the Anderson model, these sub-gap states are poles of the Nambu Green's function

$$\hat{G}^r_\sigma(t,t') \equiv -i\theta(t-t')\langle[\Psi_\sigma(t), \Psi^\dagger_\sigma(t')]_+\rangle$$

$$\hat{G}^r_\sigma(\omega)^{-1} = \hat{G}^{r(0)}_\sigma(\omega)^{-1} - \hat{\Sigma}_\sigma(\omega)$$

Note that in the presence of finite Hubbard U this Anderson impurity problem is highly non-trivial: competition between superconductivity, quantum fluctuations, Coulomb Blockade and Kondo physics.

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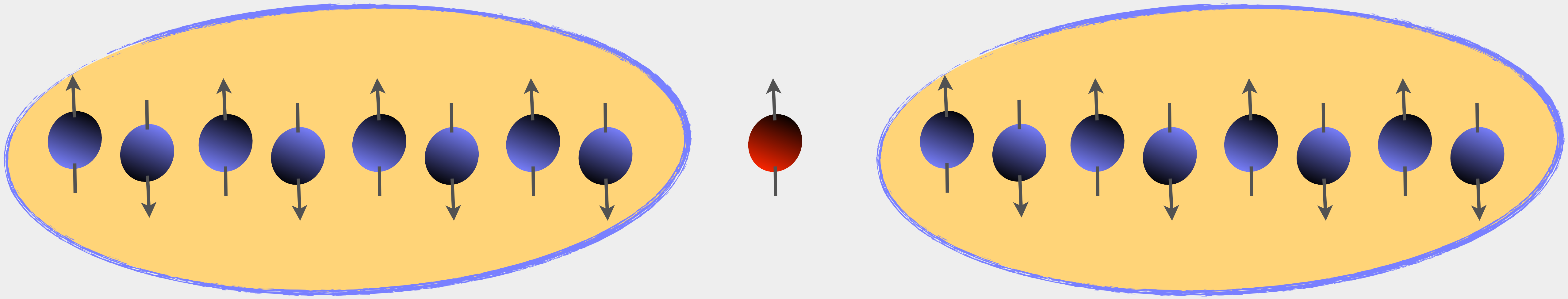
Kondo + superconductivity

Starting in the doublet phase, Kondo singlets will compete versus BCS singlets as quantum fluctuations become important!!!

$$\Delta >> T_K$$



Kondo + superconductivity

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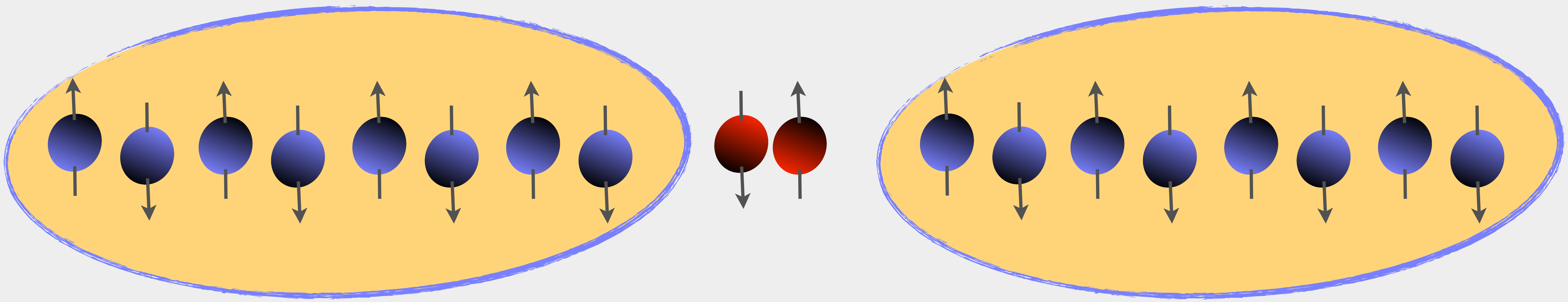
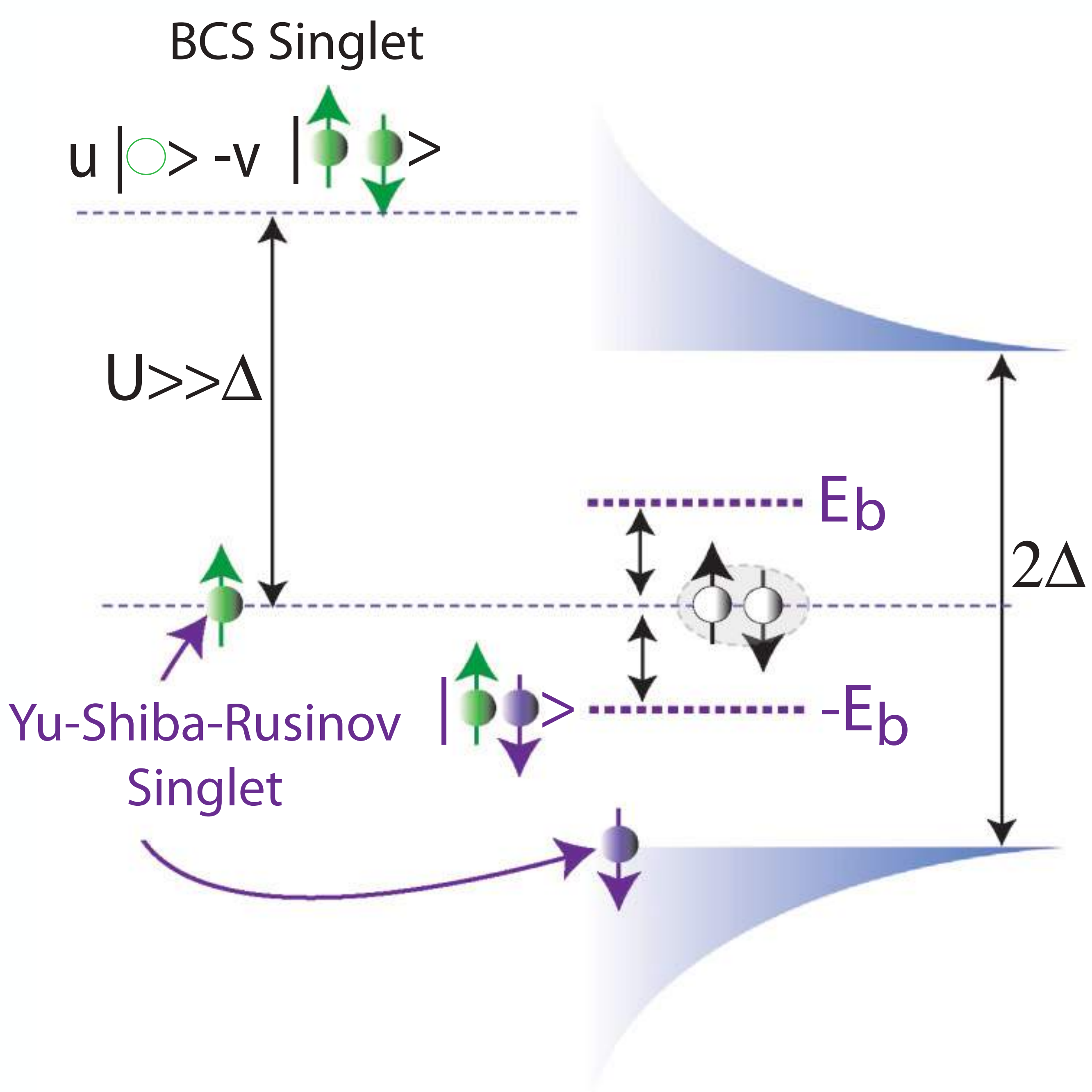


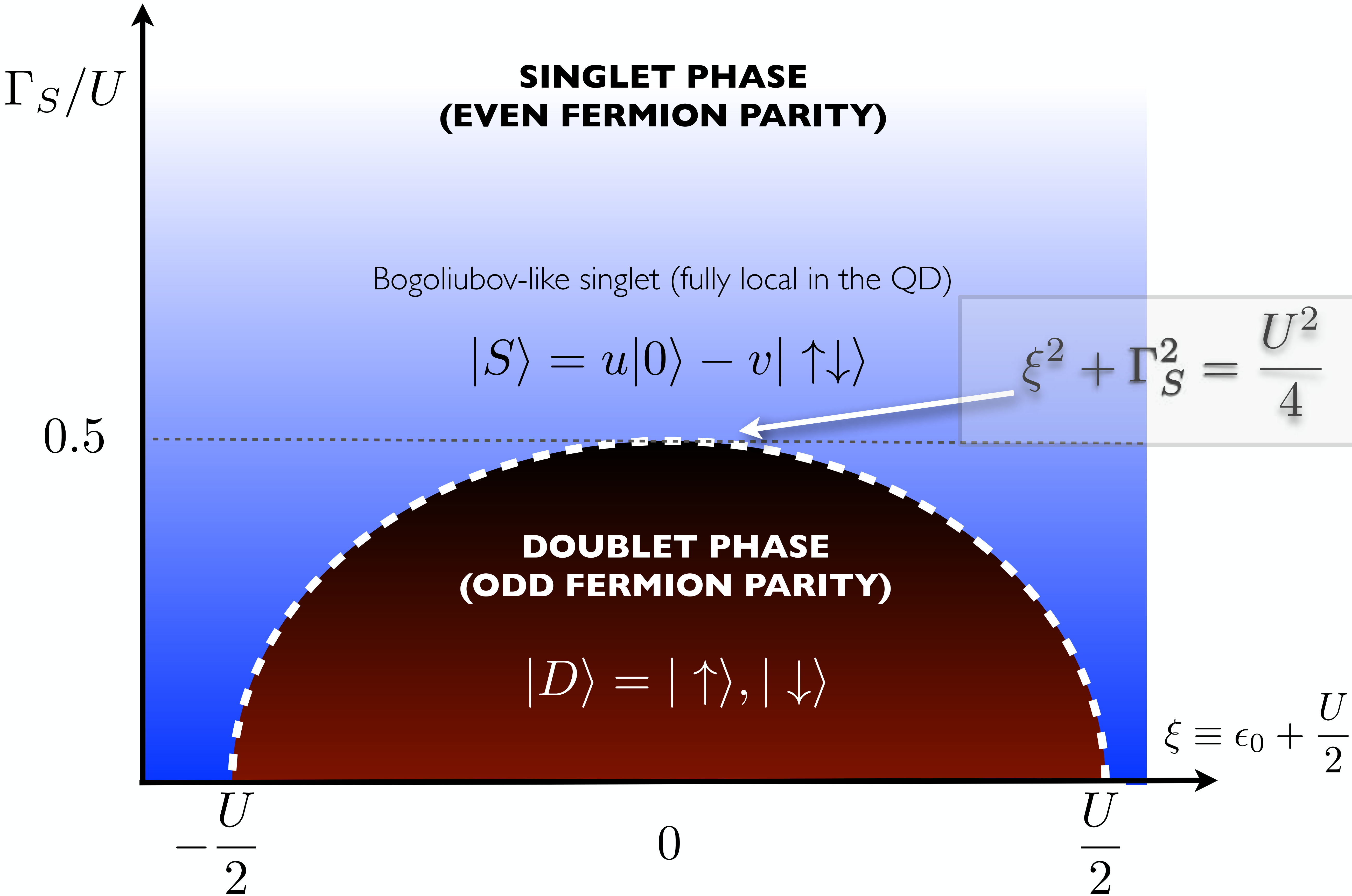
Quantum phase transition: ground state changes from doublet to singlet when

$$\Delta \sim T_K$$

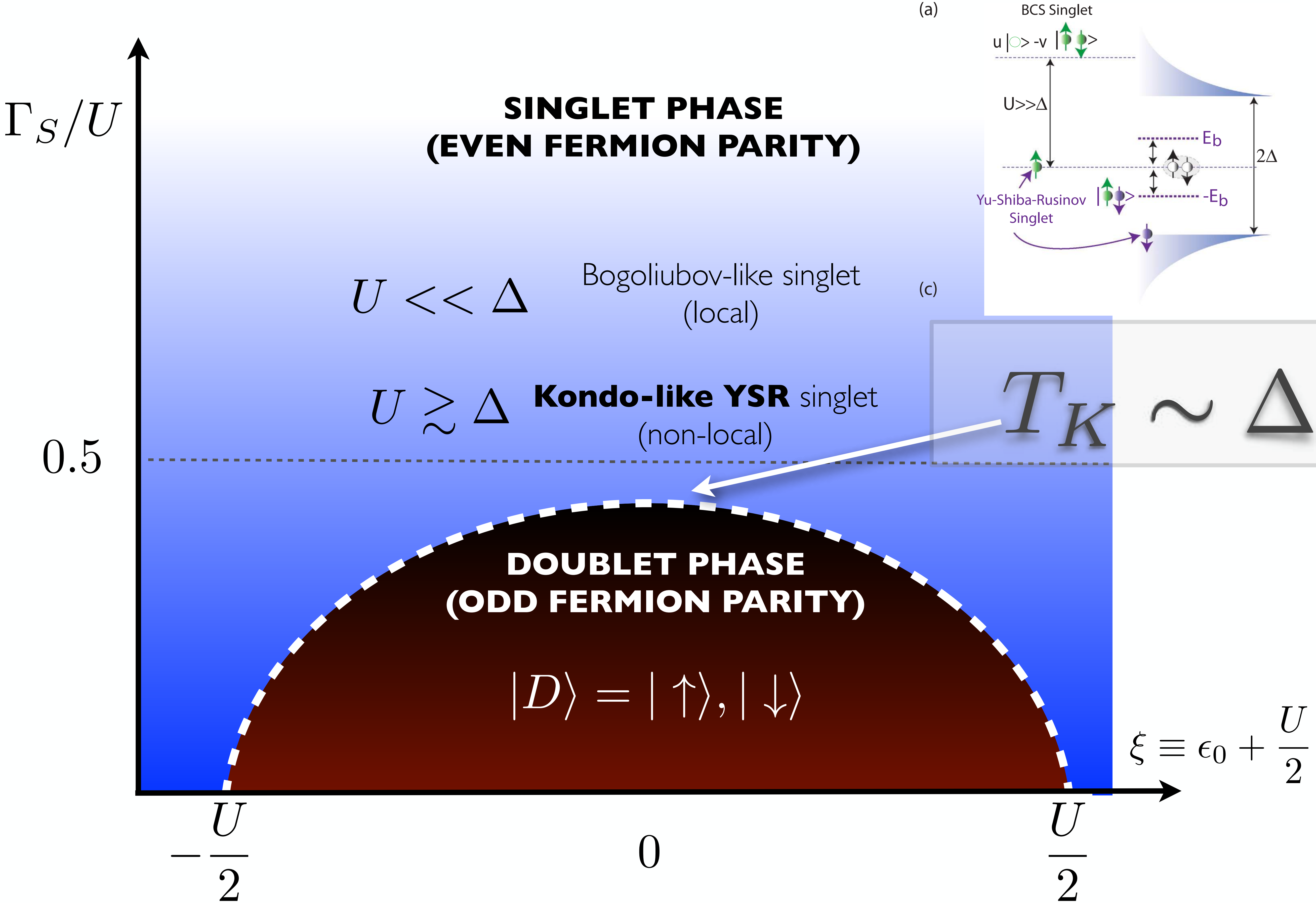
Quantum fluctuations ⇒ Kondo screening of doublets (=YSR singlets). The ground state parity is governed by the competition of different energy scales



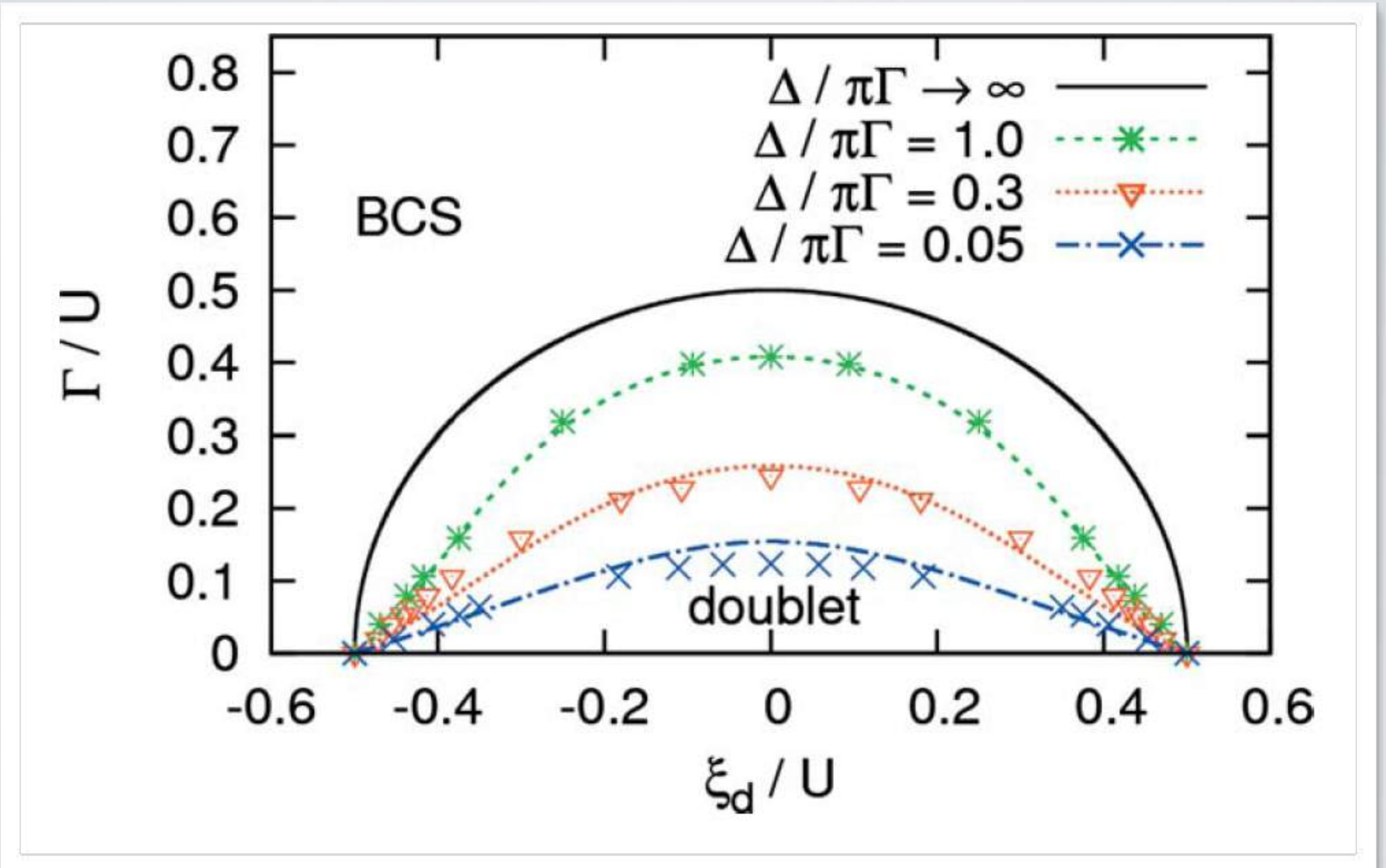
Exact phase diagram in the "large gap limit": Coulomb blockade and BCS pairing compete in fixing fermion parity of the ground state.



Finite Δ: quasiparticle excitations above the gap induce Kondo correlations

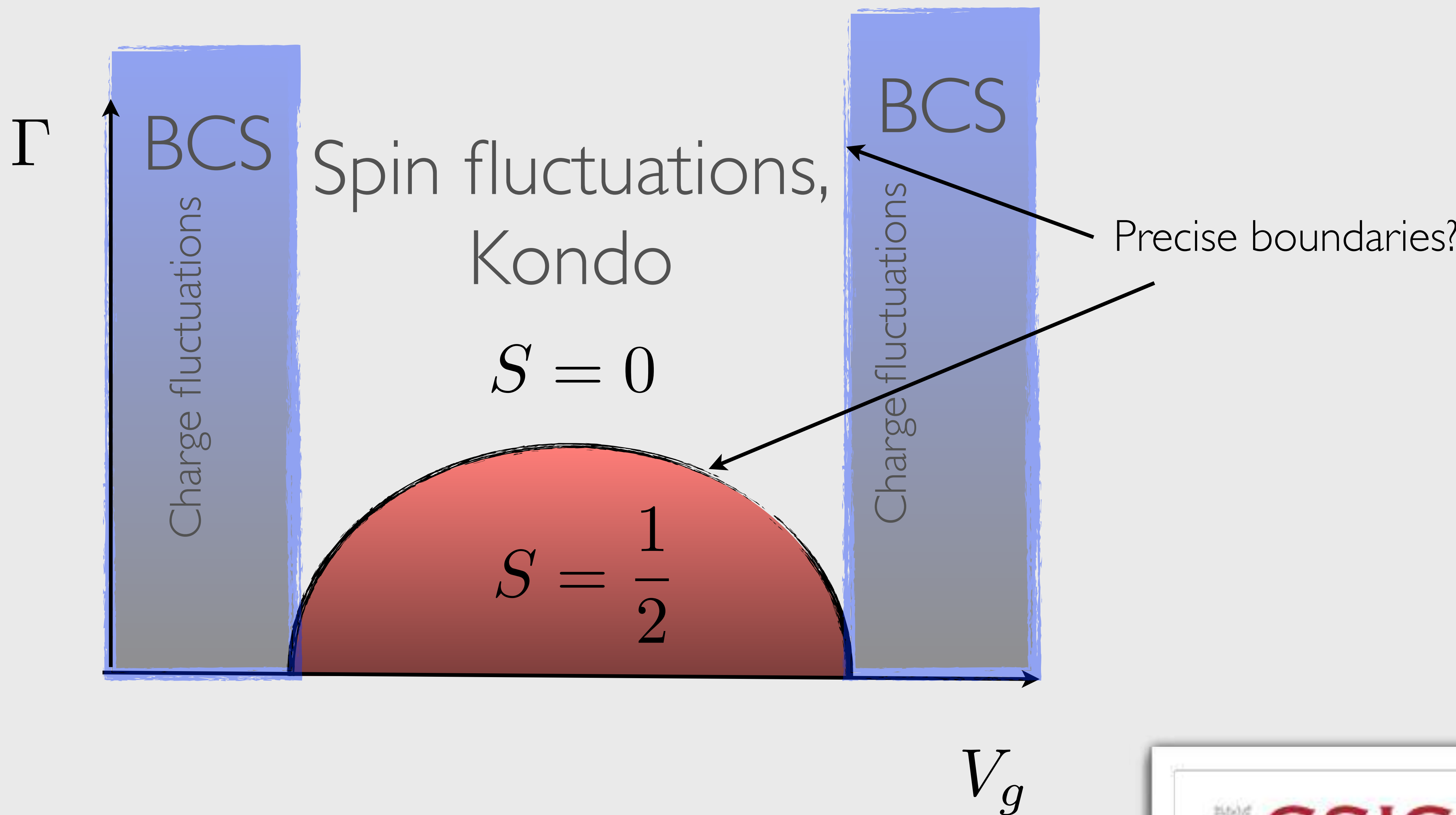


Phase diagram

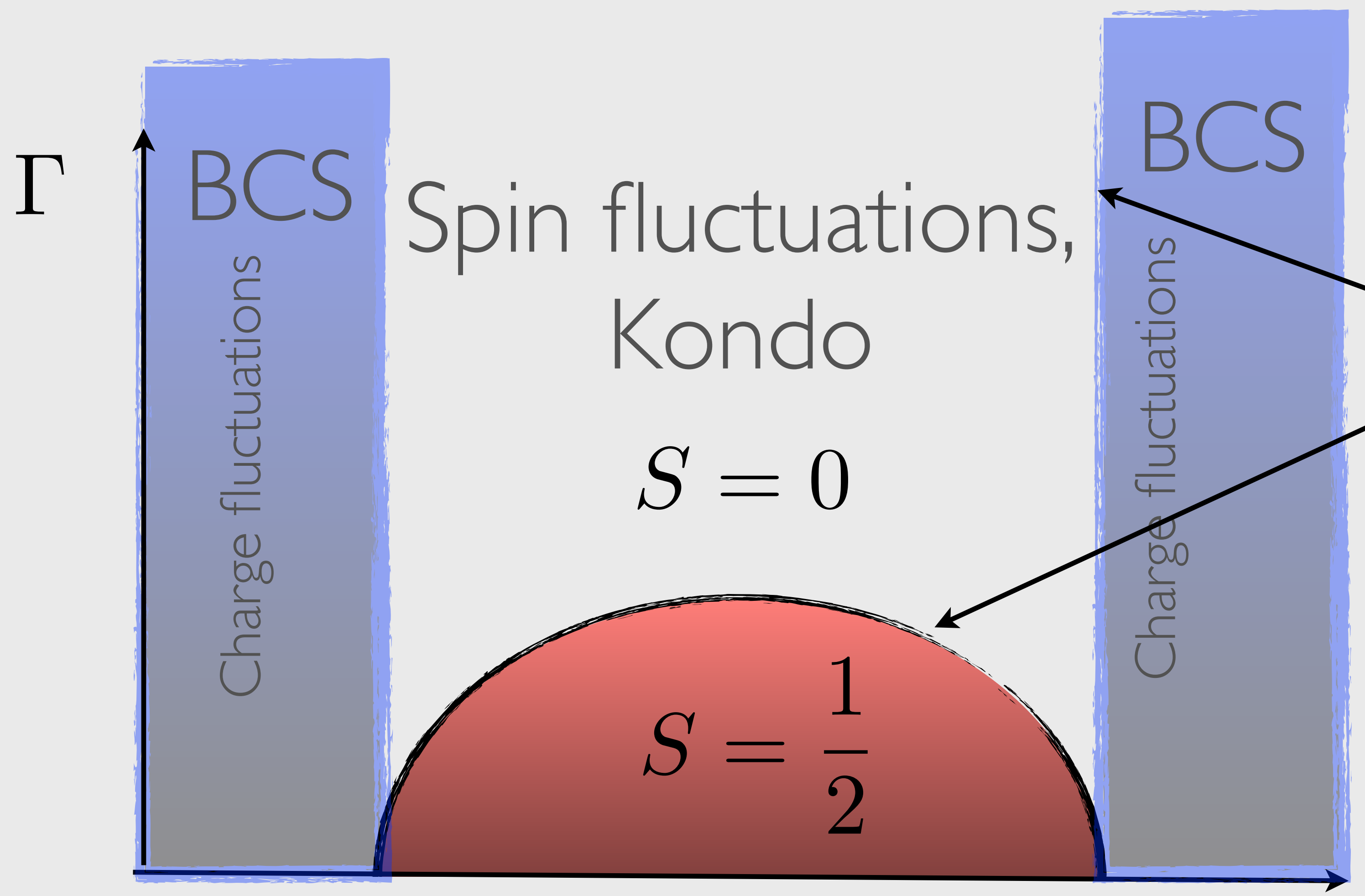


J.Bauer, A.Oguri,and A. C. Hewson, J. Phys. :Condens.Matter19 ,486211 (2007).
T. Meng, S. Florens and P. Simon PRB 79, 224521 (2009), etc...

Shiba versus Kondo, doublets versus singlets, and all this...still an open problem



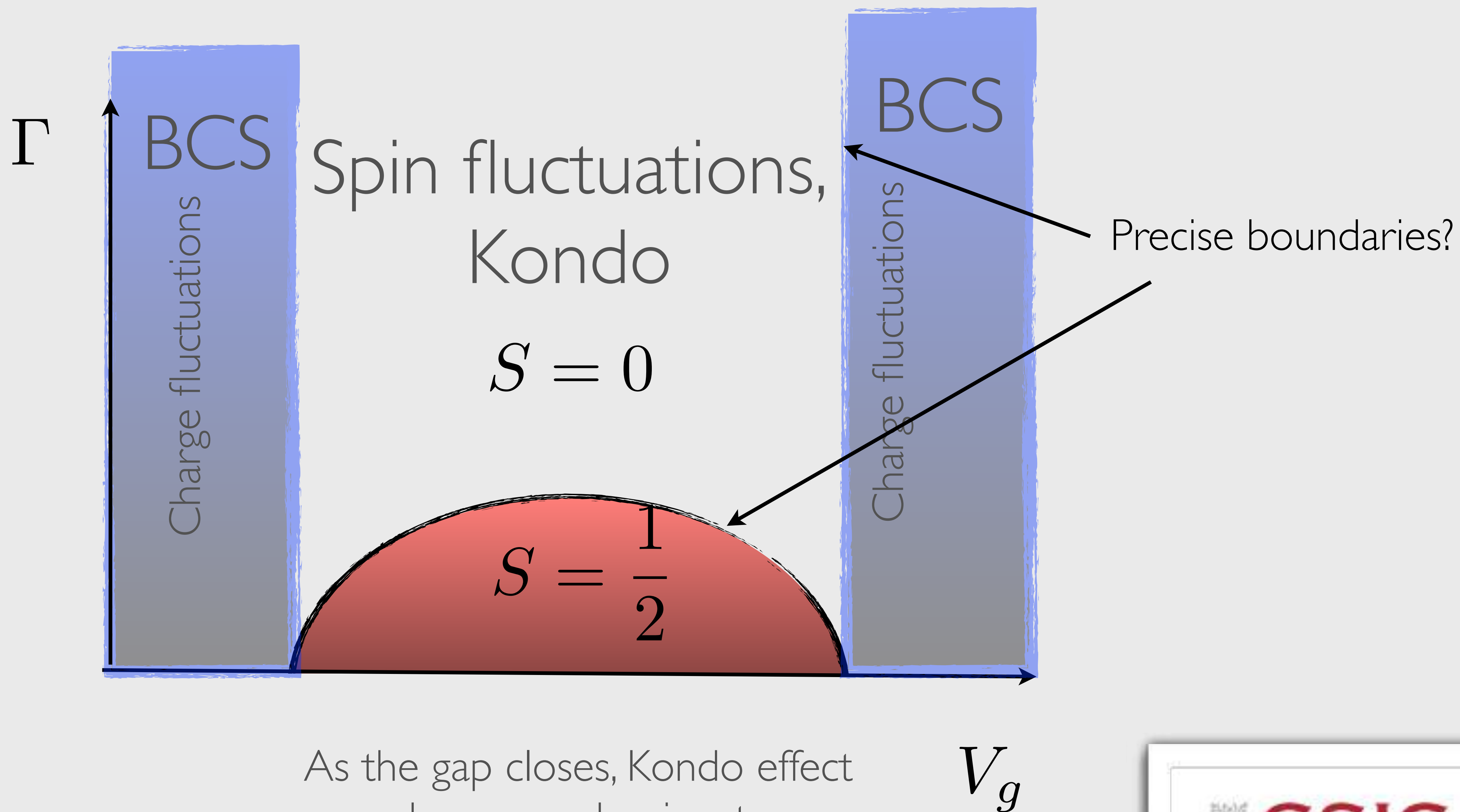
Shiba versus Kondo, doublets versus singlets, and all this...still an open problem



As the gap closes, Kondo effect becomes dominant

CSIC

Shiba versus Kondo, doublets versus singlets, and all this...still an open problem



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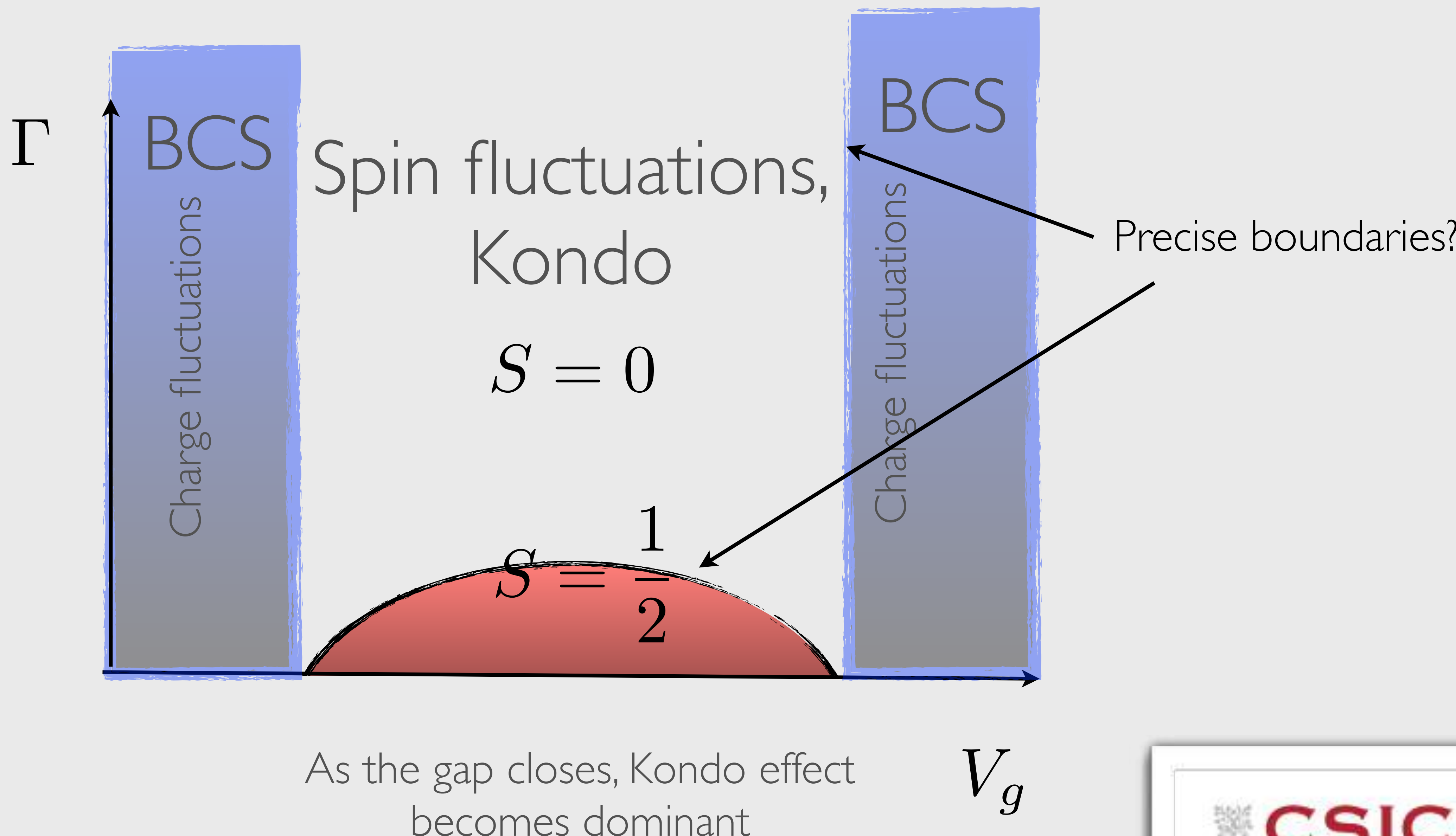


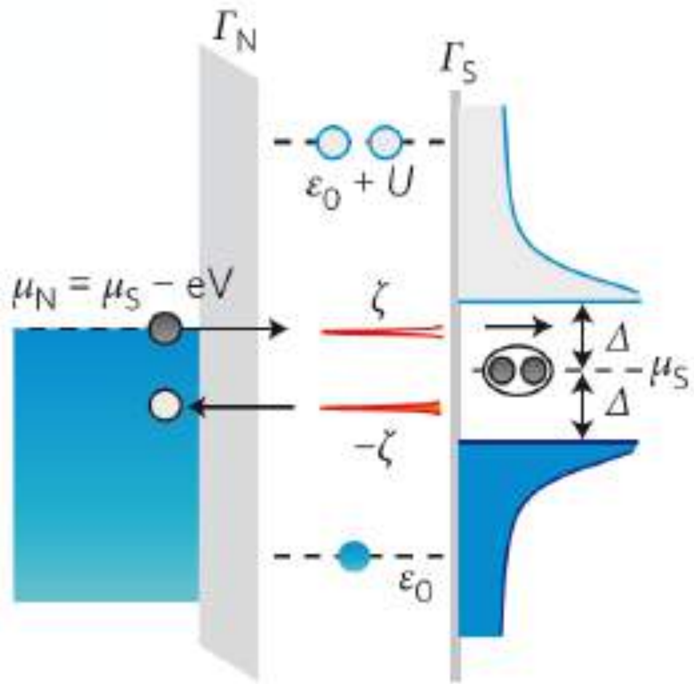
Shiba versus Kondo, doublets versus singlets, and all this...

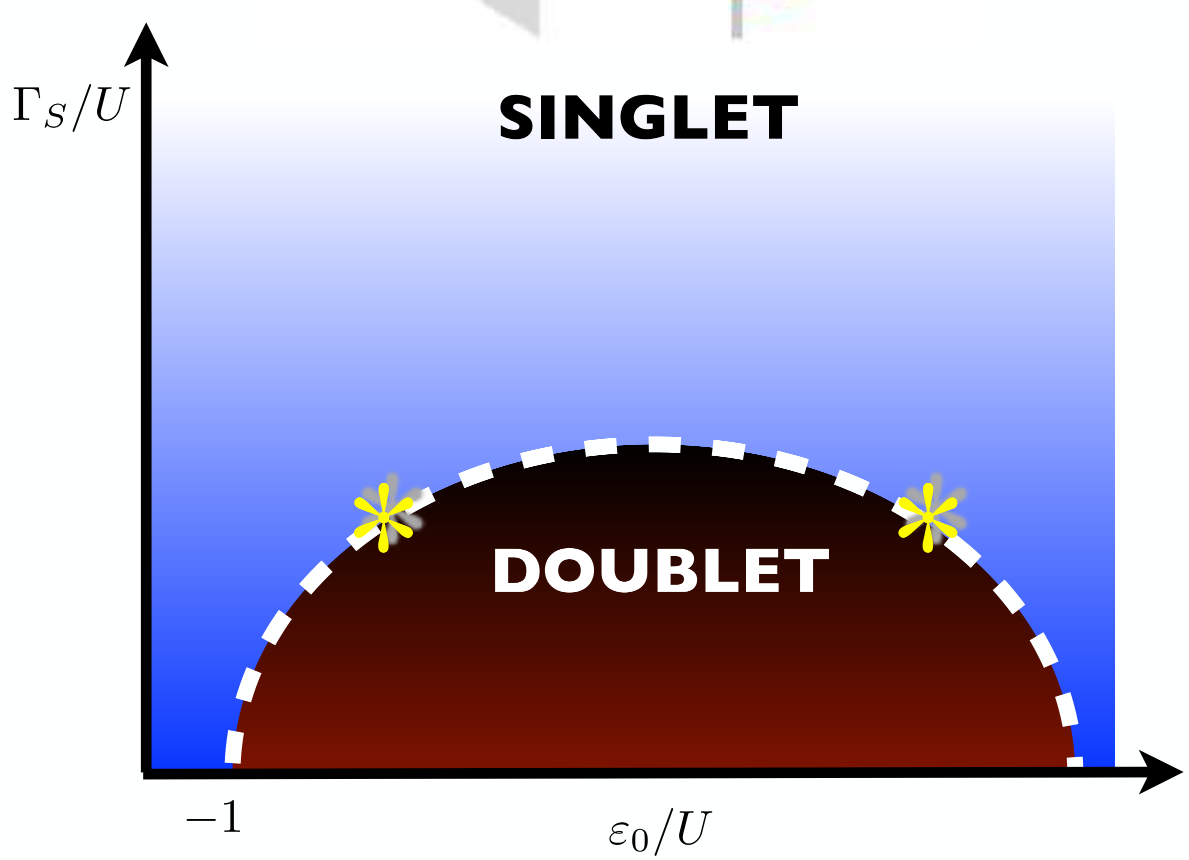
Hybrid systems composed of superconductors and some kind of quantum dot (nanotubes, nanowires, etc) are ideal testbed for all these physical effects!!!

nature nanotechnology | ARTICLES
PUBLISHED ONLINE: 15 DECEMBER 2013 | DOI: 10.1038/NNANO.2013.267

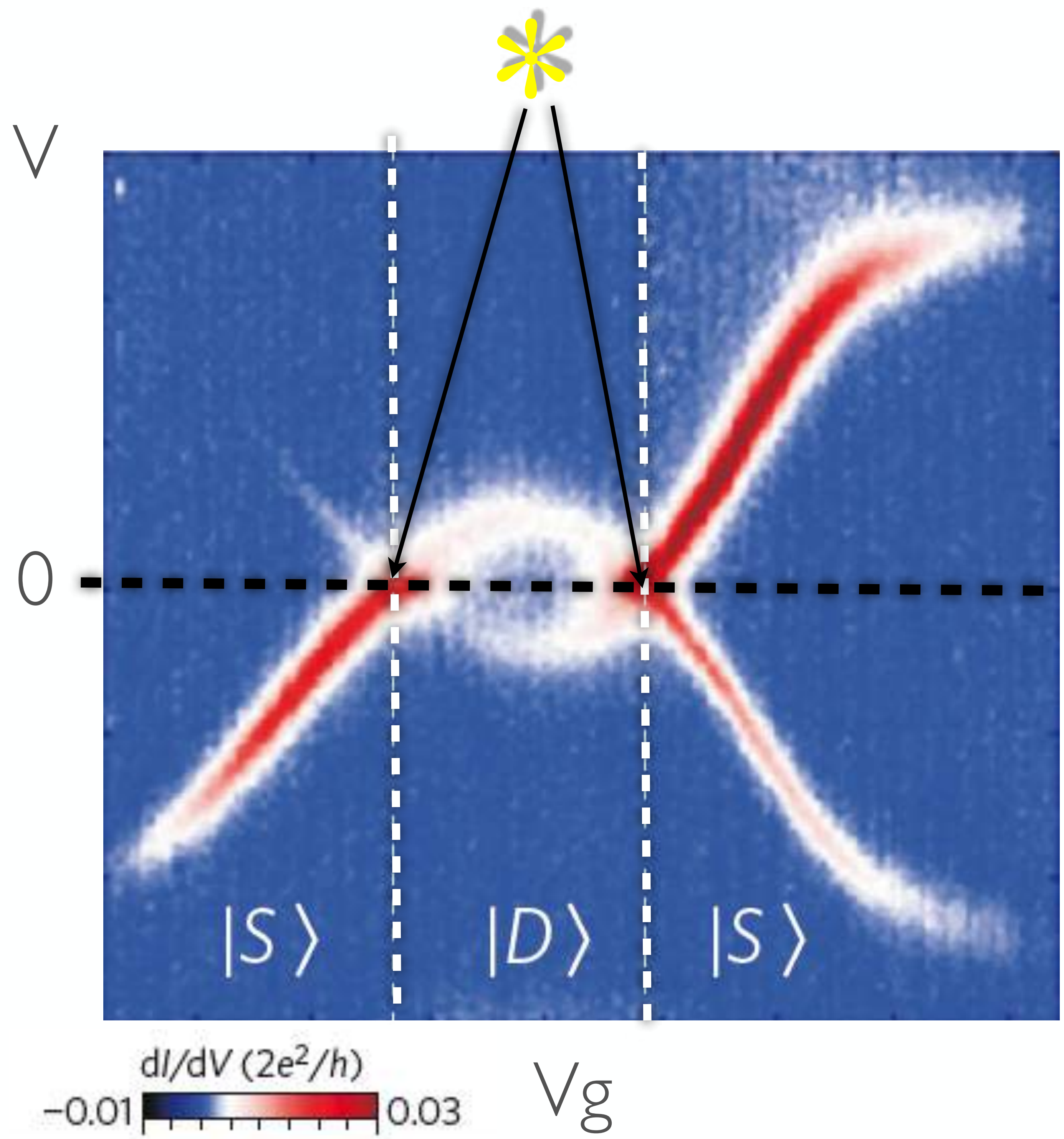
Spin-resolved Andreev levels and parity crossings in hybrid superconductor–semiconductor nanostructures

Eduardo J. H. Lee[1], Xiaocheng Jiang[2], Manuel Houzet[1], Ramón Aguado[3], Charles M. Lieber[2] and Silvano De Franceschi[1]*





Experimentally, this is seen as resonates in Andreev conductance (red lines) that **cross zero voltage**



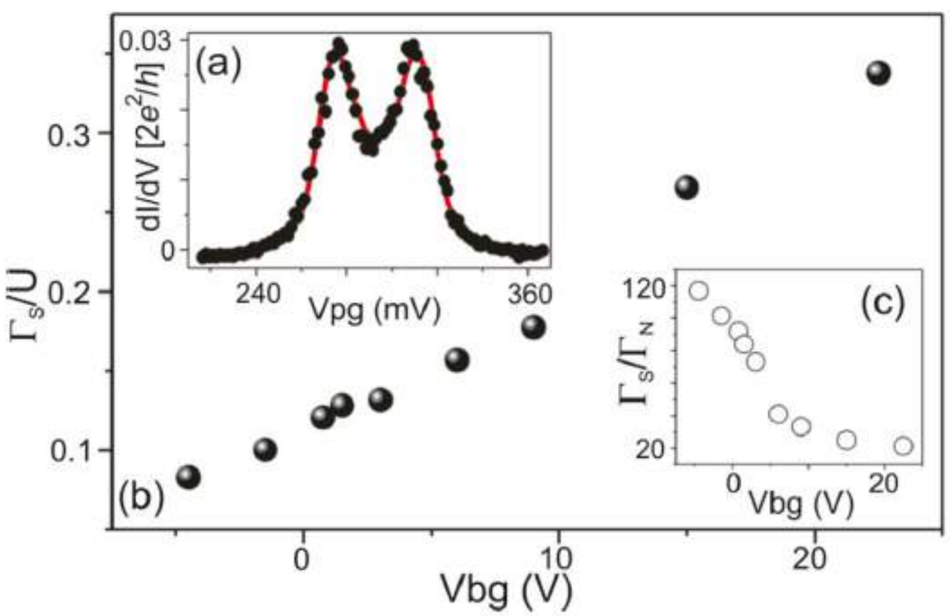
Quantum phase transition: when the **subgap states cross zero energy**, the ground state changes parity

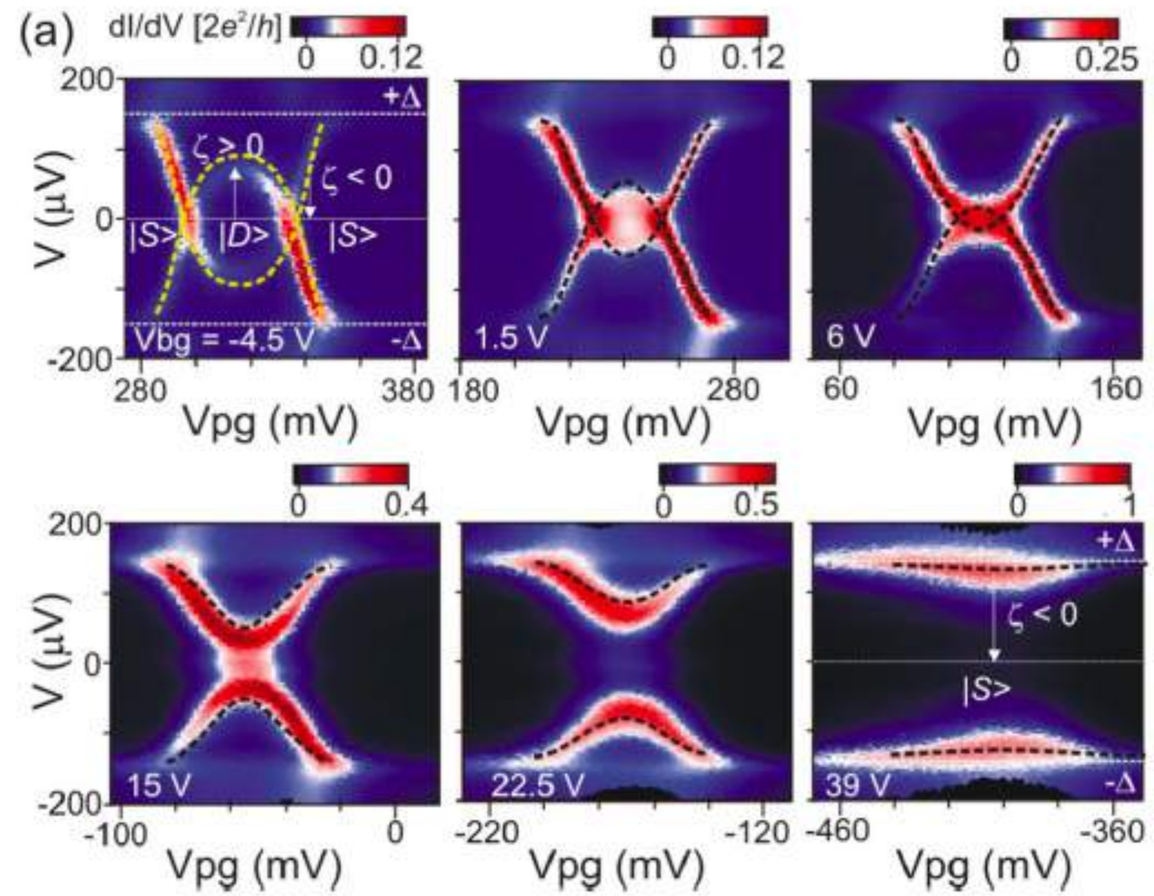
KONDO CORRECTIONS

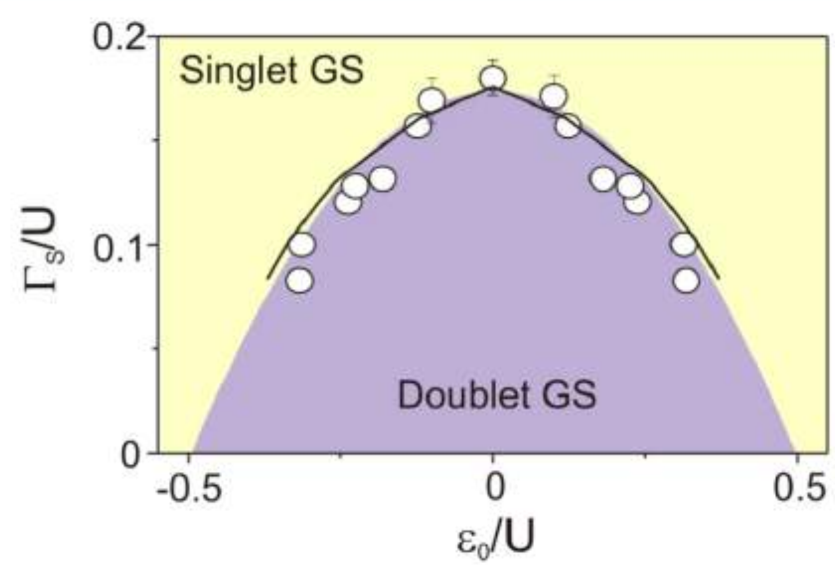
PHYSICAL REVIEW B **95**, 180502(R) (2017)

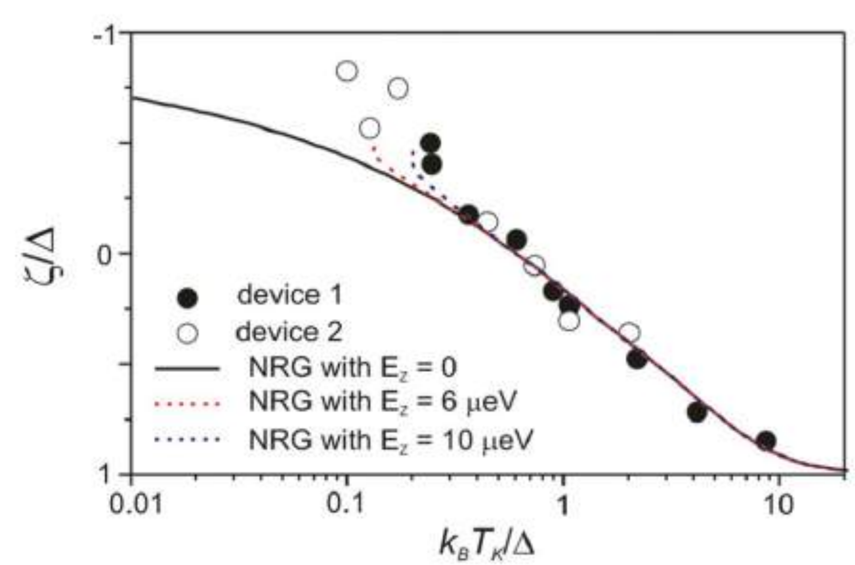
Scaling of subgap excitations in a superconductor-semiconductor nanowire quantum dot

Eduardo J. H. Lee,[1,*] Xiaocheng Jiang,[2] Rok Žitko,[3] Ramón Aguado,[4] Charles M. Lieber,[2] and Silvano De Franceschi[1,†]





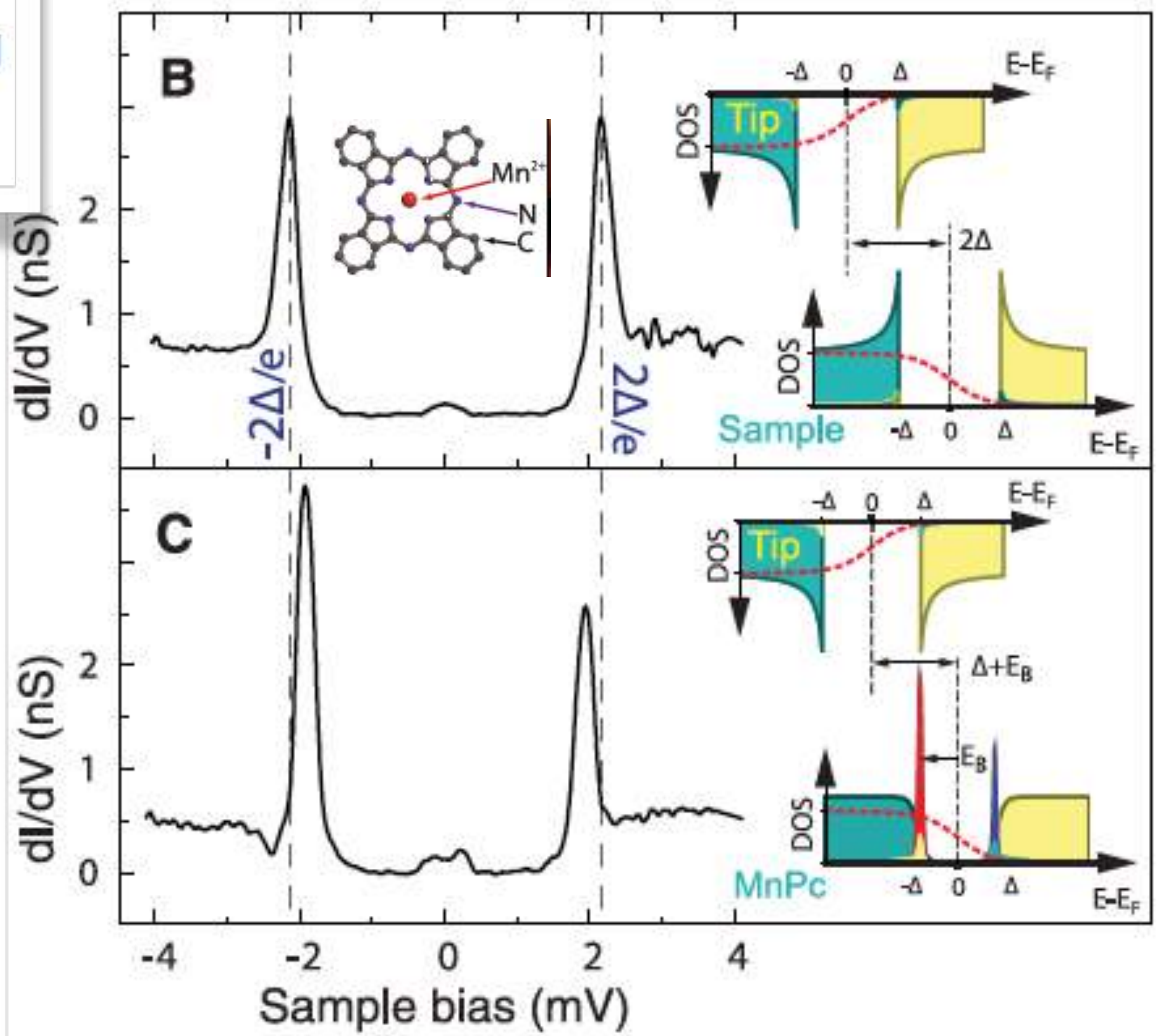




Similar physics in STM experiments

Competition of Superconducting Phenomena and Kondo Screening at the Nanoscale

K. J. Franke,* G. Schulze, J. I. Pascual



940 20 MAY 2011 VOL 332 SCIENCE www.sciencemag.org

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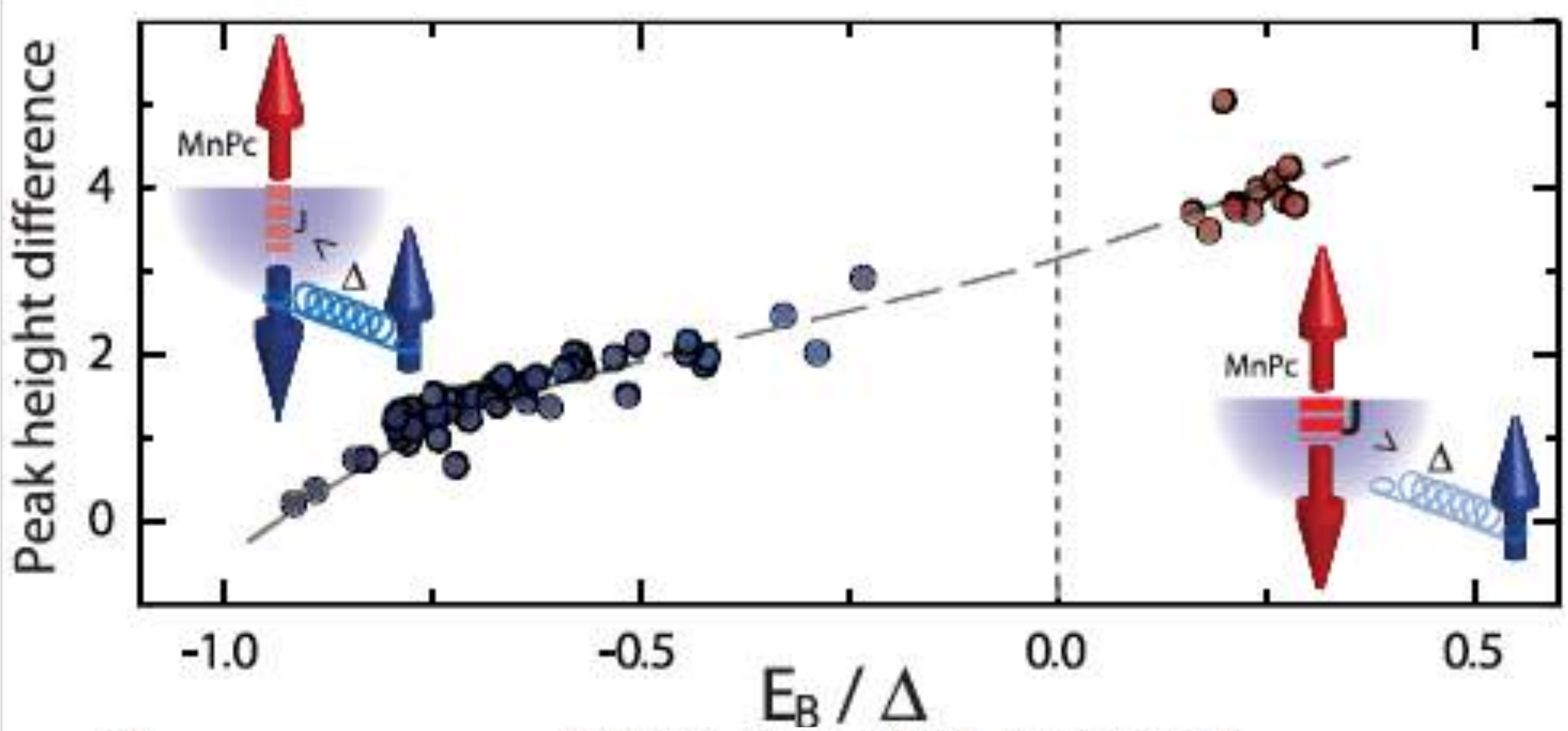

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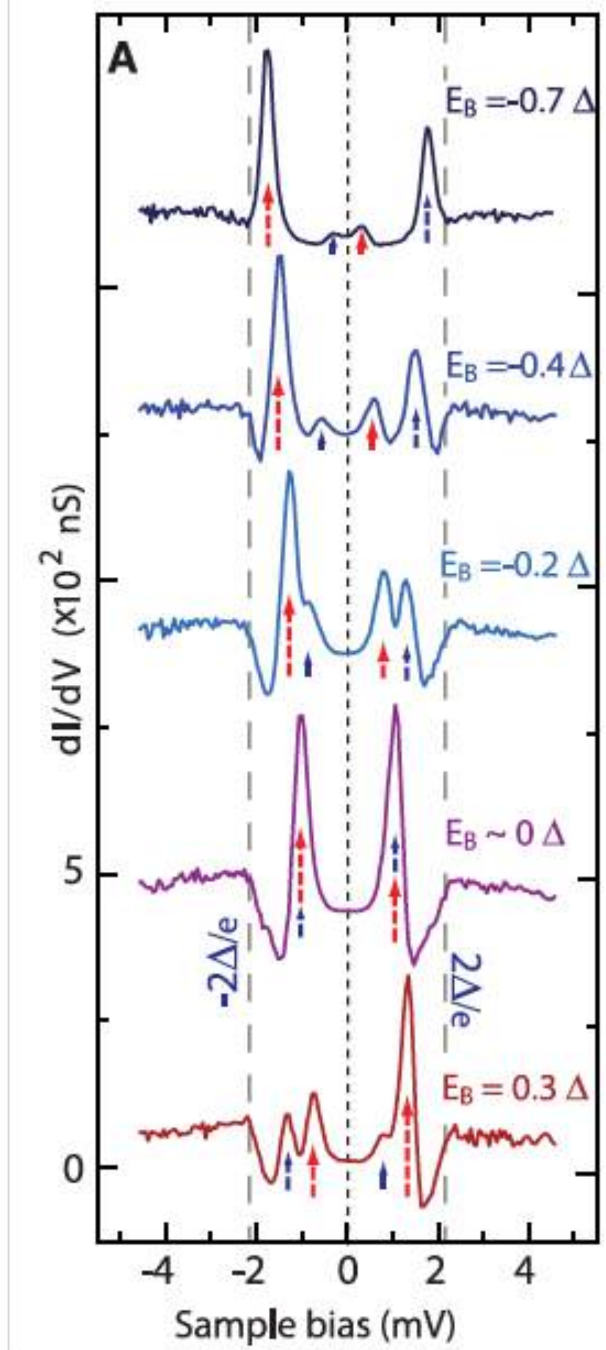

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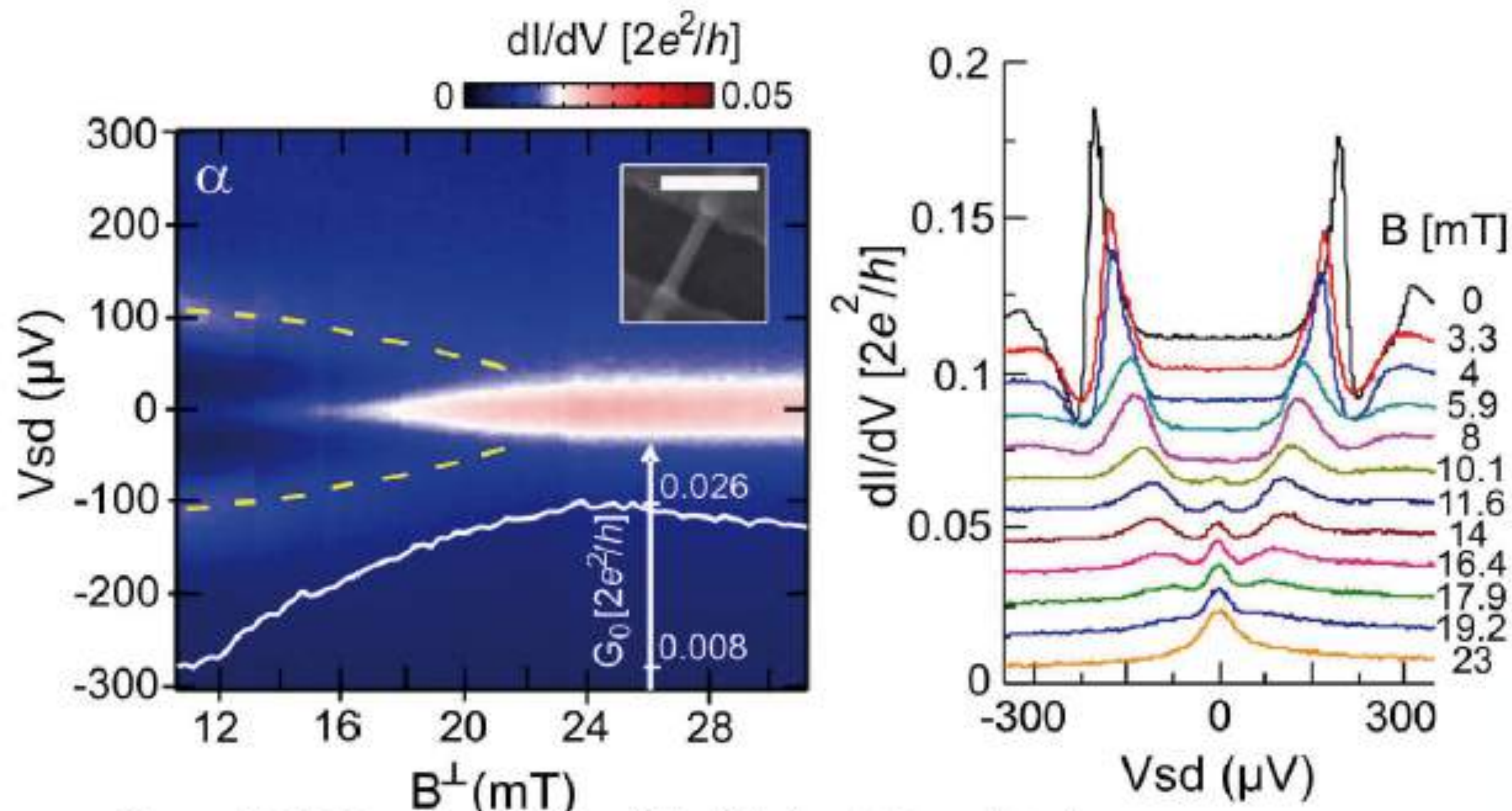
Starting from the doublet phase with YRS states inside the gap it is possible to increase the role of Kondo fluctuations by closing the gap with a magnetic field

PRL 109, 186802 (2012) PHYSICAL REVIEW LETTERS week ending 2 NOVEMBER 2012

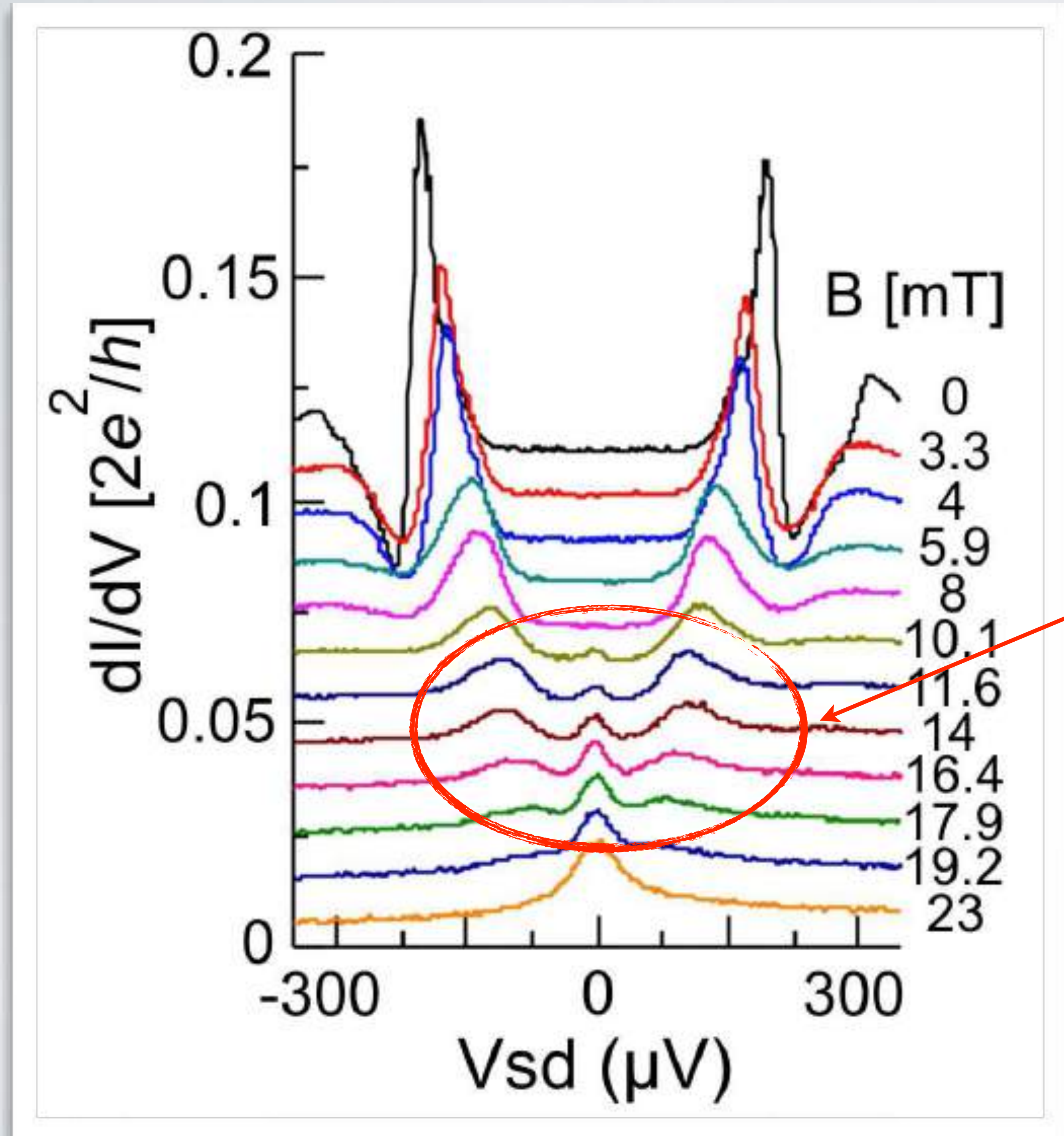
Zero-Bias Anomaly in a Nanowire Quantum Dot Coupled to Superconductors

Eduardo J. H. Lee,[1] Xiaocheng Jiang,[2] Ramón Aguado,[3] Georgios Katsaros,[1,*]
Charles M. Lieber,[2] and Silvano De Franceschi[1,†]



$T_K \approx 0.56$ K $\quad (\Gamma_L/\Gamma_R) \approx 6.6 \times 10^{-3}$

Starting from the doublet phase with YRS states inside the gap it is possible to increase the role of Kondo fluctuations by closing the gap with a magnetic field



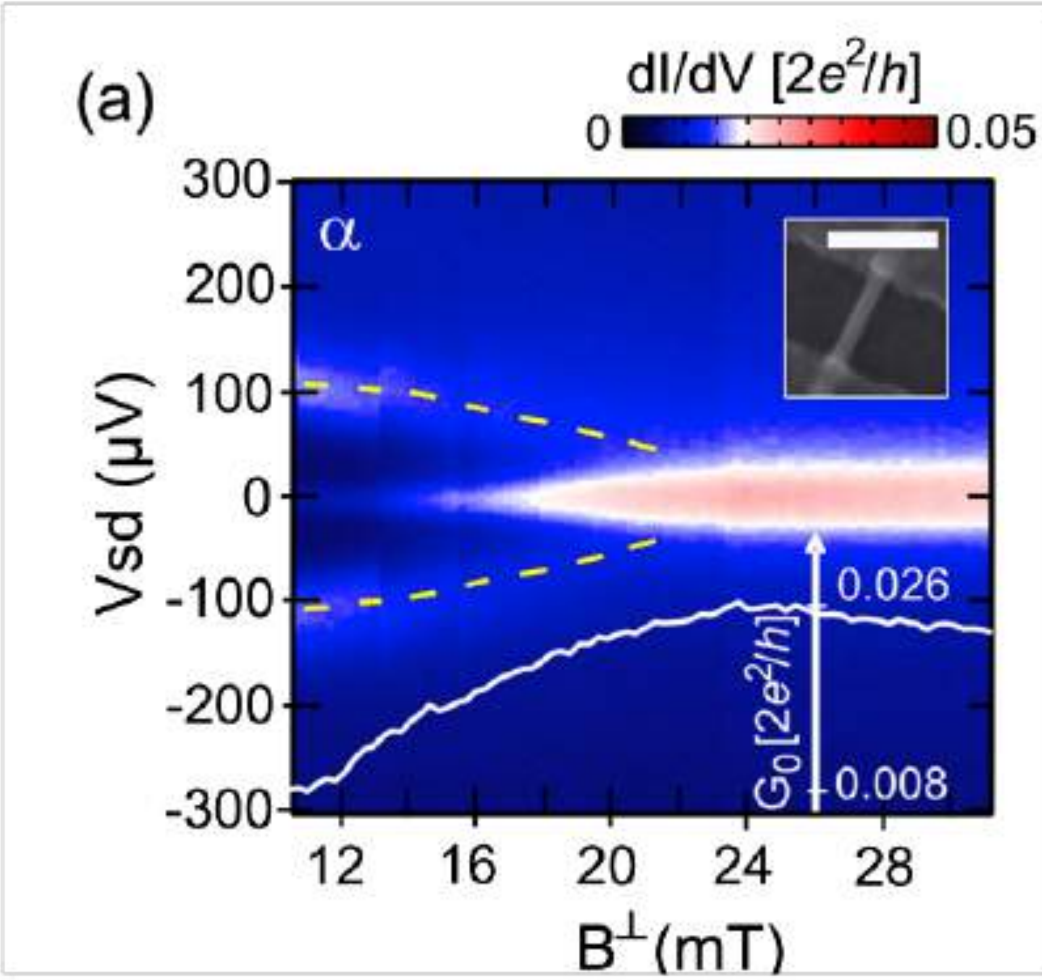
Coexistence of Kondo and induced BCS pairing!!

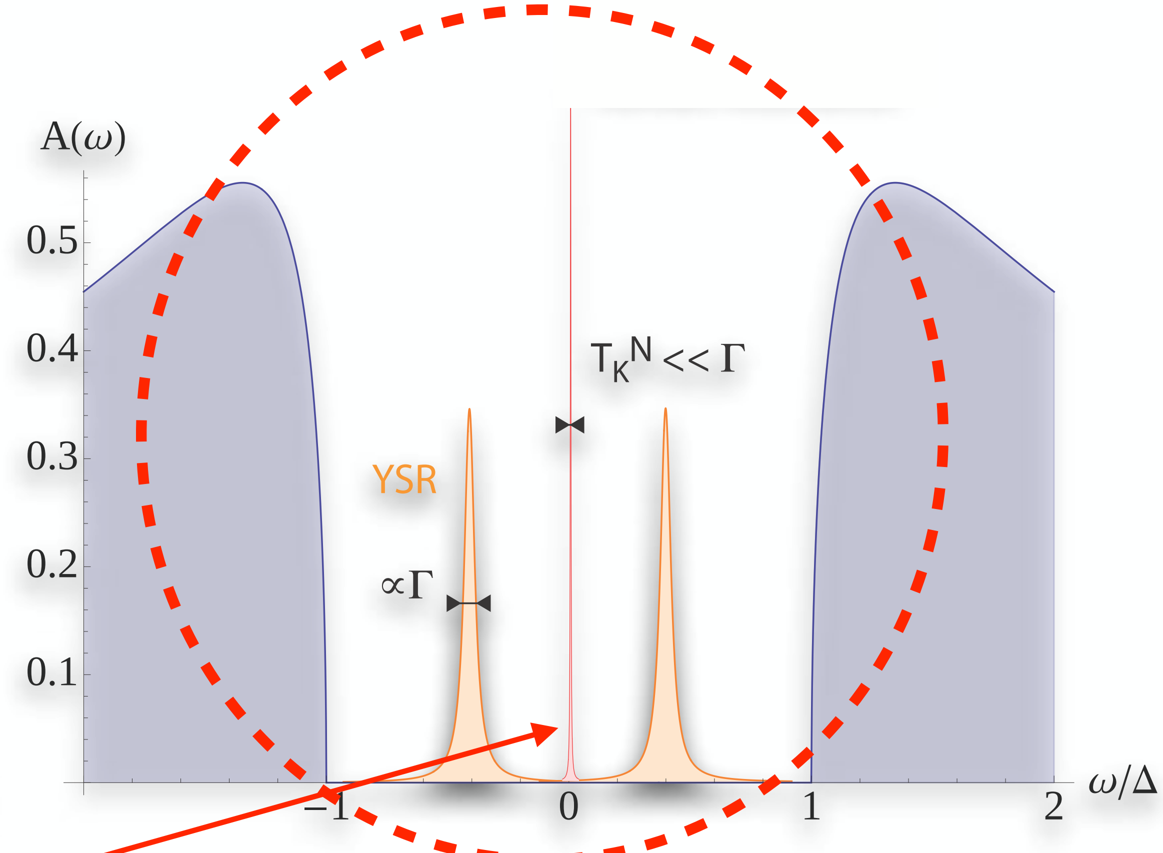
Kondo peak coexists with Superconductivity: Lee et al, PRL, 109,186802 (2012)

- **Zitko, Lim, López and Aguado, PRB, 91, 045441 (2015).**

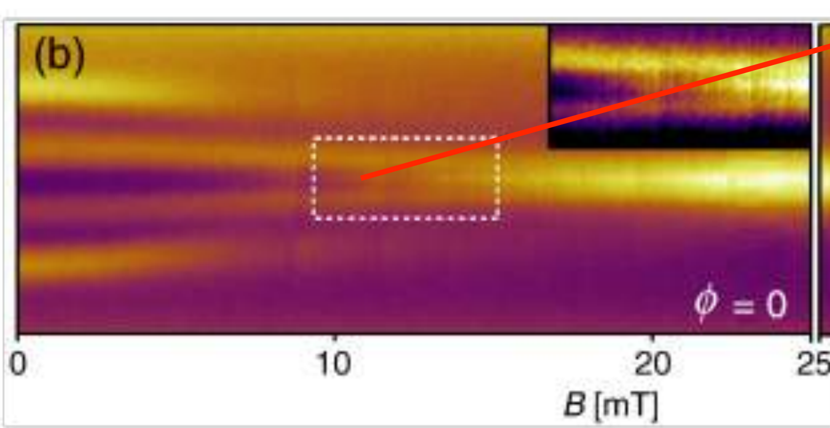
Effect of normal-state tunneling probe on the density of states

$$\Gamma_N \neq 0$$

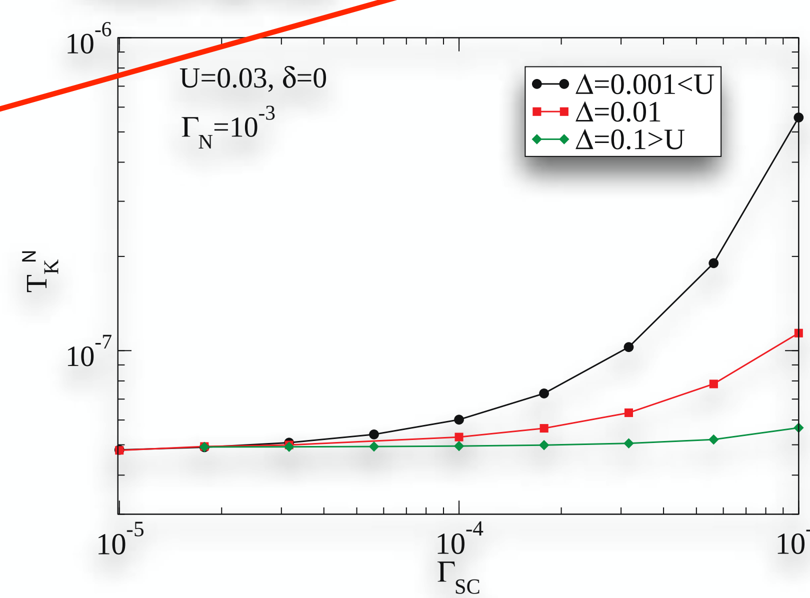




"Needle" Kondo resonance



Charles Marcus' lab PRL, 110, 217005, 2013



Schrieffer-Wolff: coupling to S may enhance this Kondo coupling! (confirmed by NRG)

Shiba versus Kondo, doublets versus singlets, and all this...still an open problem

This competition of different physical regimes and competing effects is very relevant for current experimental efforts towards detecting Majorana fermions in hybrid semiconductor/superconductor systems (Next Talk).

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