Emergence of Quantum Phases in Novel Materials

VII Edition ICMM-CSIC Postgraduate Course

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CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

BIBLIOGRAPHY

"Introduction to Many-body physics" Piers Coleman

"Introduction to superconductivity" Tinkham

SPC history:

"Superconductivity: a very short introduction" S. Blundell

Collection of reviews

- Conventional SPC "Superconductivity" Edited by Parks. 1968.
- Conventional and unconventional SPC "Superconductivity" 2008 (Fe SPC not included)

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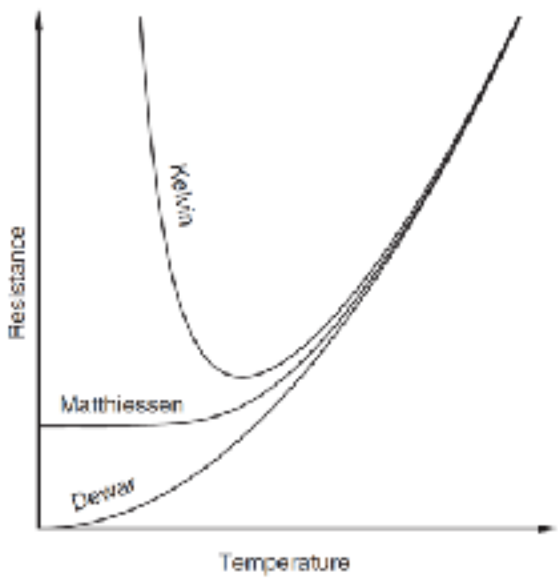
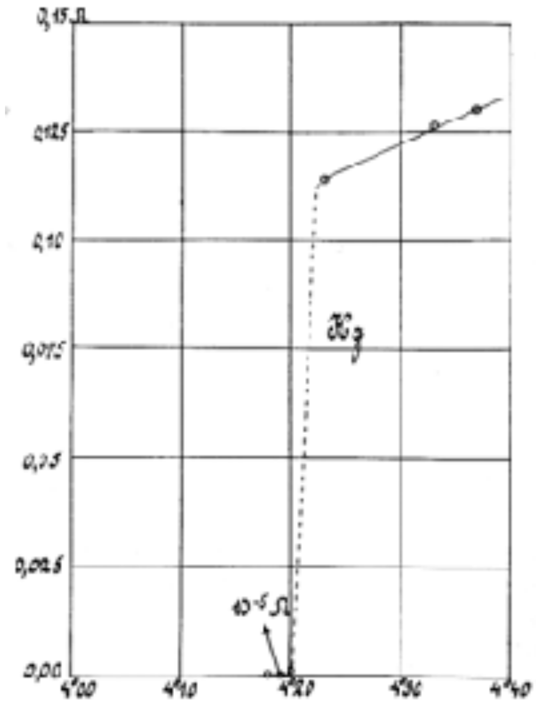
THE DISCOVERY (1911)

Resistivity in metals at low T at the turn of the 20th century





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Heike Kamerlingh Onnes liquefied He:
1st person to measure low T resistivity

S. Blundell “Superconductivity: a very short introduction”

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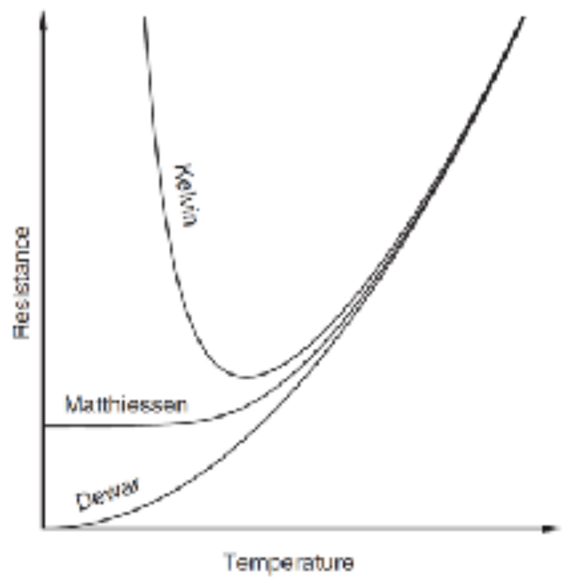
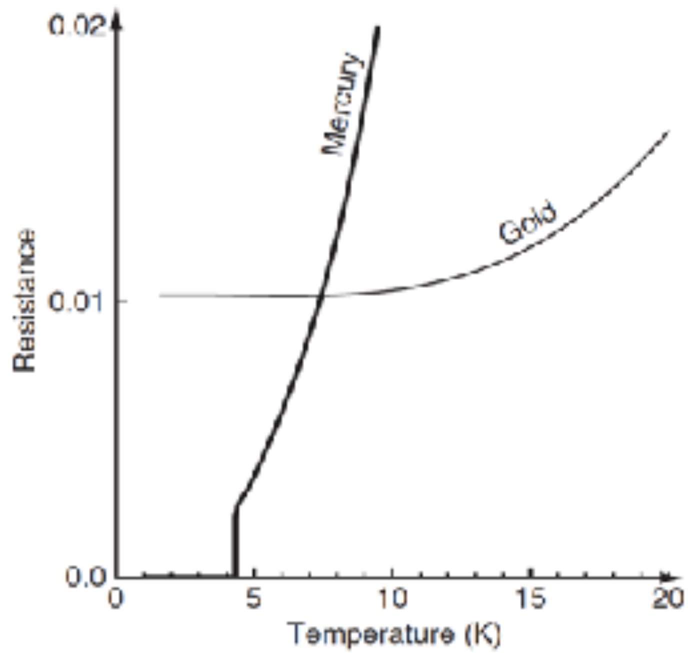
Resistivity in metals at low T at the turn of the 20th century





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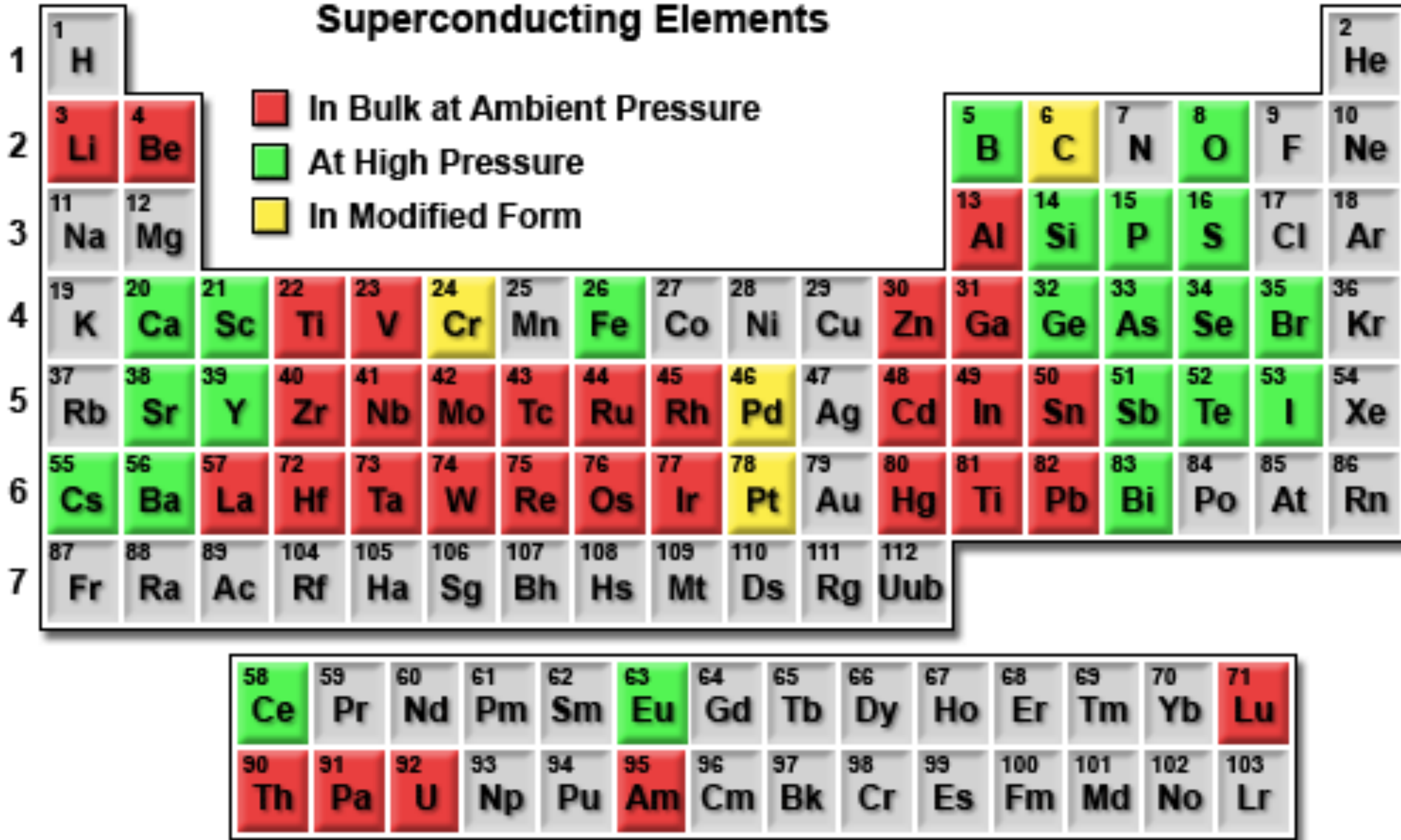

http://www.physics.buffalo.edu/

And many compounds!

https://www.webelements.com/periodicity/superconductivity_temp/

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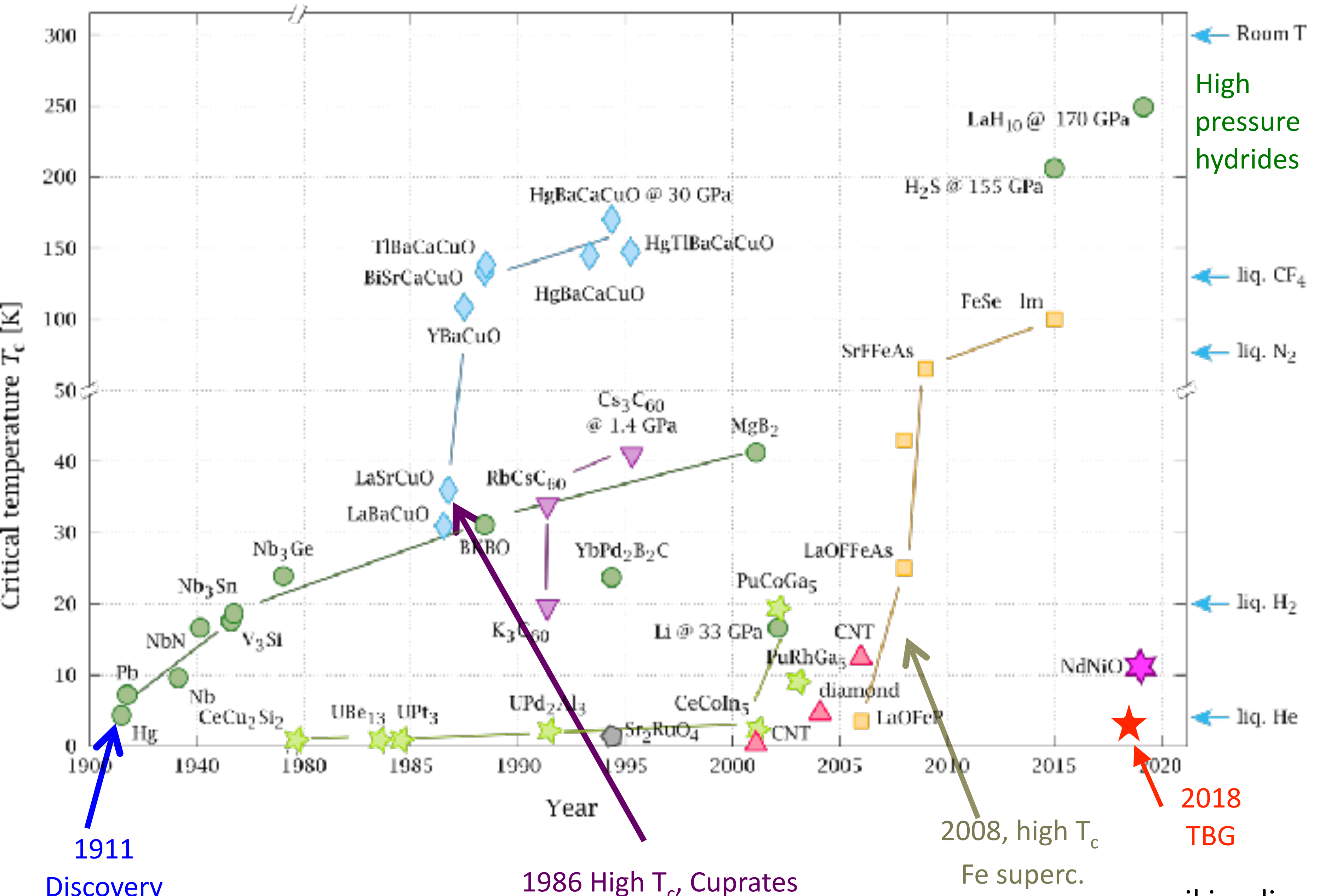

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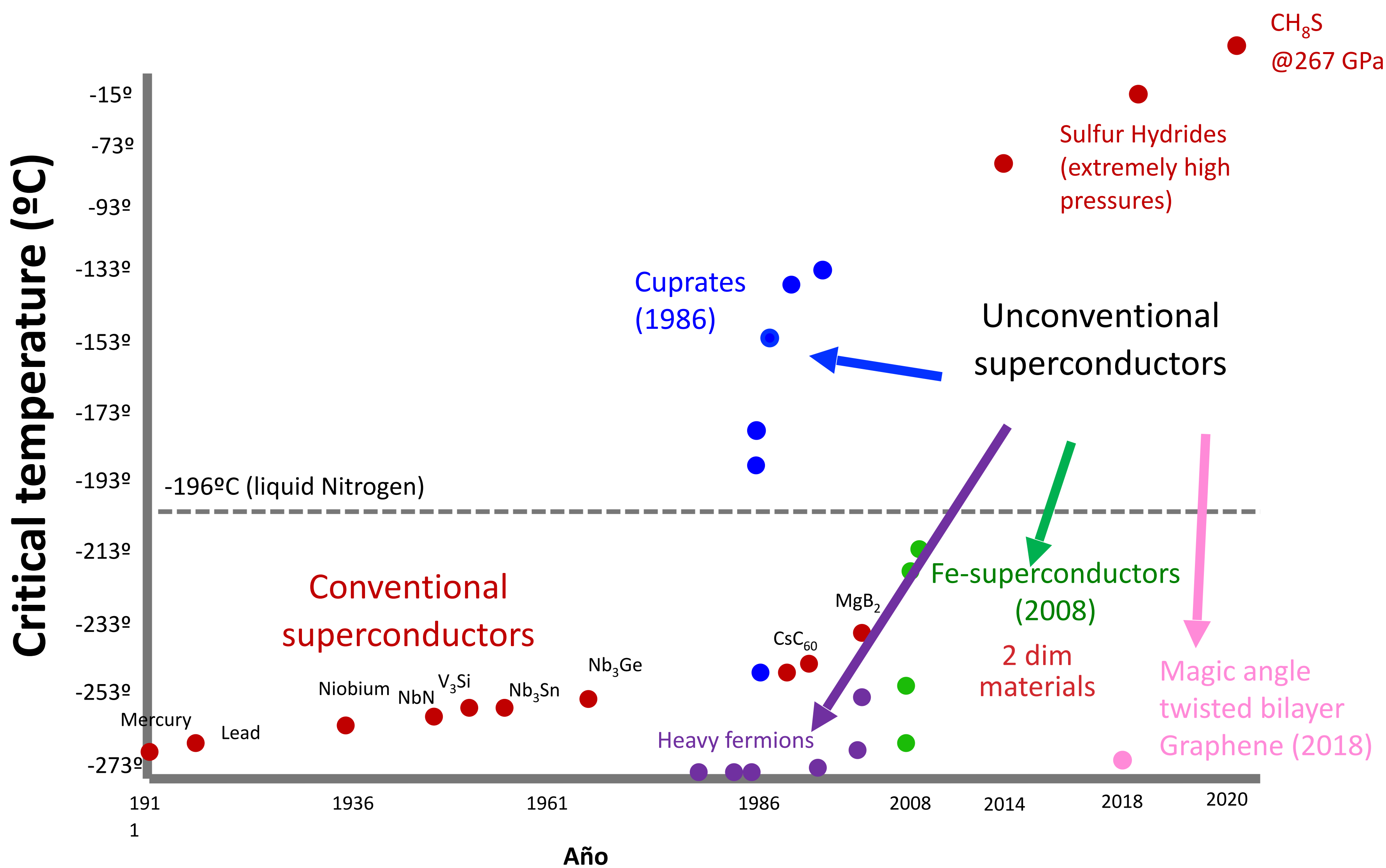

Critical temperature (ºC)
-15º
-73º
-93º
-133º
-153º
-173º
-193º
-213º
-233º
-253º
-273º
-196ºC (liquid Nitrogen)
CH_8S
@267 GPa
Sulfur Hydrides
(extremely high
pressures)
Cuprates
(1986)
Unconventional
superconductors
Conventional
superconductors
Fe-superconductors
(2008)
2 dim
materials
Magic angle
twisted bilayer
Graphene (2018)
Mercury
Lead
Niobium
NbN
V_3Si
Nb_3Sn
Nb_3Ge
CsC_{60}
MgB_2
Heavy fermions
1911
1936
1961
1986
2008
2014
2018
2020
Año

OUTLINE

- Superconductivity I
  - Properties (zero resistivity, Meissner effect)
  - Understanding (pairing, BCS, Ginzburg-Landau)
  - Electron-phonon interaction (conventional superconductivity)
- Superconductivity II (unconventional superconductivity)
  - What are the new issues.
  - What are (some of) the proposals.

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A BIT OF (THE EARLY) HISTORY

1911: Discovery

1933: Meissner effect

1935: London Equations

1937: Superfluidity in ^{4}He

1950: Isotope effect

Electron-electron interaction mediated by phonons

Ginzburg-Landau

1953: Pippard’s coherence length

1955: Idea of gap (Bardeen)

Bardeen-Pines effective interaction

1956: Cooper pairs

Abrikosov extension of Ginzburg-Landau (vortices)

1957: BCS

1959: Gor’kov derived Ginzburg-Landau theory from BCS

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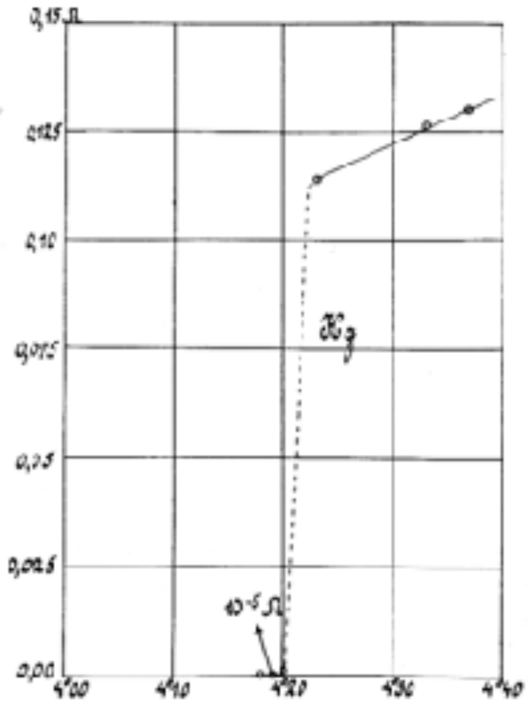
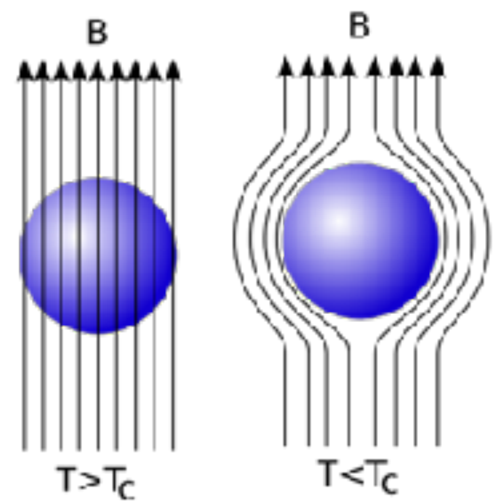
SUPERCONDUCTORS ARE NOT PERFECT CONDUCTORS



http://commons.wikimedia.org/

New lead: The Meissner-Ochsenfeld effect (1933): SUPERCONDUCTORS ARE PERFECT DIAMAGNETS



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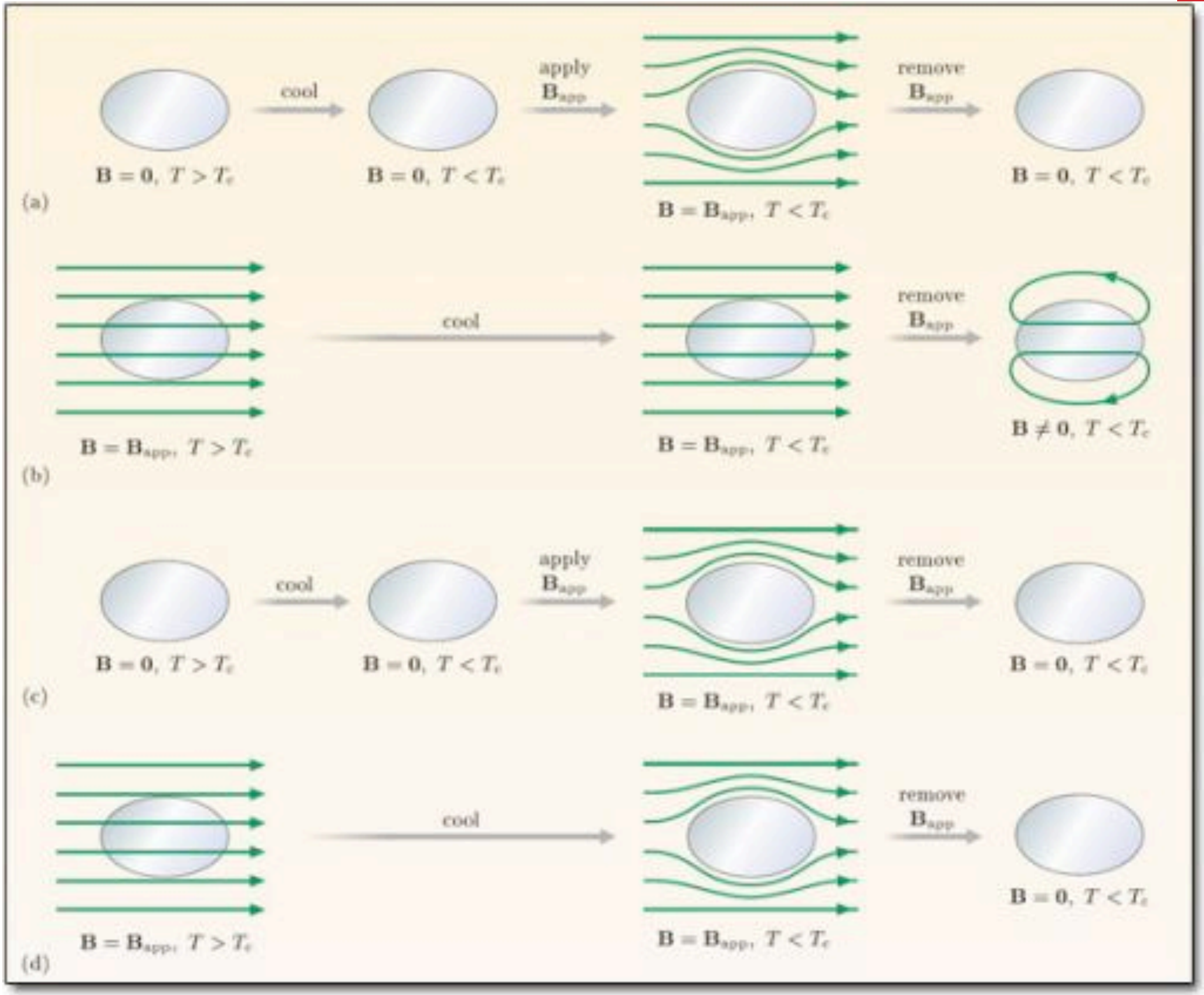
Perfect conductor

Superconductor

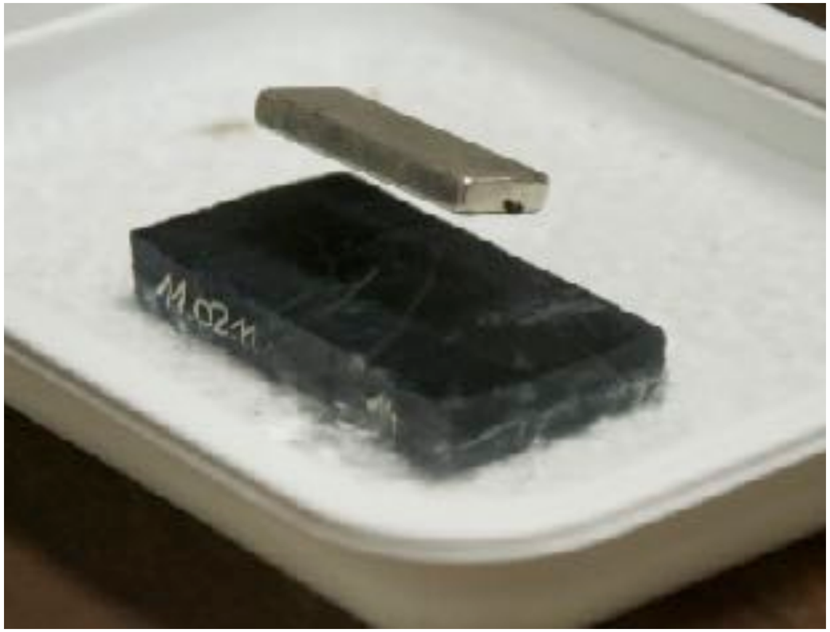


www.open.edu: Superconductivity

THE MEISSNER-OCHSENFELD EFFECT (1933)

Superconductors expel

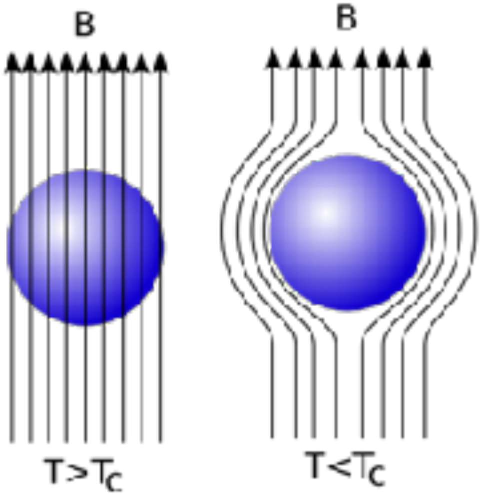
magnetic fields



www.icmm.csic.es/superconductividad

Note: this picture corresponds to the mixed state (see later)

Superconductors produce supercurrents that completely screen out the magnetic field.



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LONDON EQUATIONS (1935)

(phenomenological approach)

Ohm’s law

(a relation between current and electric field)

$\Big\downarrow$ London equations

A new relation

between electric current and magnetic field

(inspiration: Meissner effect)

LONDON EQUATIONS (1935)

(phenomenological approach)

Superconductivity is a macroscopic quantum phenomenon

Long range order of the momentum vector:

A condensation in momentum space
(= zero net momentum)
rather than in real space

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LONDON EQUATIONS (1935)

Without applied magnetic field, the ground state has zero net momentum (condensation in momentum space). If <p>=0 also under an applied field (the wave-function is “rigid”):

$$\vec{p} = m\vec{v} + e\vec{A}/c$$

$$\Big\downarrow \langle p \rangle = 0$$

$$\vec{J}_s = n_s e \langle \vec{v}_s \rangle = \frac{-n_s e^2 \vec{A}}{mc} = \frac{-\vec{A}}{\Lambda c}$$

$$\Lambda = \frac{m}{n_s e^2}$$

n_s is the density of superconducting electrons

Tinkham’s book, Coleman’s book

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LONDON EQUATIONS

Tinkham's book, Coleman's book

$$\vec{J}_s = \frac{-\vec{A}}{\Lambda c}$$

Time derivative

Curl

PERFECT CONDUCTIVITY: an electric field accelerates the electrons (in Ohm's law, an electric field sustains their velocity against resistance).

$$\boxed{\vec{E} = \frac{\partial}{\partial t}\left(\Lambda \vec{J}_s\right)}$$

SUPERCURRENT SCREENS MAGNETIC FIELD

$$\boxed{\vec{h} = -c\nabla \times \left(\Lambda \vec{J}_s\right)}$$

$$\Lambda = \frac{m}{n_s e^2}$$

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LONDON EQUATIONS

Tinkham's book, Coleman's book

Superconducting electrons do not suffer any scattering. From the Drude model for electrical conductivity

$$m\frac{d\vec{v}}{dt} = e\vec{E} - \cancel{\frac{m\vec{v}}{\tau}}$$

Accelerative supercurrent

$$\boxed{\vec{E} = \frac{\partial}{\partial t}\left(\Lambda \vec{J}_s\right)}$$

LONDON EQUATIONS

Tinkham's book, Coleman's book

SUPERCURRENT SCREENS MAGNETIC FIELD

$$\vec{h} = -c\nabla \times \left(\Lambda \vec{J}_s\right)$$

+MAXWELL EQUATION

$$\nabla \times \vec{h} = 4\pi \vec{J}_s / c$$

$$\nabla^2 \vec{h} = \frac{\vec{h}}{\lambda^2} \longrightarrow \vec{h}(r) = \vec{h}(0) e^{-r/\lambda}$$

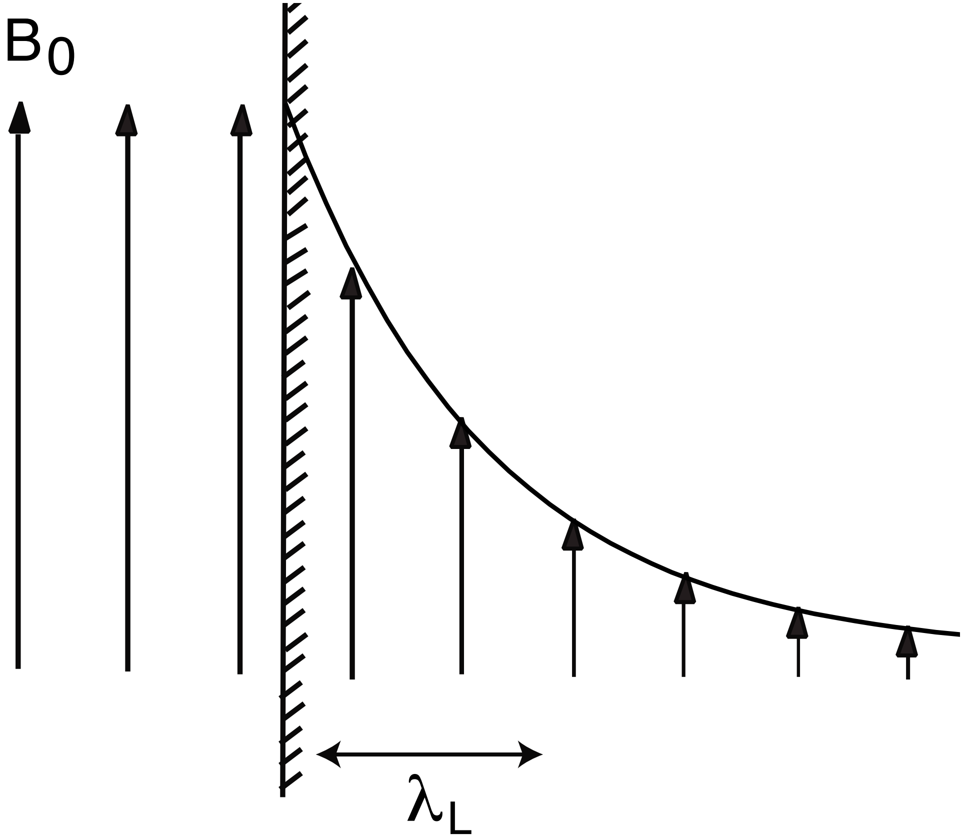
$$\Lambda = \frac{m}{n_s e^2}$$

$$\lambda^2 = mc^2 / 4\pi n_s e^2$$

$$\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} = -\nabla^2 \vec{B}$$

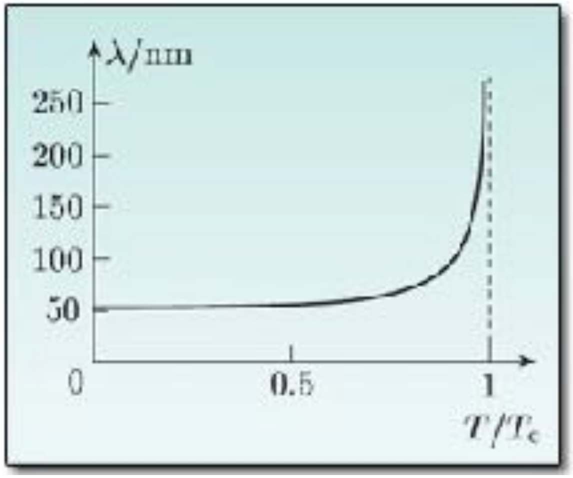
LONDON EQUATIONS

$$\vec{h}(r) = \vec{h}(0)e^{-r/\lambda}$$



LONDON PENETRATION LENGTH

$$\lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}$$



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PIPPARD'S COHERENCE LENGTH

From the uncertainty principle:

Only e^- within $k_B T_c$ of E_F participate in superconductivity

$$\Delta p \approx \frac{k_B T_c}{v_F} \qquad \Delta x \gtrsim \frac{\hbar}{\Delta p} \approx \frac{\hbar v_F}{k_B T_c} \sim \xi_0$$

From a non-local generalization of the London equations

ξ_0: wave-function size

$$\xi_0 = 0.15 \frac{\hbar v_F}{k_B T_c}$$

2 characteristic length scales: penetration and coherence lengths

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^{4}HE SUPERFLUIDITY (1937)

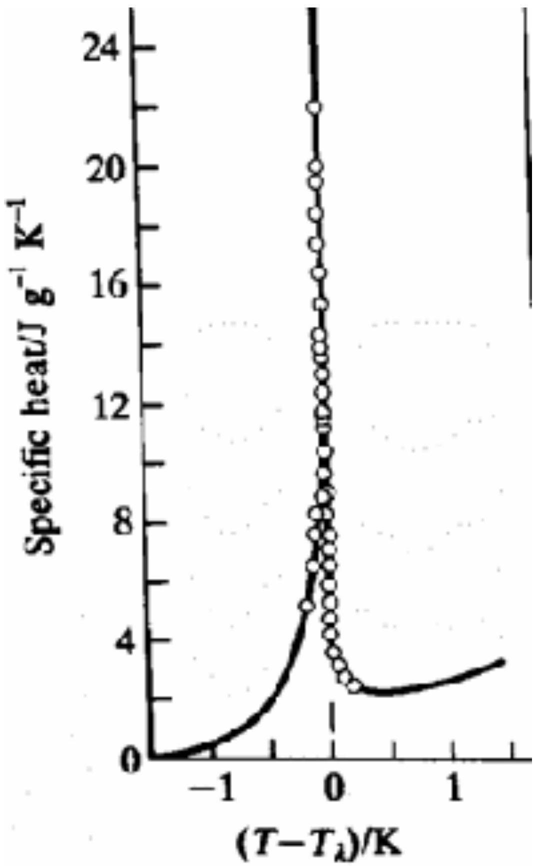
^{4}He liquifies at 4.2K

(He I: normal quantum liquid)

At 2.17K, transition to superfluid (He II).

λ transition.

Superfluidity $\rightarrow$Viscosity=0.



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^{4}HE SUPERFLUIDITY (1937)

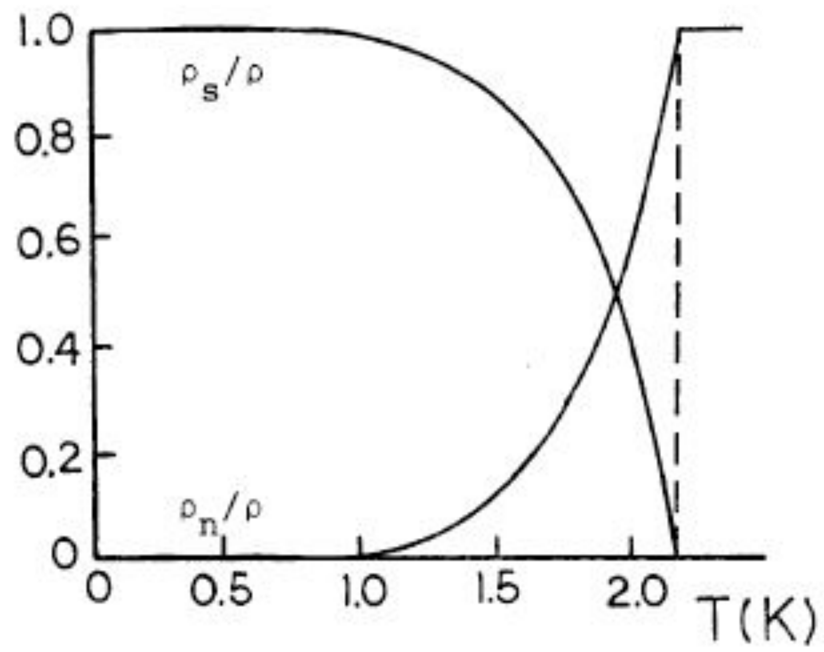
http://www.qm.phy.cam.ac.uk/teaching/

2 fluid model

- normal fluid
- superfluid

Relation to Bose Einstein condensation:

In BEC: no interactions.
In superfluid there are interactions (superfluid density is reduced)



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SUPERFLUIDITY & SUPERCONDUCTIVITY

A superconductor is a charged superfluid.

Note: What is condensing in the superconductor??
Electrons are fermions, not bosons!!
Before 1957, a microscopic theory was still missing.

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LANDAU TRANSITION THEORY U(1)

Coleman's book

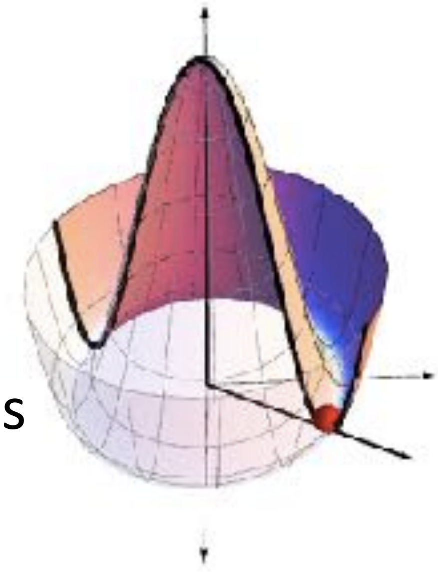
Macroscopic theory in terms of order parameters. Valid close to the critical point. The order parameter description is valid for lengths larger than the coherence length.

$$f[\Psi] = r(\Psi^*\Psi) + \frac{u}{2}(\Psi^*\Psi)^2$$

$$\Psi \equiv \Psi_1 + i\Psi_2 \equiv |\Psi| e^{i\phi}$$

For $T<T_c$

$$\Psi = \sqrt{\frac{|r|}{u}} e^{i\phi}$$



A well-defined phase breaks the U(1) continuous symmetry (spontaneous symmetry breaking)

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GINZBURG-LANDAU

(allows for inhomogeneous order parameters)

Coleman's book

New gradient terms. For a superfluid (no charge):

$$f_{GL}[\Psi, \nabla\Psi] = \frac{\hbar^2}{2m} |\nabla\Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4$$

London's phenomenological wavefunction is the complex order parameter.

$$\Psi(x) = |\Psi(x)| e^{i\phi(x)}$$

$$n_s = |\Psi|^2$$

GINZBURG-LANDAU

(allows for inhomogeneous order parameters)

Coleman's book

New gradient terms. For a superfluid (no charge):

$$f_{GL}[\Psi,\nabla\Psi]=\frac{\hbar^2}{2m}|\nabla\Psi|^2+r|\Psi|^2+\frac{u}{2}|\Psi|^4$$

The rigidity of the order parameter is in the gradient term

$$\nabla\Psi=(\nabla|\Psi|+i|\Psi|\nabla\phi)\,e^{i\phi}$$

$$f_{GL}=\frac{\hbar^2}{2m}|\Psi|^2(\nabla\phi)^2+[\frac{\hbar^2}{2m}(\nabla|\Psi|)^2+r|\Psi|^2+\frac{u}{2}|\Psi|^4]$$

Phase stiffness

GINZBURG-LANDAU
(allows for inhomogeneous order parameters)

Coleman's book

New gradient terms. For a superfluid (no charge):

$$f_{GL}[\Psi, \nabla\Psi] = \frac{\hbar^2}{2m}|\nabla\Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4$$

Length scale related to the gradient term: **correlation length** (length scale of the amplitude fluctuations).

At T=0, it is the Pippard's coherence length.

$$\xi(T) = \xi(0)\left|1 - \frac{T}{T_c}\right|^{-1/2} ; \xi(0) = \sqrt{\frac{\hbar^2}{2maT_c}}$$

GINZBURG-LANDAU

Phase stiffness

$$f_{GL} = \frac{\rho_\phi}{2}(\nabla\phi)^2 + \text{constant}$$

Amplitude fluctuations are confined to scales shorter than ξ. For longer lengths those terms are constant.

Calculate the current

$$\vec{J} = -i\frac{\hbar}{2m}\left(\psi^* \overrightarrow{\nabla}\psi - \overrightarrow{\nabla}\psi^*\psi\right)$$

Using:

$$\psi(x) = \sqrt{n_s(x)}e^{i\phi(x)}$$

$$\vec{J}_s = n_s\frac{\hbar}{m}\nabla\phi$$

Superfluid velocity

GINZBURG LANDAU (now with charge)

$$F[\Psi,\mathbf{A}]=\int d^d x\left[\frac{\hbar^2}{2M}|(\nabla-\frac{ie^*}{\hbar}\mathbf{A})\Psi|^2+r|\Psi|^2+\frac{u}{2}|\Psi|^4+\frac{(\nabla\times\mathbf{A})^2}{2\mu_0}\right]$$

The free energy is gauge invariant (any change in the phase of the order parameter can be compensated by a gauge transformation)

$$\psi(x)\to\psi(x)e^{i\alpha(x)} \qquad \mathbf{A}\to\mathbf{A}+\frac{\hbar}{e^*}\nabla\alpha$$

The electromagnetic part of the Free energy has an associated length: the London penetration length λ.

(Note e*. e*=2e only after BCS theory)

GINZBURG LANDAU (now with charge)

$$\psi(x) \rightarrow \psi(x)e^{i\alpha(x)} \qquad \mathbf{A} \rightarrow \mathbf{A} + \frac{\hbar}{e^*}\nabla\alpha$$

By absorbing the phase of the order parameter, the phase stiffness appears as a mass term (gapped excitation).

ANDERSON-HIGGS MECHANISM

(see section 11.6.2 of Coleman's book for a demonstration)

(in a neutral superfluid you have Goldstone (massless) modes)

GINZBURG LANDAU (now with charge)

$$F[\Psi, \mathbf{A}] = \int d^d x \left[\frac{\hbar^2}{2M} |(\nabla - \frac{ie^*}{\hbar}\mathbf{A})\Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4 + \frac{(\nabla \times \mathbf{A})^2}{2\mu_0} \right]$$

Now the current is:

$$\mathbf{J}(x) = \frac{e^*\hbar}{M}|\Psi|^2 \nabla\phi - \frac{e^{*2}}{M}|\psi|^2 \mathbf{A} = e^* n_s \frac{\hbar}{M}\left(\vec{\nabla}\phi - \frac{e^*}{\hbar}\mathbf{A}\right) = e^* n_s \mathbf{v}_s$$

An external vector potential produces superflow

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D.C. JOSEPHSON EFFECT (1962)

A zero voltage supercurrent should flow between two superconducting electrodes connected by a "weak link".

$$I_s = I_c \sin \Delta\phi$$

Critical current at the junction

Difference in the phase of the Ginzburg-Landau wave-functions

A.C. JOSEPHSON EFFECT

An applied voltage leads to an oscillatory current

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GINZBURG LANDAU (now with charge)

$$F[\Psi, \mathbf{A}] = \int d^d x \left[\frac{\hbar^2}{2M} |(\nabla - \frac{ie^*}{\hbar}\mathbf{A})\Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4 + \frac{(\nabla \times \mathbf{A})^2}{2\mu_0} \right]$$

The length scale for A: London penetration length λ.

The length scale for $(\nabla\psi)^2$ is the coherence length ξ.

Ginzburg Landau parameter: λ/ξ.

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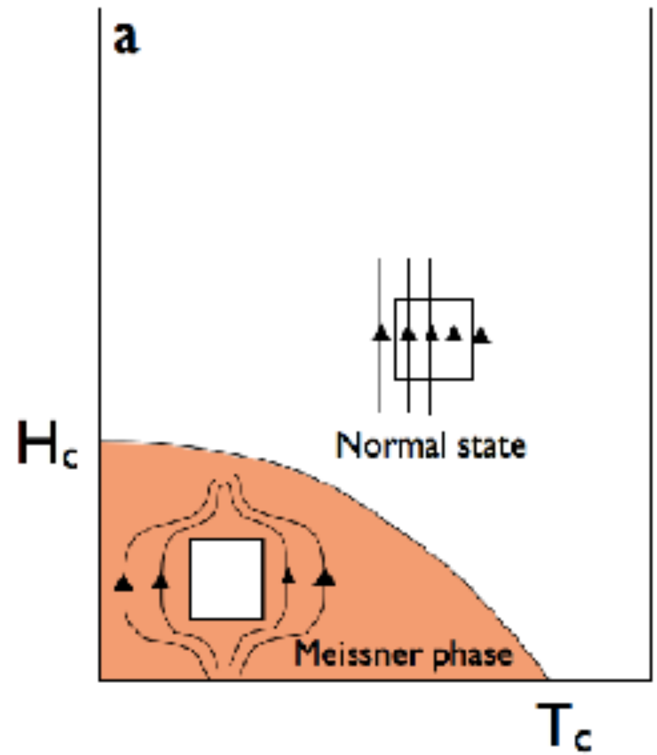
CRITICAL FIELD H_C

There is a maximum value of magnetic field H_c that can be applied before superconductivity is destroyed. There is also a critical current I_c.

First superconductors discovered had a low H_c. This is bad news if you want to build magnets with superconducting cables.

Some superconductors (alloys) didn't exclude the magnetic field completely and had much larger critical fields (1930's).



Coleman's book

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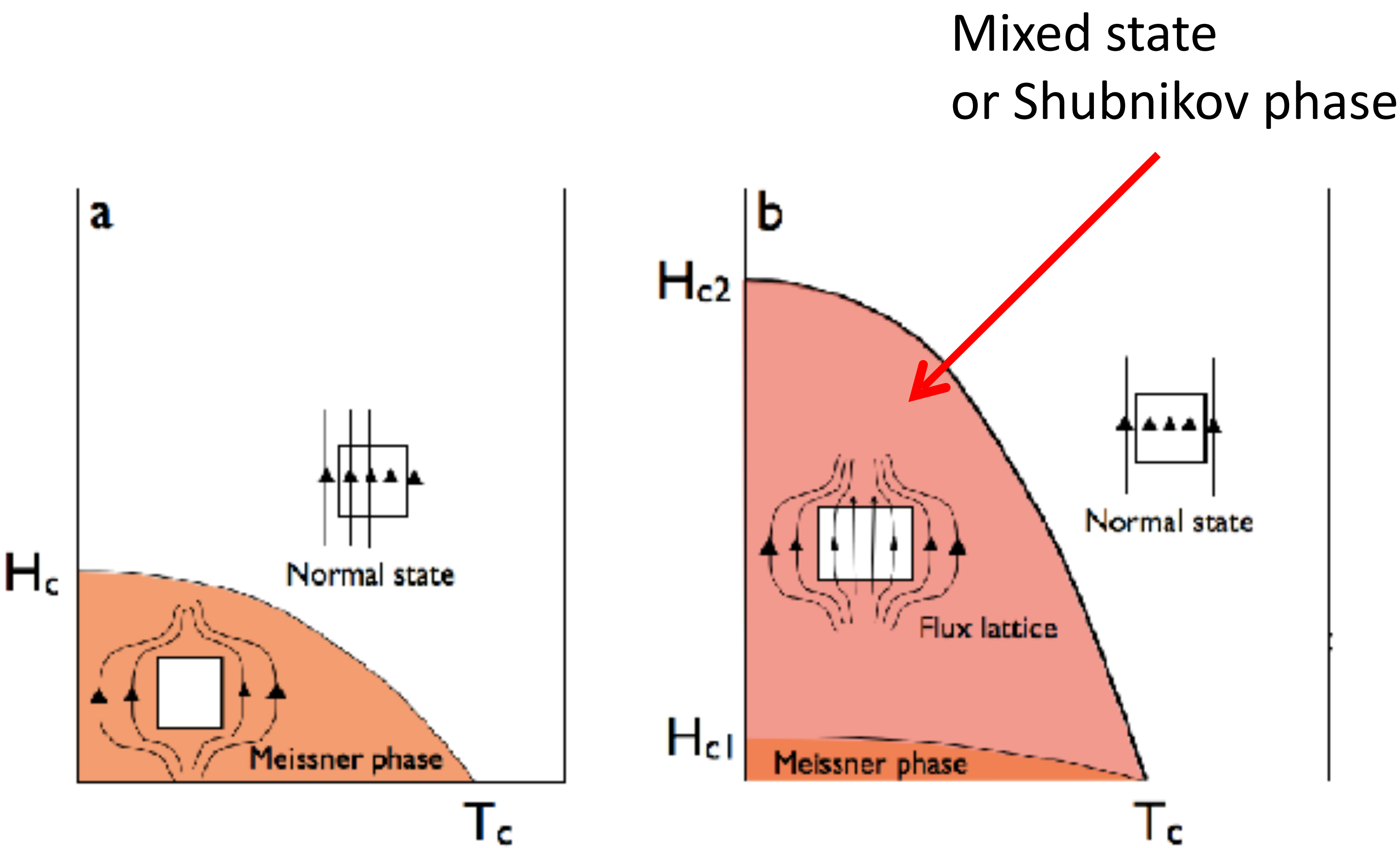
CRITICAL FIELD H_C



Coleman's book

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TYPE I/TYPE II SUPERCONDUCTORS

The first superconductors were all Type I (λ<<ξ).

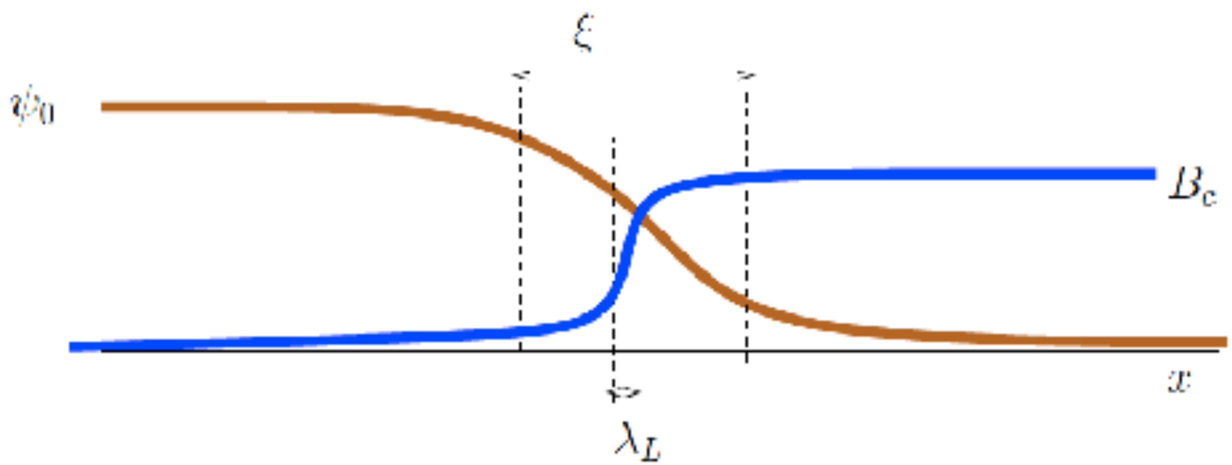
Abrikosov explored the opposite possibility.

London penetration length

$$\lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}$$

Correlation length

$$\xi(T) = \xi(0)\left|1 - \frac{T}{T_c}\right|^{-1/2};\ \xi(0) = \sqrt{\frac{\hbar^2}{2maT_c}}$$



Coleman's book

TYPE I/TYPE II SUPERCONDUCTORS

The first superconductors were all Type I (λ<<ξ).

Abrikosov explored the opposite possibility.

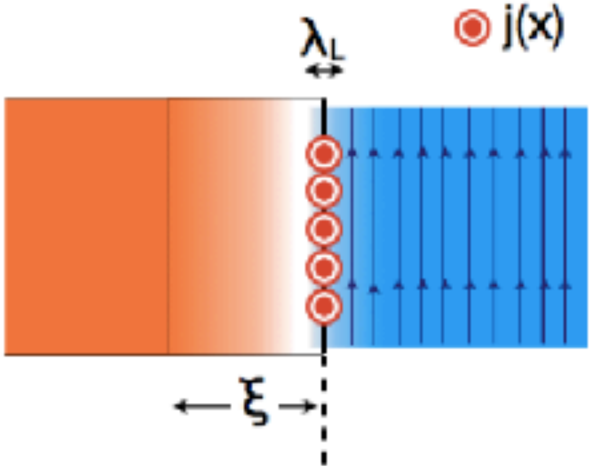
London penetration length

$$\lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}$$

London and Ginzburg-Landau considered λ<ξ.
The supercurrent that screens the applied magnetic field occurs in a thin layer.

The correlation length

$$\xi(T) = \xi(0)\left|1 - \frac{T}{T_c}\right|^{-1/2};\ \xi(0) = \sqrt{\frac{\hbar^2}{2maT_c}}$$



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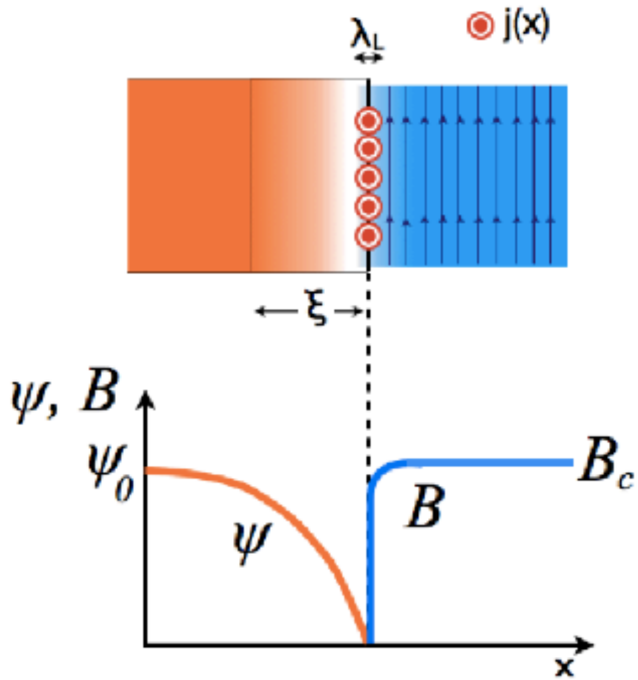
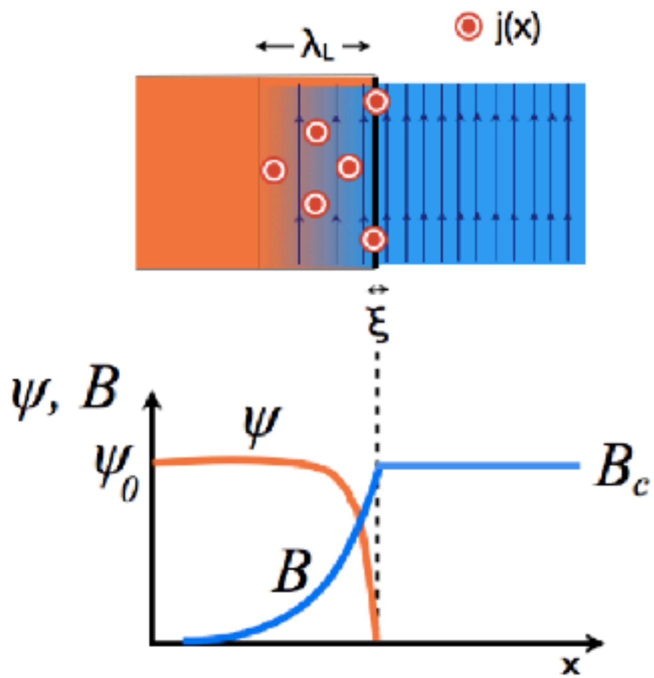
TYPE I/TYPE II SUPERCONDUCTORS

Type I





Type II

Coleman's book

Modifying the wave-function costs a lot of energy. Not much energy gain by allowing H in. N/S interfaces are costly. At H_c there is a first order transition into the normal state.

Modifying the wave-function doesn't cost too much. You gain energy by allowing the magnetic field in. N/S interfaces are favoured. Two critical fields. **Vortices**.

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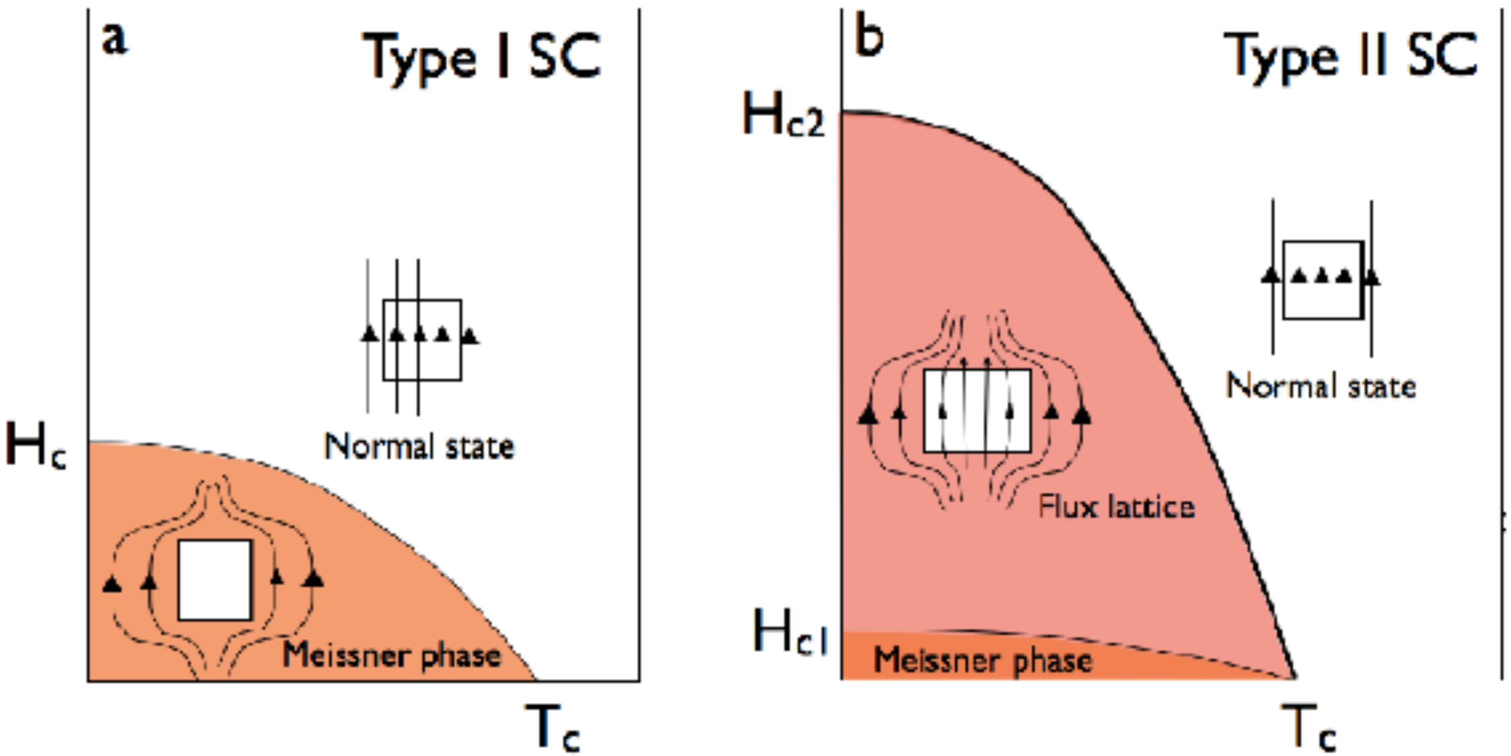
CRITICAL FIELD H_C

$$\frac{\lambda_L}{\xi} < \frac{1}{\sqrt{2}} \qquad \frac{\lambda_L}{\xi} > \frac{1}{\sqrt{2}}$$



$$B_{c2} = \frac{\phi_0}{2\pi\xi^2} \qquad B_{c1} = \frac{\phi_0}{2\pi\lambda^2} \qquad B_{c1}B_{c2} = B_c^2$$

Coleman's book

SOME EXAMPLES

http://www.qm.phy.cam.ac.uk/teaching/

Compound	T_c [K]	ξ [nm]	λ [nm]	$\kappa = \lambda/\xi$	
Al	1.18	1600	50	0.03	type-I
Pb	7.19	83	39	0.47	type-I
Nb	9.25	40	44	1.1	border
Nb_3Sn	18.2	3.6	124	34	type-II
$YBa_2Cu_3O_{7-\delta}$	90	(1.5)	(130)	(87)	type-II

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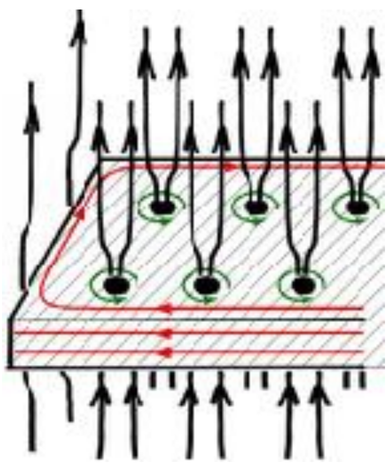
VORTICES

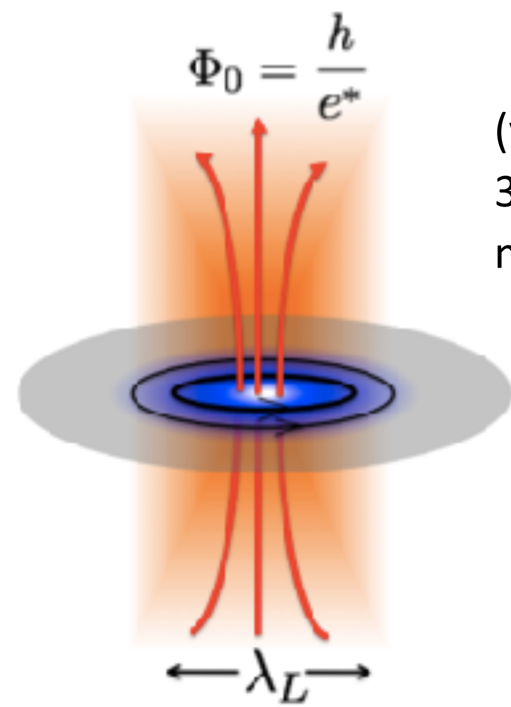
In type II superconductors, part of the field can go through the superconductor. A supercurrent circulates around the magnetic flux to screen it: Vortices.



http://www.supraconductivite.fr/

Superconductivity is suppressed at the vortex core

The flux is quantized



(vortex in 3dim XY model)

Coleman's book

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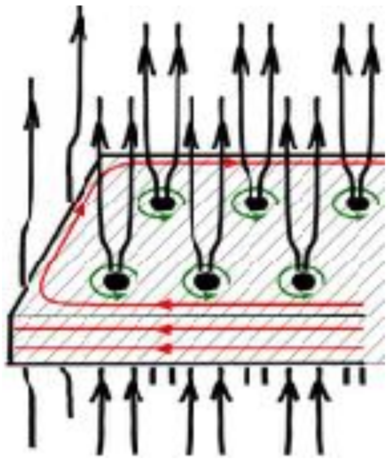
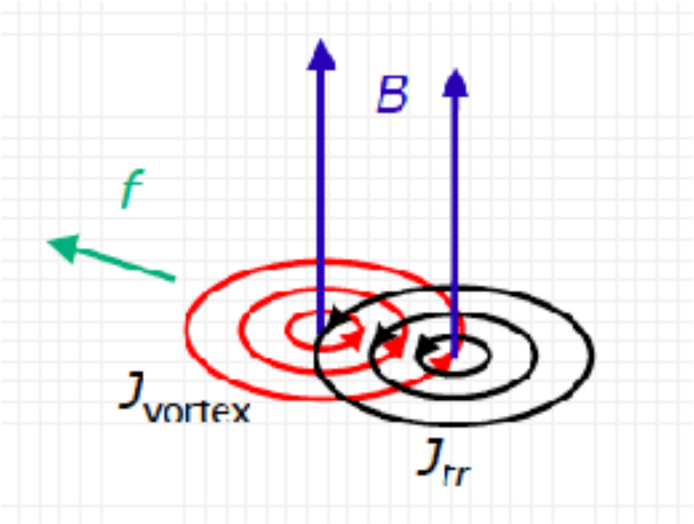
VORTICES

In type II superconductors, part of the field can go through the superconductor. A supercurrent circulates around the magnetic flux to screen it: Vortices.



http://www.supraconductivite.fr/



Coleman's book

Vortices repel each other
(they have the same topological charge)

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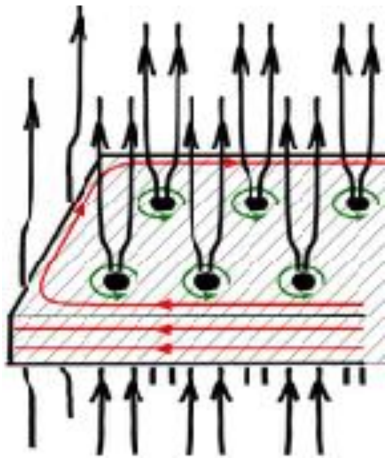
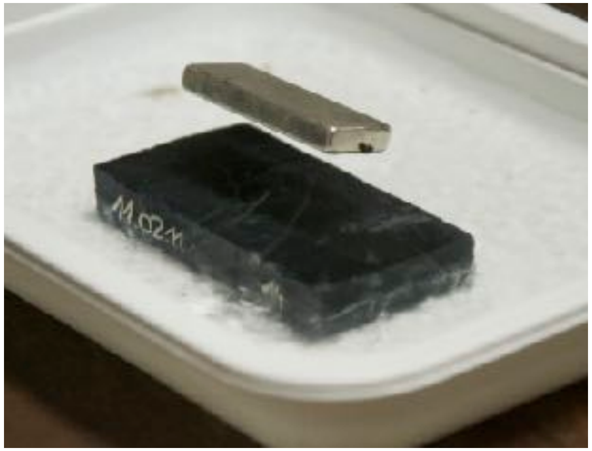
VORTICES

In type II superconductors, part of the field can go through the superconductor. A supercurrent circulates around the magnetic flux to screen it: Vortices.



http://www.supraconductivite.fr/

Pinning!!



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PINNING

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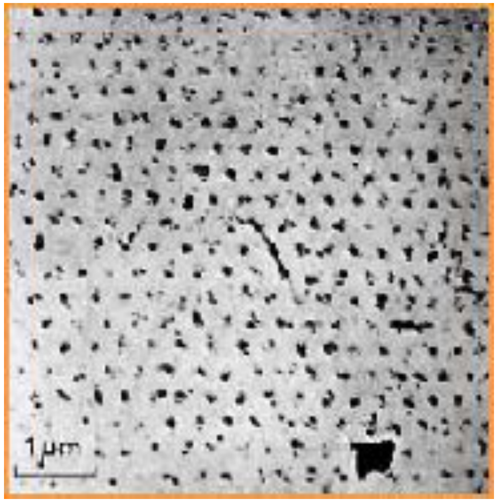
wp.icmm.csic.es/superconductividad/videos

VORTICES

In type II superconductors, part of the field can go through the superconductor. A supercurrent circulates around the magnetic flux to screen it: Vortices.
From London eqs: the magnetic flux is quantized. Vortices order in a regular array.

First observation of an array of vortices



Physics Letters 24A, 526 (1967)

Supercurrent exert forces on vortices leading to dissipation. Impurities pin the vortices so they cannot move.

TOWARDS A MICROSCOPIC MODEL

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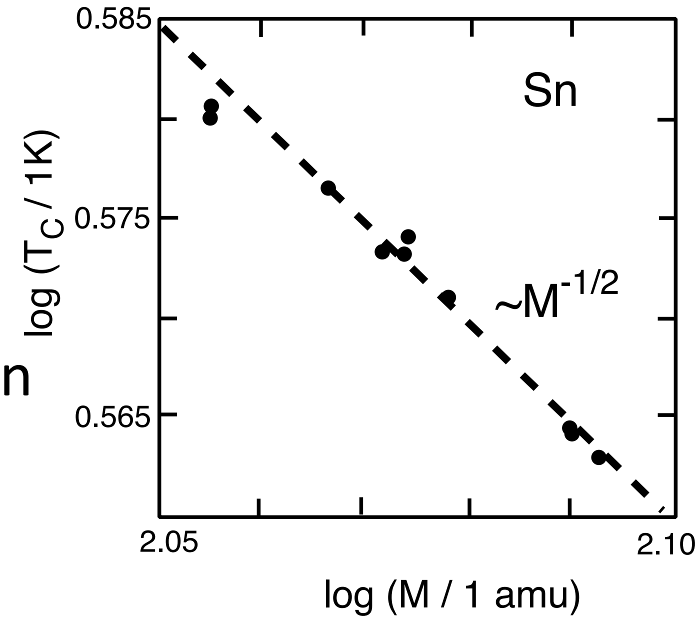
ISOTOPE EFFECT

(C. Reynolds et al, E. Maxwell)

Evidence that superconductivity was related to electron-phonon interaction



$$T_c \propto \omega_D \approx \frac{1}{\sqrt{M}}$$

Independently, Fröhlich suggested that SPC occurs due to electron-lattice interaction. Also Bardeen, independently, as soon as he knew about the isotope effect.

$$V_{\text{eff}}(\mathbf{k},\mathbf{k}') = -\, g^2_{\mathbf{k}-\mathbf{k}'} \frac{2\omega_{\mathbf{k}-\mathbf{k}'}}{\omega^2_{\mathbf{k}-\mathbf{k}'} - (\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'})^2}$$

Formulation of the electron-phonon interaction. Leads to attractive energy for low-energy transfer.

THE IDEA OF A GAP

Bardeen proposed that a gap in the spectrum would lead to the rigidity of the wave-function. Some experiments were showing a sign of a gap (activated specific heat).

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EFFECTIVE INTERACTION

Bardeen-Pines effective interaction:

screened Coulomb
+
retarded electron-phonon interaction

Bare Coulomb

$$V(r) = e^2/r \rightarrow V_q = 4\pi e^2/q^2$$

Screened Coulomb

$$V(r) = \frac{e^2 e^{-\kappa r}}{\epsilon r} \rightarrow V_q = \frac{4\pi e^2}{\epsilon(q^2+\kappa^2)}$$

Coulomb is always repulsive

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EFFECTIVE INTERACTION

Electron-phonon interaction. Phonon of frequency ω_q.

$$V_q + |M_q|^2 \left(\frac{1}{\hbar\omega - \hbar\omega_q} + \frac{1}{-\hbar\omega - \hbar\omega_q} \right) = V_q + \frac{2|M_q|^2}{\hbar} \frac{\omega_q}{\omega^2 - \omega_q^2}$$

Change in E
for the e-

M_q: electron-phonon matrix element.
If electron-phonon interaction is too weak (good metals), no superconductivity arises

Modification of the dielectric screening by a moving particle in a lattice that can be deformed.

Overscreening: The effective electron-electron interaction can be negative (attractive)

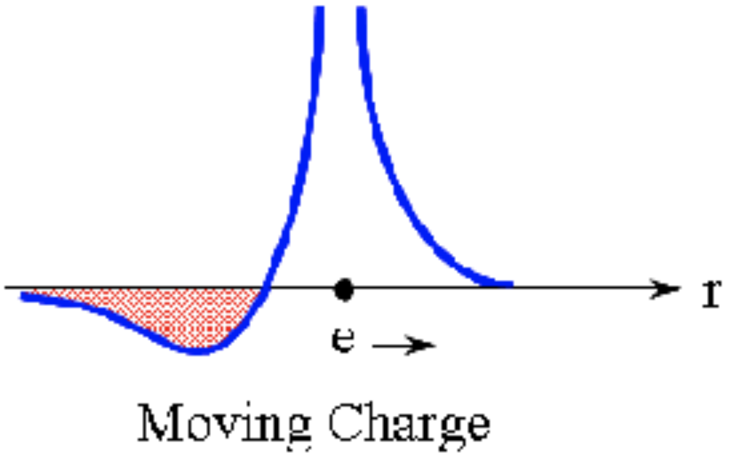
Key issue: the dynamics of the phonons is much slower than the electrons'

EFFECTIVE INTERACTION

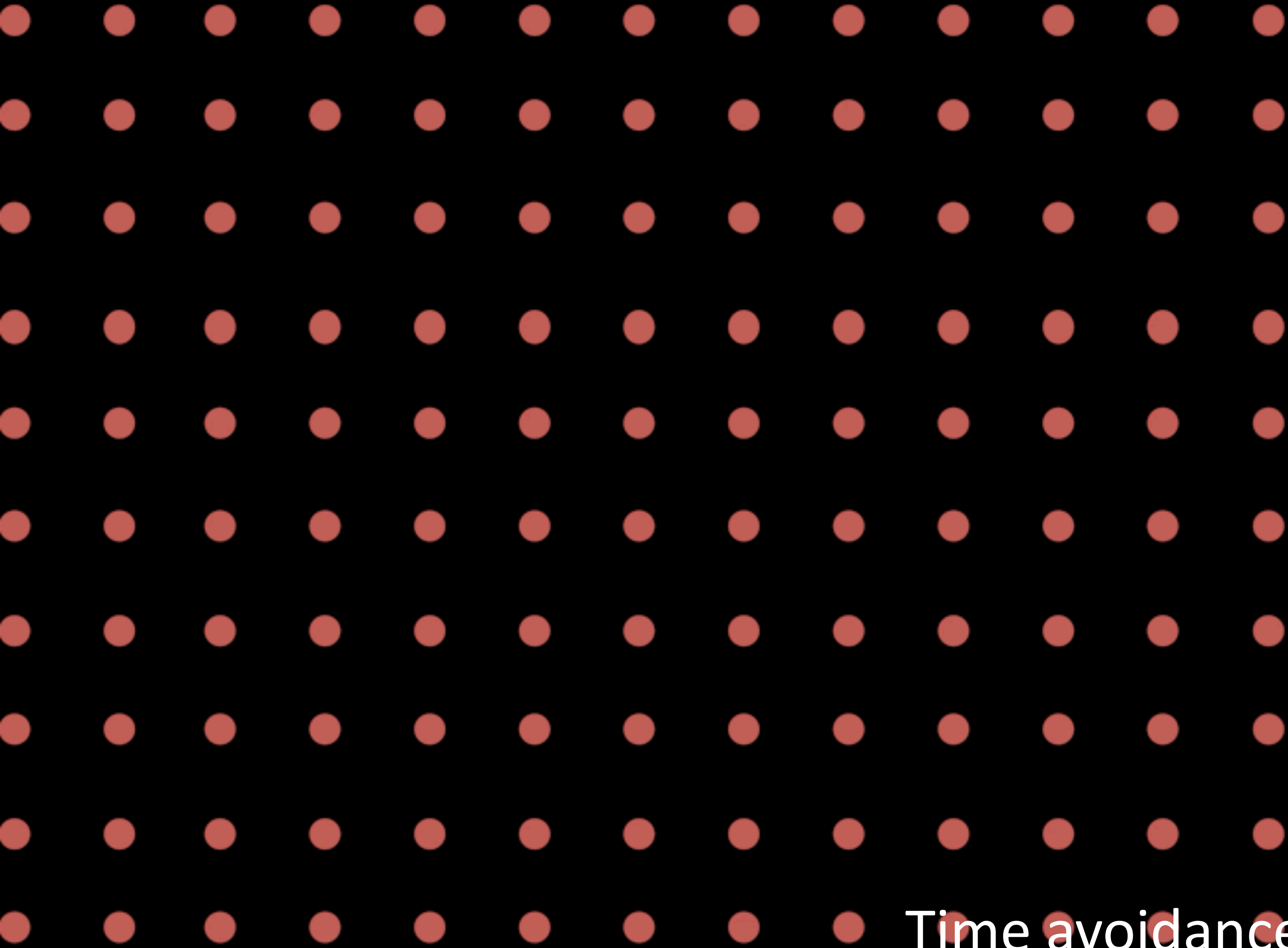
Include electron-phonon interaction (must be important due to isotope effect). Phonon of frequency ω_q.

$$V_q + |M_q|^2 \left(\frac{1}{\hbar\omega - \hbar\omega_q} + \frac{1}{-\hbar\omega - \hbar\omega_q} \right) = V_q + \frac{2|M_q|^2}{\hbar} \frac{\omega_q}{\omega^2 - \omega_q^2}$$

Attraction if $\omega < \omega_q$



http://www.qm.phy.cam.ac.uk/teaching/



Time avoidance

BCS theory (1957)

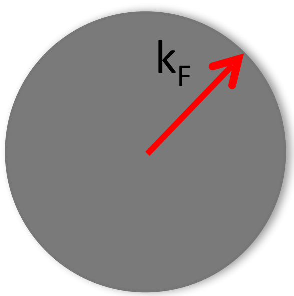
Bardeen-Cooper-Schrieffer

Interaction-pairs-wavefunction

Nobel prize 1972

COOPER PAIRS (1956)

Arbitrarily weak attractions can bind pairs of electrons on the Fermi surface (free particles would need an energy minimum).



Add two electrons on top of the Fermi sea

$$|\Psi\rangle = \Lambda^\dagger |FS\rangle$$

Assume zero momentum and singlet pairing (expected lowest energy)

With pair creation operator

$$\Lambda^\dagger = \sum_{\mathbf{k}} \phi_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}$$

pair wavefunction
(s-wave symmetry for conventional superconductors)

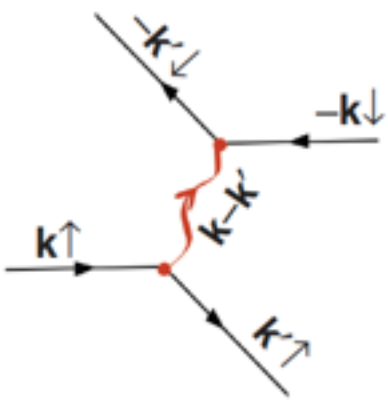
COOPER PAIRS (1956)

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \hat{V}$$

Define: $|\mathbf{k}_P\rangle \equiv c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} |FS\rangle$

$$H|\Psi\rangle = \sum_{|\mathbf{k}|>k_F} 2\epsilon_{\mathbf{k}}\phi_{\mathbf{k}}|\mathbf{k}_p\rangle + \sum_{|\mathbf{k}|,|\mathbf{k}'|>k_F} |\mathbf{k}_p\rangle\langle\mathbf{k}_p|\hat{V}|\mathbf{k}'_p\rangle\phi_{\mathbf{k}'}$$

$$E\phi_{\mathbf{k}} = 2\epsilon_{\mathbf{k}}\phi_{\mathbf{k}} + \sum_{|\mathbf{k}'|>k_F} \langle\mathbf{k}_p|\hat{V}|\mathbf{k}'_p\rangle\phi_{\mathbf{k}'}$$



Coleman's book

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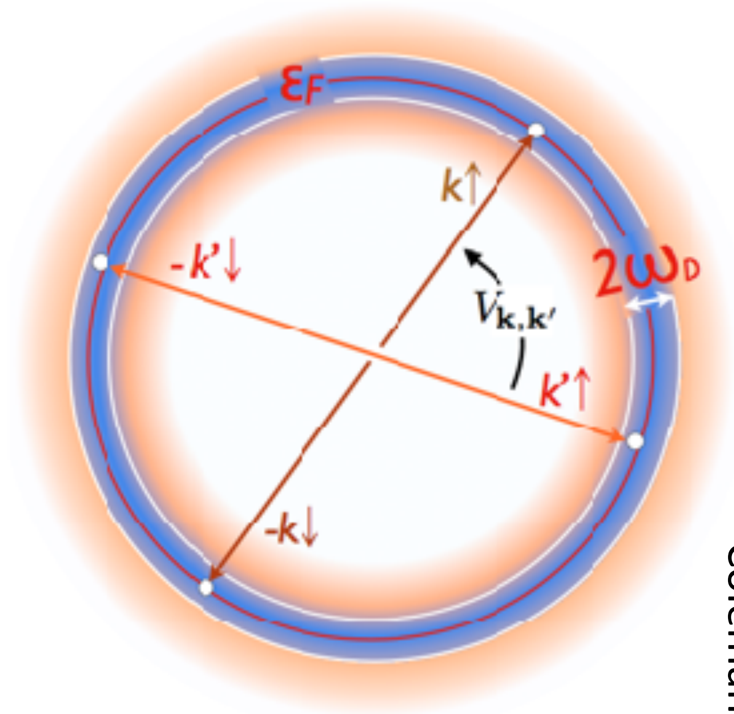
COOPER PAIRS (1956)

Simplified model interaction: only attractive within an energy ω_D of the Fermi energy



$$V_{\mathbf{k},\mathbf{k}'} = \begin{cases} -g_0/V & (|\epsilon_{\mathbf{k}}|,\ |\epsilon_{\mathbf{k}'}| < \omega_D) \\ 0 & (\text{otherwise}). \end{cases}$$

Coleman's book

COOPER PAIRS (1956)

Coleman's book

$$E\phi_{\mathbf{k}} = 2\epsilon_{\mathbf{k}}\phi_{\mathbf{k}} + \sum_{|\mathbf{k}'|>k_F} \langle \mathbf{k}_p | \hat{V} | \mathbf{k}'_p \rangle \phi_{\mathbf{k}'}$$

$$\downarrow$$

$$(E - \epsilon_{\mathbf{k}})\phi_{\mathbf{k}} = -\frac{g_0}{V} \sum_{0<\epsilon_{\mathbf{k}'}<\omega_D} \phi_{\mathbf{k}'}$$

$$\phi_{\mathbf{k}} = -\frac{g_0/V}{E - 2\epsilon_{\mathbf{k}}} \sum_{0<\epsilon_{\mathbf{k}'}<\omega_D} \phi_{\mathbf{k}'} \quad \cdots\cdots \quad E = -2\omega_D e^{\frac{-2}{g_0 N(0)}}$$

Fermi sea is unstable to the formation of Cooper pairs

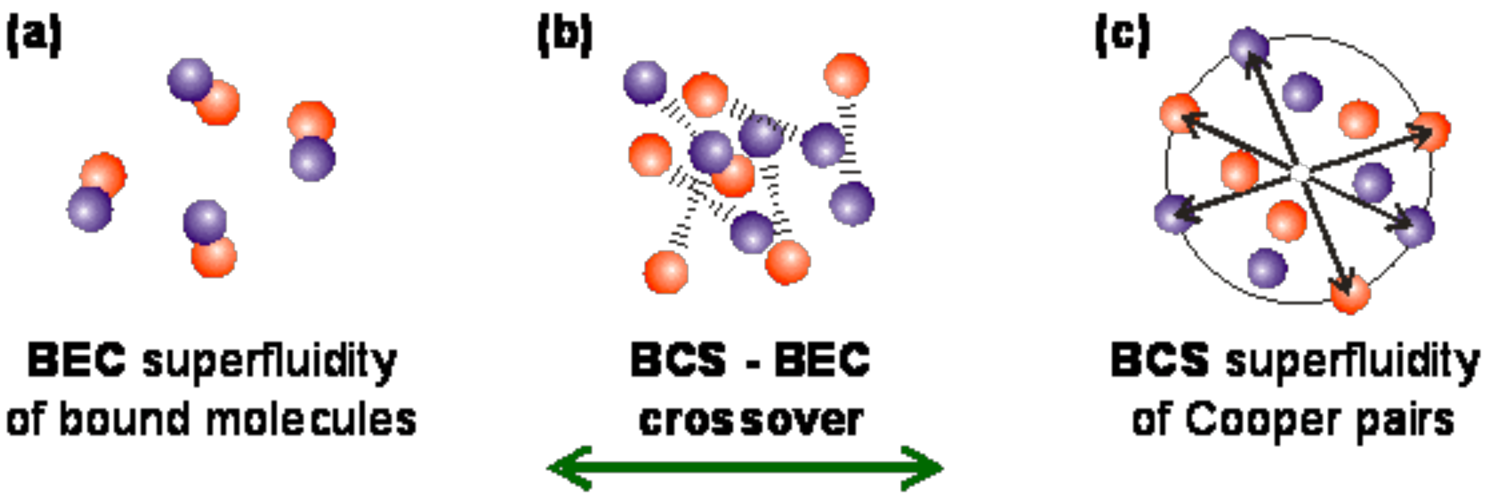
- Cooper pair is a bound state beneath the particle-hole continuum.
- Cooper pairs can condensate.

COOPER PAIRS (1956)

The degeneracy of the Fermi sea N(0) implies pairs form even for a very weak pairing interaction

$$E = -2\omega_D e^{\frac{-2}{g_0 N(0)}}$$

- The Cooper pair has a small smearing in k space →a large extension (coherence length) in real space (10^4 Å for Al).
- Pairs overlap.



http://www.qm.phy.cam.ac.uk/teaching/

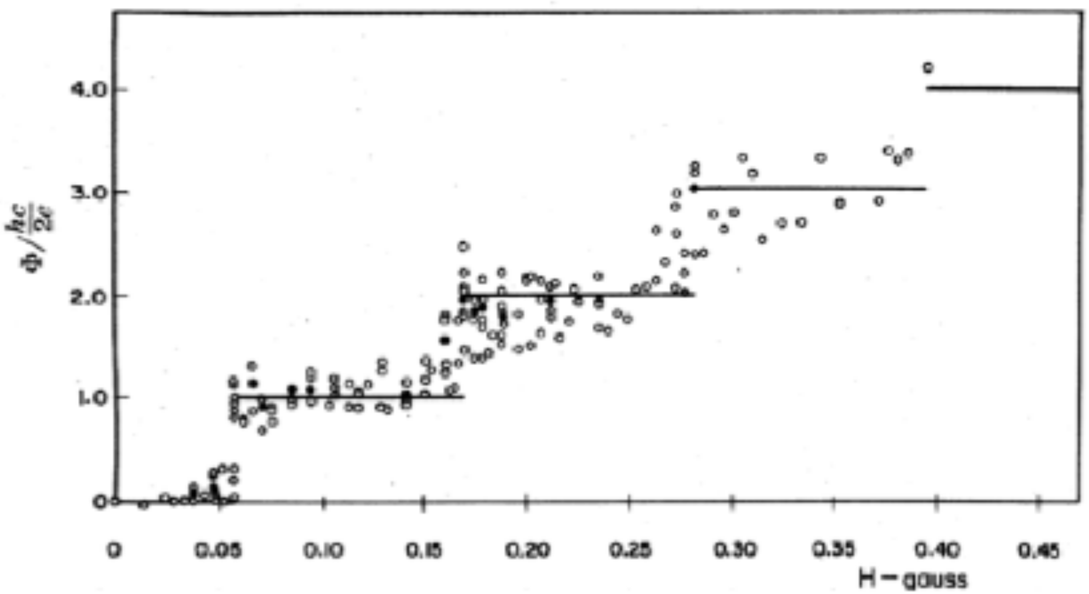
http://users.physics.harvard.edu/~greiner/fermicondensates.html

OBSERVATION OF COOPER PAIRS

The vortex magnetic flux is quantized (London, 1950)

$$\Phi_0 = \frac{h}{e^*} \equiv \frac{h}{2e}$$

Magnetic flux in a cylinder provides a proof for the charge of the Cooper pair.



Phys. Rev. Lett. 7, 43–46 (1961)

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SCHRIEFFER: BCS WAVE-FUNCTION

Coherent superposition of Cooper pair operators

$$|\Psi_{BCS}\rangle = \exp[\Lambda^{\dagger}]\,|0\rangle \qquad \Lambda^{\dagger} = \sum_{\mathbf{k}} \phi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}$$

$$|\Psi_{BCS}\rangle = \prod_{\mathbf{k}} \exp[\phi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}]\,|0\rangle = \prod_{\mathbf{k}} (1 + \phi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow})\,|0\rangle$$

(all higher power orders are 0)

And the BCS Hamiltonian

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

PAIRING HAMILTONIAN

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

Mean field approximation

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$$

Tinkham's book

Mean field Hamiltonian

$$H = \sum_{k\sigma} \epsilon_k c^{\dagger}_{k\sigma} c_{k\sigma} - \sum_{k} \left(\Delta_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} + \Delta^*_k c_{-k\downarrow} c_{k\uparrow} \right)$$

PAIRING HAMILTONIAN

Mean field Hamiltonian

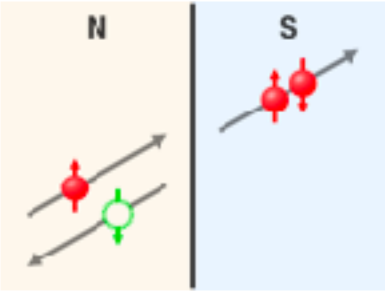
$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - \sum_k \left(\Delta_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \Delta^*_k c_{-k\downarrow} c_{k\uparrow} \right)$$

Pair creation: $e^- + e^- \rightarrow \mathrm{pair}^{2-}$

Note $c_{-\mathbf{k}\downarrow} = h^\dagger_{\mathbf{k}\downarrow}$ is a hole creation operator

Andreev reflection:

$$e^- \rightarrow \mathrm{pair}^{2-} + h^+$$



wikipedia

Coleman's book

EXCITATION SPECTRUM

Diagonalize (Bogoliubov transformation)

$$\alpha^\dagger_{k\uparrow} = c^\dagger_{k\uparrow} u_k + c_{-k\downarrow} v_k$$
$$\alpha_{-k\downarrow} = c_{-k\downarrow} u^*_k - c^\dagger_{k\uparrow} v^*_k$$

$$H = \sum_{k\sigma} E_k \alpha^\dagger_{k\sigma} \alpha_{k\sigma}$$

$$E_{\mathbf{k}} = \sqrt{\epsilon^2_{\mathbf{k}} + |\Delta_{\mathbf{k}}|^2}$$

Tinkham's book, Coleman's book

The self consistent eq for the gap is:

$$\Delta_{\mathbf{k}} = -\frac{1}{2} \sum_{\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon^2_{\mathbf{k}'} + |\Delta_{\mathbf{k}'}|^2}} V_{\mathbf{k},\mathbf{k}'}$$

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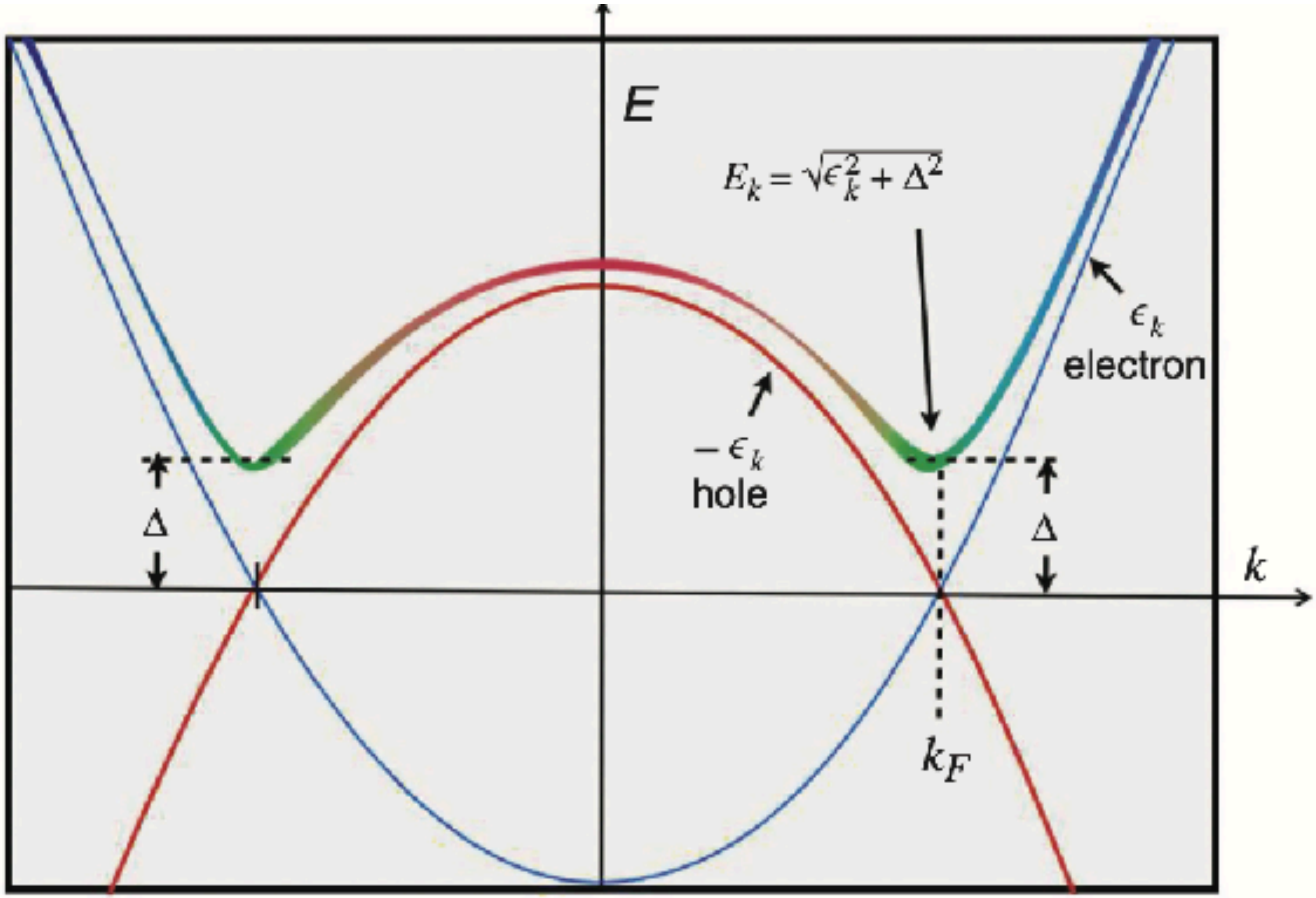
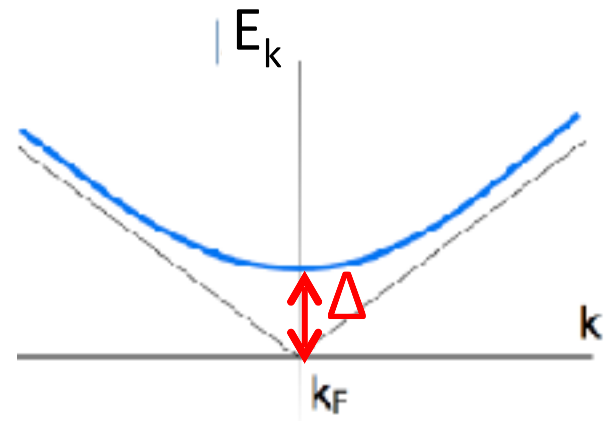


Excitation spectrum. Andreev scattering mixes electron and hole excitation spectra producing the gap Δ.

Coleman’s book

Excitation energy of quasiparticles

$$E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}|^2}$$

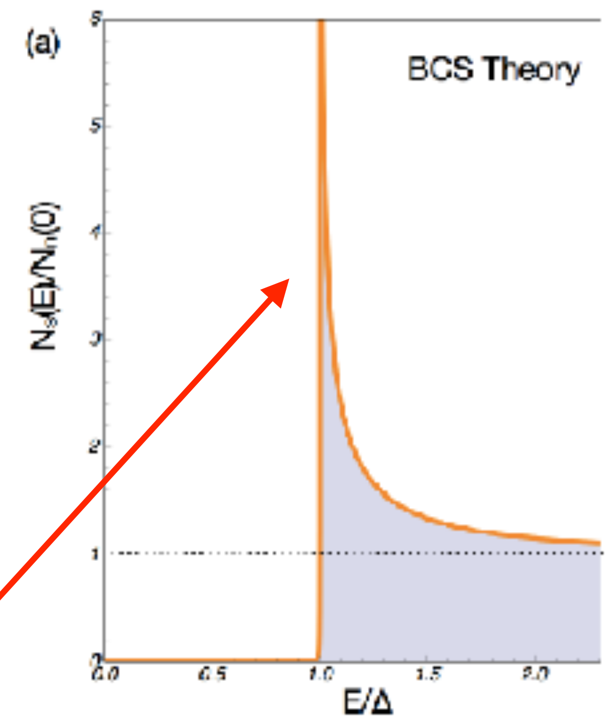


Coleman's book

Density of Bogoliubov quasiparticles

$$N_s(E) = N_n(0)\frac{d|\epsilon_{\mathbf{k}}|}{dE_{\mathbf{k}}} =$$

$$= N_n(0)\left(\frac{E}{\sqrt{E^2 - |\Delta|^2}}\right)\theta(E - |\Delta|)$$



Coherence peak

BCS THEORY

Explicit equation for Δ (T=0)

$$\Delta_{\mathbf{k}} = -\frac{1}{2}\sum_{\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}} V_{\mathbf{k},\mathbf{k}'}$$

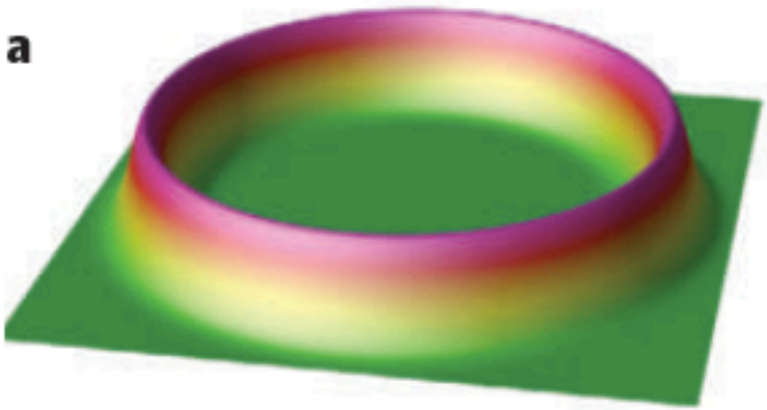
Within BCS $V_{kk'} = -V$ within $\hbar\omega_D$ of E_F

Then $\Delta_{kk'} = \Delta$ (isotropic)

$$\Delta \approx 2\hbar\omega_D e^{-1/N(0)V}$$

Isotropic pairing function: s-wave

a



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BCS THEORY

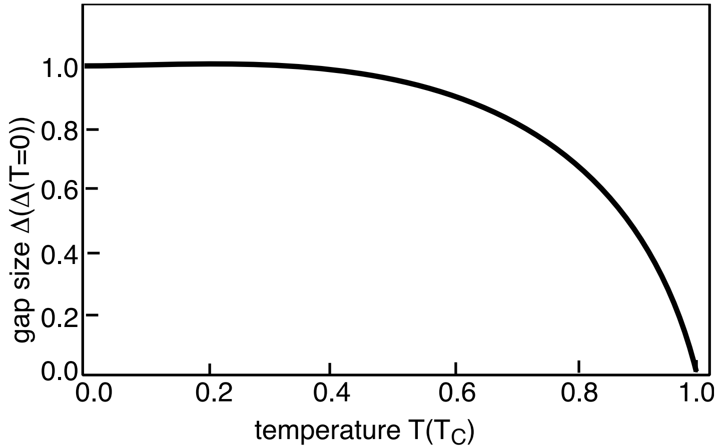
For finite T

$$\Delta_{\mathbf{k}} = -\frac{1}{2}\sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}} \tanh\left(\frac{\beta}{2}\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}\right)$$

For weak coupling superconductors

$$\frac{\Delta(T)}{\Delta(0)} \sim 1.76\left(1 - \frac{T}{T_c}\right)^{1/2}$$

Universal Δ(T) curve
in the weak coupling limit



Tinkham's book

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MIGDAL-ELIASHBERG
(INCLUSION OF RETARDATION EFFECTS)

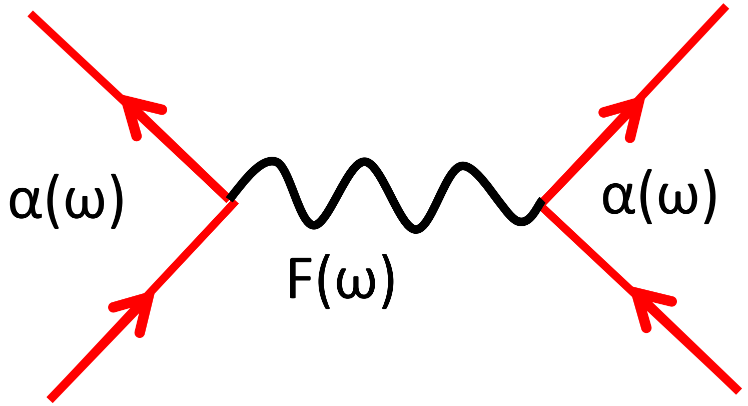
BCS theory included retardation effects in a minimal way by introducing an energy cut-off: the Debye frequency.

Eliashberg treated the electron-phonon interaction including correctly the retardation effects.

$$T_c = F(\alpha^2 F(\omega), \mu^*)$$

Electron-phonon spectral function

Coulomb pseudopotential



Migdal (1957): vertex corrections $O(m/M)^{1/2}$ can be dropped (equivalent to adiabaticity condition)

MIGDAL-ELIASHBERG
(INCLUSION OF RETARDATION EFFECTS)

BCS theory included retardation effects in a minimal way by introducing an energy cut-off: the Debye frequency.

Eliashberg treated the electron-phonon interaction including correctly the retardation effects.

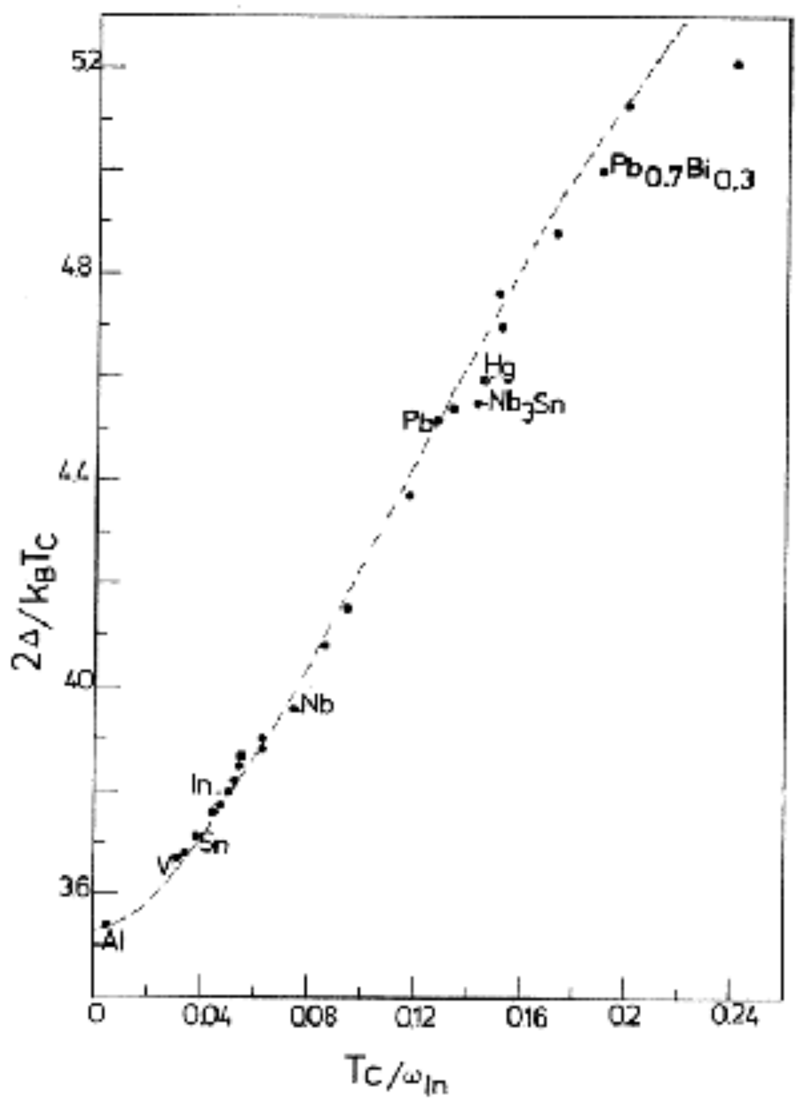
$$T_c = F(\alpha^2 F(\omega), \mu^*) \qquad \Delta = G(\alpha^2 F(\omega), \mu^*)$$

$$\frac{2\Delta}{k_B T_c} = 3.53 \left[1 + 12.5 \left(\frac{T_c}{\omega_{ln}} \right)^2 \log \left(\frac{\omega_{ln}}{2T_c} \right) \right]$$

Coupling strength

PRB 29, 184 (1984)

MIGDAL-ELIASHBERG



PRB 29, 184 (1984)

$$\frac{2\Delta(0)}{k_BT_c} = 3.53$$

MAX T_C FOR PHONON MEDIATED SUPERCONDUCTIVITY

McMillan PR 167, 331

$$T_c \propto \langle \omega_{ph} \rangle \exp[-1/(\lambda - \mu^*)]$$

Input: phonon spectrum

With the electron phonon coupling constant

$$\lambda = 2\int_0^\infty \frac{\alpha^2 F(\omega)}{\omega}$$

TABLE VIII. The predicted maximum superconducting transition temperature for four classes of materials found from the observed T_c and λ and Fig. 11, together with the observed T_c.

Pure metals

Alloys

Metal	T_c (°K)	λ	T_c^{max} (°K)	Observed maximum T_c	Material
Pb	7.2	1.3	9.2	8.8	Pb–Bi
Nb	9.2	0.82	22	10.8	Zr–Nb
V_3Si	17	0.82	40		
Nb_3Sn			28	20	Nb_3Al–Nb_3Ge

Note:
MgB_2 Tc=40K

Canfield and Crabtree, Physics Today 2003.

MgB_2 (2001)

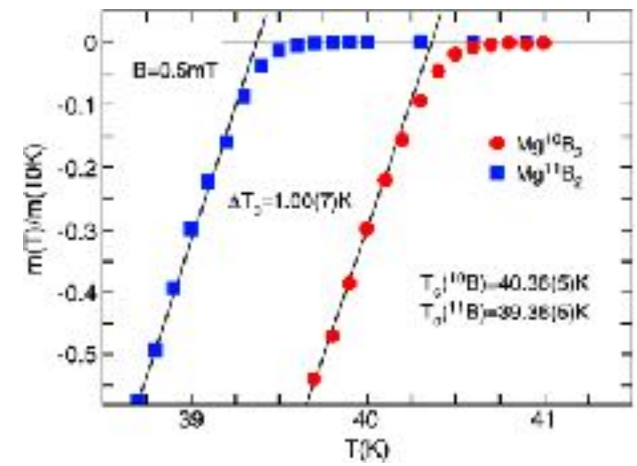
T_c=39 K.

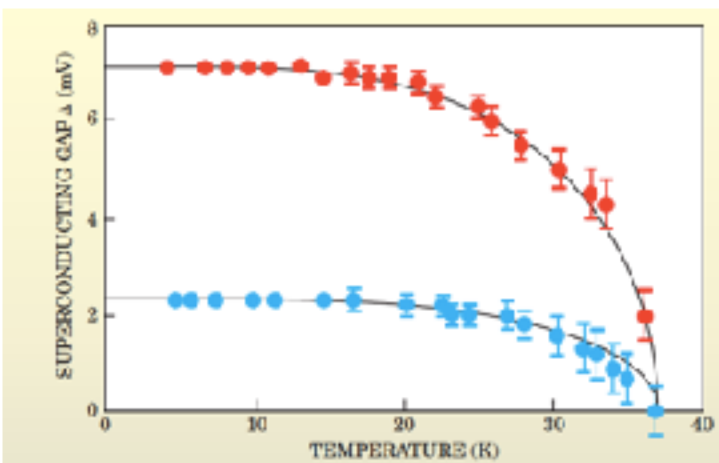
MULTIBAND SUPERCONDUCTOR

Conventional SPC.
2 differentiated bands.
One very strong phonon (in the B plane) coupled to the conduction electrons.

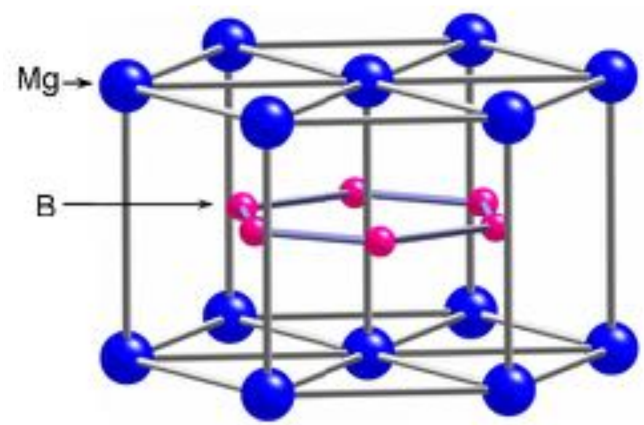
Isotope effect

JPCM 24 233201





2 energy gaps with different electron-phonon strengths

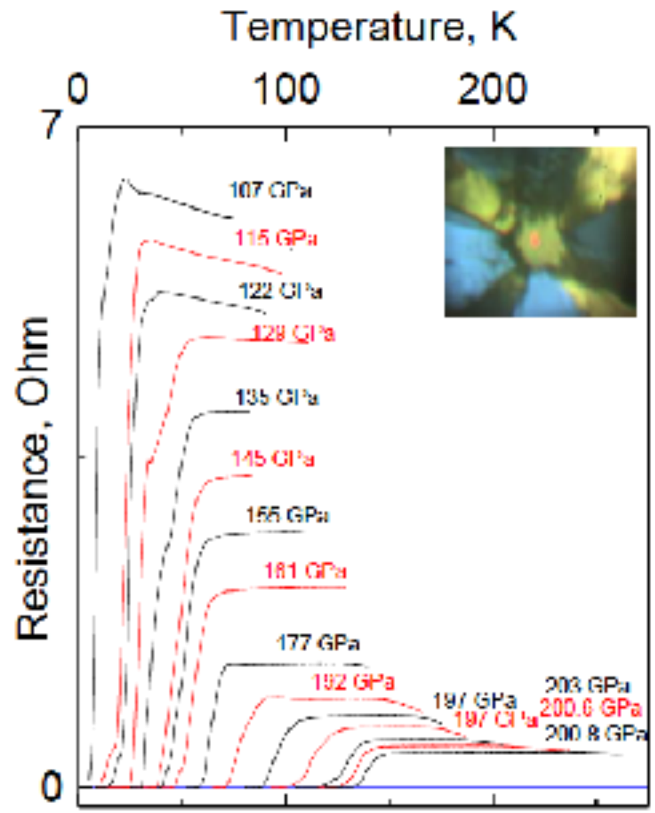


DOI: 10.5772/17117

RECORD T_C FOR ELECTRON-PHONON MEDIATED SUPERCONDUCTIVITY

H_2S T_c=203 K (at 200 GPa); Nature 525, 73 (2015)

LaH_{10} T_C=250 K at 170 GPa; Nature 569, 528 (2019)

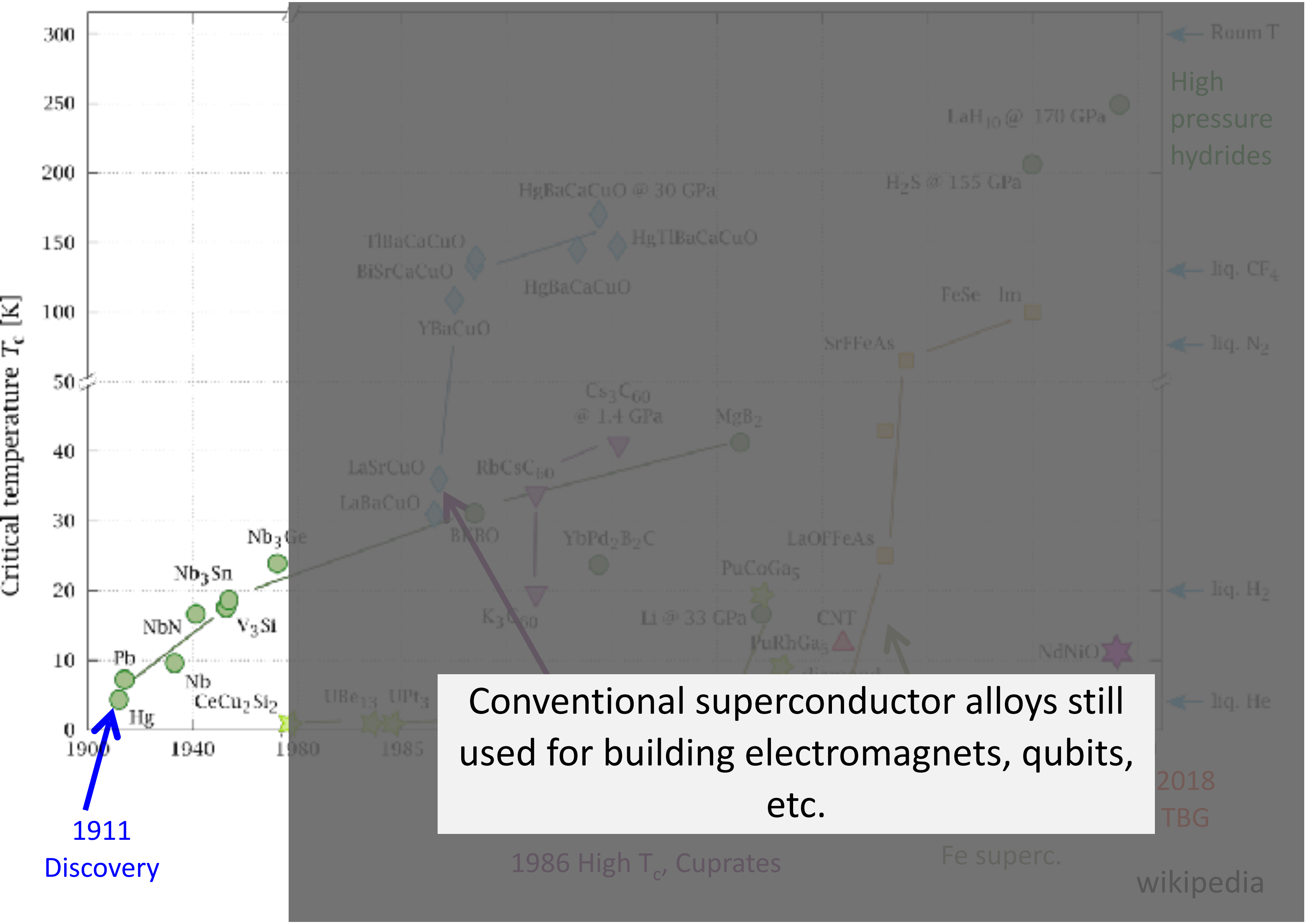


H_2S leads to H_3S under high pressures.

Hydrogen atoms are light → high phonon frequencies expected. Large Tc predicted
Ashcroft PRL 21, 1748 (1968); 21, 1748 (2004)

Role of anharmonic phonons.
Errea et al PRL 114, 157004

Recent developments by Ranga Dias (closer to ambient conditions) under discussion.

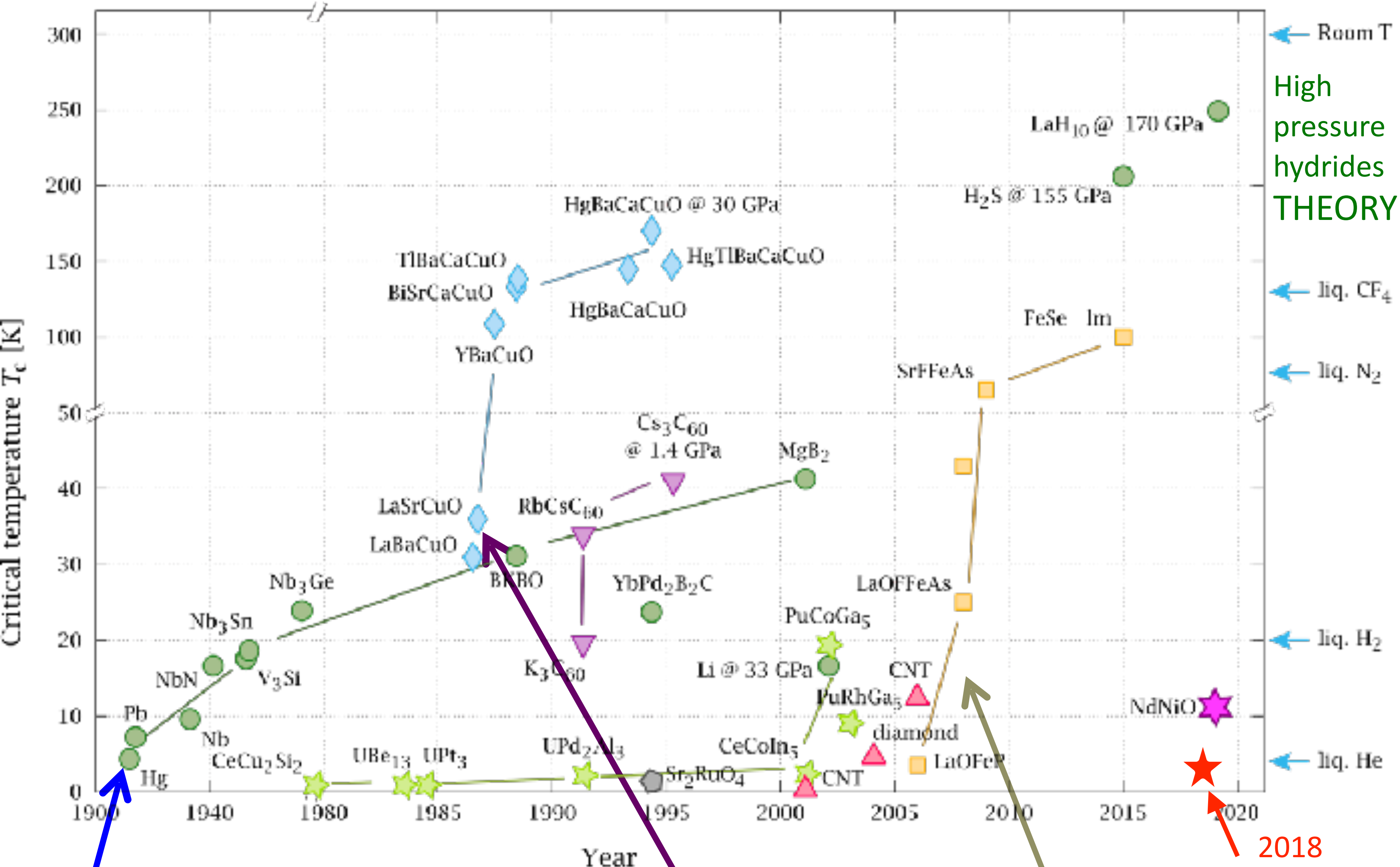


Conventional superconductor alloys still used for building electromagnets, qubits, etc.

Mazin, Nature 464, 183 (2010)

MATTHIAS'S RULES (1950's)

- High symmetry (cubic best) is good
- High density of electronic states is good.
- Stay away from oxygen
- Stay away from magnetism (PM impurities destroy superconductivity)
- Stay away from insulators
- Stay away from theorists



1911
Discovery

1986 High T_c, Cuprates

OXIDE, ANTIFERROMAGNET

PARENT COMPOUND: INSULATOR

2008, high T_c

Fe superc

ANTIFERRO

2018

TBG

wikipedia

OUTLINE

- **Superconductivity**
  - Properties (zero resistivity, Meissner effect)
  - Understanding (pairing, BCS, Ginzburg-Landau)
  - Electron-phonon interaction (conventional superconductivity)
- **Unconventional superconductivity (unsolved)**
  - What are the new issues.
  - What are the proposals.

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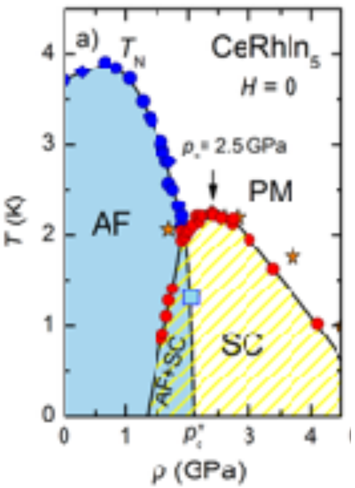
MATTHIAS'S RULES ?

Theory predicted superconductivity in hydrogen compounds (Ashcroft) and semiconductors ($SrTiO_3$, Cohen).

AF supercond.

Heavy fermions



AF+SPC

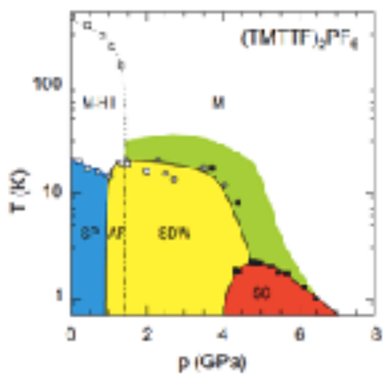
Coexistence

Oxides

Insulators

Magnetism

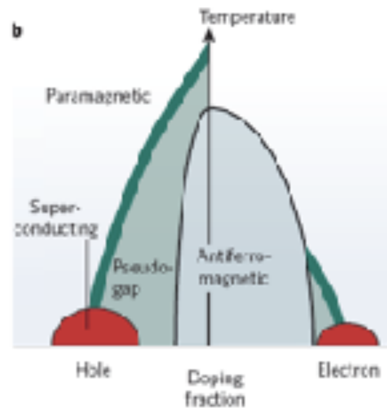
Organics



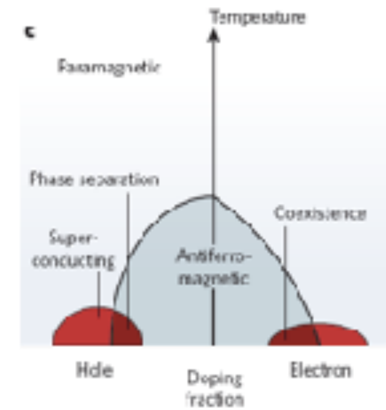
Eur. Phys. JB 21, 175

Cuprates

(high Tc)

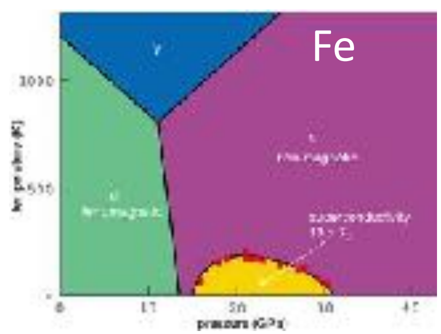


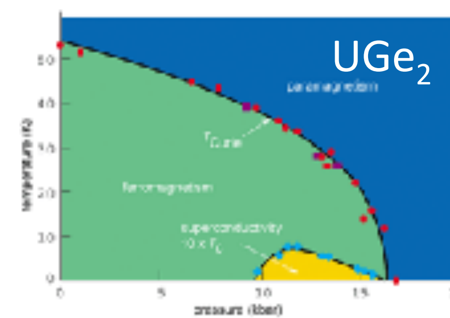
Fe-superconductors



Nature 464, 183 (2010)

FM superconductors





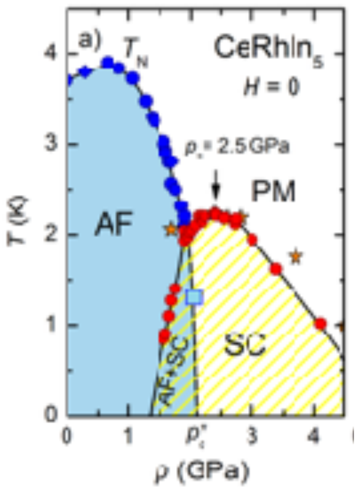
Physics World, Jan 2002

UNCONVENTIONAL SUPERCONDUCTORS

Not driven by conventional phonons.

AF supercond.

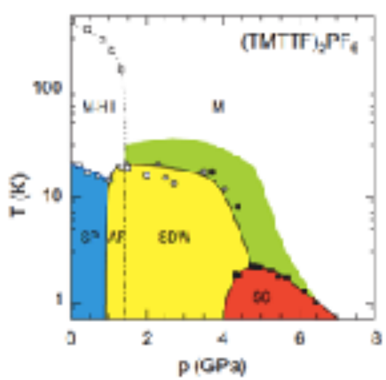
Heavy fermions



AF+SPC Coexistence

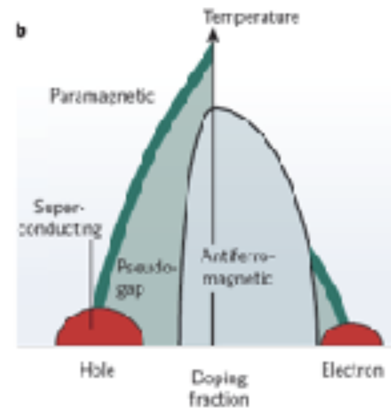
Oxides
Insulators
Magnetism

Organics

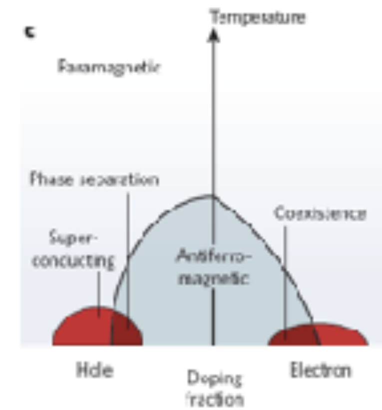


Eur. Phys. JB 21, 175

Cuprates (high Tc)

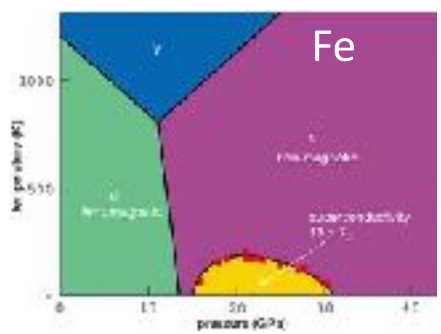


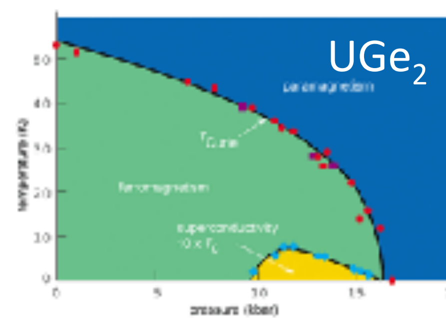
Fe-superconductors



Nature 464, 183 (2010)

FM superconductors





Physics World, Jan 2002

UNCONVENTIONAL SUPERCONDUCTORS

The "normal" state is more complicated

- Proximity or coexistence with magnetism
- Strong correlations.
  - Competing orders (stripes).
  - Is there a Fermi surface? Doped Mott insulator. Non-Fermi liquid behaviour. Pseudogap phases.
- Low dimensionality, anisotropies, nematicity.

UNCONVENTIONAL SUPERCONDUCTORS

<u>The superconducting state is different</u>

The pairing function Δ_k may

- Be non-isotropic (including nodes, sign changes)
- Have a finite orbital momentum (p- or d-wave symmetry)
- Be spin-triplet (p-wave symmetry)

- $\lambda >> \xi$ (type II)
- Anisotropies

$$\frac{2\Delta}{k_B T_c} >> 3.53$$

Many theories.
2 distinct approaches to the problem:

- Stay within BCS (need a Fermi surface!) but a new pairing (glue) mechanism is needed (maybe spin fluctuations, some kind of electron-phonon interaction).

- Start from the Mott state (no boson exchange required) and see how to gain energy from pairing
  - Resonating valence bonds
  - Kinetic energy driven
  - Quantum criticality…

Many theories.
2 distinct approaches to the problem:

- Stay within BCS (need a Fermi surface!) but a new pairing (glue) mechanism is needed (maybe spin fluctuations, some kind of electron-phonon interaction).

  A: We know how to deal with it.
  D: Usually there is no Fermi surface.

Many theories.
2 distinct approaches to the problem:

- Start from the Mott state
  - Resonating valence bonds
  - Kinetic energy driven
  - Quantum criticality…

A: It seems, in principle, more self-consistent.
D: We need to properly treat the Mott state first!!

FIRST APPROACH

Assume BCS is valid for non-conventional superconductors.

We need some attractive interaction (but no phonons!)
+Very strong electron-electron repulsive interaction.
Is there a way around it??

PAIRING SYMMETRY

$$\psi(\vec{r}_1 s_1, \vec{r}_2 s_2) = \varphi(\vec{r}_1, \vec{r}_2)\chi(s_1, s_2)$$

Spatial Spin

Pair wavefunction must be antisymmetric

Spin singlet →even parity orbital wave function s, d
Spin triplet → odd parity orbital wave-function p, f

Singlet SPC

s-wave: conventional spc
d-wave: cuprates
d-wave/d-wave: Fe scp

p-wave (triplet SPC):

- Superfluidity in ^3He
- UPt_3, UTe_2
- Indications in some multilayered graphene systems.
- Proximity effects.

Rev. Mod. Phys. 69, 645

SUPERFLUIDITY IN ^{3}HE (1972)

T_c=2.7 mK

Pairing cannot be mediated by the lattice. Nuclear forces are strongly repulsive in the core → no s-wave possible. Need of wavefunctions that vanish at $r \rightarrow 0$.

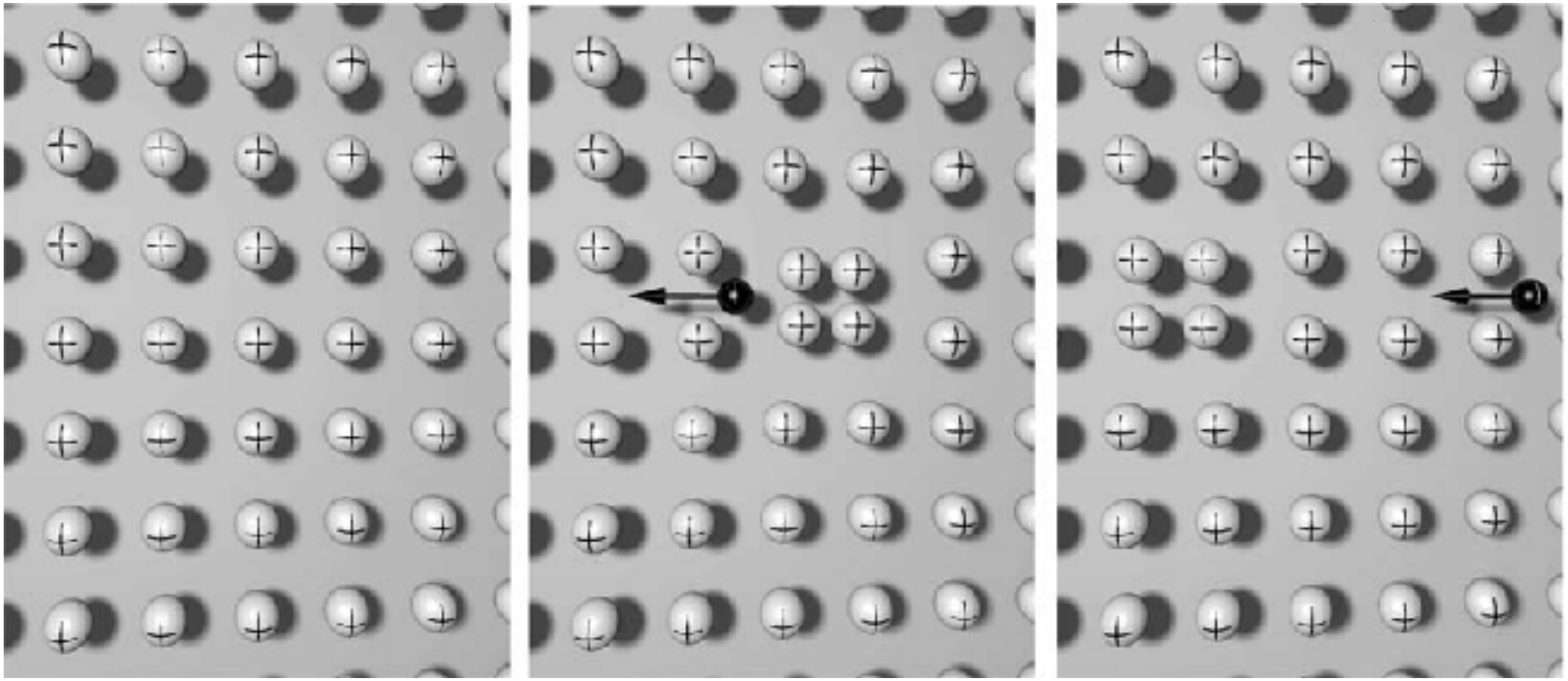
High angular momentum pairing (p-wave) was proposed as a way to overcome the short range repulsion (Pitaevskii 1959).

Glue: One possibility is mediation by ferromagnetic spin fluctuations: FM paramagnons (FM fluctuations suppress s-wave and enhance p-wave pairing).

SUPERFLUIDITY IN ^{3}HE

Attractive interactions by ferromagnetic fluctuations:

FM clouds are formed which attract the ^{3}He quasiparticles (something like magnetic polarons instead of lattice polarons)



Blundell's book

Lattice polarons

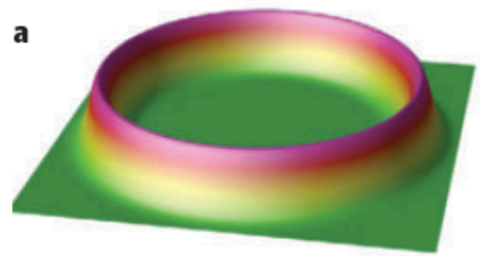
S-WAVE

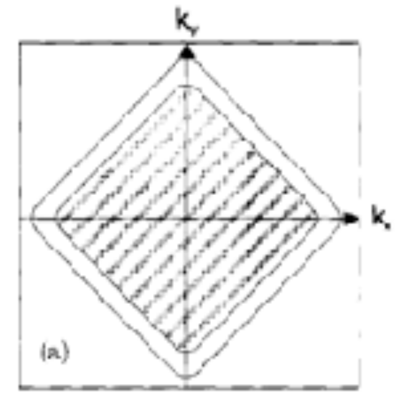
Gap equation from BCS (T=0)

$$\Delta_k = -\frac{1}{\Omega}\sum_{kk'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}}$$

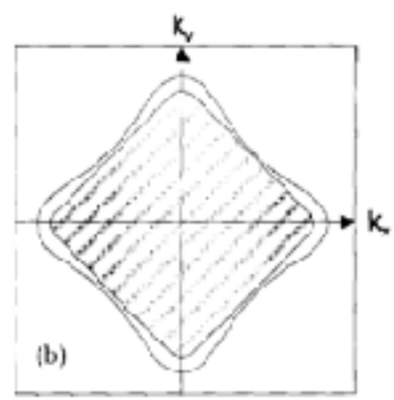
For $V_{kk'}$ constant and attractive: isotropic gap $\Delta_k=\Delta$

s wave gap
(spherical symmetry)





More generally, s-wave gap may be anisotropic with no sign changes.



Scalapino, Phys. Rep. 250,329 (1995)

M.J. Calderón calderon@icmm.csic.es

D-WAVE

Gap equation from BCS (T=0)

$$\boxed{\Delta_k = -\frac{1}{\Omega}\sum_{kk'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}}}$$

Repulsive $V_{kk'}$ ⟷ Anisotropic Δ_k with sign change!

$$\mathrm{sign}(\Delta_{\mathbf{k}}) = -\,\mathrm{sign}(V_{\mathbf{k},\mathbf{k}'})\mathrm{sign}(\Delta_{\mathbf{k}})$$

An anisotropic pair potential leads to an anisotropic gap

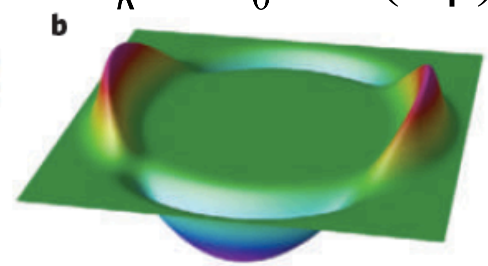
$$V_{kk'} = -V_0 \gamma_k \gamma_{k'}$$

$$\Delta_k = \gamma_k \Delta_0$$

form factor

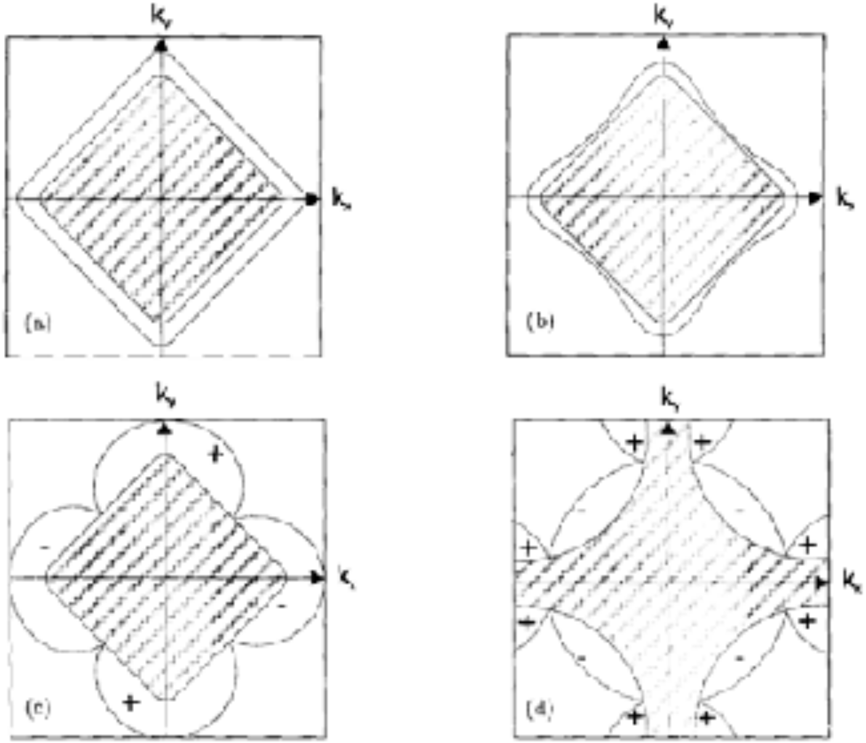
For instance, d-wave

$$\Delta_k = \Delta_0 \cos(2\phi)$$

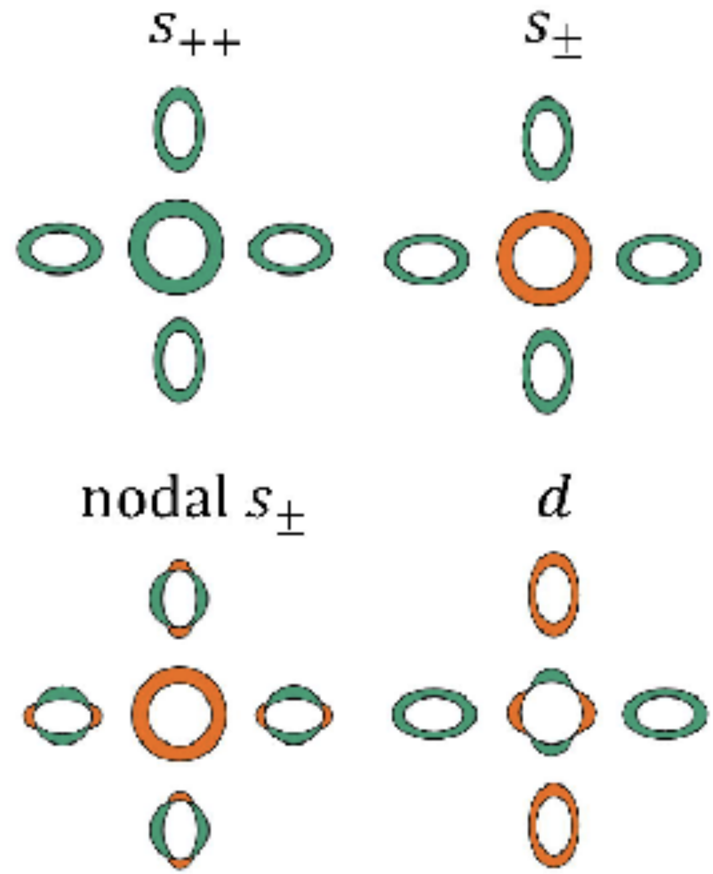


The gap has nodes and sign changes

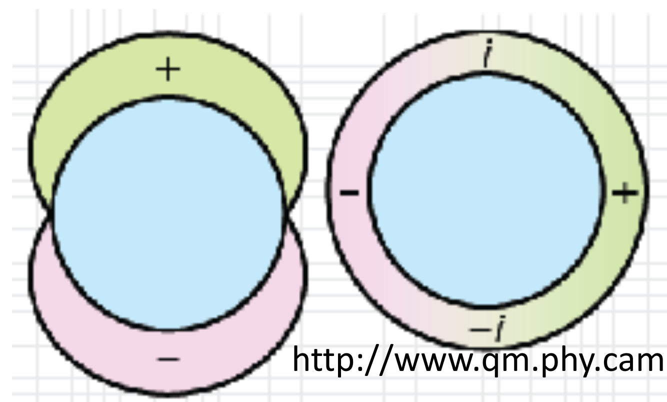
GAP SYMMETRIES...



Scalapino, Phys. Rep. 250,329 (1995)



Hirshfield et al. 1106.3712



http://www.qm.phy.cam.ac.uk/teaching/

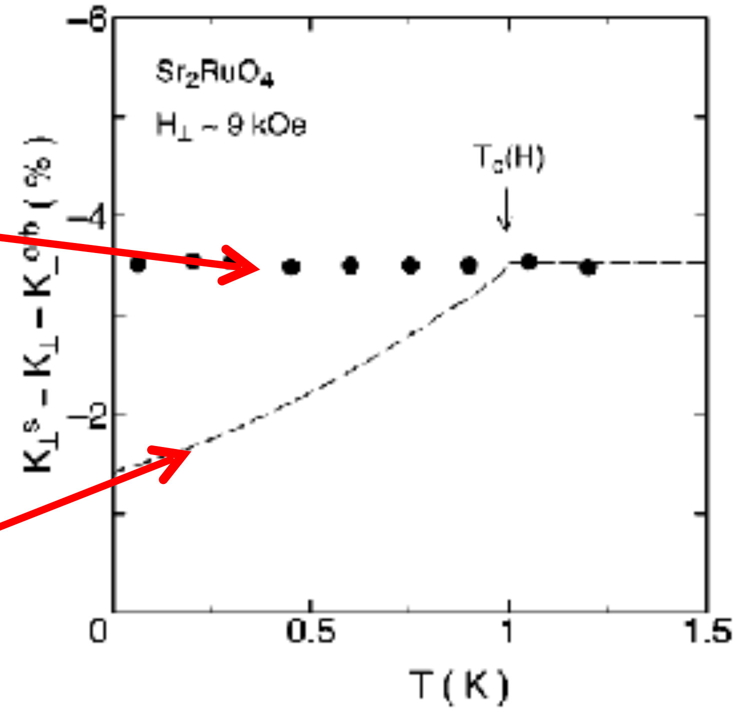
Non-isotropic gaps lead to sensitivity of unconventional SPC to non-magnetic impurities

M.J. Calderón calderon@icmm.csic.es

SINGLET VERSUS TRIPLET

From Knight shift experiments
(nuclear magnetic resonance experiment)

triplet

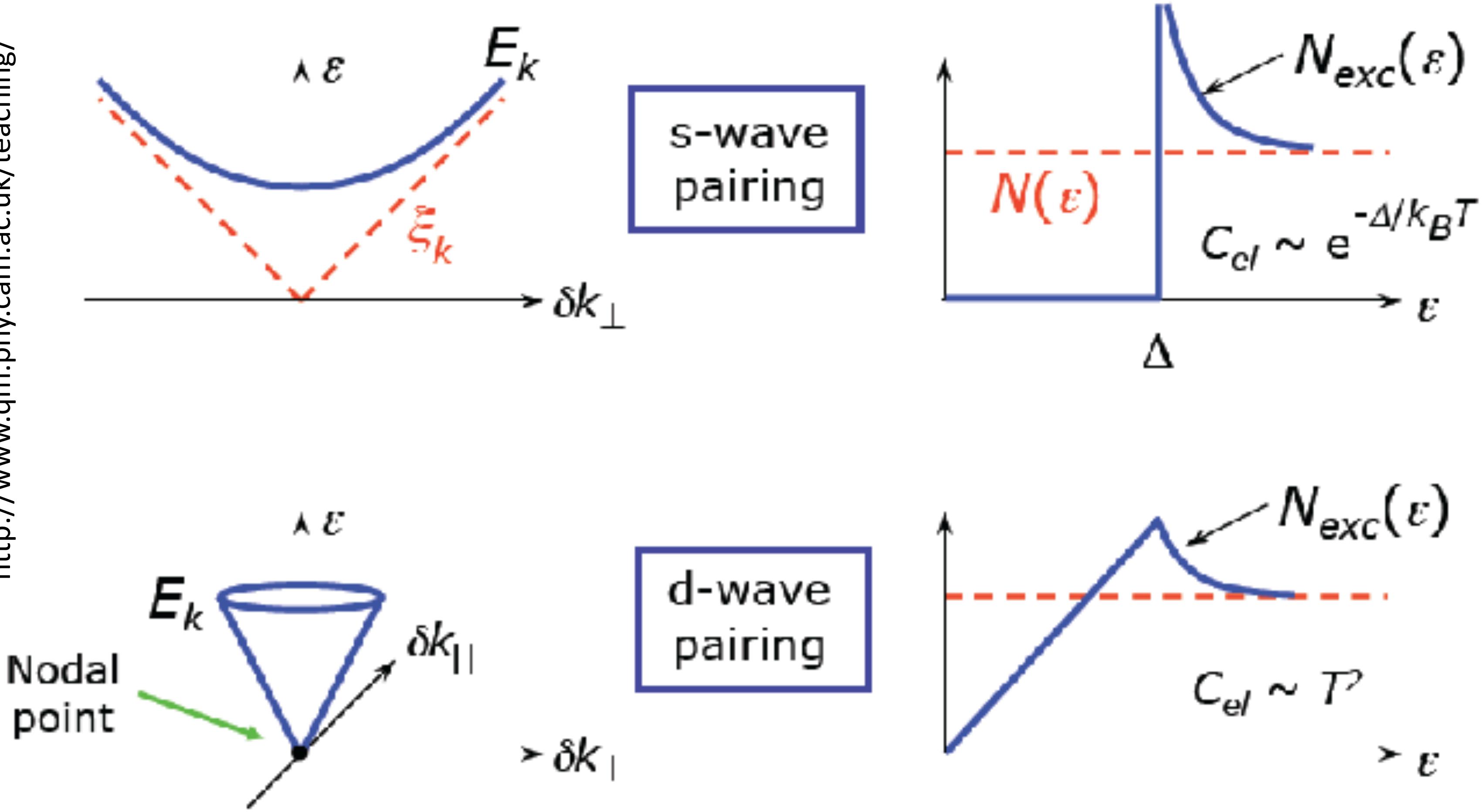


Singlet
(spin quenching at low T)

PRB 63, 060507 (2001)

(note Sr_2RuO_4 is currently known not to be a triplet superconductor)

NODES VERSUS NODELESS



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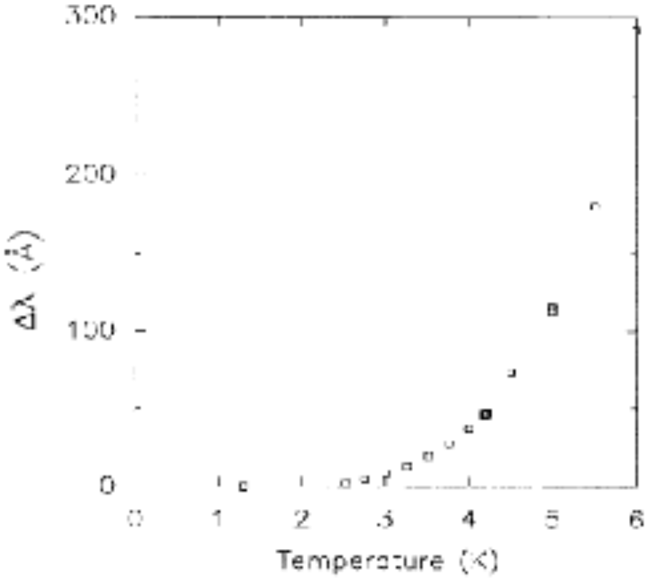
Without nodes: activated behavior (λ, specific heat…)
With nodes: power law behaviors

Gap without nodes

London penetration length within BCS

$$\frac{\Delta\lambda(T)}{\lambda(0)} \sim 3.33 \left(\frac{T_c}{T}\right)^{1/2} \exp(-1.76 T_c/T)$$

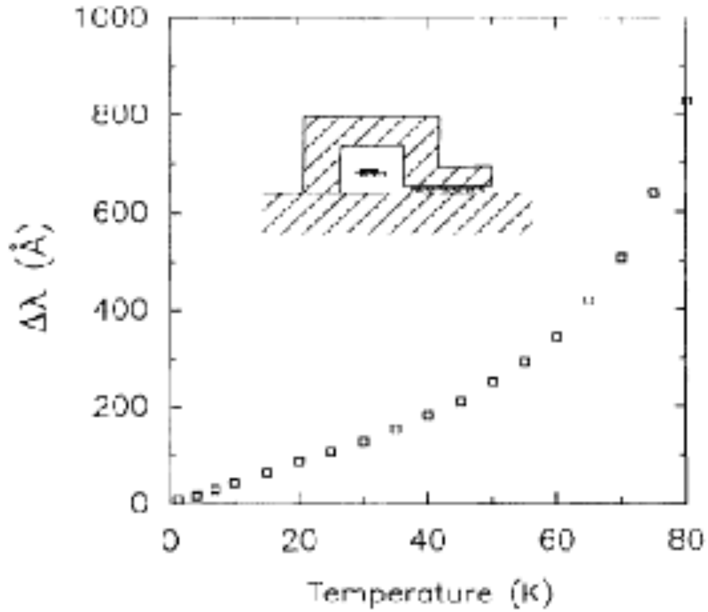


$Pb_{0.95}Sn_{0.05}$

PRL 70, 3999 (1993)

Gap with nodes

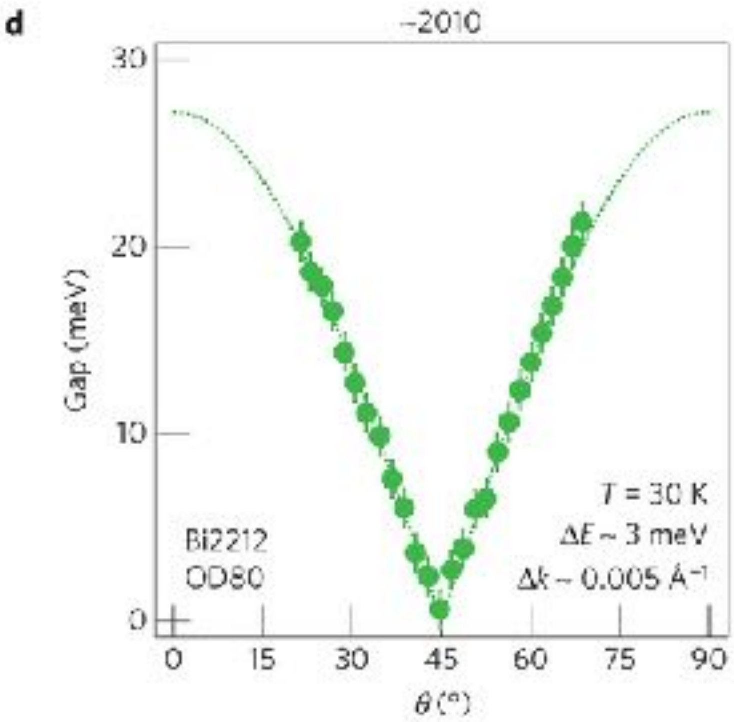
Power law dependencies



$YBa_2Cu_3O_{6.95}$

(no phase information)

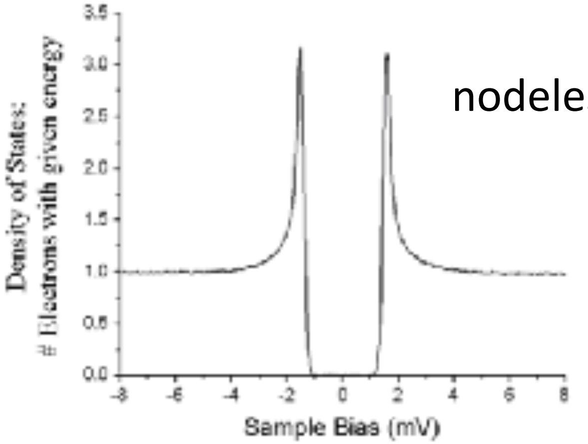
ARPES



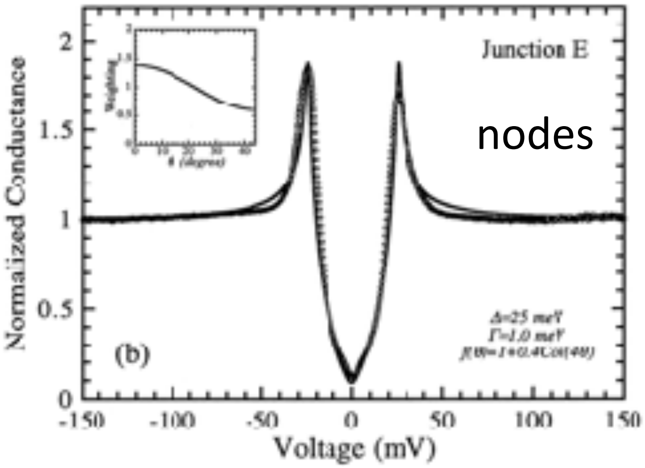
Nature Physics 10, 483–495 (2014)

(no phase information)

TUNNELING SPECTROSCOPY



nodeless



nodes

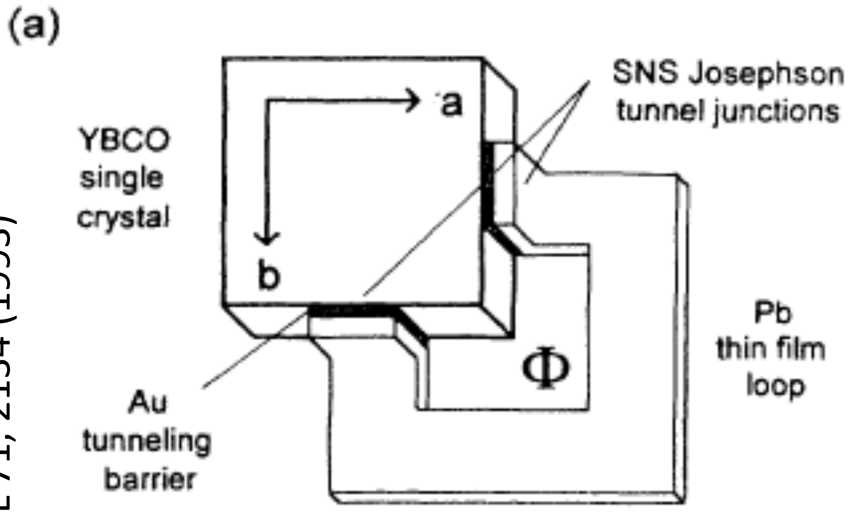
www.personal.psu.edu/ewh10

Physica C 320, 9

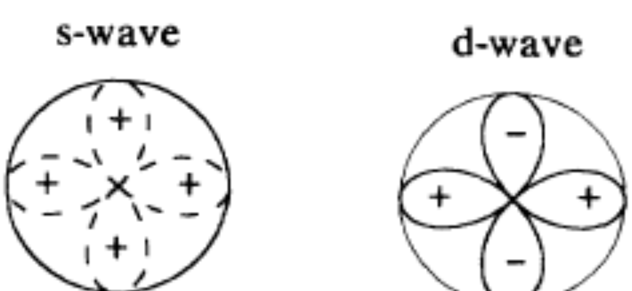
SENSITIVITY TO THE PHASE: JOSEPHSON EFFECT

$$I_s = I_c \sin\Delta\varphi$$

PRL 71, 2134 (1993)

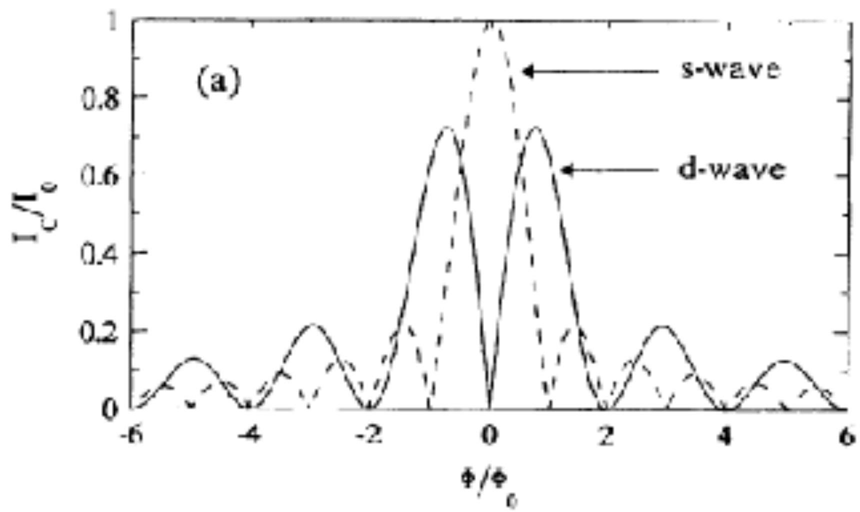


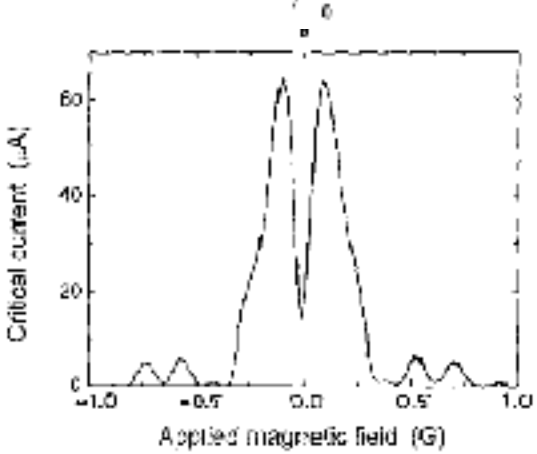
$$\phi_a - \phi_b + 2\pi\Phi/\Phi_0 + \delta_{ab} = 0$$



$\delta_{ab} = 0$ $\delta_{ab} = \pi$

Calculated critical current

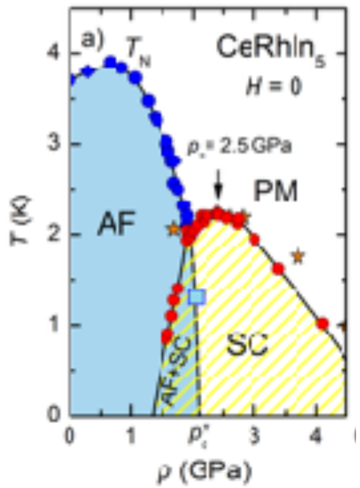




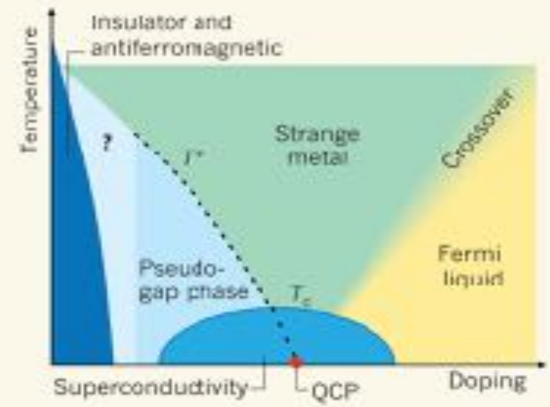
MATERIALS

SOME TYPICAL PHASE DIAGRAMS

Heavy fermions

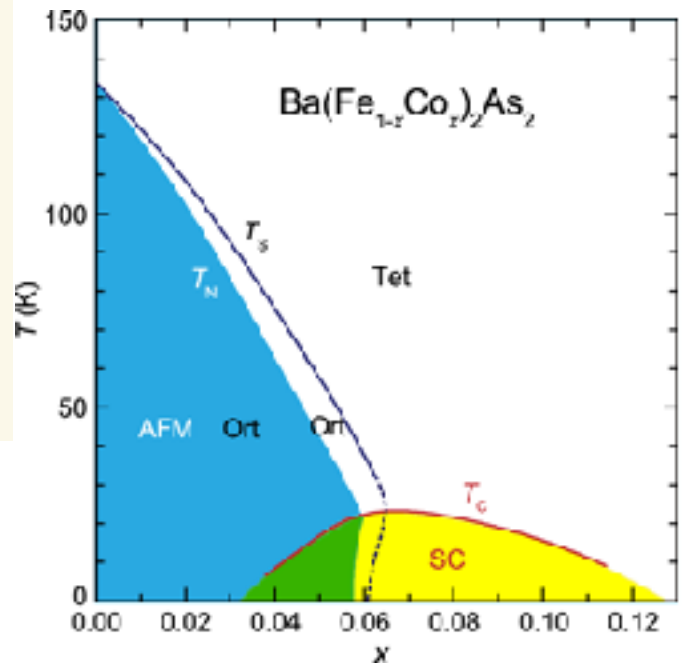


Cuprates



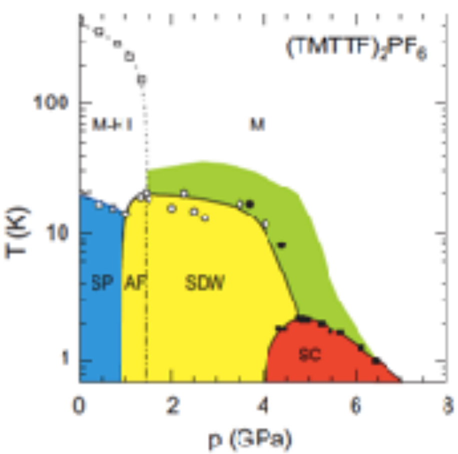
Nature 468, 184–185

Fe-superconductors



Nandi et al, PRL 104, 057006 (2010)

Organics



In common:
(AF)magnetic
phases

Eur. Phys. JB 21, 175

HEAVY FERMIONS (1979)

"Our experiments demonstrate for the first time that superconductivity can exist in a metal in which many-body interactions, probably magnetic in origin, have strongly renormalized the properties of the conduction-electron gas."

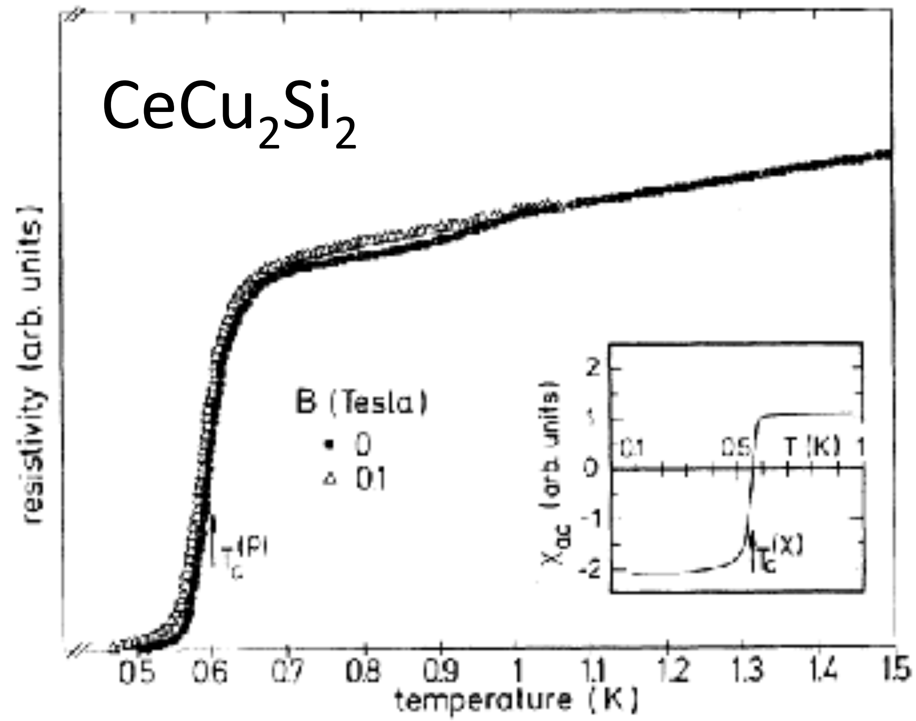
PRL 43, 1892 (1979)

Coexisting AF + SPC

Reentrant SPC due to competition with Kondo

Quantum criticality

Nat. Phys. 4, 186



PRL 43, 1892 (1979)

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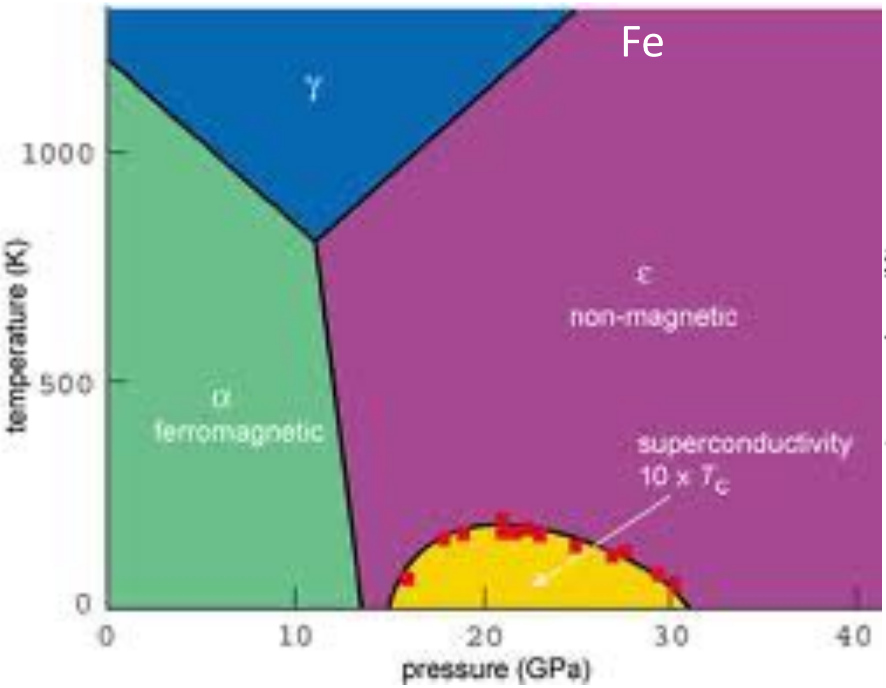
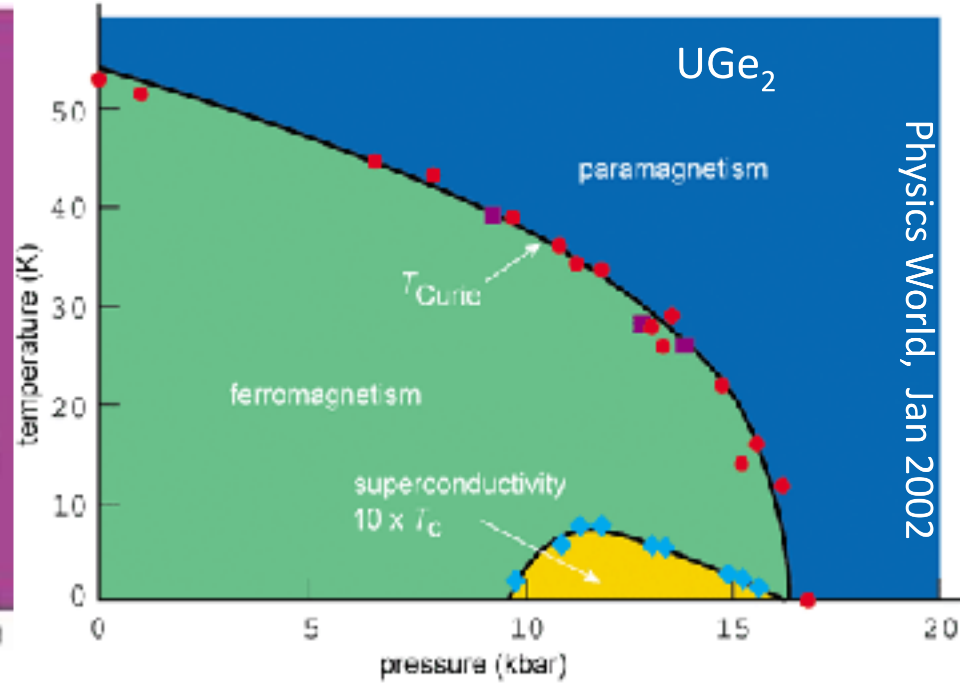
FERROMAGNETIC SUPERCONDUCTORS





Proximity of quantum critical point can lead to coexistence

PRL 94, 097003 (2005)

Possible triplet pairing

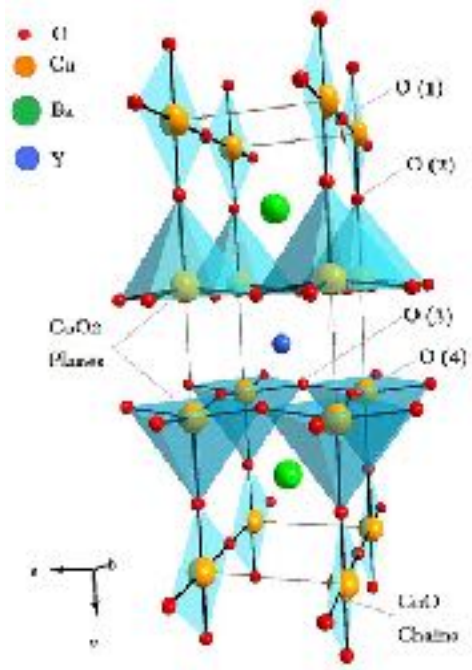
UPt_3, UTe_2 Science 365, 684 (2019)

URhGe Science 309, 1343 (2005)

UCoGe Nature Comm 8, 14480 (2017)

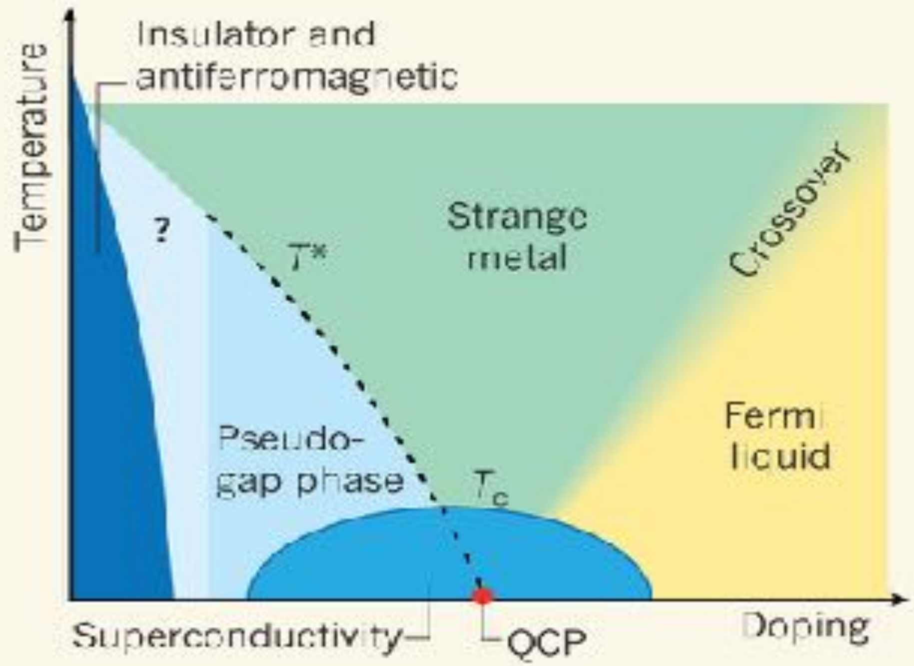
CUPRATES

- Layers of CuO_2. Different related structures.
- But note!: SPC requires coherence in 3dim.
- Highest Tc 134K (at ambient pressure).
- Tc increases with number of CuO2 planes in the unit cell (up to n=3).
- Pairs were found to be singlets.
- d-wave pairing was proposed in the cuprates early on. Scalapino, Phys. Rep. 250, 329 (1995)
- Undoped cuprates are correlated insulators and AF (π,π).



wikipedia

CUPRATES: COMPLEX "NORMAL" STATE

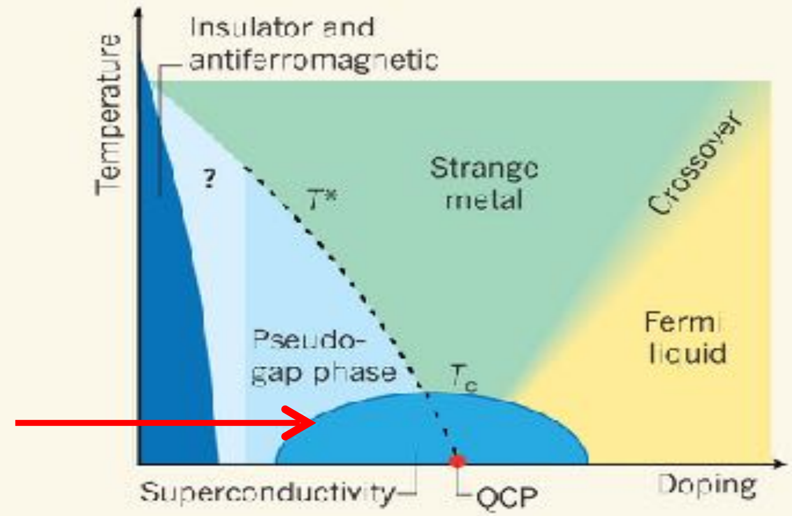


Nature 468, 184–185

PSEUDOGAP

Spin quenching sets up at T*.

underdoped



Nature 468, 184–185

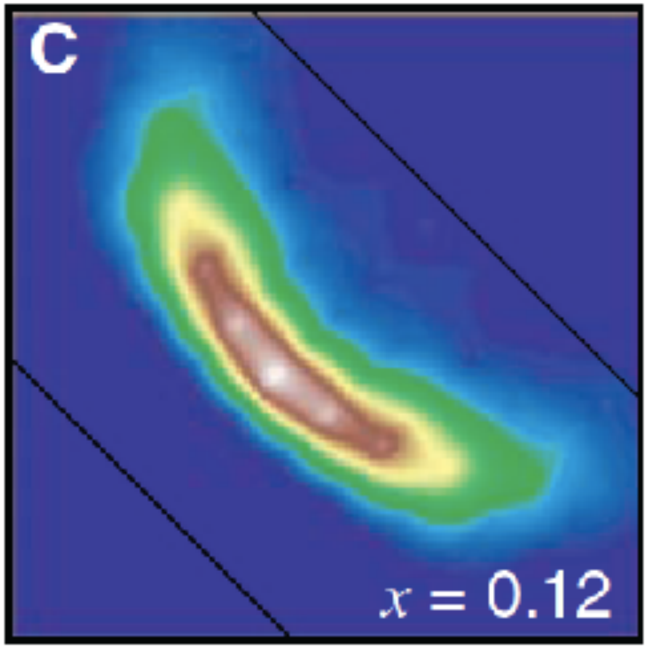
Origin?: spin-singlet formation (Anderson), pairing with short range order (preformed pairs), antiferromagnetic fluctuations, charge density wave

Is it due to fluctuations or is it a new phase (with a related broken symmetry)? Transition or crossover?

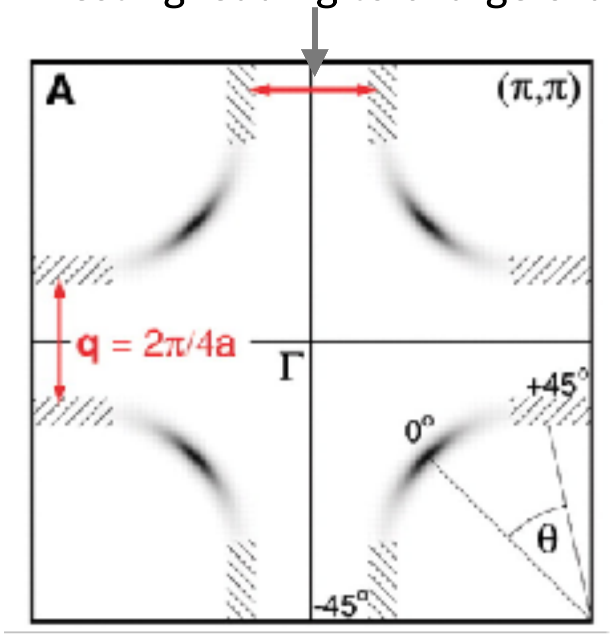
PSEUDOGAP

In other words:
Is it a precursor or a competing phase?

ARPES



Nesting leading to charge order

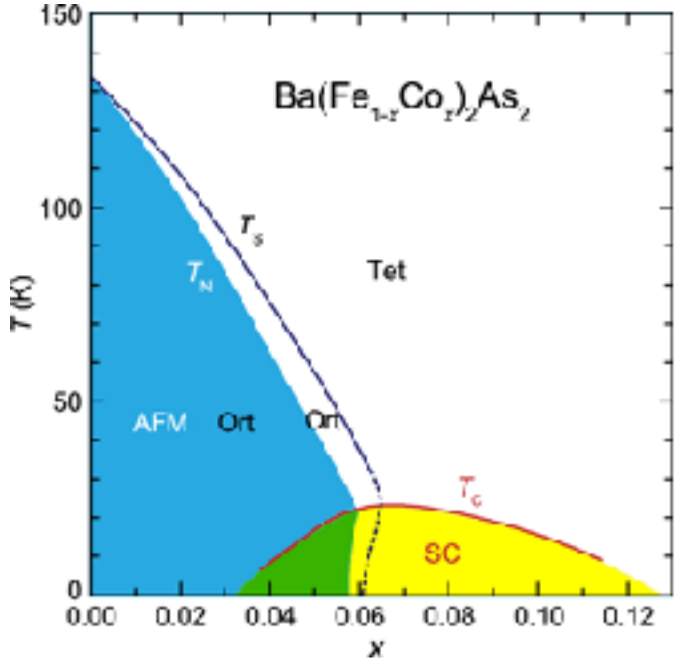


Science 307, 901

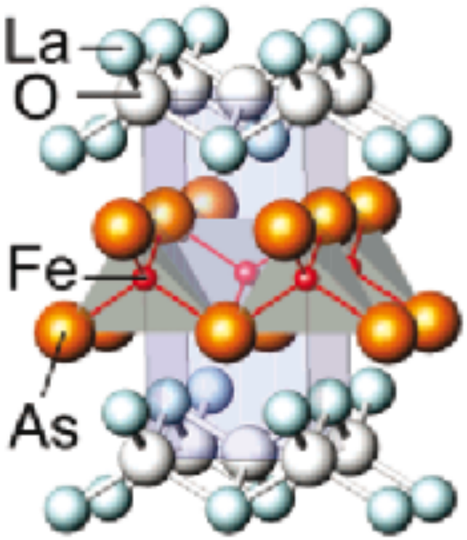
Norman et al, Adv. Phys. 54, 715 (2005)
http://www.msd.anl.gov/files/msd/cuprates-columbia.pdf

FE BASED SUPERCONDUCTORS

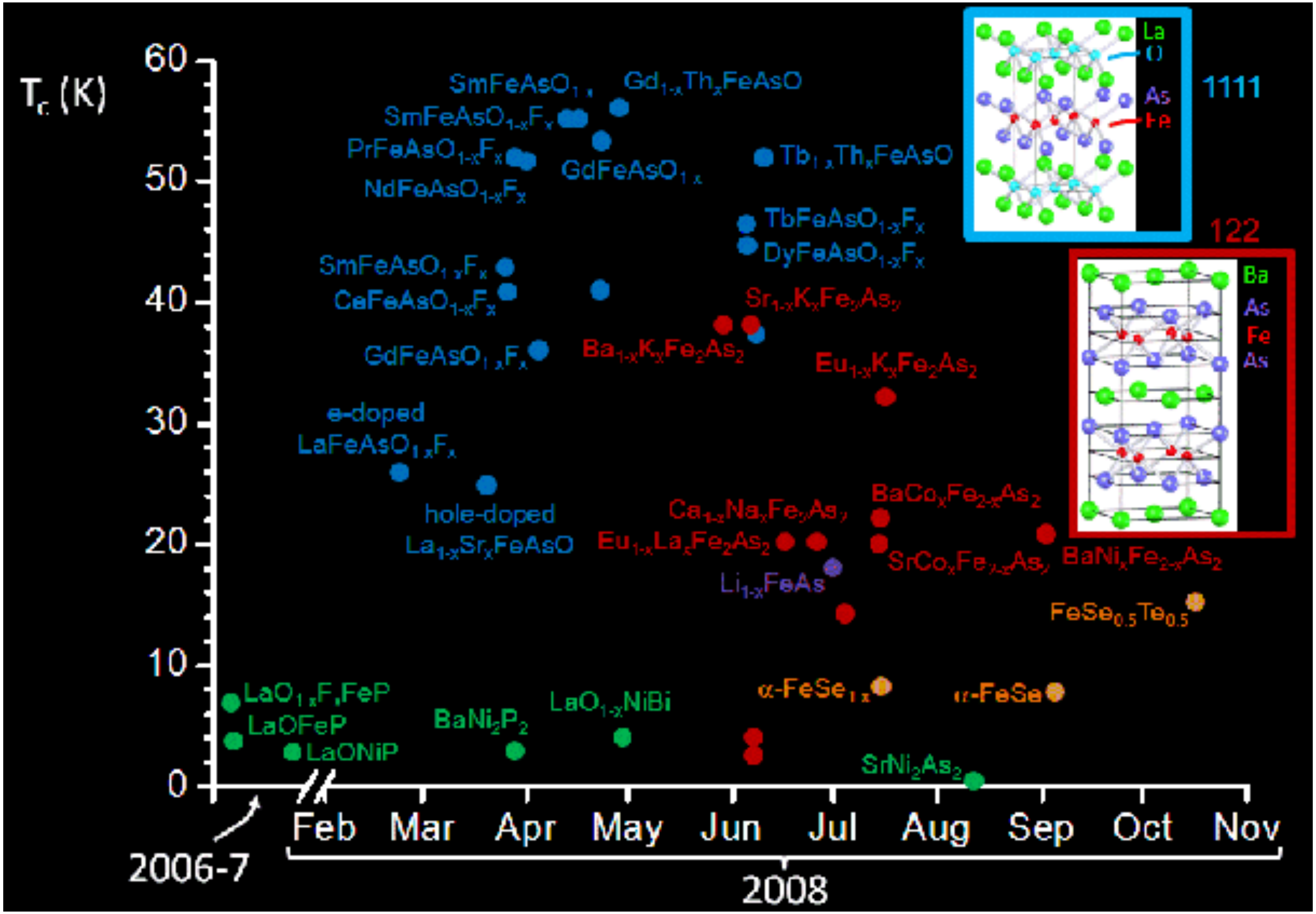
Many different families discovered, all sharing a Fe plane



Nandi et al, PRL 104, 057006 (2010)

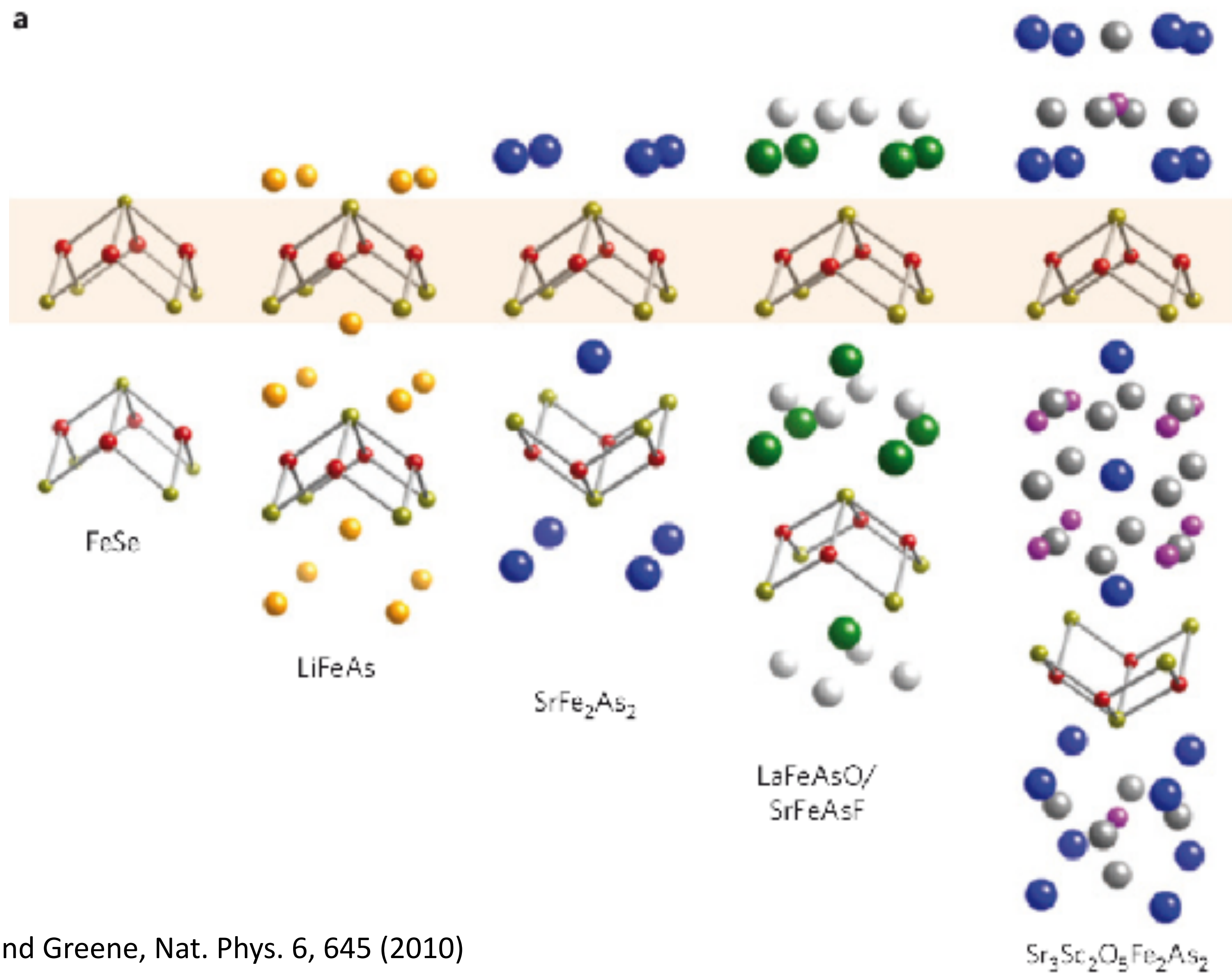


An important breakthrough because it could help understand cuprates



T_c (K)
SmFeAsO$_{1-x}$
Gd$_{1-x}$Th$_x$FeAsO
SmFeAsO$_{1-x}$F$_x$
PrFeAsO$_{1-x}$F$_x$
GdFeAsO$_{1-x}$
Tb$_{1-x}$Th$_x$FeAsO
NdFeAsO$_{1-x}$F$_x$
TbFeAsO$_{1-x}$F$_x$
DyFeAsO$_{1-x}$F$_x$
SmFeAsO$_{1-x}$F$_x$
CeFeAsO$_{1-x}$F$_x$
Sr$_{1-x}$K$_x$Fe$_2$As$_2$
GdFeAsO$_{1-x}$F$_x$
Ba$_{1-x}$K$_x$Fe$_2$As$_2$
Eu$_{1-x}$K$_x$Fe$_2$As$_2$
e-doped
LaFeAsO$_{1-x}$F$_x$
hole-doped
La$_{1-x}$Sr$_x$FeAsO
Ca$_{1-x}$Na$_x$Fe$_2$As$_2$
BaCo$_x$Fe$_{2-x}$As$_2$
Eu$_{1-x}$La$_x$Fe$_2$As$_2$
SrCo$_x$Fe$_{2-x}$As$_2$
BaNi$_x$Fe$_{2-x}$As$_2$
Li$_{1-x}$FeAs
FeSe$_{0.5}$Te$_{0.5}$
LaO$_{1-x}$F$_x$FeP
LaOFeP
LaONiP
BaNi$_2$P$_2$
LaO$_{1-x}$NiBi
α-FeSe$_{1-x}$
α-FeSe
SrNi$_2$As$_2$
0
10
20
30
40
50
60
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
2006-7
2008
1111
La
O
As
Fe
122
Ba
As
Fe
As

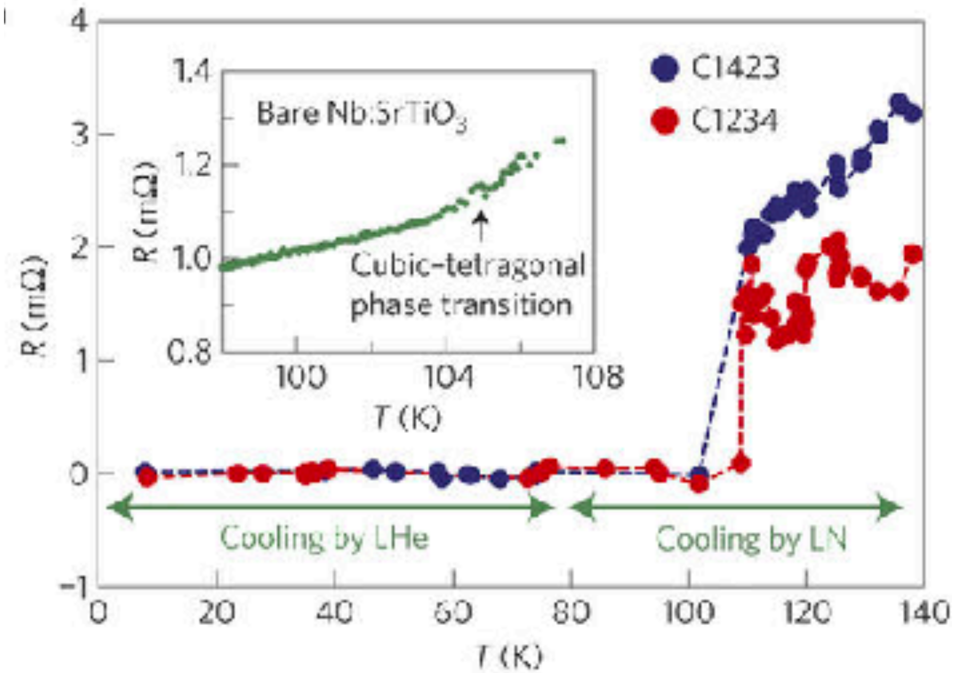
Fe-As or Fe-Se planes



Paglione and Greene, Nat. Phys. 6, 645 (2010)

HIGHEST T_C IN Fe SUPERCONDUCTORS

Single layer FeSe on doped $SrTiO_3$



Nature Materials 14, 285–289 (2015)

Fe BASED SUPERCONDUCTORS

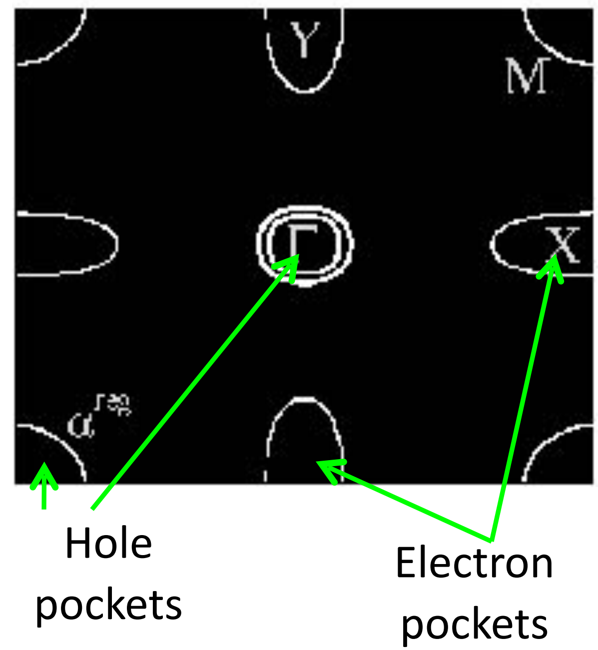
Differences with cuprates:

- The AF state is metallic (not Mott insulator): Hund metal (correlations).
- Multiorbital system (more than 1 gap possible)
- SPC can be achieved without chemical doping.
- More 3dim-like (less anisotropy in c-direction)

Proposed mechanisms for superconductivity

- Spin fluctuations (π,0)
- Orbital fluctuations
- Possible role of nematicity

Extended BZ (1 Fe unit cell)



Gap symmetry

Family	Full gap	Highly anisotropic	Strong nodal
1111	$PrFeAsO_{1-y}$[52K] [293] SmFeAs(O,F)[55K] [295]	LaFeAs(O,F)[26K] [214] NdFeAs(O,F) [214]	LaFePO[6K] [203, 204, 294]
122	$(Ba,K)Fe_2As_2$[40K] [146, 236, 296, 242] $Ba(Fe,Co)_2As_2$ [OP,23K] [238, 208]	$Ba(Fe,Co)_2As_2$ [OD] [238, 241]* $Ba(Fe,Ni)_2As_2$ [297]* $Ba(Fe,Co)_2As_2$ [UD] [241]*	KFe_2As_2 [4K] [211, 309] $BaFe_2(As,P)_2$[OP,31K] [205, 149] $(Ba,K)Fe_2As_2$ [UD] [242]
111	LiFeAs [18K] [298, 258]		LiFeP [6K] [299]
11		Fe(Se,Te) [27K] [231, 246]	

arXiv:1106.3712

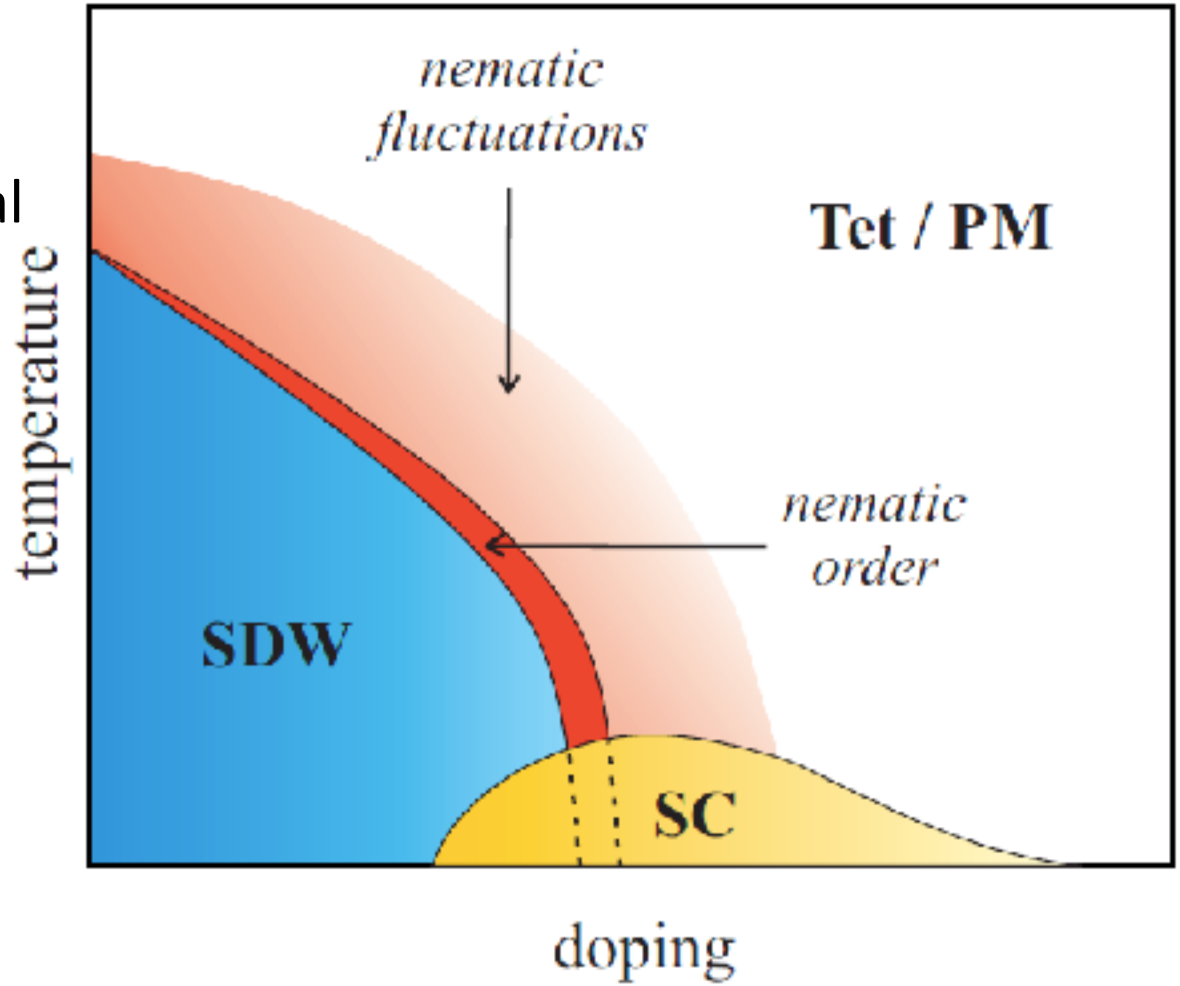
Fe BASED SUPERCONDUCTORS

Role of <u>nematicity</u>?

Breaks the tetragonal symmetry.

Related to magnetic fluctuations.

Nematic order postulated for the pseudogap in cuprates.



http://www.ifsc.usp.br/coloquio/2013/Fernandes.pdf

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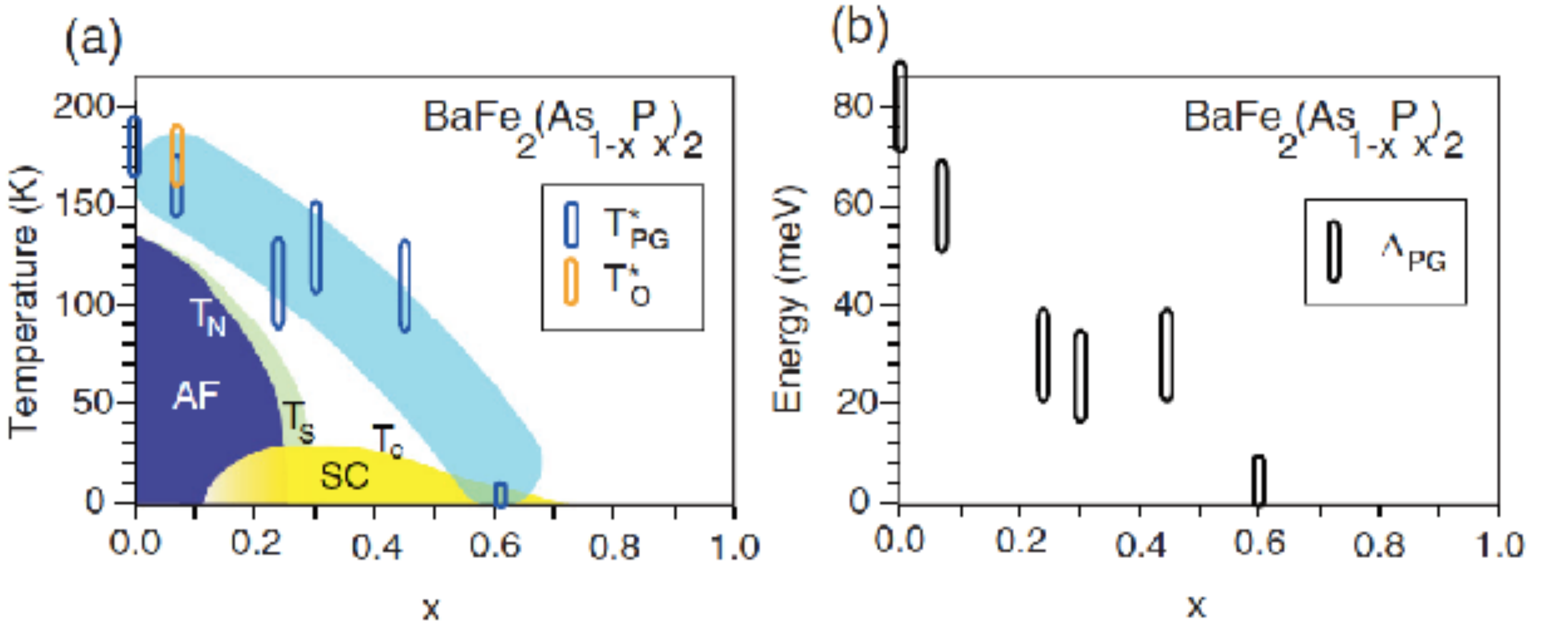
Fe BASED SUPERCONDUCTORS

Pseudogap

Gap opening due to spin fluctuations: Nat. Comm. **2**, 392 (2011)



Accompanied by orbital ordering. PRB 89, 045101 (2014)

MECHANISMS

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SPIN-FLUCTUATION MECHANISM

Spin fluctuations play the role of phonons (à la BCS).

The “normal” state can be described as a nearly AF Fermi liquid (unconventional Fermi liquid close to an AF instability).

Note: calculating the effective interaction is not trivial because vertex corrections can be important (Migdal’s theorem doesn’t apply)

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SPIN-FLUCTUATION MECHANISM

If the magnetic susceptibility has a peak at q (remember nesting) the interaction is also peaked at q and positive.

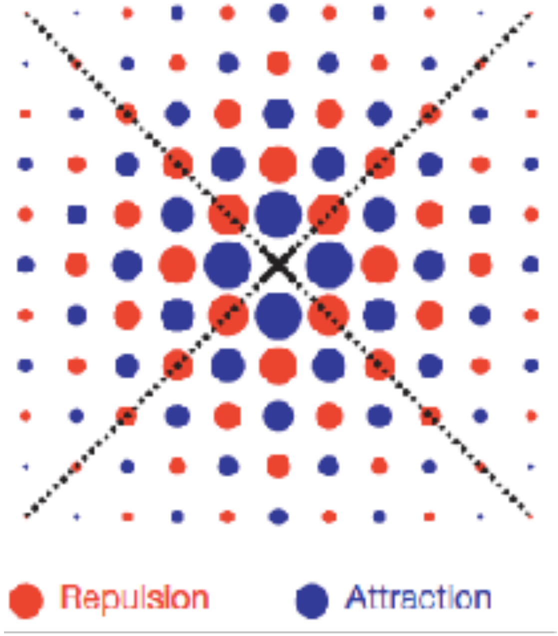
$$\Gamma_s(\boldsymbol{k},\boldsymbol{k}') = \frac{3}{2}U^2\frac{\chi_0(\boldsymbol{q})}{1-U\chi_0(\boldsymbol{q})}$$

If you look at the interaction in real space

It changes sign with position!!

Attractive interaction by space avoidance

q=(π,π)

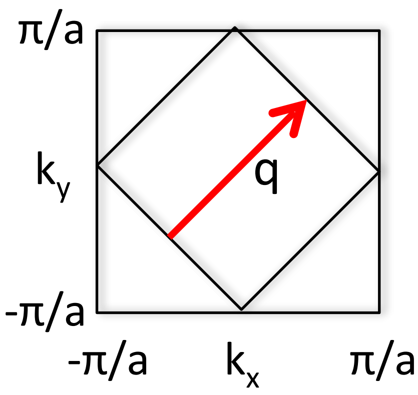


Nature 450, 1177 (2007)

Scalapino, Phys. Rep. 1995
Hirshfield et al. 1106.3712

SPIN-FLUCTUATION MECHANISM

Square lattice at half-filling:



At q=(π,π) the spin susceptibility is maximal (nesting)

$$V_{k,k'} = V_{k-k'} = V_q \propto \chi_q > 0 \qquad q = (\pi,\pi)$$

To fullfil the gap equation, you need an anisotropic gap such that

$$\Delta_k = -\frac{1}{\Omega}\sum_{kk'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}}$$

$$\mathrm{sign}(\Delta_{\mathbf{k}}) = -\,\mathrm{sign}(V_{\mathbf{k},\mathbf{k}'})\mathrm{sign}(\Delta_{\mathbf{k}})$$

d-wave

$$\boxed{\begin{aligned} \Delta_k &= \frac{\Delta_0}{2}(\cos k_x - \cos k_y) \\ \Delta_{k+(\pi,\pi)} &= \frac{\Delta_0}{2}(-\cos k_x + \cos k_y) = -\Delta_k \end{aligned}}$$

Scalapino, Phys. Rep. 1995
Hirshfield et al. 1106.3712

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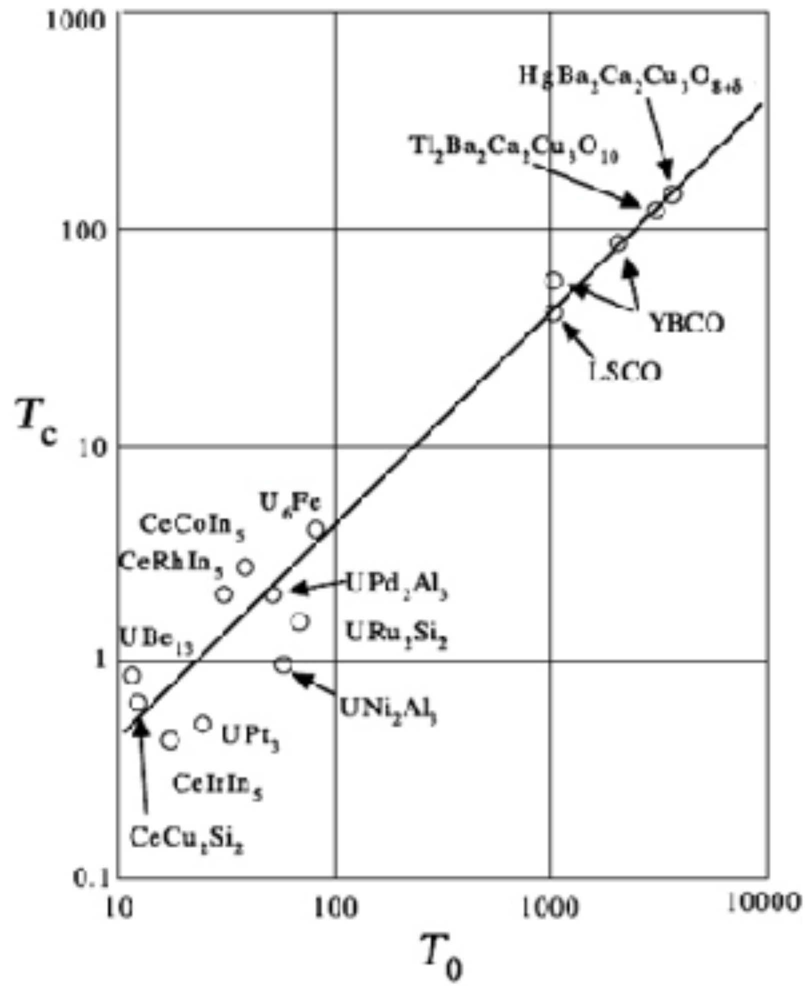
SPIN-FLUCTUATION MECHANISM

Correlation between Tc and a typical energy scale of spin fluctuations.

In the Scalapino approach, the t-J model is the basis for understanding.



Moriya-Ueda, Rep. Prog. Phys. 66, 1299 (2003)

Scalapino, Rev. Mod. Phys. 84, 1383 (2012)

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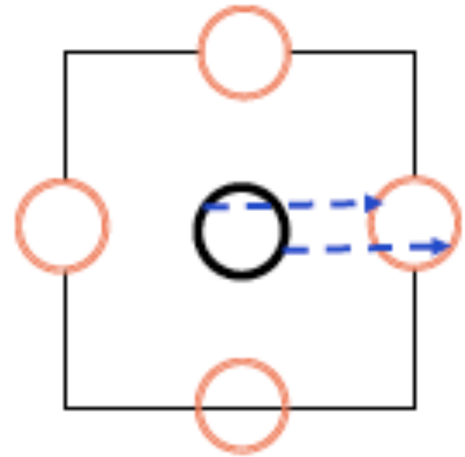
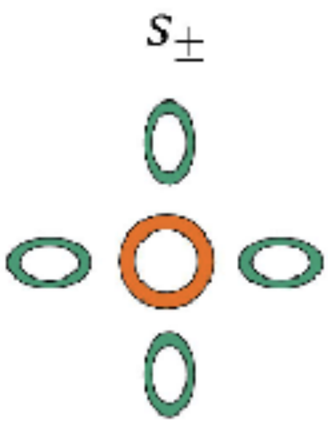
SPIN-FLUCTUATION MECHANISM

For a murtiorbital system (as Fe-superconductors)

For instance, spin fluctuations related to nesting between electron and hole pockets: q=(π,0)





This would lead to an $s_{\pm}$

(as the order parameter averages to zero on the Fermi surface, you also get "Coulomb avoidance")

arXiv:0901.4790

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ELECTRON-PHONON

Not discarded!

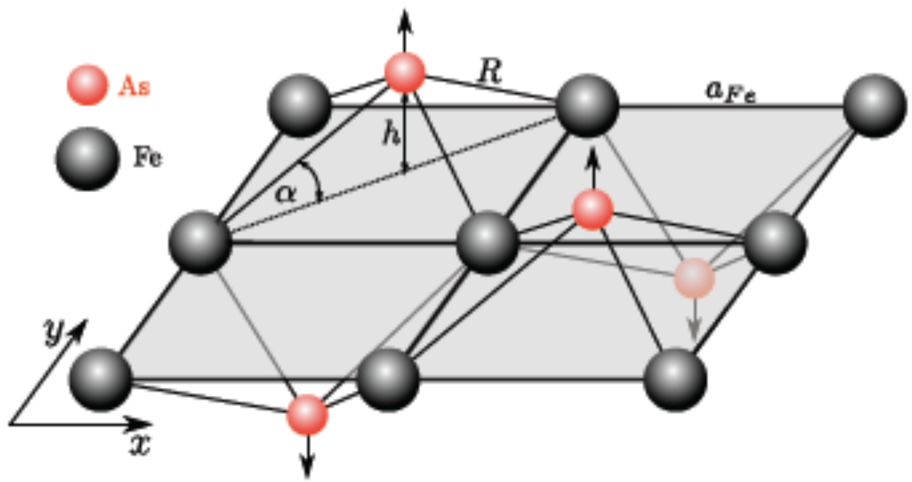
For cuprates: Phys. Rev. Lett. 105, 257001

Maybe coupled to other degrees of freedom?

For Fe-superconductors

Spin-phonon?

Adv. in Cond. Matt. Phys. 2010, 164916 (2010)



PRB 88, 165106 (2013)

Orbital fluctuations induced by electron-phonon.

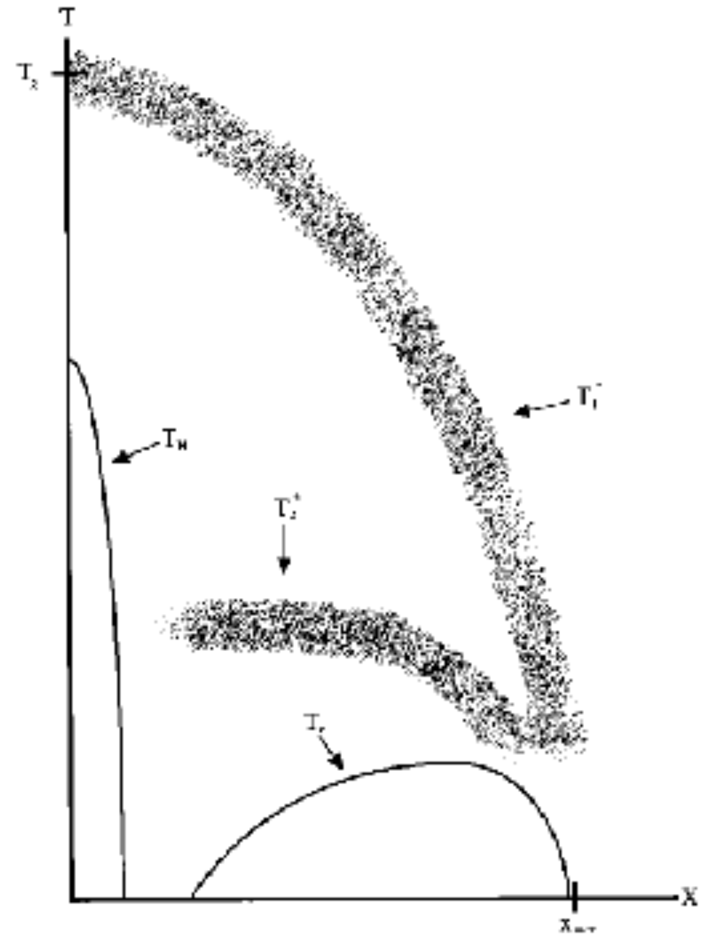
PRL 104, 157001

Interplay of many degrees of freedom may be relevant.

KINETIC ENERGY DRIVEN MECHANISM

Carlson et al, Chapter 21

Different energy scales involved for underdoped cuprates



PRB 56, 6120

T*: Phase transition or crossover
(no exp. evidence of phase transition)

$$\mathbf{T_c < T^*_{pair} < T^*_{stripe}}$$

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KINETIC ENERGY DRIVEN MECHANISM

Carlson et al, Chapter 21

Different energy scales involved for underdoped cuprates

At T^*_{stripe}: stripe formation. Stripes are rivers of charge where holes can move (gain kinetic energy in 1dim). In between, AF regions where the carriers are localized.

At T^*_{pair}: local pairing (spin-gap) within the 1DEGs (stripes). Pairs can tunnel to neighboring 1DEG (which in principle has another k_F). This way the system gains kinetic energy in a perpendicular direction as well. There is finite Δ but not fixed phase.

At T_c: phase coherence sets in (Josephson coupling between stripes) $\rightarrow$ superconductivity.

$$T_c < T^*_{pair} < T^*_{stripe}$$

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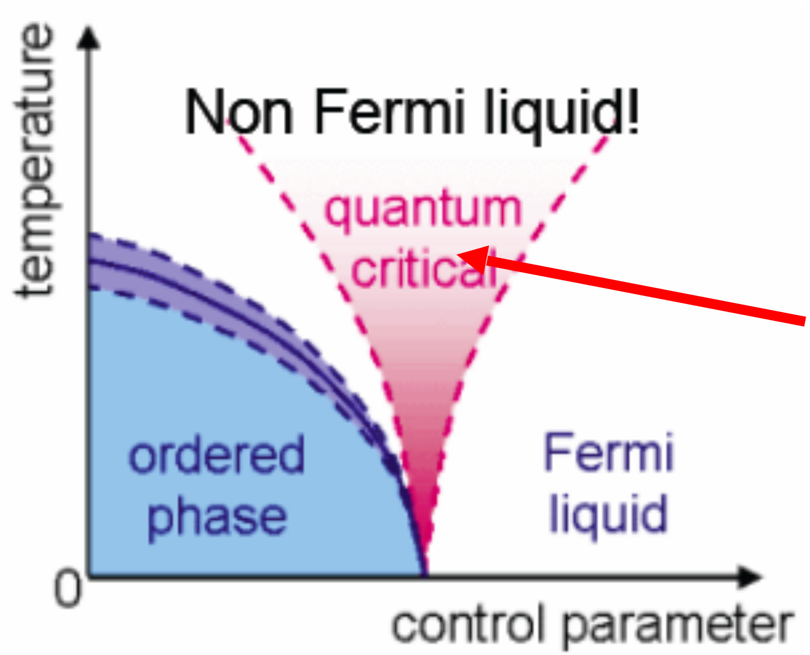
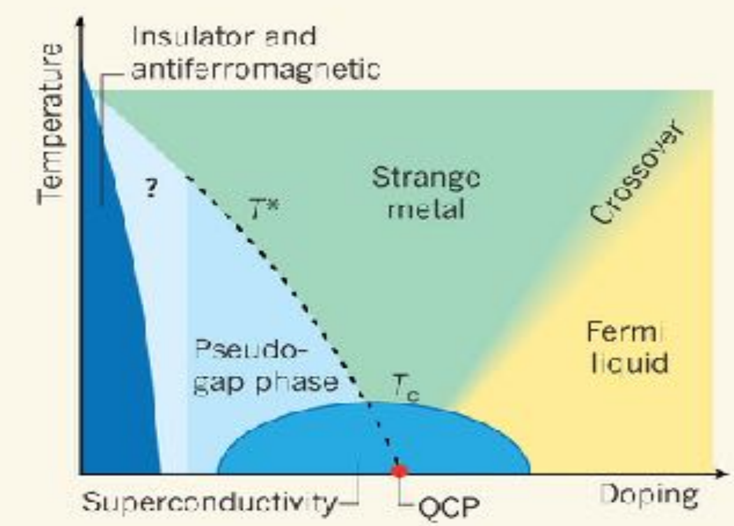
QUANTUM PHASE TRANSITIONS



Region in which termal and quantum fluctuations are equally important

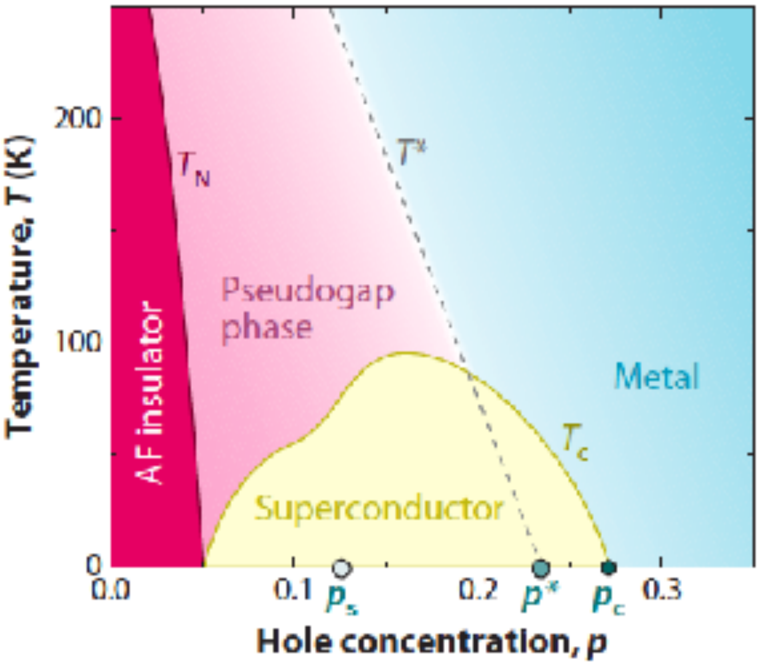
Transition at T=0 with consequences for T>0.

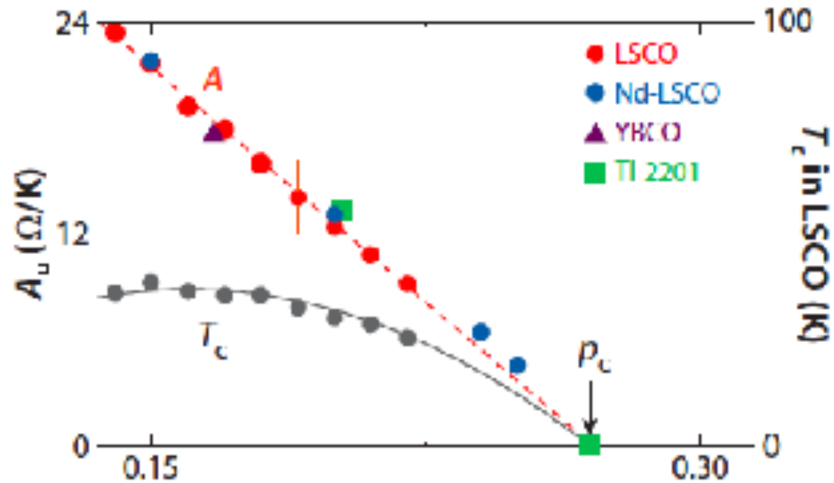
Divergence of time coherence.



Nature 468, 184–185

Taillefer Annu Rev Cond Matter Phys (2010)





Resistivity linear with T above the spc phase appears at p_c (doping at the Quantum Critical Point).

Linear T resistivity coming from fluctuations in the pseudogap phase (spin density wave order).

In organic spc and Fe pnictides, similar phenomena coming from spin fluctuations.

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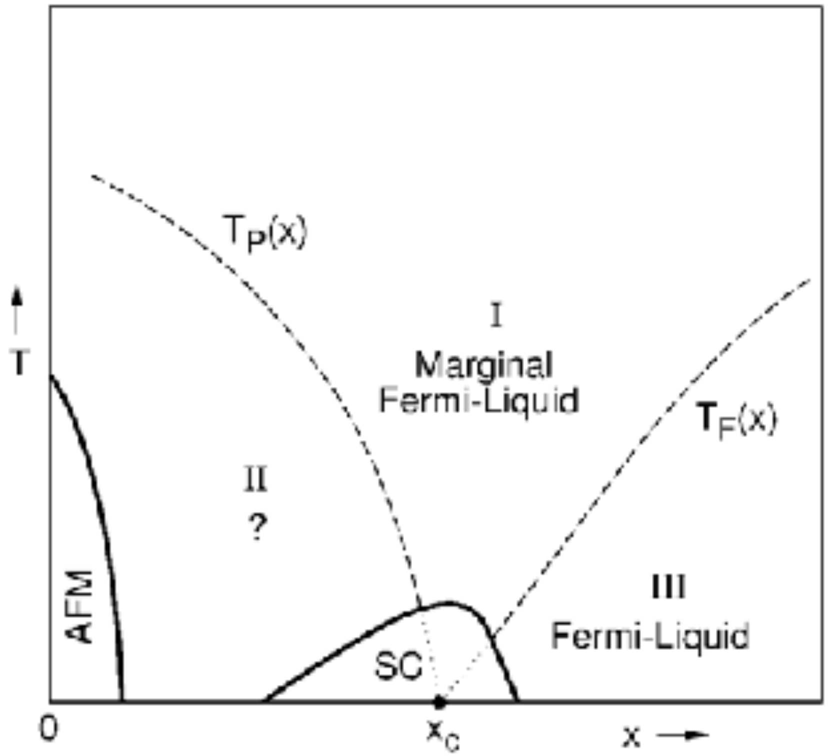
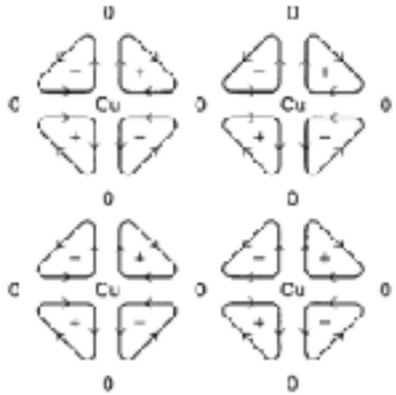
HIDDEN ORDER

Idea: the pseudogap phase is in fact a gapped phase (pseudo in experiment due to imperfections or dynamic effects)



PRL 83, 3538

Circulating currents, PRB 55, 14554



Charge order PRL 87, 056401

d-density wave PRB 63, 094503

Nematic phase arXiv:1404.0362

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RESONATING VALENCE BONDS

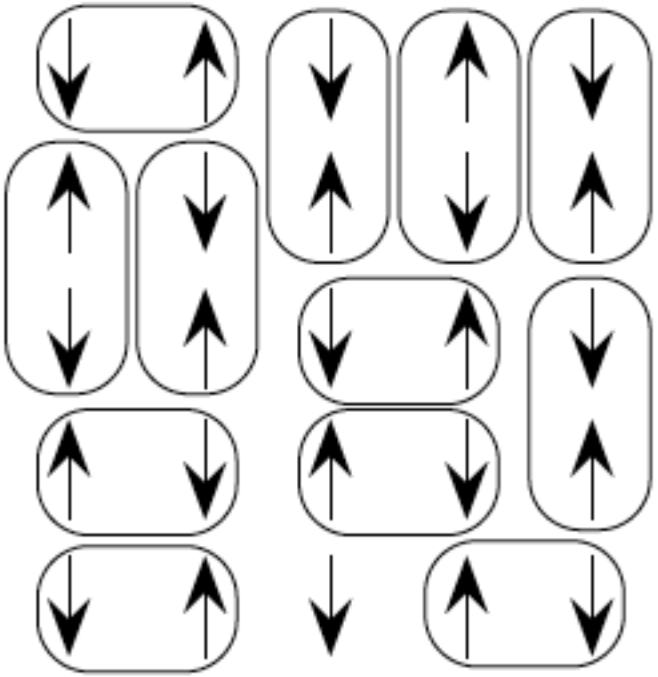
(Anderson)

Mottness from the start.

Pairing scale very large
(related to T*)

Quantum fluctuations destroy long range order leading to a spin liquid of singlets.

However, spin liquid not found on cuprates.



http://www.msd.anl.gov/files/msd/cuprates-columbia.pdf

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POST HIGH T_C PRINCIPLES TO FIND SPC

MAZIN, NATURE 464, 183 (2010)

- Layered structures
- Carrier density should not be too high (compared to conventional metals)
- Transition metals of the fourth period (3d) are good
- Magnetism is essential
- Proper Fermi surface geometry is essential (in relation to spin excitations)

Corollary: work with solid state chemists (you need complex chemical compounds)

.... Really?

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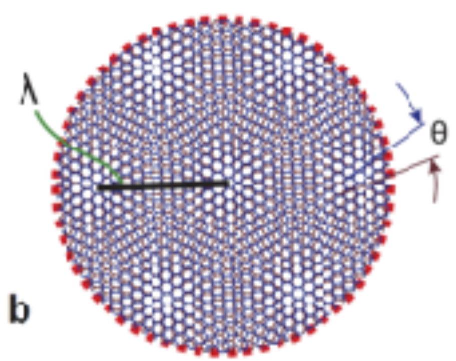
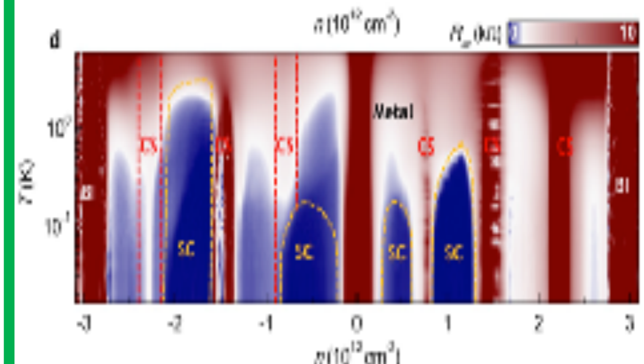
SUPERCONDUCTIVITY IN MULTILAYERED GRAPHENE (2018-)

Moiré systems:

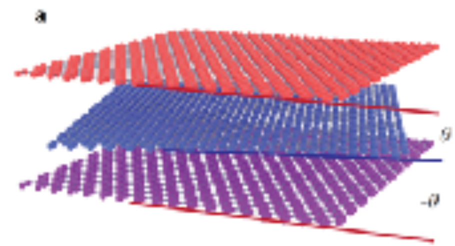
Twisted bilayer graphene

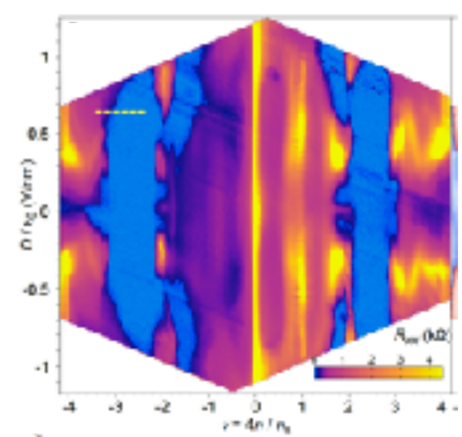




Cao et al, Nature 556, 80 (2018); Nature 556, 43 (2018)
Lu, et al Nature 574, 653 (2019)

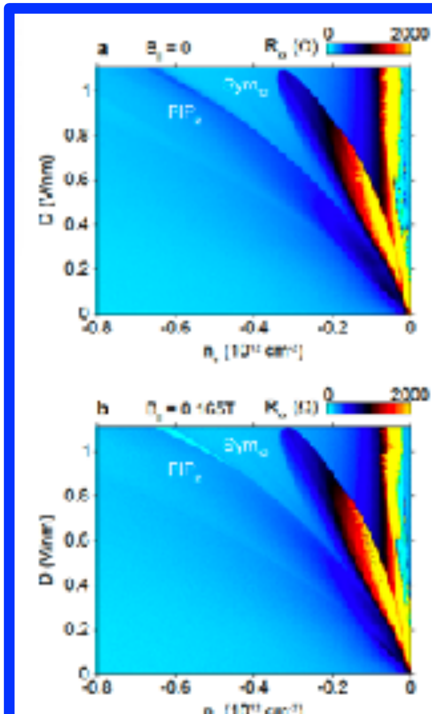
Mirror-symmetric twisted trilayer, quatrilayer and pentalayer graphene





Park et al, Nature 590, 249 (2021)
Zhang et al, Science 377, 1538 (2022)
Park et al Nat. Mater. 21, 877 (2022)

Non-moiré systems:



Bernal bilayer: Zhou et al Science 375, 774 (2022)

ABC trilayer: Zhou et al, Nature 598, 429; 598, 434 (2021)

OUTLINE

- **Superconductivity**
  - Properties (zero resistivity, Meissner effect)
  - Understanding (pairing, BCS, Ginzburg-Landau)
  - Electron-phonon interaction (conventional superconductivity)
- **Unconventional superconductivity (unsolved)**
  - What are the new issues.
  - What are the proposals.

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