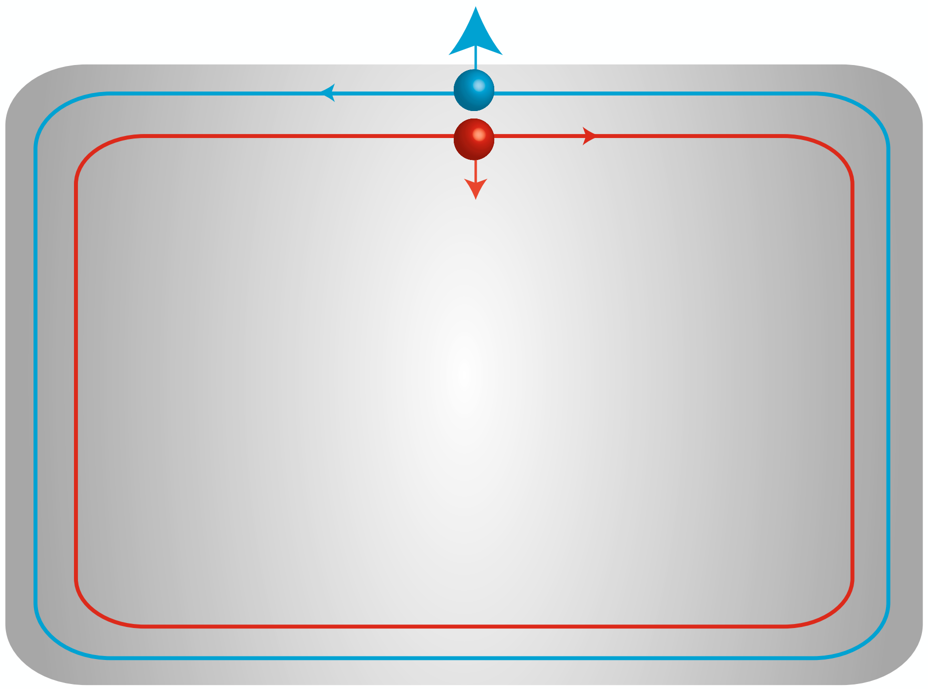
As you have seen in previous lectures:

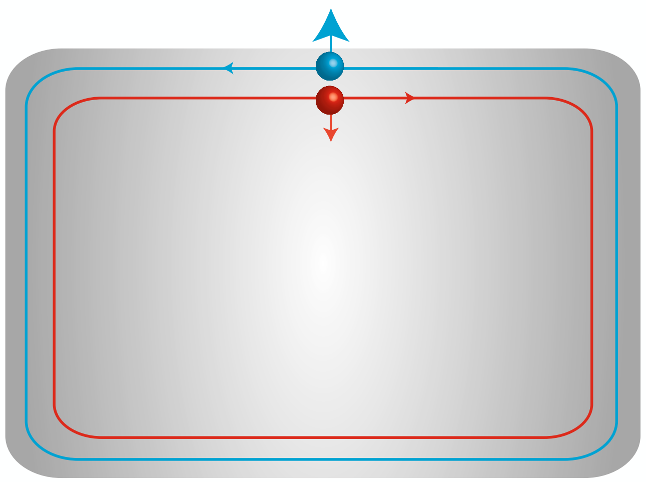
- The main propery of 2D TIs is the presence of counterpropagating spin-filtered edge states (helical states) that are connected by time-reversal symmetry.

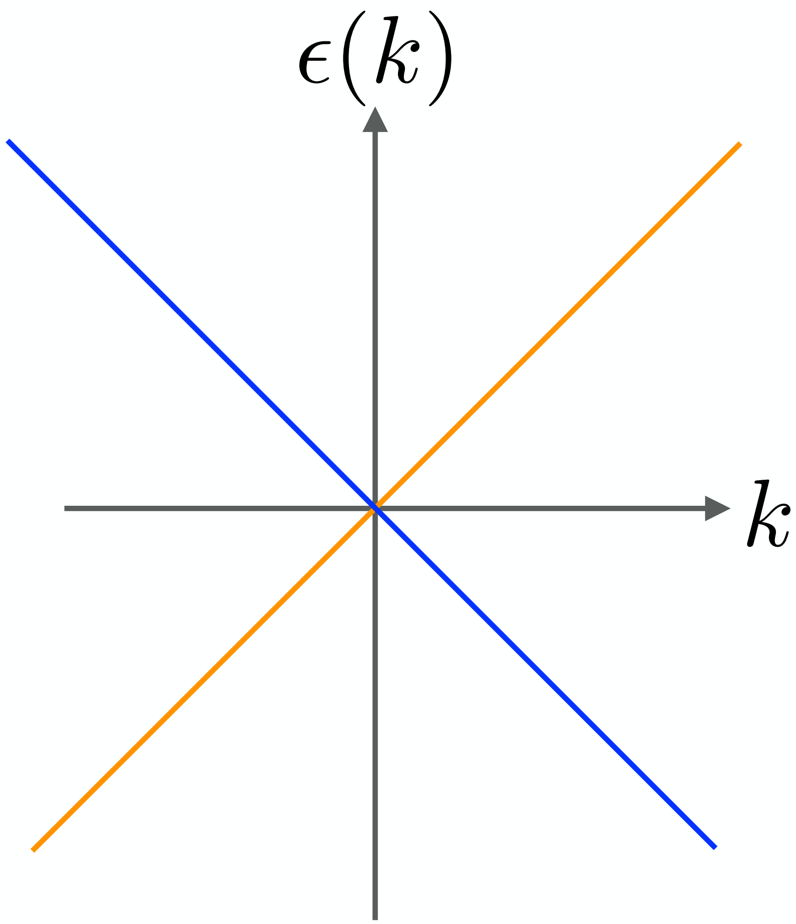
- The bulk is gapped.

- Counterpropagating edge modes are Kramer's partners.



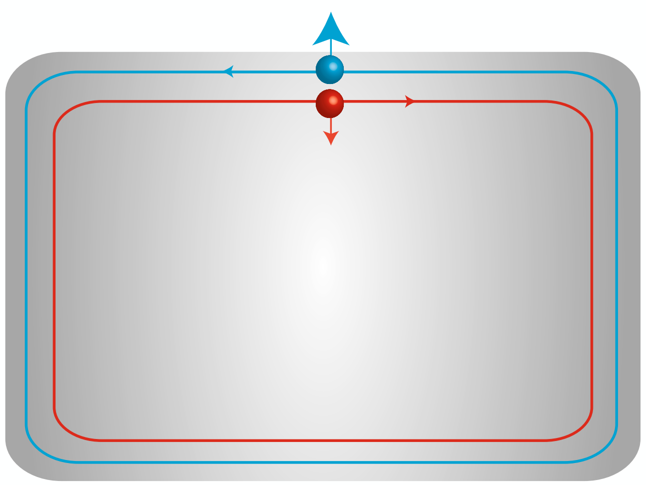
$$H_{2DTI} = \int dx \Psi^\dagger (-iv\partial_x \sigma^z - \mu)\Psi$$

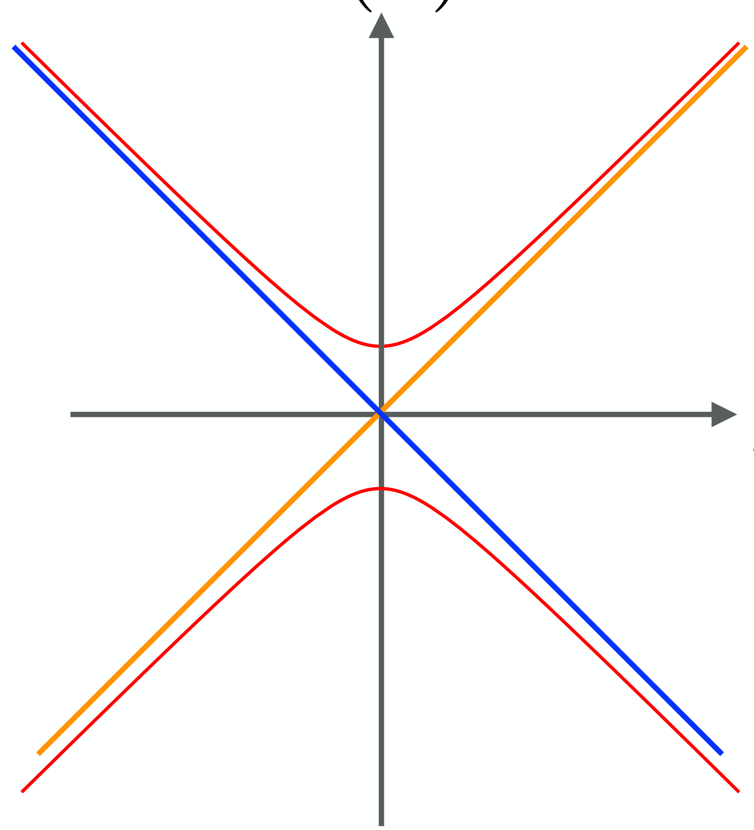




$$H_{2DTI} = \int dx \Psi^\dagger (-iv\partial_x \sigma^z - \mu)\Psi$$

Physical realisations for engineering p-wave topological superconductors



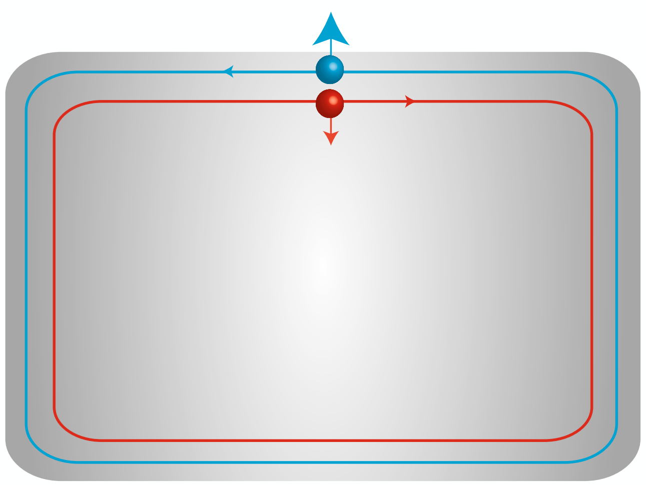


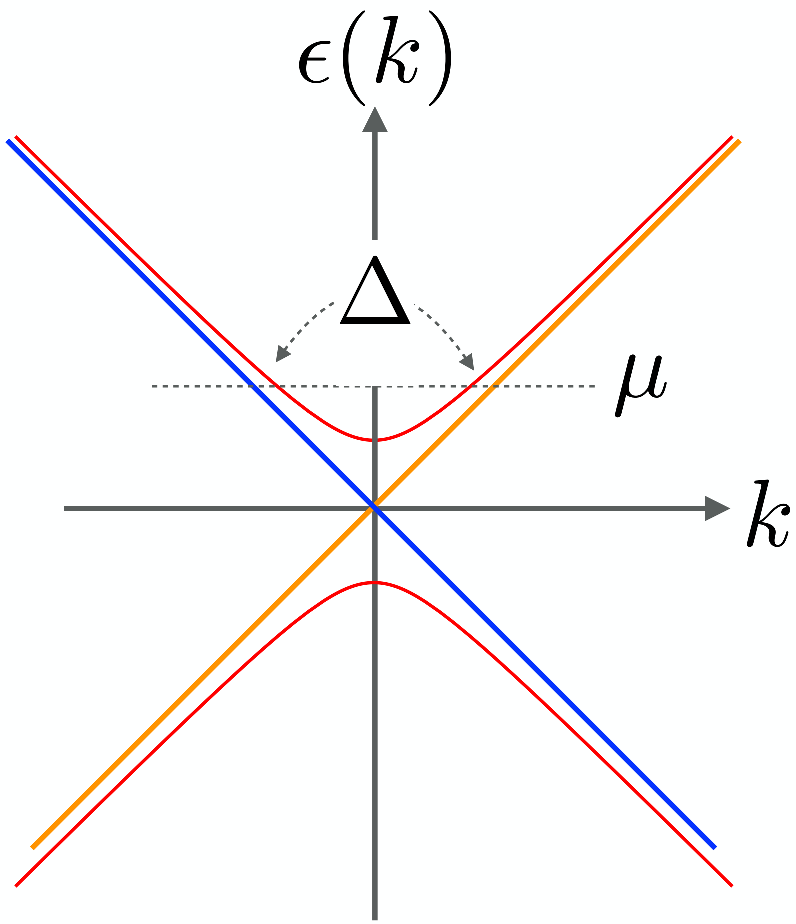
$$H_Z = -h \int dx \Psi^\dagger \sigma^x \Psi$$

A gap opens owing to a mass term: Zeeman interaction that breaks time-reversal symmetry. Edge spectrum becomes

$$\epsilon_\pm(k) = -\mu \pm \sqrt{(vk)^2 + h^2}$$

$$H_{2DTI} = \int dx \Psi^\dagger (-iv\partial_x \sigma^z - \mu) \Psi$$





$$H_Z = -h \int dx \Psi^\dagger \sigma^x \Psi$$

A gap opens owing to a mass term: Zeeman interaction that breaks time-reversal symmetry. Edge spectrum becomes

$$\epsilon_\pm(k) = -\mu \pm \sqrt{(vk)^2 + h^2}$$

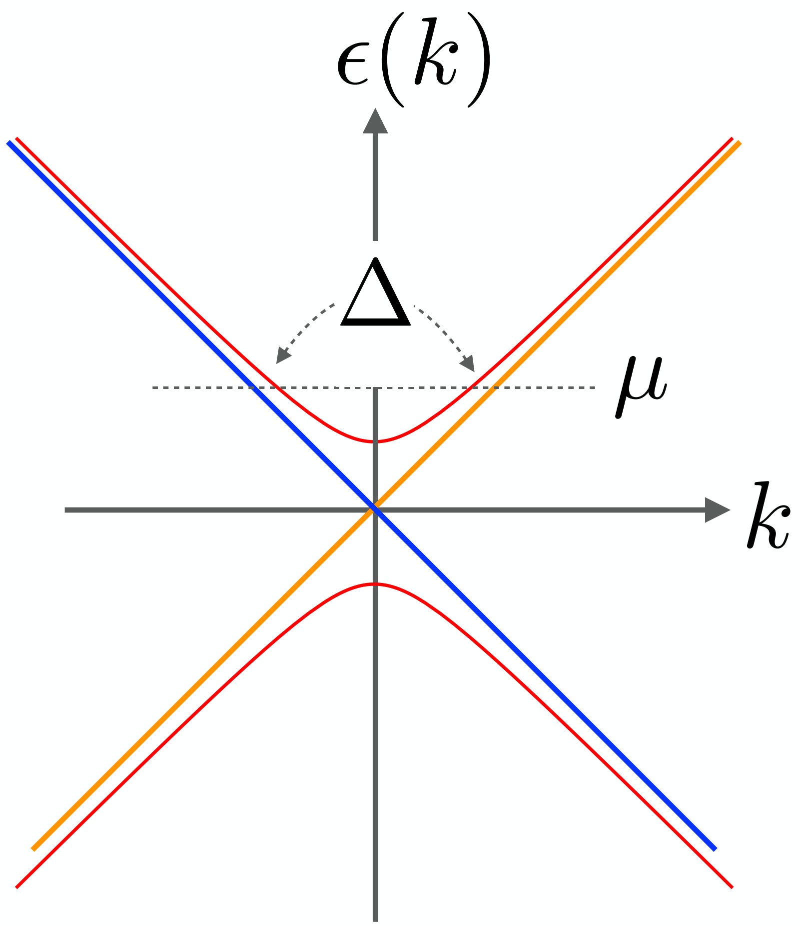
$$H_{2DTI} = \int dx \Psi^\dagger(-iv\partial_x \sigma^z - \mu)\Psi$$

$$H_\Delta = \int dx \Delta(\Psi_\uparrow^\dagger \Psi_\downarrow^\dagger + h.c)$$

Important: pairing comes from s-wave proximity effect

The Hamiltonian in the new basis acquires p-wave pairing terms!!

$$H_\pm = \int \frac{dk}{2\pi} \{\epsilon_+(k)\psi_+^\dagger(k)\psi_+(k) + \epsilon_-(k)\psi_-^\dagger(k)\psi_-(k)$$
$$+ \frac{\Delta_p}{2}[\psi_+(-k)\psi_+(k) + \psi_-(-k)\psi_-(k) + h.c] + \frac{\Delta_s}{2}[\psi_-(-k)\psi_+(k) + h.c]\}$$

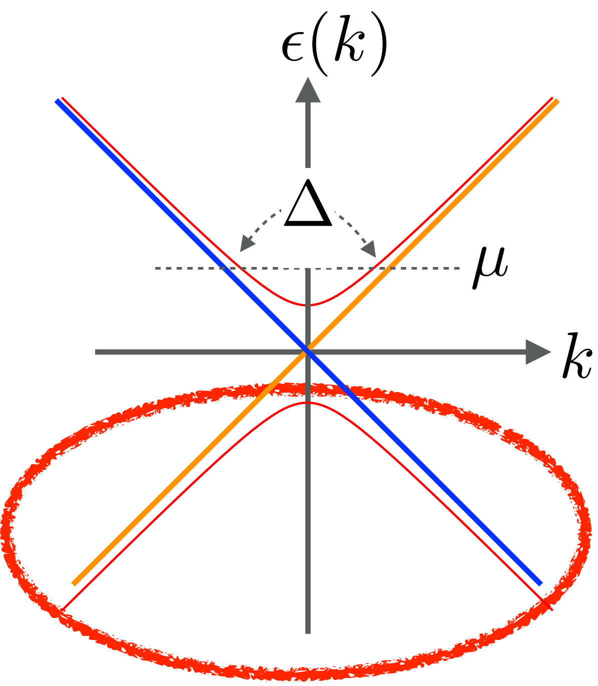


$$\Delta_p(k) = \frac{vk\Delta}{\sqrt{(vk)^2 + h^2}}$$

$$\Delta_s(k) = \frac{h\Delta}{\sqrt{(vk)^2 + h^2}}$$

$$\epsilon_\pm(k) = -\mu \pm \sqrt{(vk)^2 + h^2}$$

Fu and Kane, PRB **79**, 161408(R), 2009



Project out the lowest band

$$\psi_- \rightarrow 0$$

$$\epsilon_+(k\rightarrow 0) \approx -(\mu - h) + \frac{v^2}{2h}k^2 \equiv -\mu_{eff} + \frac{k^2}{2m_{eff}}$$

$$\Delta_p(k\rightarrow 0) \approx \frac{v\Delta}{h}k \equiv \Delta_{eff}k$$

$$H_{eff} = \int dx[\psi_+^\dagger(-\frac{\partial_x^2}{2m_{eff}} - \mu_{eff})\psi_+] + \frac{\Delta_{eff}}{2}(-\psi_+ i\partial_x\psi_+ + H.c.)]$$

Kitaev's model for 1D spinless p-wave superconductor, which is topological for

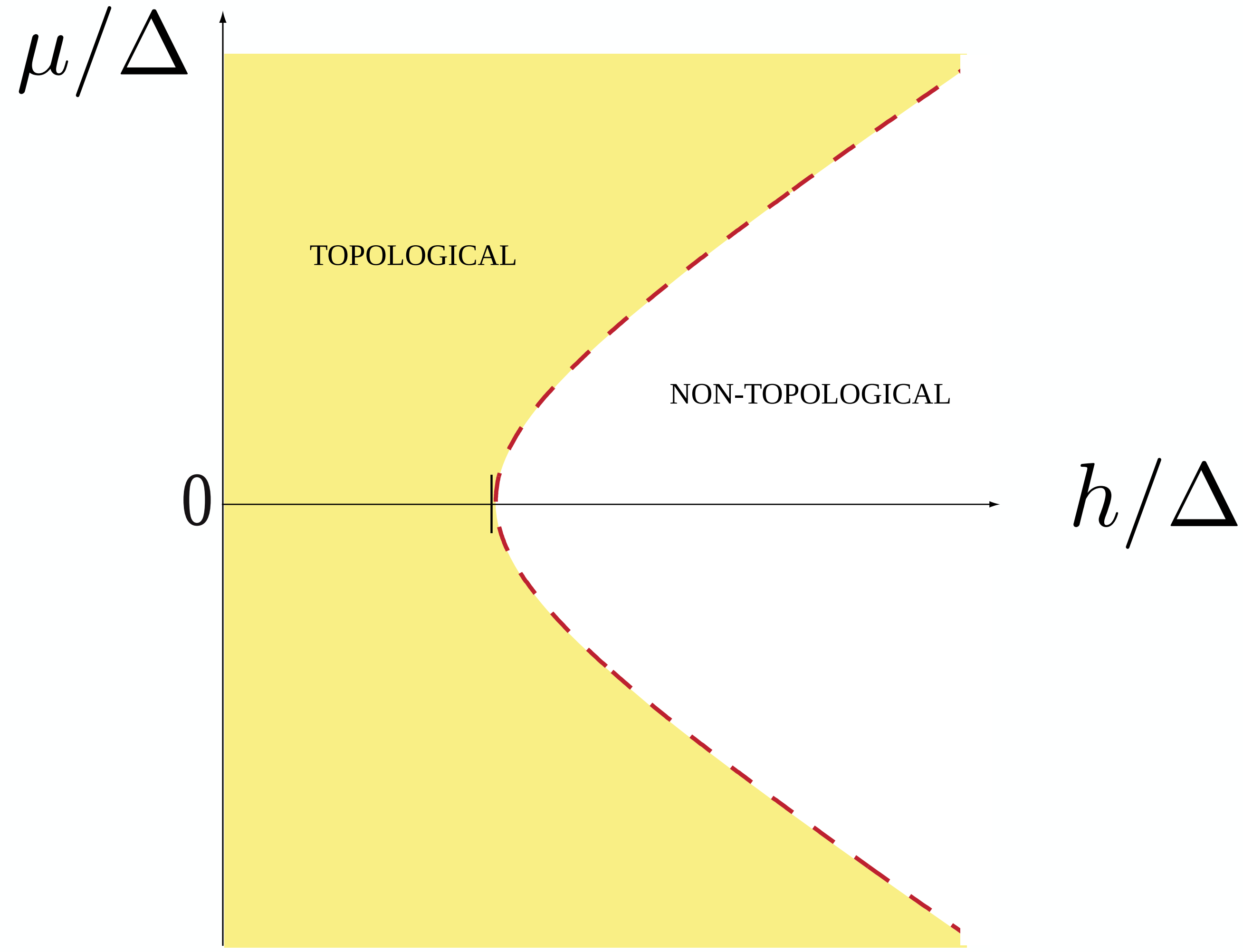
$$|\mu| \gtrsim h$$

$$E_{\pm}(k) = \sqrt{\Delta^2 + \frac{\epsilon_+^2 + \epsilon_-^2}{2} \pm (\epsilon_+ - \epsilon_-)\sqrt{\Delta_s^2 + \mu^2}}$$

$$E_{\pm}(k=0) = 0$$

Gap closing

$$h^2 = \Delta^2 + \mu^2$$

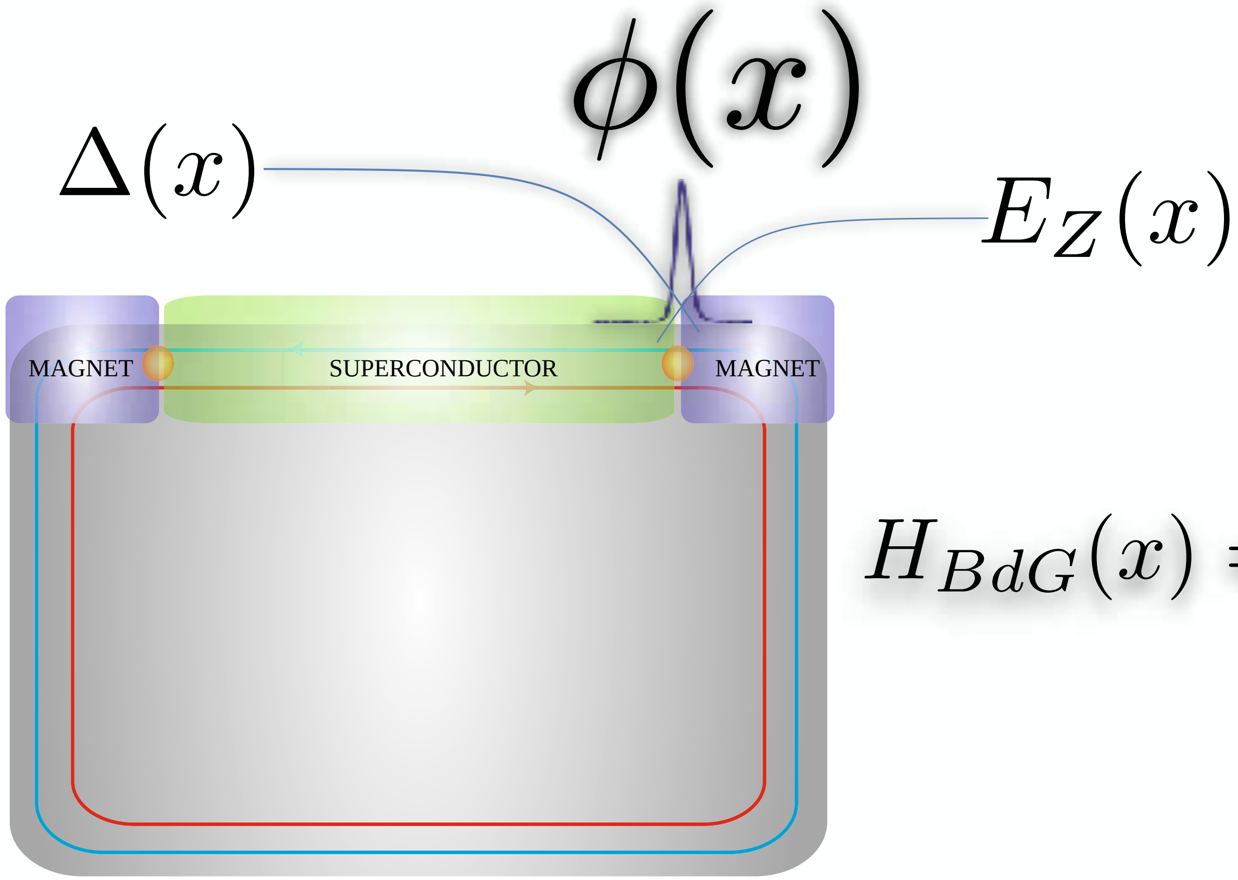


$$\Delta_p(k) = \frac{vk\Delta}{\sqrt{(vk)^2 + h^2}}$$

$$\Delta_s(k) = \frac{h\Delta}{\sqrt{(vk)^2 + h^2}}$$

$$\epsilon_{\pm}(k) = -\mu \pm \sqrt{(vk)^2 + h^2}$$

Fu and Kane, PRB **79**, 161408(R), 2009



$$H_{BdG}(x) = -iv_F\partial_x\sigma^x\tau^z + E_Z(x)\sigma^z + \Delta(x)\tau^x.$$

$$H_{BdG}(x)\Phi_0(x) = 0 \quad \Rightarrow \quad \Phi_0(x) = \frac{\phi(x)}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\phi(x) \sim e^{-\frac{1}{v}\int_0^x dx'[E_Z(x')-\Delta(x')]}$$

The envelope function is localised **at the boundary between the mass terms**

Fu and Kane, PRB **79**, 161408(R), 2009

Using $\hat{\Psi}(x) = \begin{pmatrix} c_\uparrow(x) \\ c_\downarrow(x) \\ -c_\downarrow^\dagger(x) \\ c_\uparrow^\dagger(x) \end{pmatrix}$ $\gamma = \int dx \Phi^\dagger(x)\hat{\Psi}(x)$

$$\gamma = \gamma^\dagger = \frac{1}{\sqrt{2}} \int dx \phi(x)[c_\downarrow(x) + c_\downarrow^\dagger(x)]$$

Self-conjugate Majorana

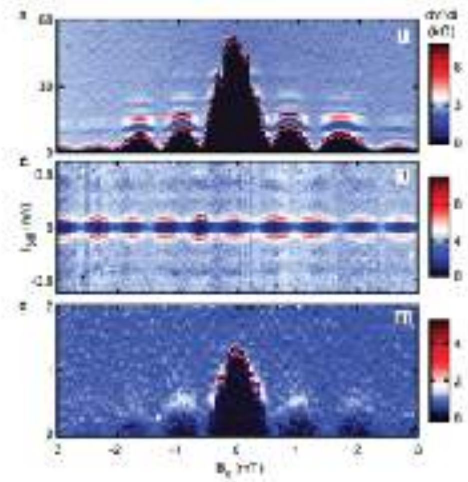
Fu and Kane, PRB **79**, 161408(R), 2009

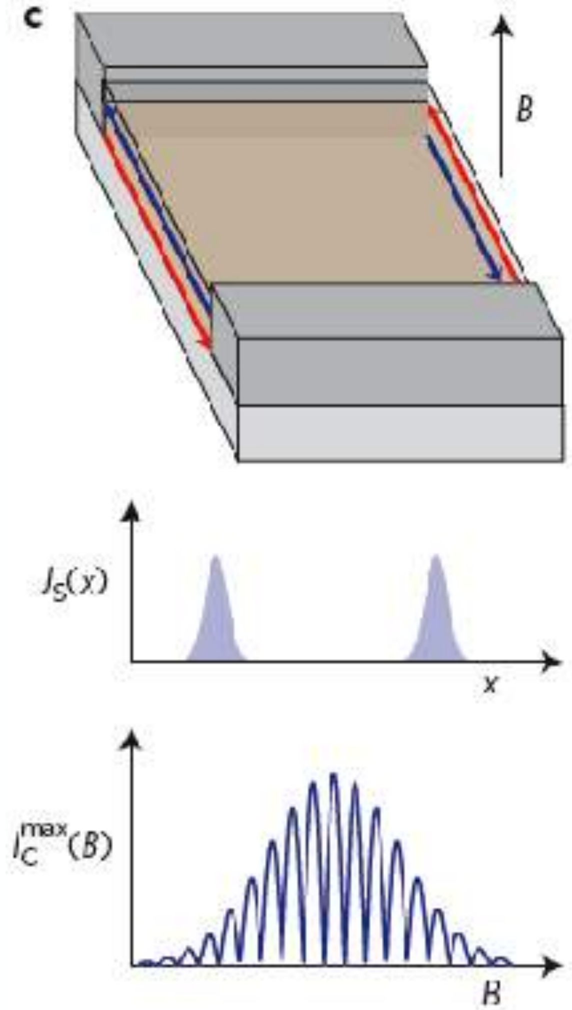
CSIC

LETTERS
PUBLISHED ONLINE: 3 AUGUST 2014 | DOI: 10.1038/NPHYS3036
nature physics

Induced superconductivity in the quantum spin Hall edge

Sean Hart[1†], Hechen Ren[1†], Timo Wagner[1], Philipp Leubner[2], Mathias Mühlbauer[2], Christoph Brüne[2], Hartmut Buhmann[2], Laurens W. Molenkamp[2] and Amir Yacoby[1*]

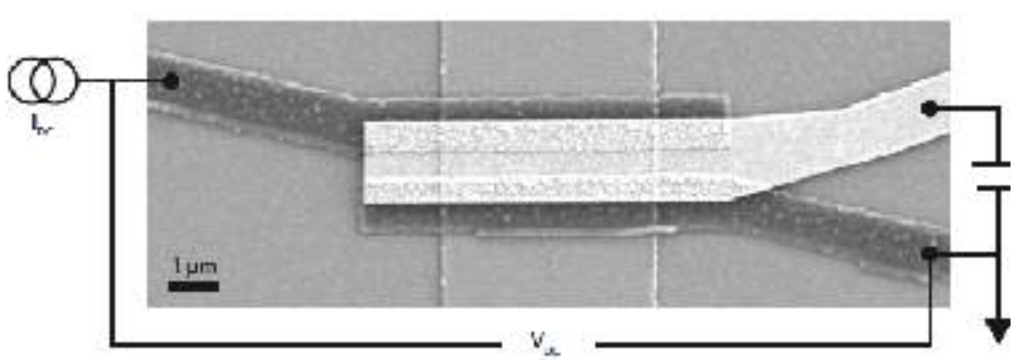


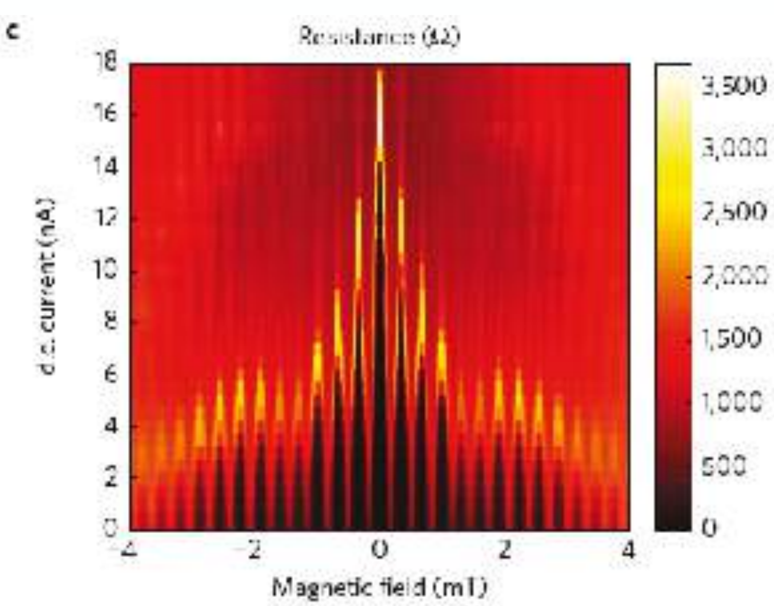


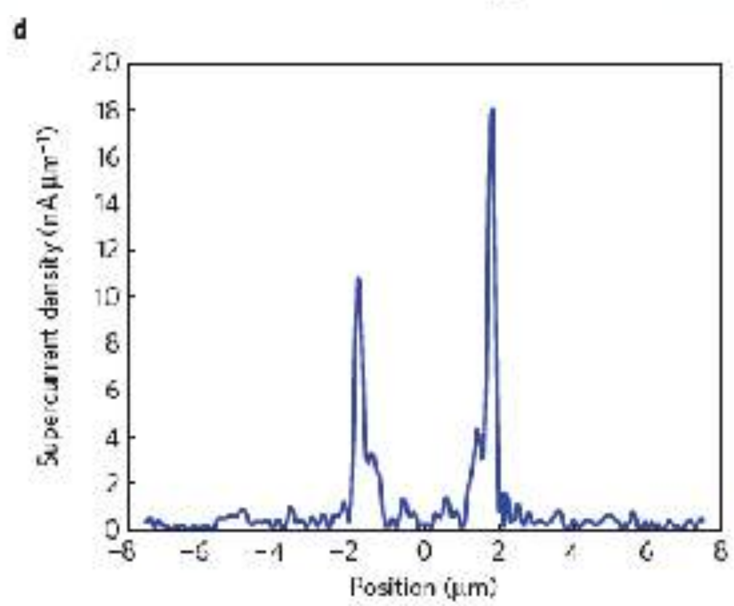
nature nanotechnology
LETTERS
PUBLISHED ONLINE: 11 MAY 2015 | DOI: 10.1038/NNANO.2015.86

Edge-mode superconductivity in a two-dimensional topological insulator

Vlad S. Pribiag[1†‡], Arjan J. A. Beukman[1‡], Fanming Qu[1‡], Maja C. Cassidy[1], Christophe Charpentier[2], Werner Wegscheider[2] and Leo P. Kouwenhoven[1*]







CSIC

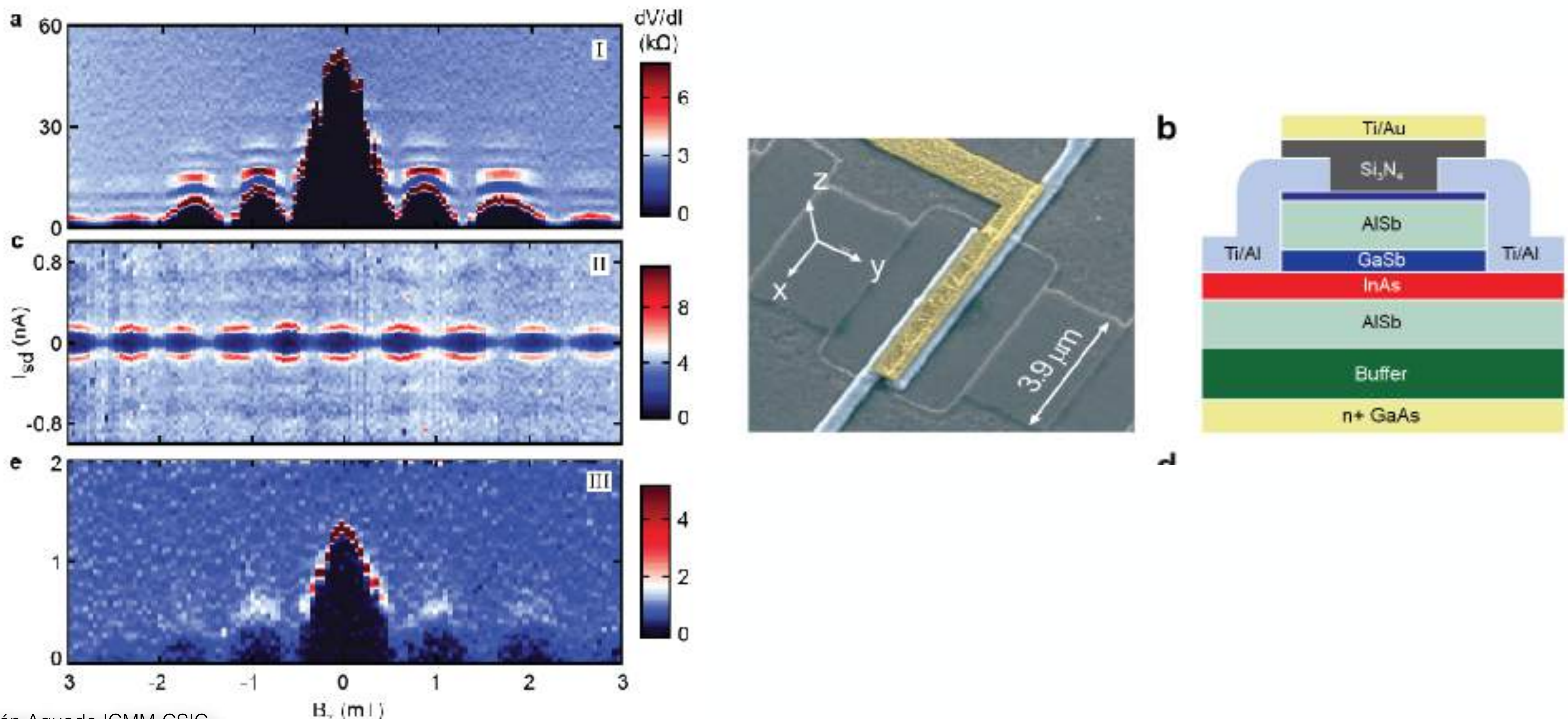
nature nanotechnology

LETTERS

PUBLISHED ONLINE: 11 MAY 2015 | DOI: 10.1038/NNANO.2015.86

Edge-mode superconductivity in a two-dimensional topological insulator

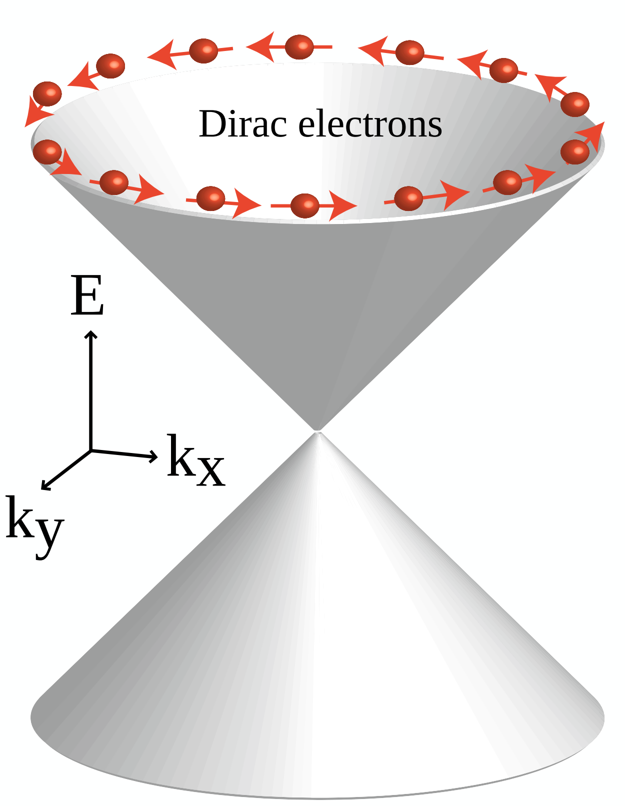
Vlad S. Pribiag[1†‡], Arjan J. A. Beukman[1§], Fanming Qu[1§], Maja C. Cassidy[1], Christophe Charpentier[2], Werner Wegscheider[2] and Leo P. Kouwenhoven[1*]



- One can also engineer systems that realize a topological phase supporting Majorana fermions in two dimensions by inducing an effective p+ip superconducting pairing in a spinless 2D electron gas. Simplest case 3D TIs:

$$H_{3DTI} = \int d^2 r \psi^\dagger [-iv(\partial_x \sigma^y - \partial_y \sigma^x) - \mu]\psi$$

$$\epsilon_\pm(\mathbf{k}) = \pm v|\mathbf{k}| - \mu$$

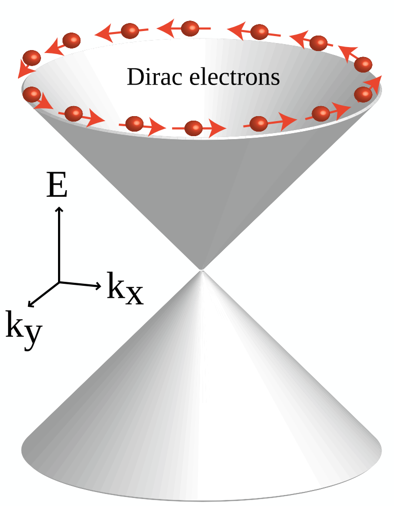


- For any chemical potential residing within the bulk gap there us only one single Fermi surface (Dirac cones non-degenerate).
- Electrons along the Fermi surface are not spin-polarized (momentum-spin locking) so p+ip pairing can be effectively induced by s-wave proximity effect.

$$E_\pm(\mathbf{k}) = \sqrt{\epsilon_\pm^2(\mathbf{k}) + \Delta^2}$$

$$H_{3DTI} = \int d^2 r \psi^\dagger [-iv(\partial_x \sigma^y - \partial_y \sigma^x) - \mu]\psi$$

$$\epsilon_\pm(\mathbf{k}) = \pm v|\mathbf{k}| - \mu$$



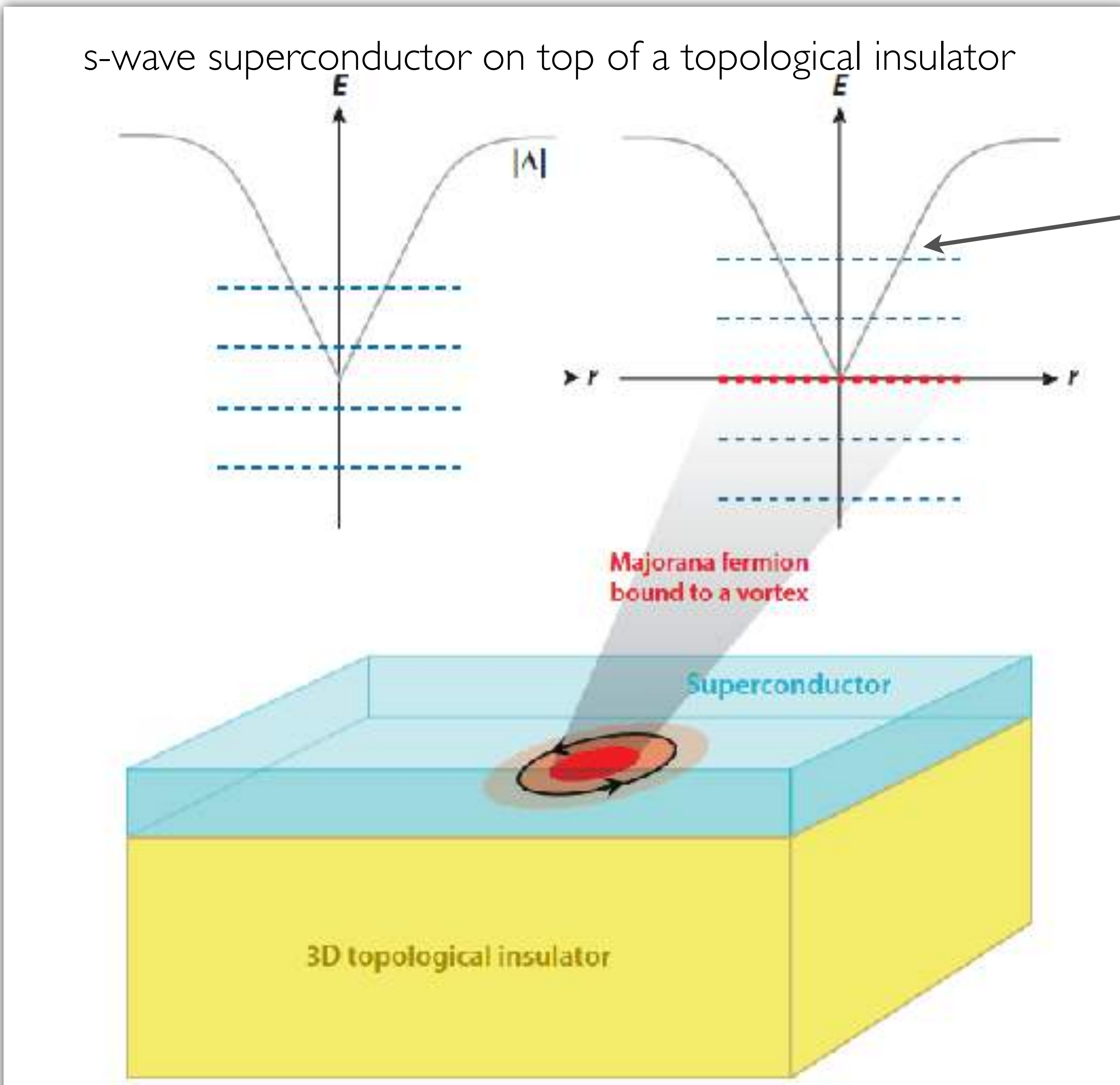
$$H_\Delta = \int d^2 r \Delta(\psi_\uparrow^\dagger \psi_\downarrow^\dagger + h.c)$$

$$H = \sum_{s=\pm} \int \frac{d^2\mathbf{k}}{2\pi} \{\epsilon_s(\mathbf{k})\psi_s^\dagger(\mathbf{k})\psi_s(\mathbf{k}) + [\frac{\Delta}{2}(\frac{k_x + ik_y}{|\mathbf{k}|})\psi_s(\mathbf{k})\psi_s(-\mathbf{k}) + h.c]\}$$

- Time-reversal breaking of any form will generate chiral Majorana edge states at the boundary between topologically superconducting and magnetically gapped regions in the surface of a 3D TI.

Fu and Kane, PRL **100**, 096407, 2008

CSIC



Fu and Kane, PRL, 100, 096407, 2008

Vortex created by a magnetic field.
This vortex traps midgap states.

$$E_n \approx (n+\alpha)\frac{\Delta^2}{E_F}, n = 0, \pm 1, ...$$

Zero-point motion, relativistic particles have

$$\alpha = 0$$

The masless 2D Dirac equation includes a zero mode solution (Majorana).

Figure from Carlo Beenakker, Annual Review Condensed Matter, 4, 113, 2013

Fu & Kane's proposal: the surface of a 3D Topological Insulator proximized with an **s-wave** superconductor is effectively a 2D p+ip superconductor. Vortices bind Majoranas

Fu and Kane, PRL, 100, 096407, 2008

$$\sigma \cdot p\psi + i\Delta\sigma_2\psi^* = i\partial_t\psi$$

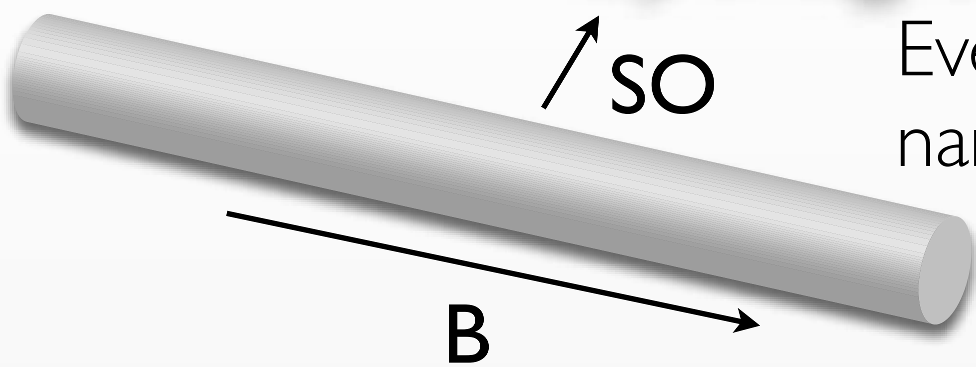
Note the analogy!!!

$$\begin{pmatrix} \sigma \cdot p & m \\ m & -\sigma \cdot p \end{pmatrix} \begin{pmatrix} \psi \\ \phi \end{pmatrix} = i\partial_t \begin{pmatrix} \psi \\ \phi \end{pmatrix}$$

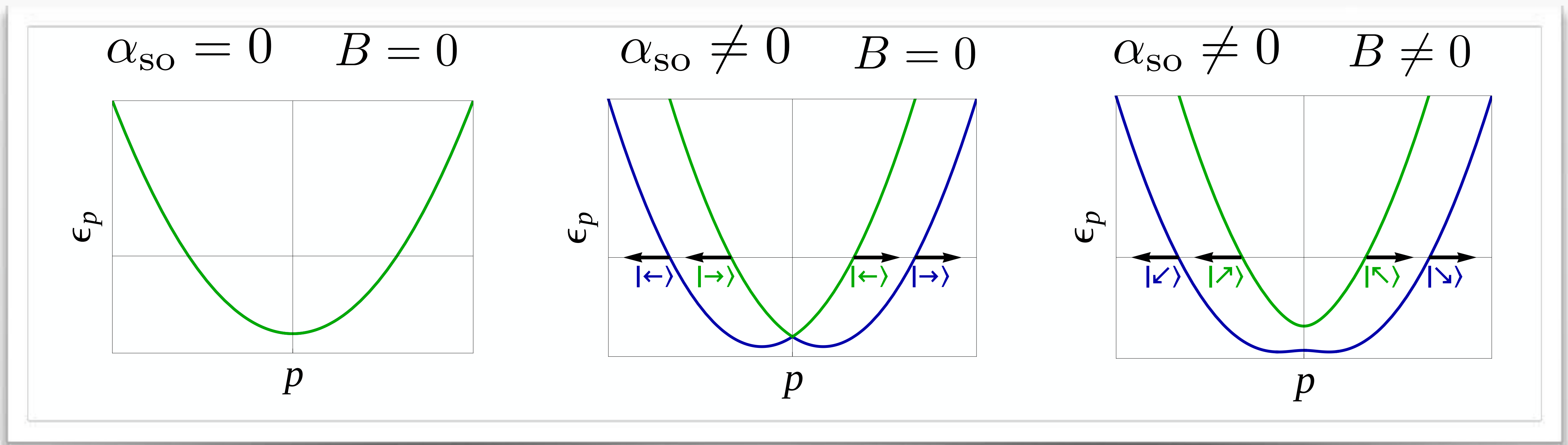
$$\sigma \cdot p\psi + im\sigma_2\psi^* = i\partial_t\psi$$

Jackiw and Rossi, Nuclear Phys. B, 190, 681, 1981

Physical realizations of one-dimensional p-wave Topological Superconductors



Even simpler: use one-dimensional semiconducting nanowires with strong Rashba spin-orbit coupling.

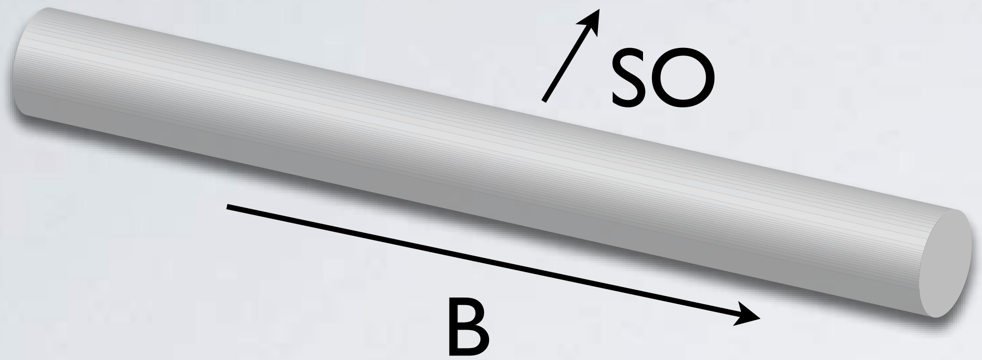


- SO splits NW states into 2 subbands of opposite helicity: + and -

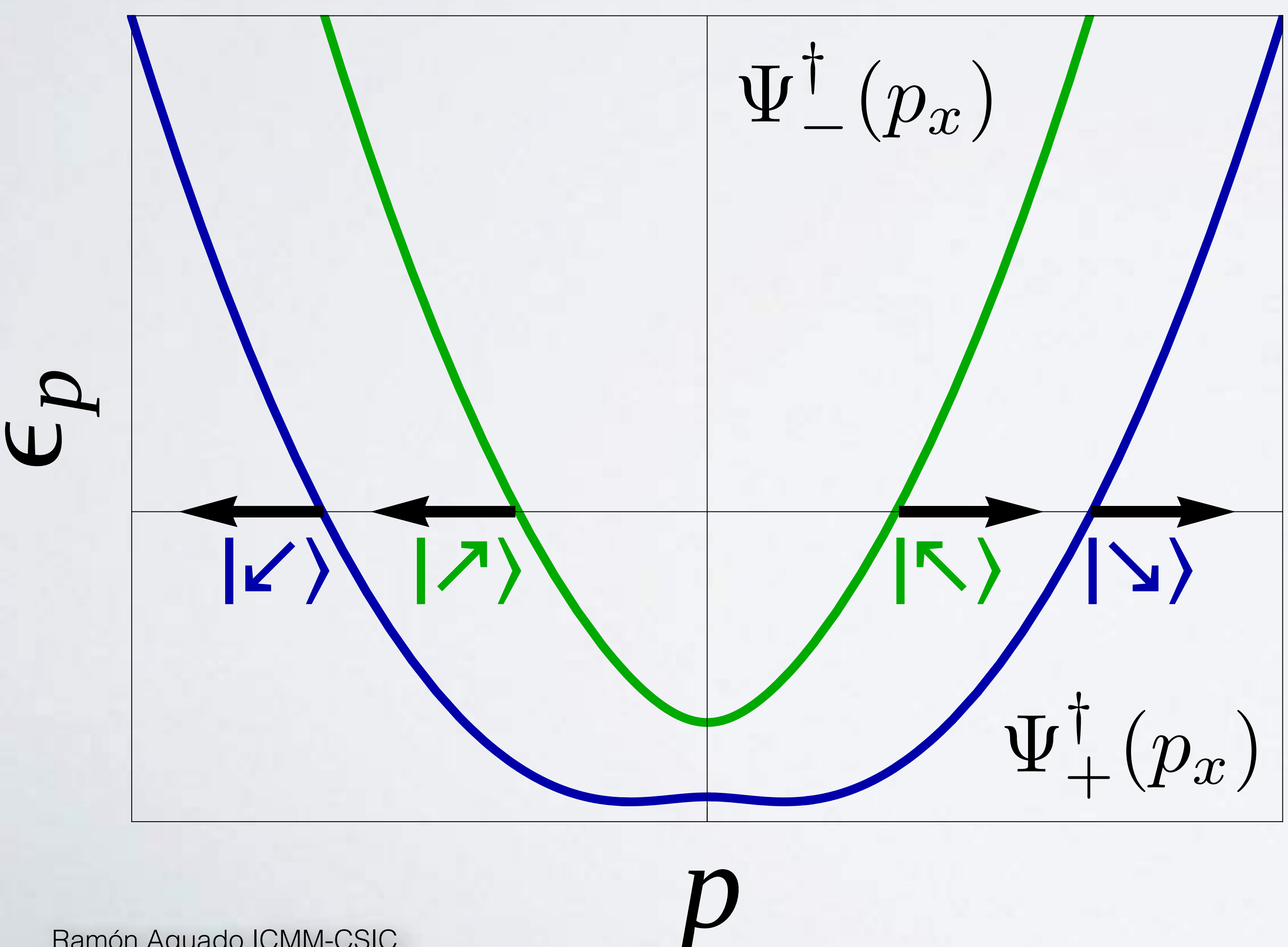
  —— $+$ —— $-$

- At finite B, +/- subbands have spins canted away from SO axis

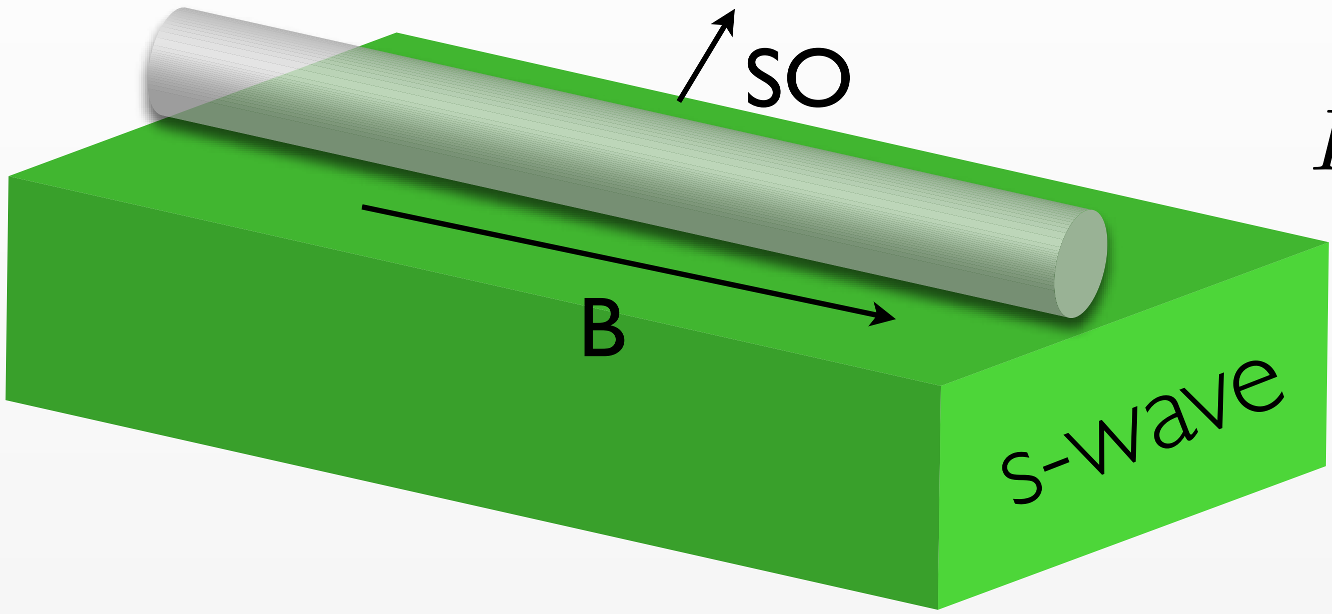
$$H_0 = \int dx \Psi^\dagger_\sigma(x) \left[-\frac{\hbar^2 \partial_x^2}{2m} - \mu - i\alpha_{SO}\partial_x \sigma^y + E_Z \sigma^x \right]_{\sigma,\sigma'} \Psi_{\sigma'}(x)$$



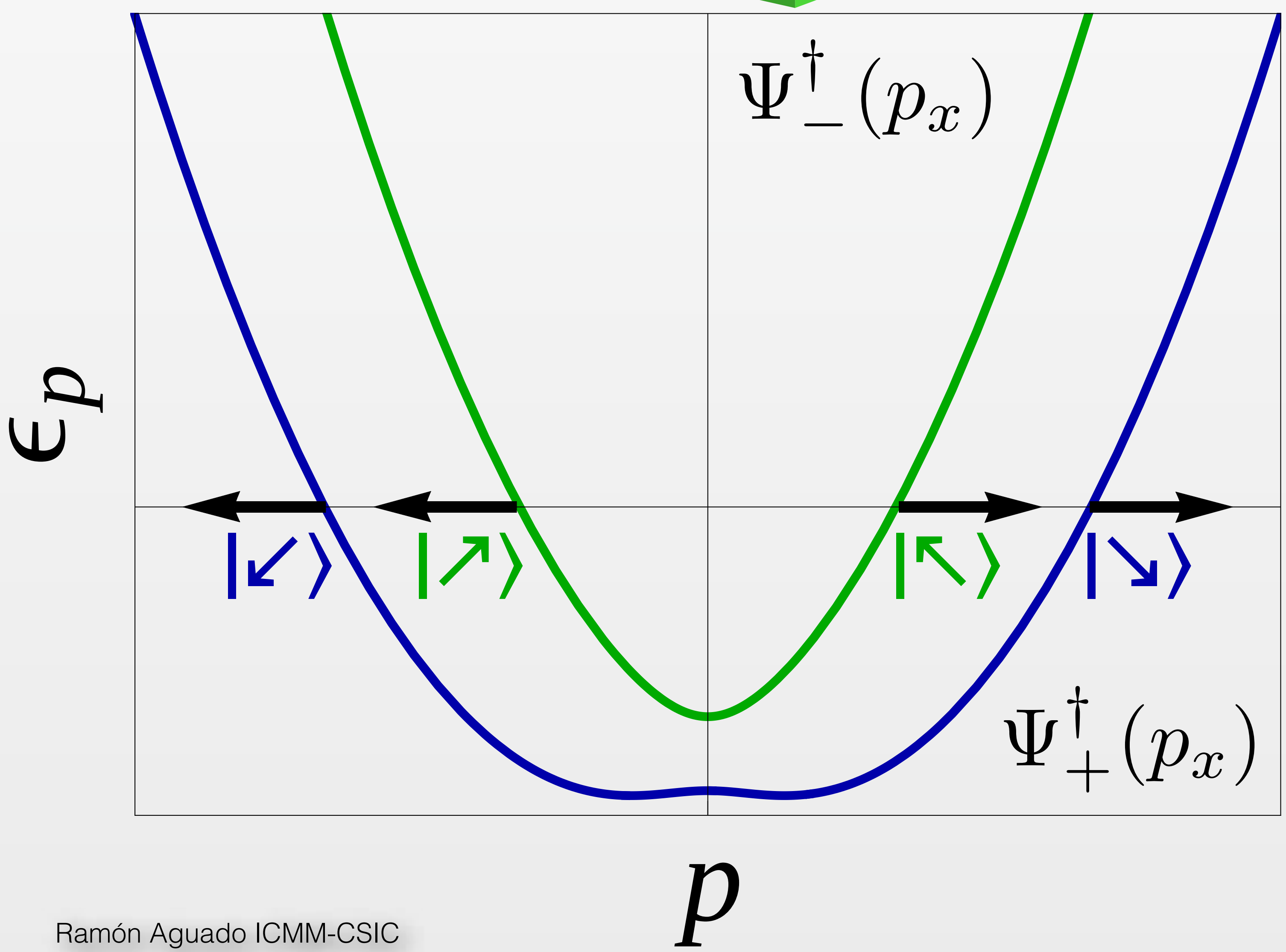
$$E_Z = \frac{1}{2} g \mu_B B$$



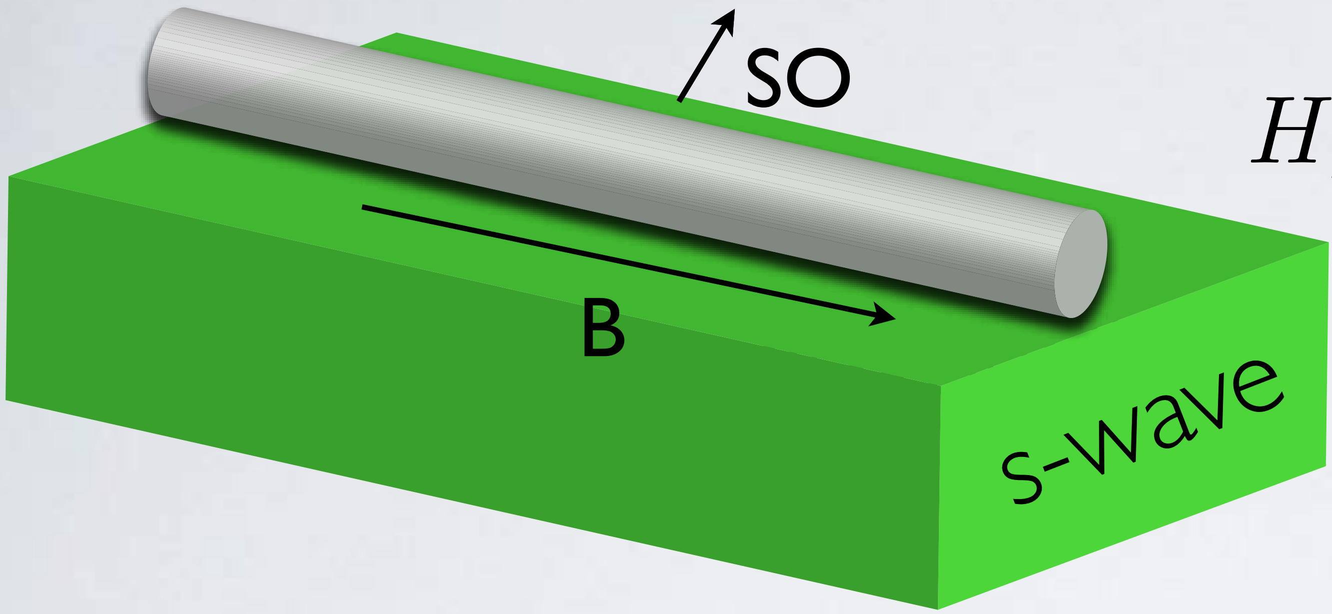
$$H_0 = \int dx \Psi^\dagger_\sigma(x) \left[-\frac{\hbar^2 \partial_x^2}{2m} - \mu - i\alpha_{SO}\partial_x \sigma^y + E_Z \sigma^x \right]_{\sigma,\sigma'} \Psi_{\sigma'}(x)$$



$$H_{SC} = \int dx \Delta [\Psi^\dagger_\uparrow(x)\Psi^\dagger_\downarrow(x) + H.c.]$$



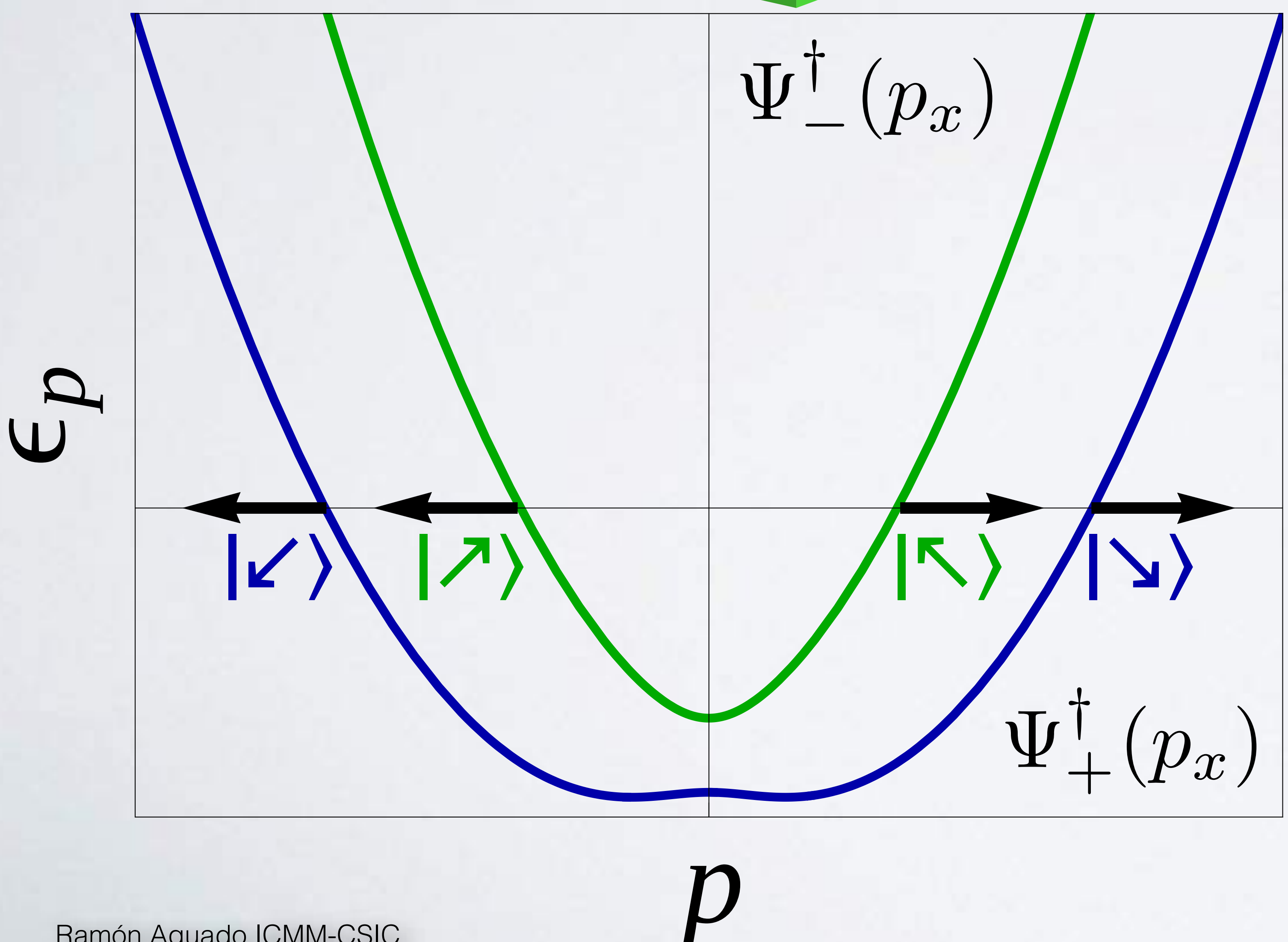
$$H_0 = \int dx \Psi^\dagger_\sigma(x)\left[-\frac{\hbar^2\partial_x^2}{2m} - \mu - i\alpha_{SO}\partial_x\sigma^y + E_Z\sigma^x\right]_{\sigma,\sigma'}\Psi_{\sigma'}(x)$$



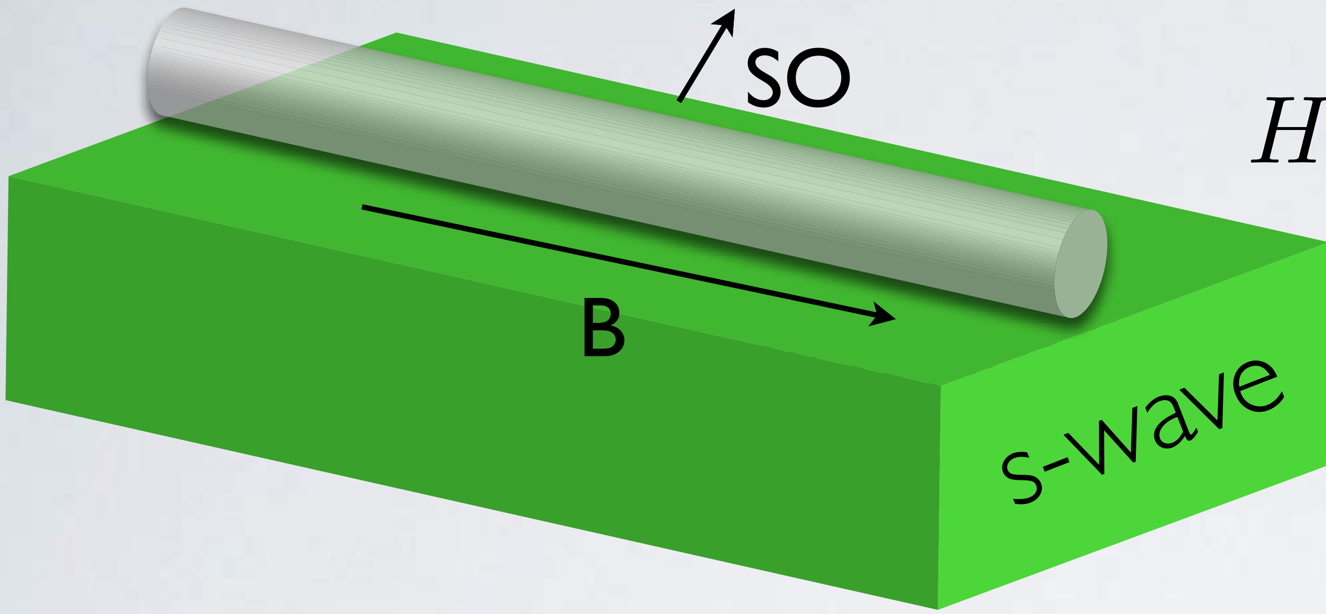
$$H_{SC} = \int dx \Delta[\Psi^\dagger_\uparrow(x)\Psi^\dagger_\downarrow(x) + H.c.]$$

Helical basis

$$\Delta_{+-}(p_x)\Psi^\dagger_+(p_x)\Psi^\dagger_-(-p_x)$$
$$\Delta_{--}(p_x)\Psi^\dagger_-(p_x)\Psi^\dagger_-(-p_x)$$
$$\Delta_{++}(p_x)\Psi^\dagger_+(p_x)\Psi^\dagger_+(-p_x)$$



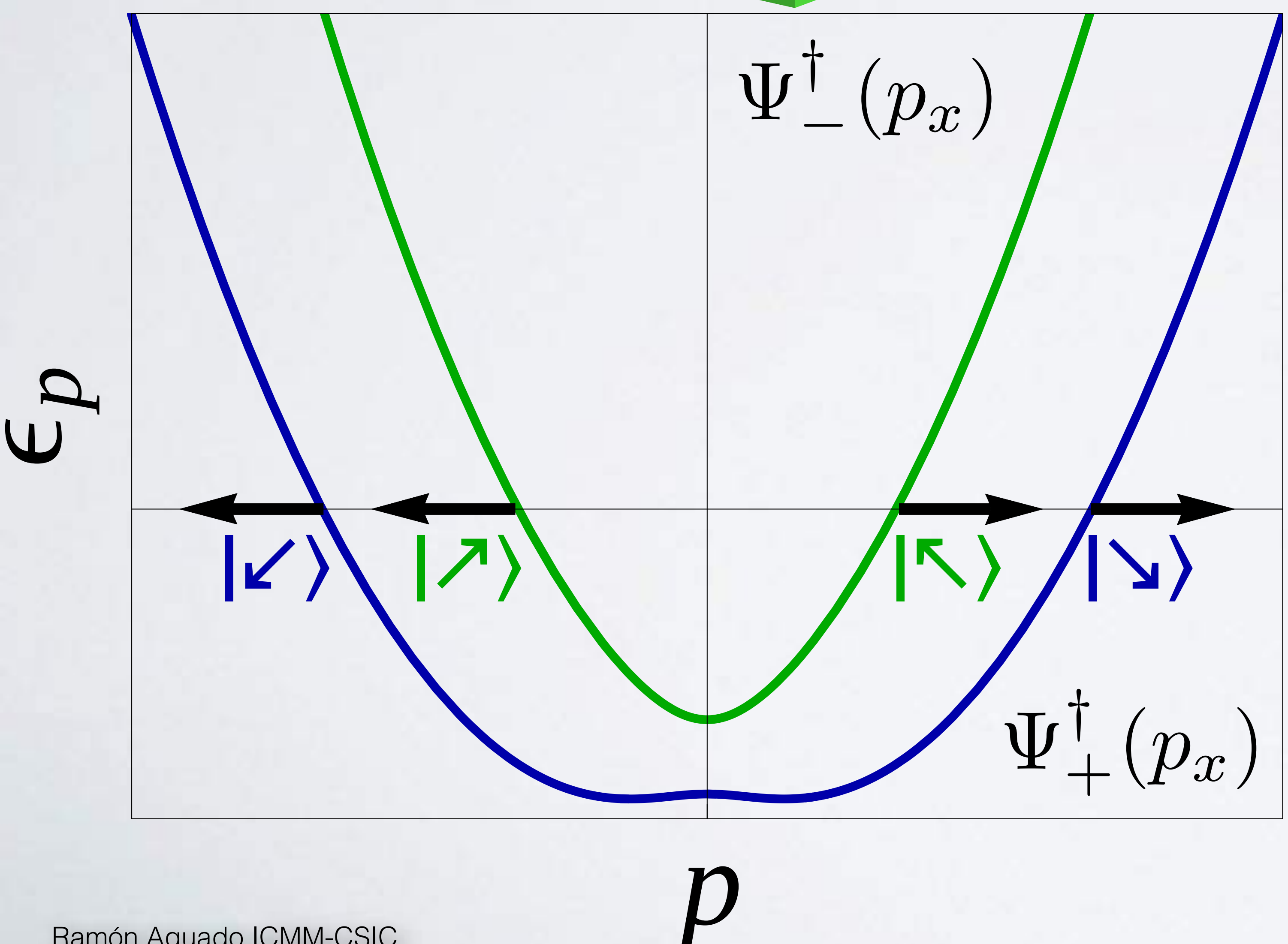
$$H_0 = \int dx \Psi^\dagger_\sigma(x) \left[-\frac{\hbar^2 \partial_x^2}{2m} - \mu - i\alpha_{SO}\partial_x \sigma^y + E_Z \sigma^x \right]_{\sigma,\sigma'} \Psi_{\sigma'}(x)$$



$$H_{SC} = \int dx \Delta [\Psi^\dagger_\uparrow(x) \Psi^\dagger_\downarrow(x) + H.c.]$$

Helical basis

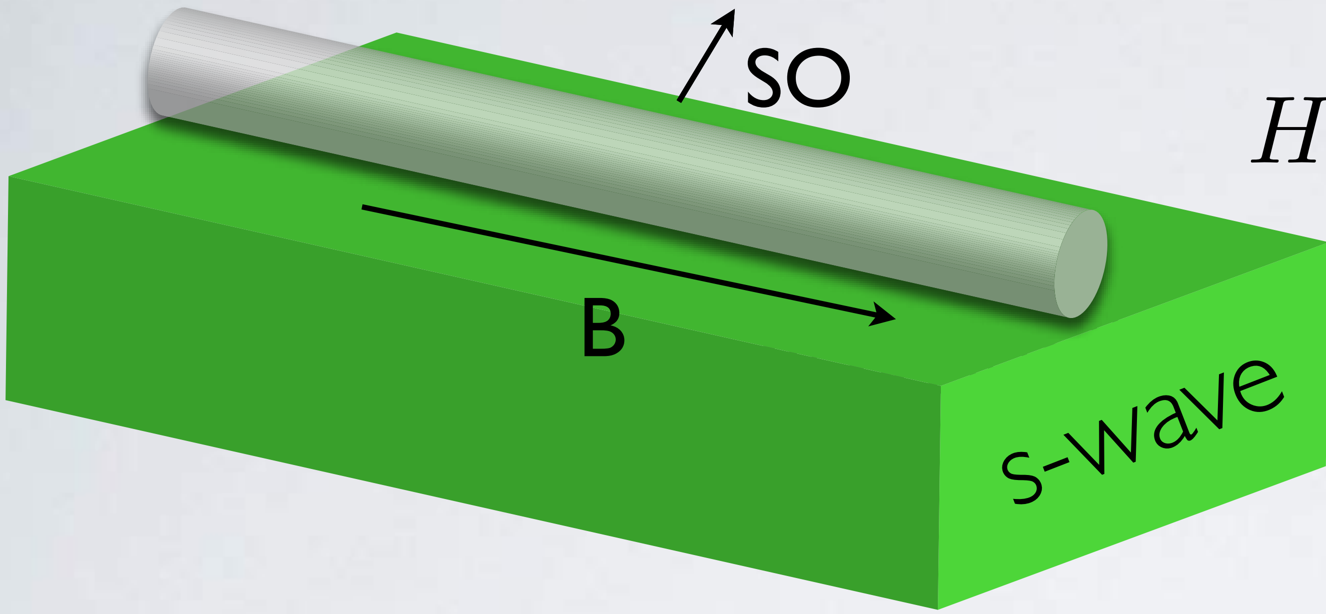
$$\Delta_{+-}(p_x)\Psi^\dagger_+(p_x)\Psi^\dagger_-(-p_x)$$
$$\Delta_{--}(p_x)\Psi^\dagger_-(p_x)\Psi^\dagger_-(-p_x)$$
$$\Delta_{++}(p_x)\Psi^\dagger_+(p_x)\Psi^\dagger_+(-p_x)$$



$$\Delta^s_{+-} = \frac{E_Z \Delta}{\sqrt{E_Z^2 + \alpha_{SO}^2 p^2}}$$

$$\Delta^p_{\mp} = \frac{\pm i\alpha_{SO} p \Delta}{2\sqrt{E_Z^2 + \alpha_{SO}^2 p^2}}$$

$$H_0 = \int dx \Psi^\dagger_\sigma(x) \left[-\frac{\hbar^2 \partial_x^2}{2m} - \mu - i\alpha_{SO}\partial_x \sigma^y + E_Z \sigma^x \right]_{\sigma,\sigma'} \Psi_{\sigma'}(x)$$



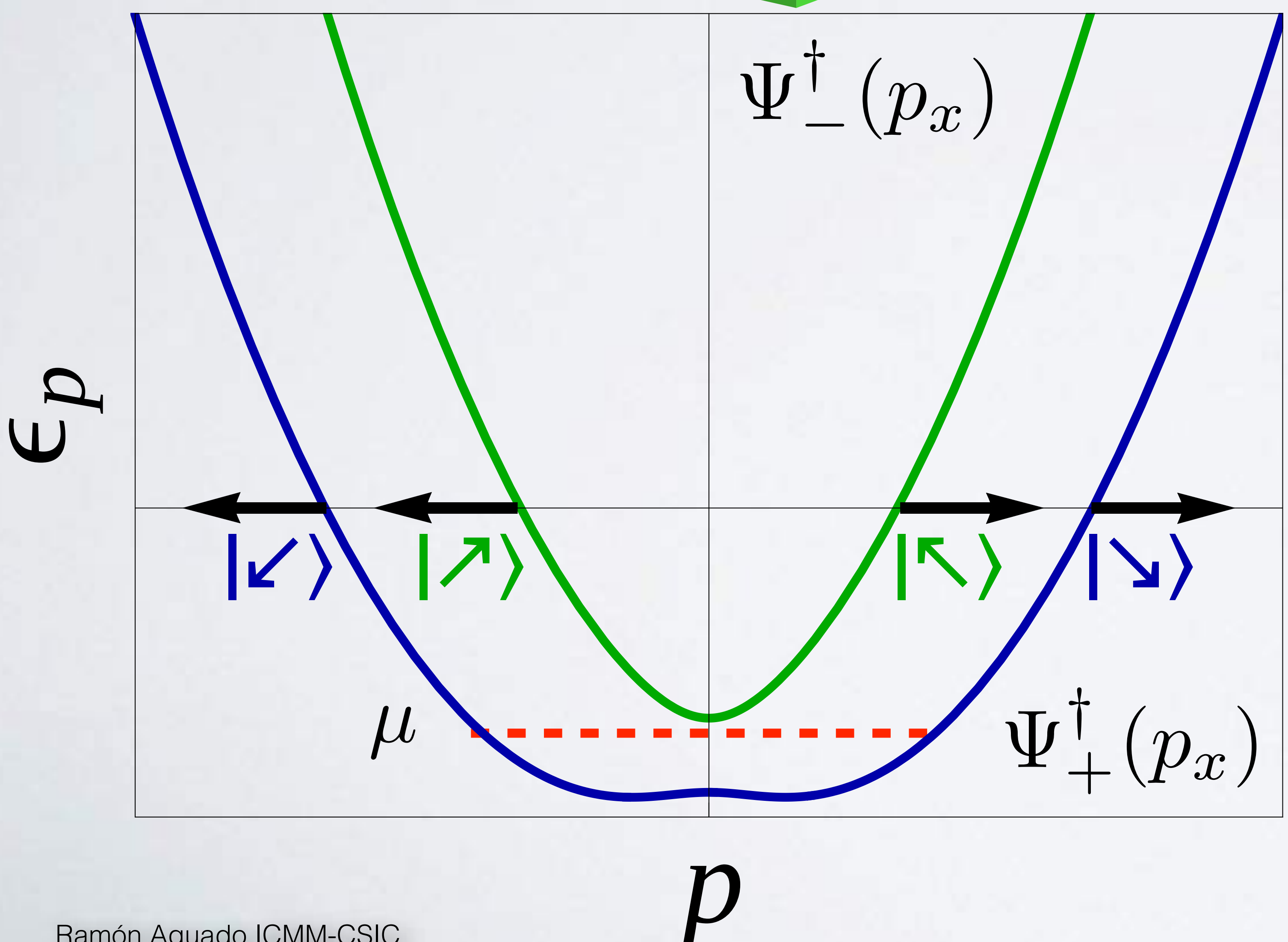
$$H_{SC} = \int dx \Delta [\Psi^\dagger_\uparrow(x) \Psi^\dagger_\downarrow(x) + H.c.]$$

Helical basis

$$\Delta_{+-}(p_x)\Psi^\dagger_+(p_x)\Psi^\dagger_-(-p_x)$$

$$\Delta_{--}(p_x)\Psi^\dagger_-(p_x)\Psi^\dagger_-(-p_x)$$

$$\Delta_{++}(p_x)\Psi^\dagger_+(p_x)\Psi^\dagger_+(-p_x)$$

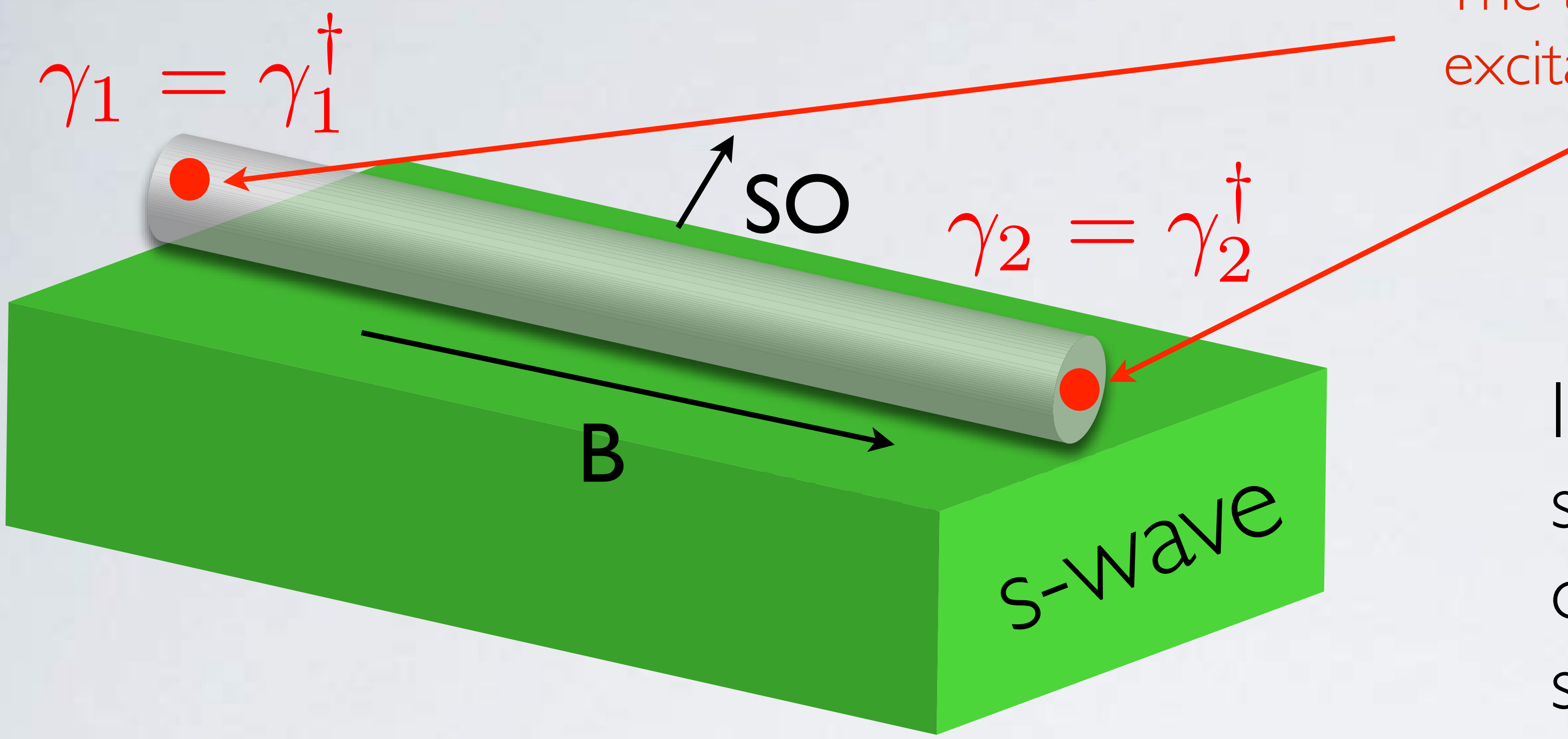


effective p-wave pairing

$$\Delta^p_{++} = \frac{i\alpha_{SO} p \Delta}{2\sqrt{E_Z^2 + \alpha_{SO}^2 p^2}}$$

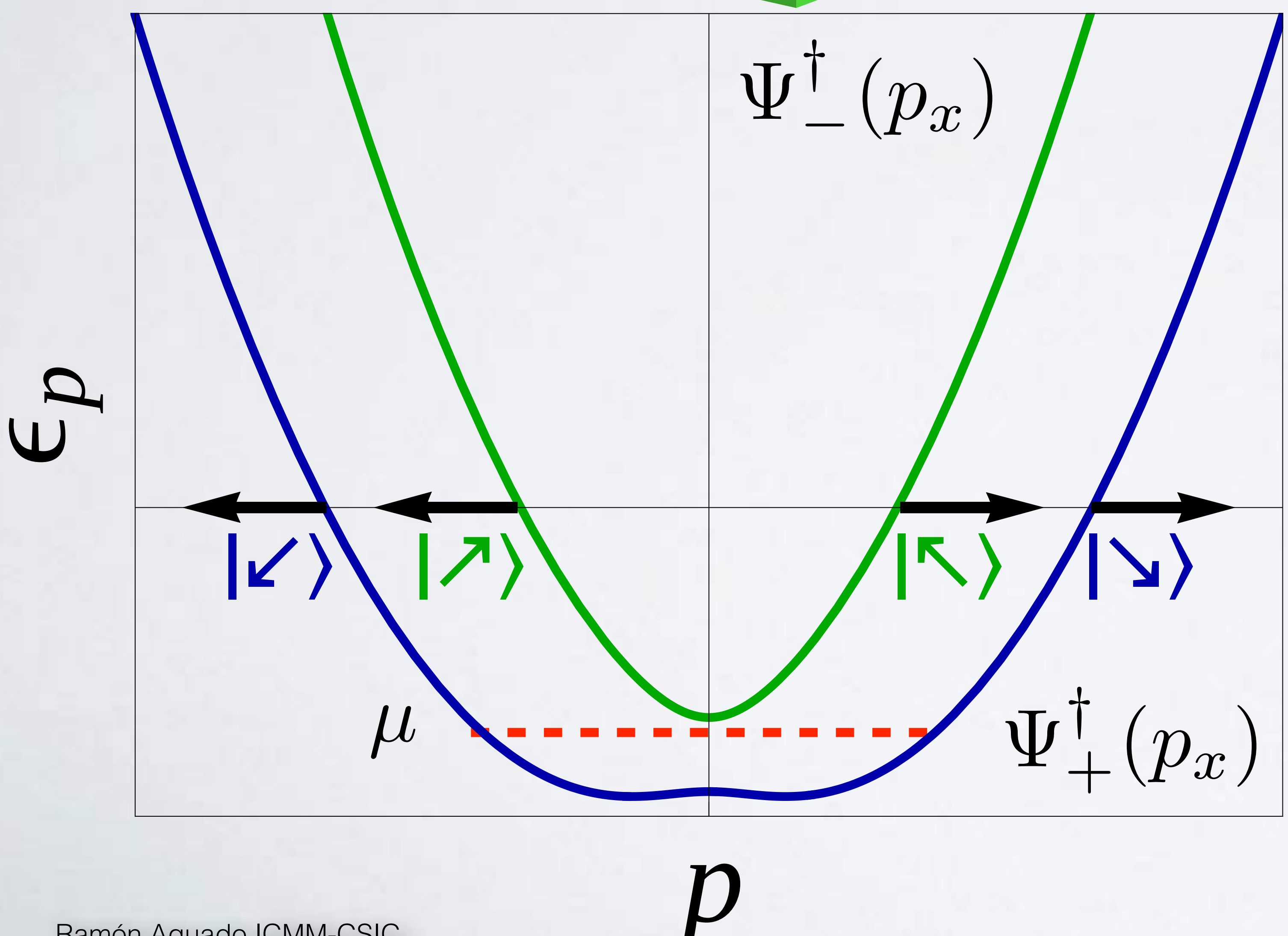
Lutchyn, Sau and Das Sarma, PRL 105, 077001 (2010)
Oreg, Refael, von Oppen, PRL, 105, 177002 (2010)

The two Majoranas constitute a <u>single</u> electronic excitation which can be <u>arbitrarily</u> delocalized.



$$2i\gamma_1\gamma_2 = 2(d^\dagger d - \frac{1}{2})$$

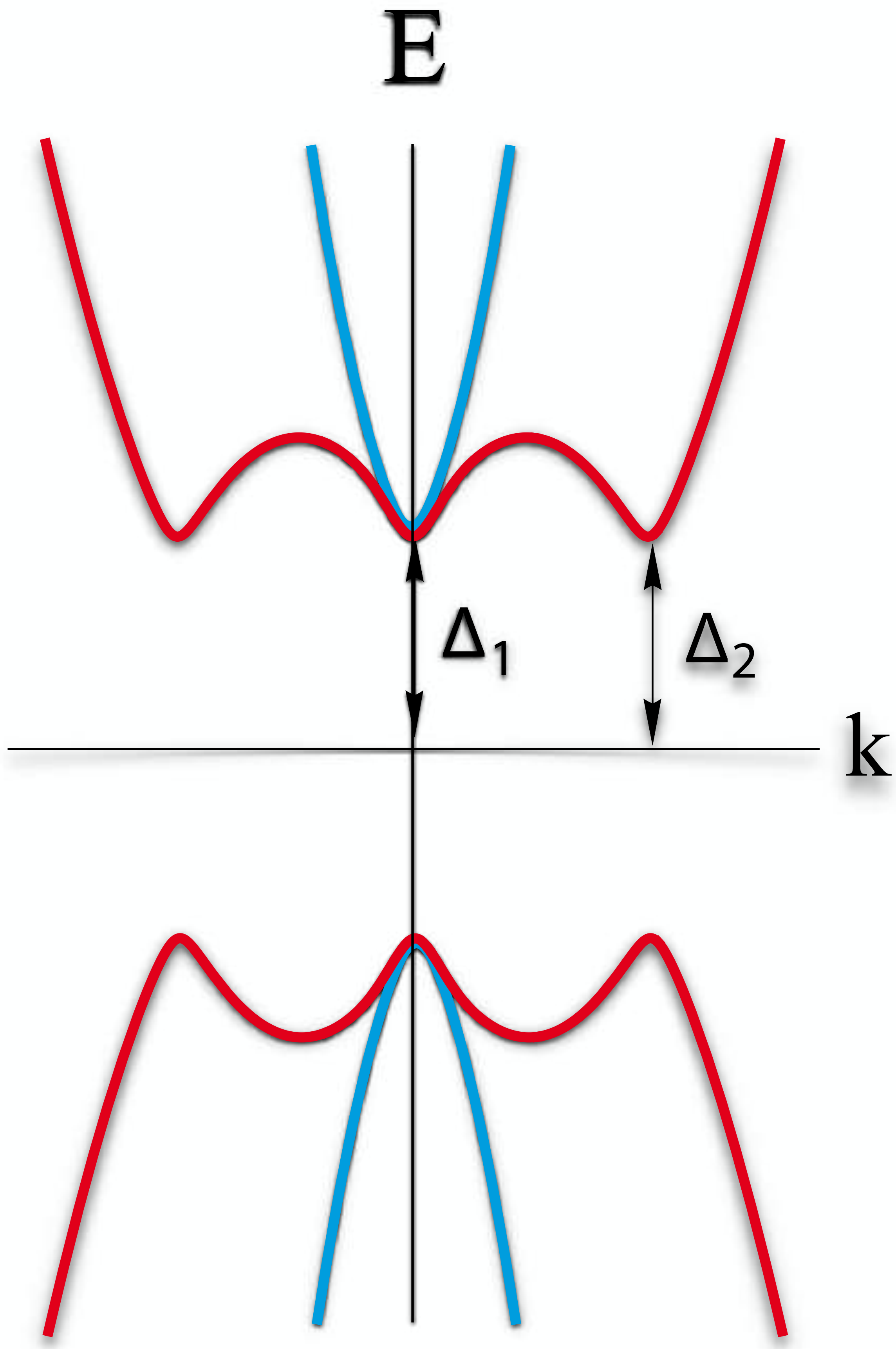
In the presence of s-wave pairing such helical nanowire is a realization of Kitaev's one-dimensional p-wave superconductor model (2001).



effective p-wave pairing

$$\Delta_{++}^p = \frac{i\alpha_{SO}p\Delta}{2\sqrt{E_Z^2 + \alpha_{SO}^2 p^2}}$$

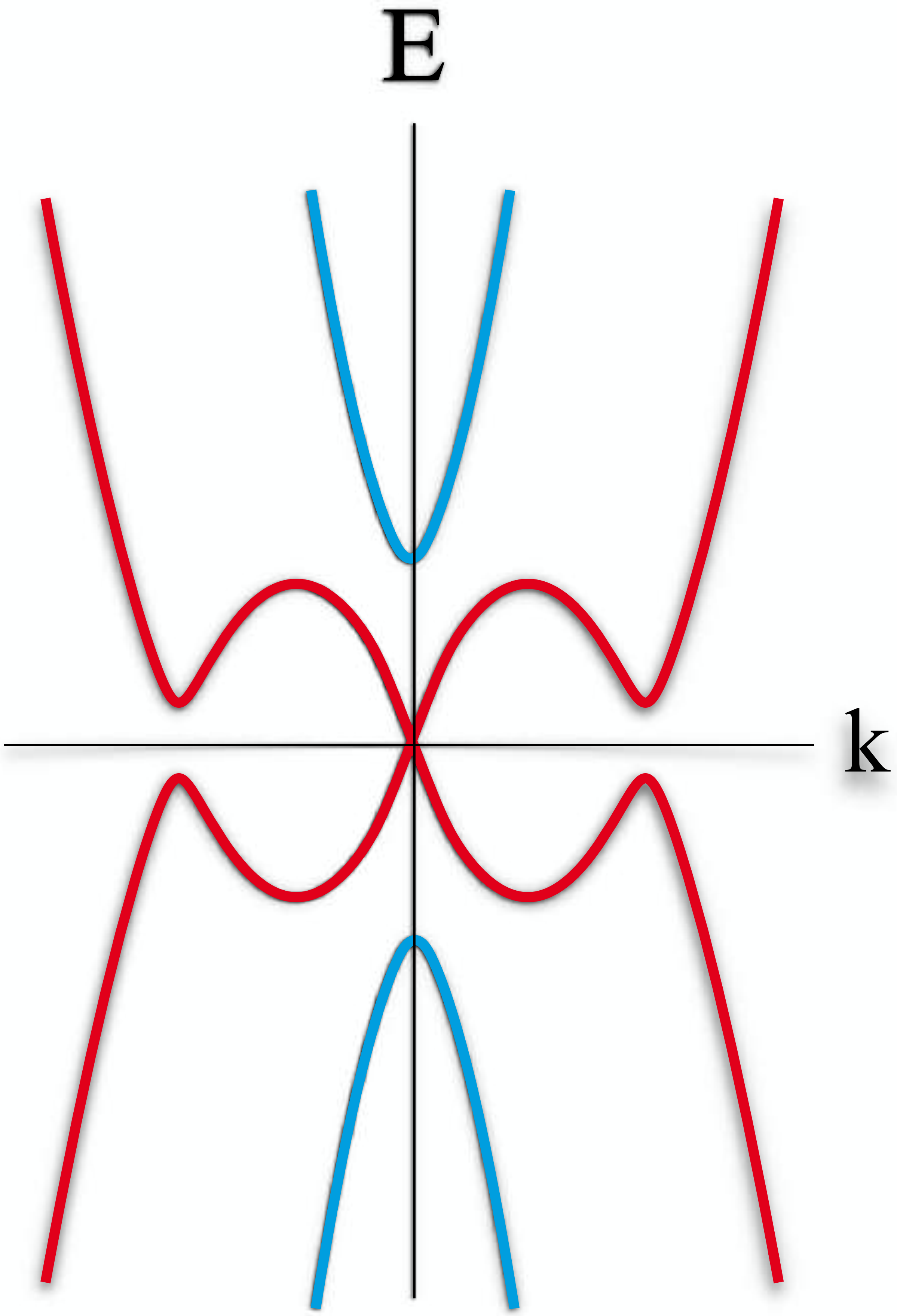
Band inversion with increasing Zeeman field



$$E_Z^c = \sqrt{\Delta^2 + \mu^2}$$

$$E_Z < E_Z^c$$

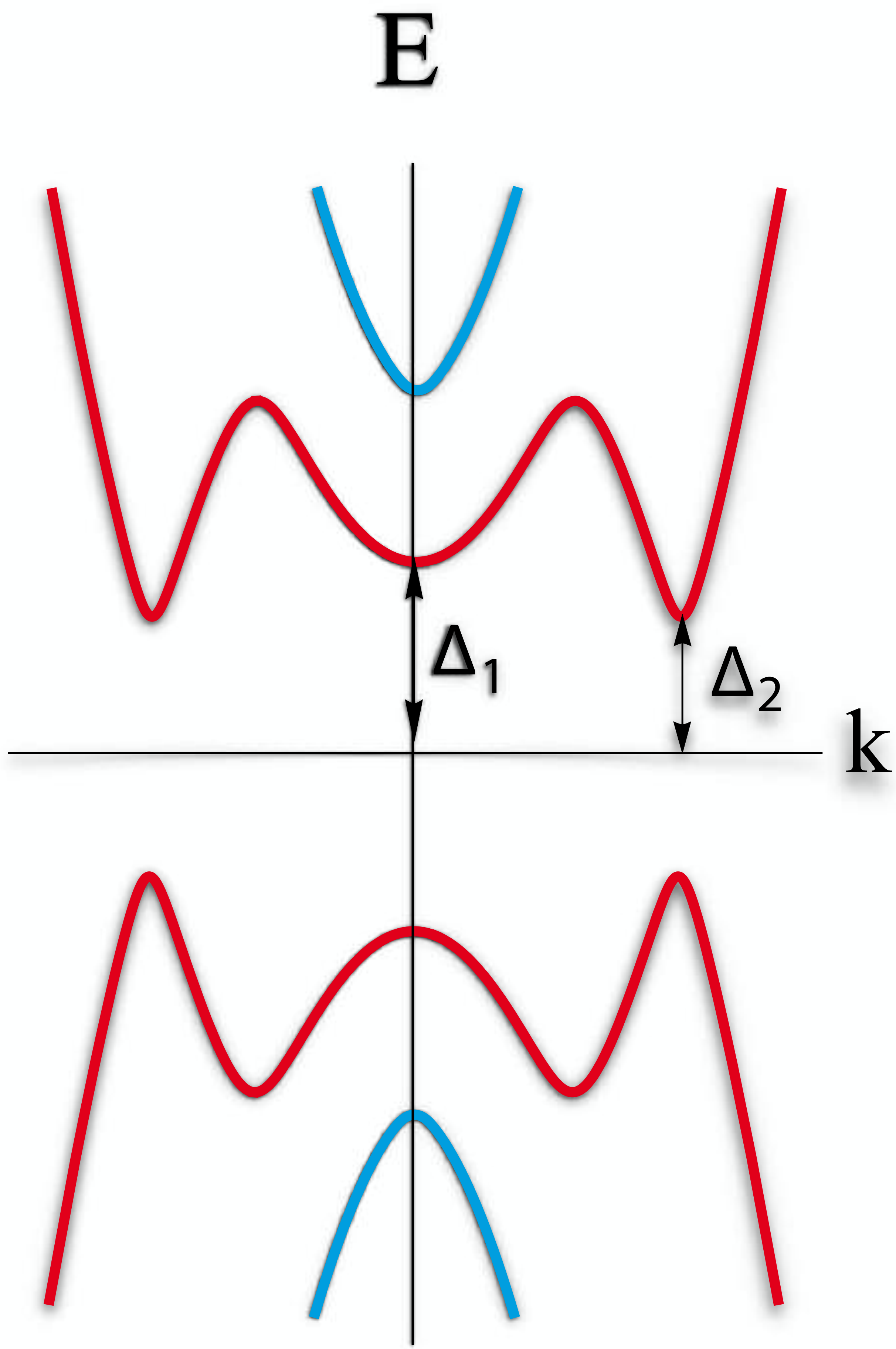
Band inversion with increasing Zeeman field



$$E_Z^c = \sqrt{\Delta^2 + \mu^2}$$

$$E_Z = E_Z^c$$

Band inversion with increasing Zeeman field

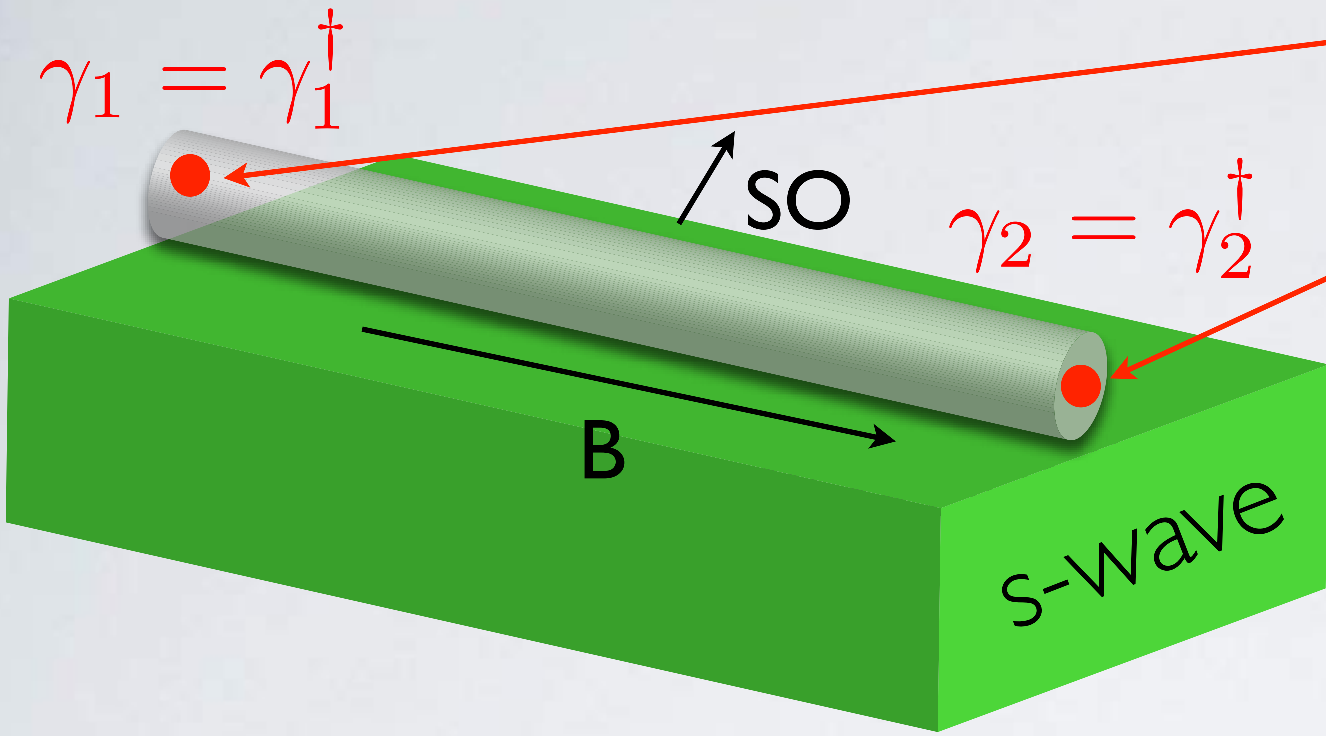


$$E_Z^c = \sqrt{\Delta^2 + \mu^2}$$

$$E_Z > E_Z^c$$

Lutchyn, Sau and Das Sarma, PRL 105, 077001 (2010)
Oreg, Refael, von Oppen, PRL, 105, 177002 (2010)

The two Majoranas constitute a <u>single</u> electronic excitation which can be <u>arbitrarily</u> delocalized. Non-Abelian statistics, relevant for topological quantum computation



$$E_Z^c = \sqrt{\Delta^2 + \mu^2}$$

$$E_Z = \frac{1}{2} g \mu_B B$$

Very attractive proposal, all the ingredients are available in the lab:

- Nanowires with strong spin-orbit coupling (In As, InSb) $\alpha_{SO} \sim 0.1 - 0.2 eV \AA$

- Large g-factors $g \sim 10 - 50$

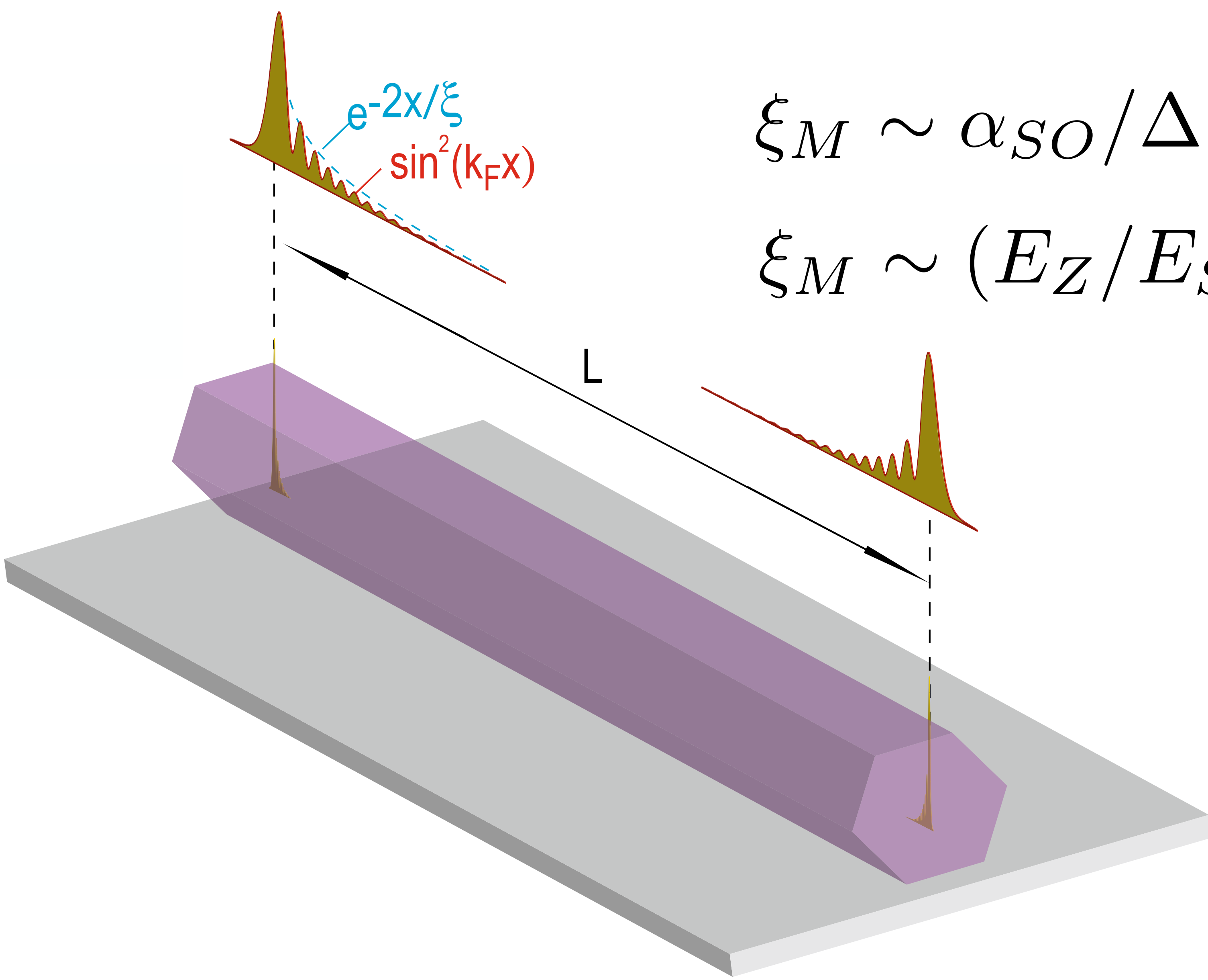
- Good proximity effect with superconductors (Niobium, Vanadium, etc) with large critical fields.

- Gate-tunable (low chemical potential) nanowires.

Finite length effects

- The Majorana wave functions exponentially decay into the bulk of the superconductor.
- This gives rise to a residual overlap between Majorana modes residing at opposite ends of the nanowire in realistic wires of finite length (deviations from zero energy, so-called *Majorana oscillations*):

$$\delta\varepsilon \sim \hbar^2 k_F \frac{e^{-2L/\xi_M}}{m\xi_M} cos(k_F L).$$

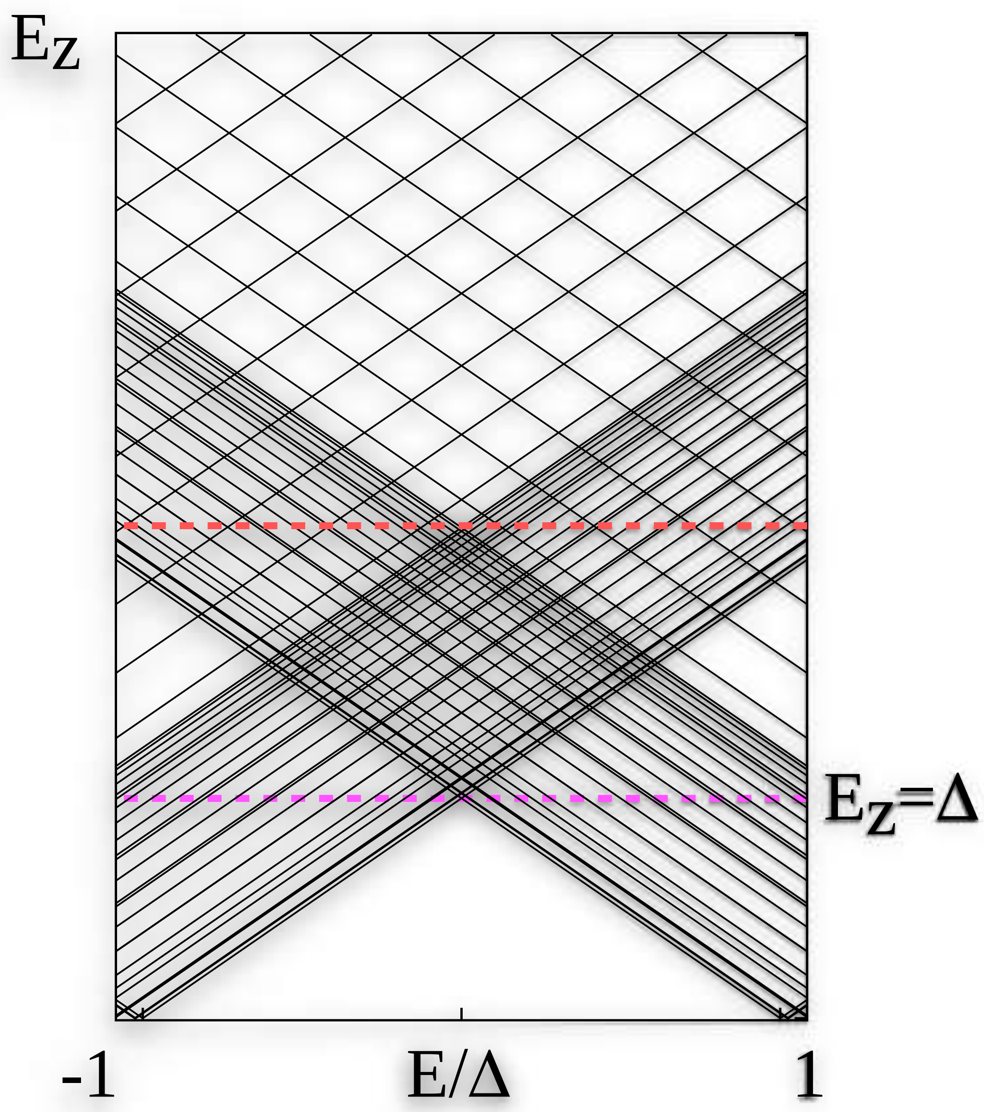


$$\xi_M \sim \alpha_{SO}/\Delta$$ Large SO limit

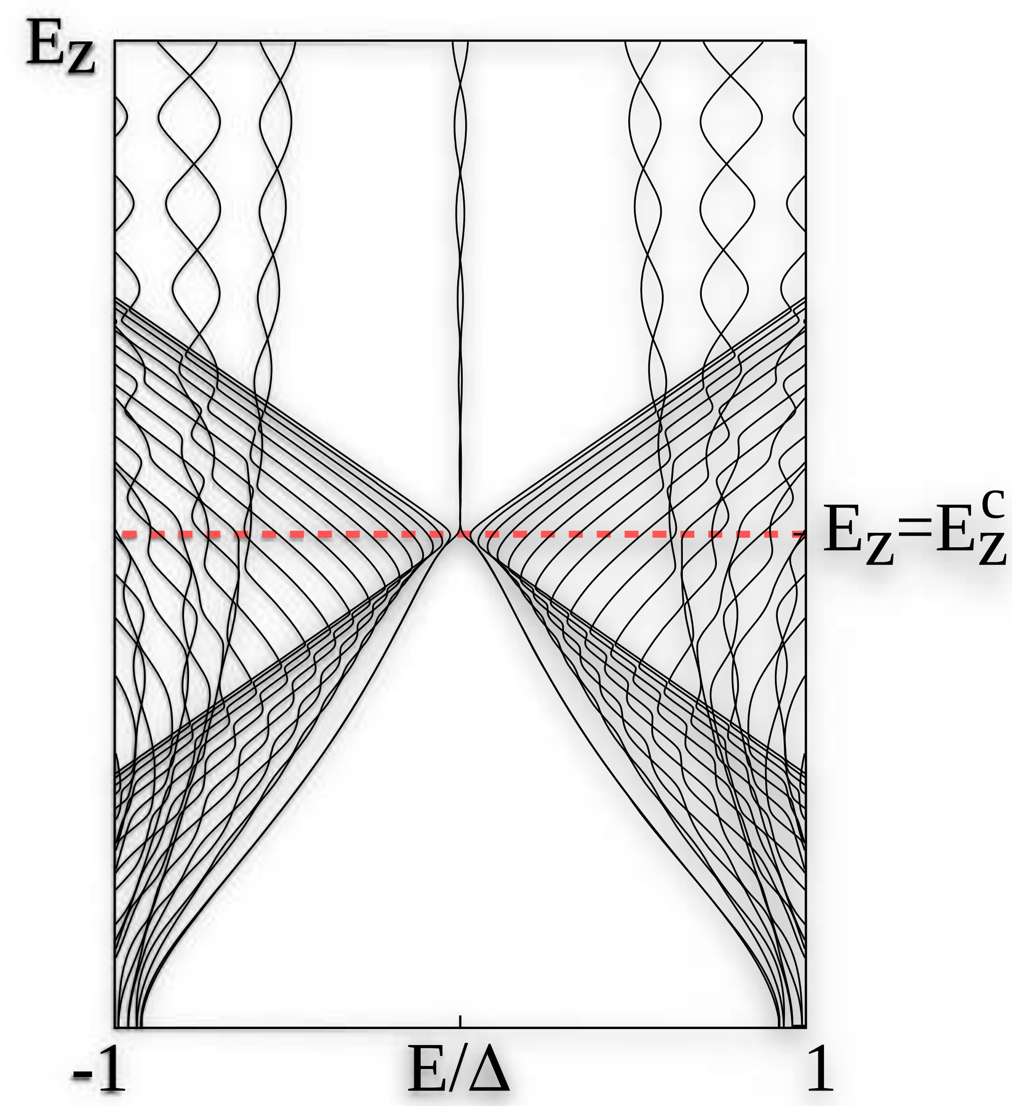
$$\xi_M \sim (E_Z/E_{SO})\alpha_{SO}/\Delta$$ Large Zeeman limit

Finite length effects

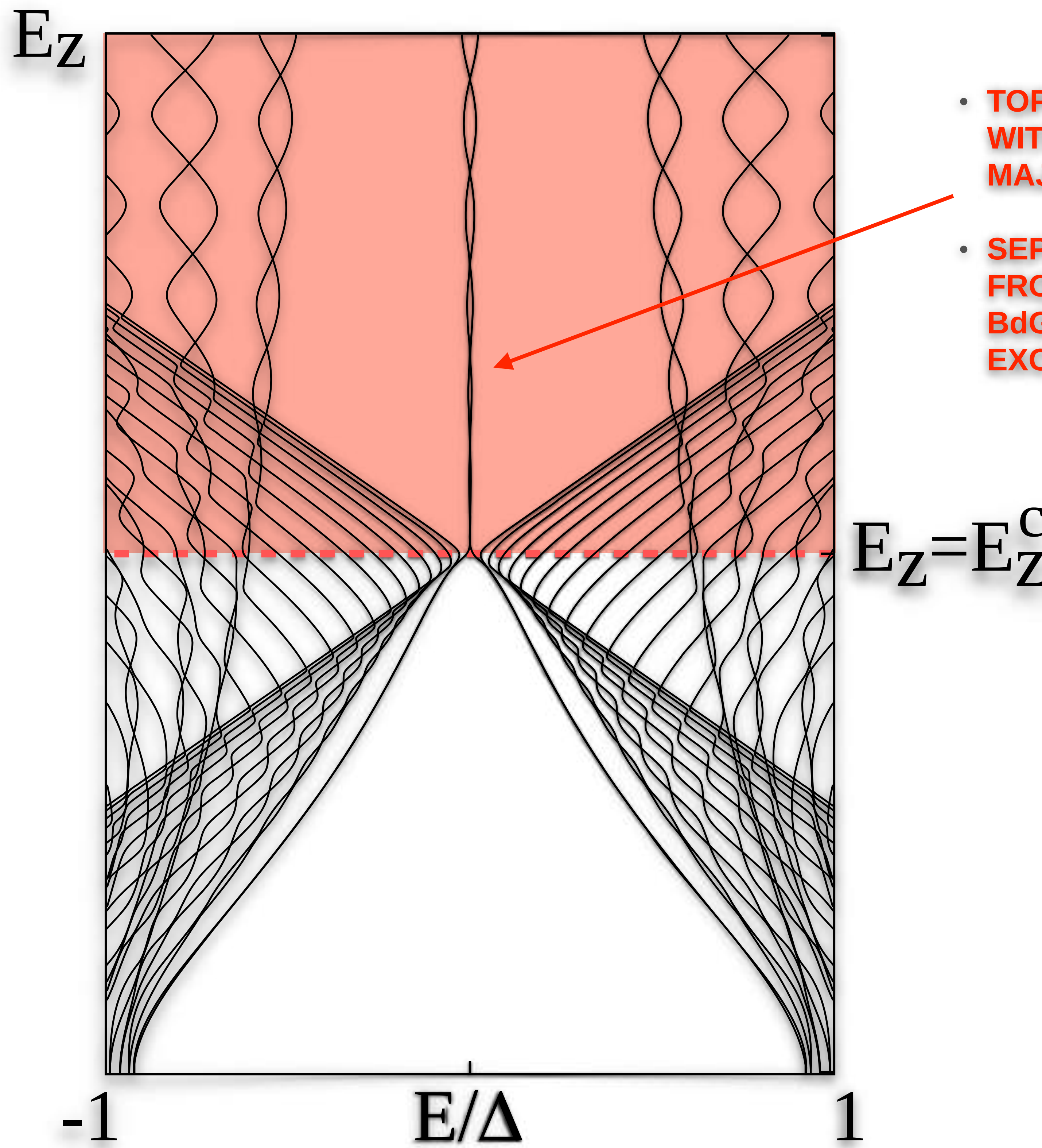
BdG spectrum without spin-orbit coupling



BdG spectrum with spin-orbit coupling

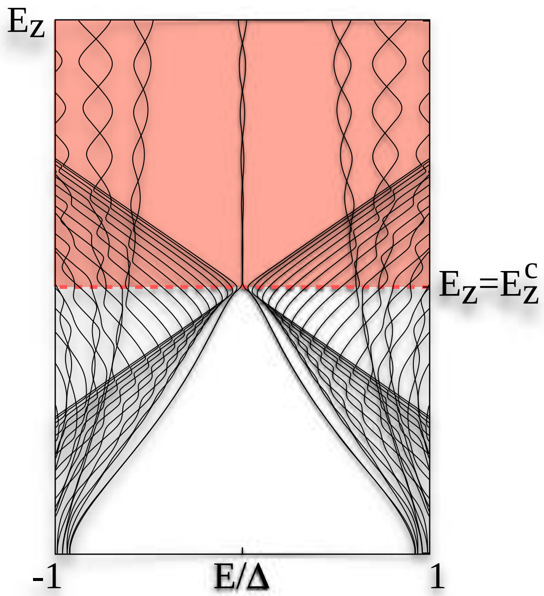


Finite length effects



- TOPOLOGICAL REGIME WITH NEAR ZERO MAJORANA MODES
- SEPARATED BY A GAP FROM THE REST OF BdG QUASIPARTICLE EXCITATIONS

HOW TO MEASURE MAJORANAS IN NANOWIRES? ANDREEV REFLECTION IN NORMAL-SUPERCONDUCTOR JUNCTIONS

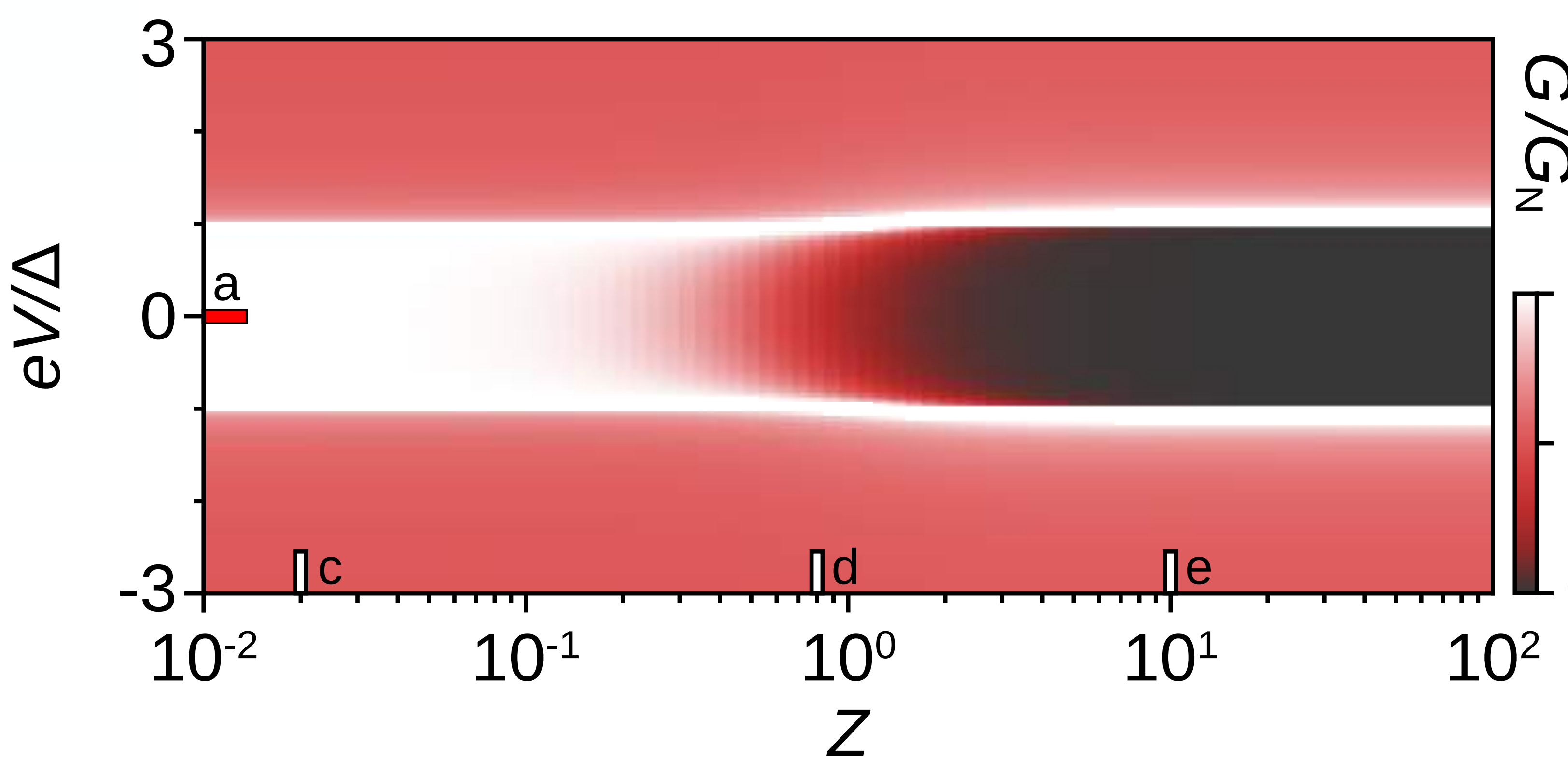


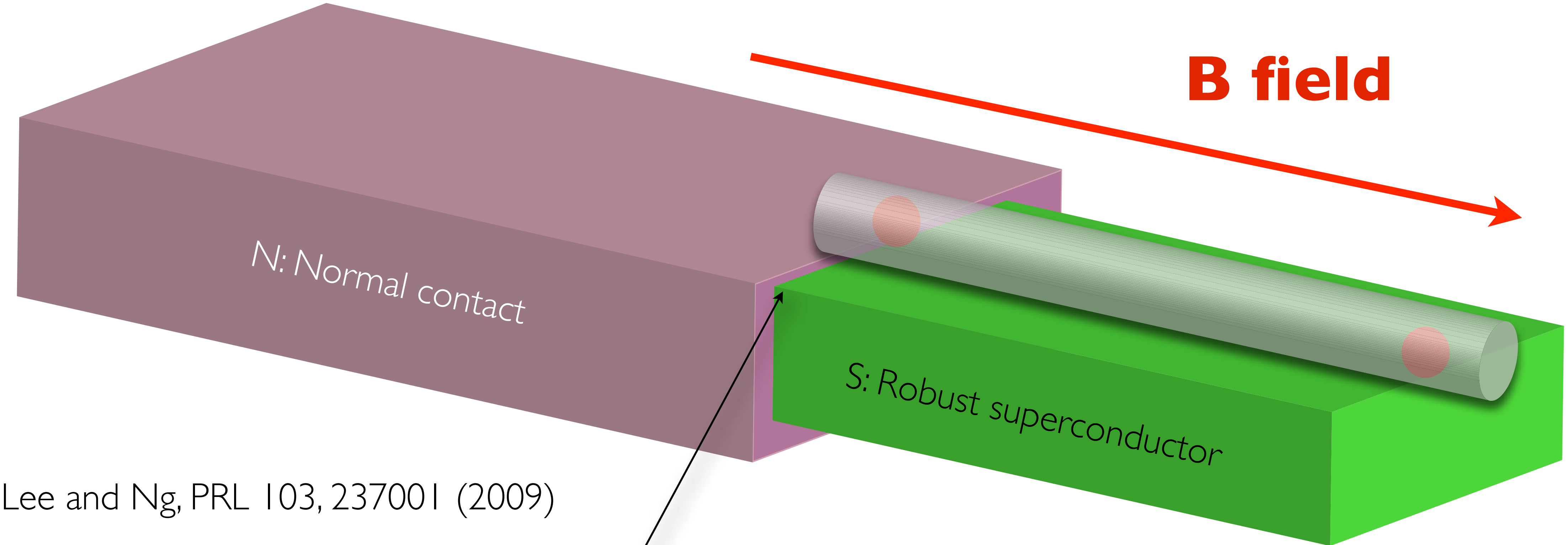
Andreev conductance

$$G = \frac{2e^2}{h} Tr[r_{eh} r_{eh}^\dagger] = \frac{2e^2}{h} \sum_{n=1}^{N} R_n$$

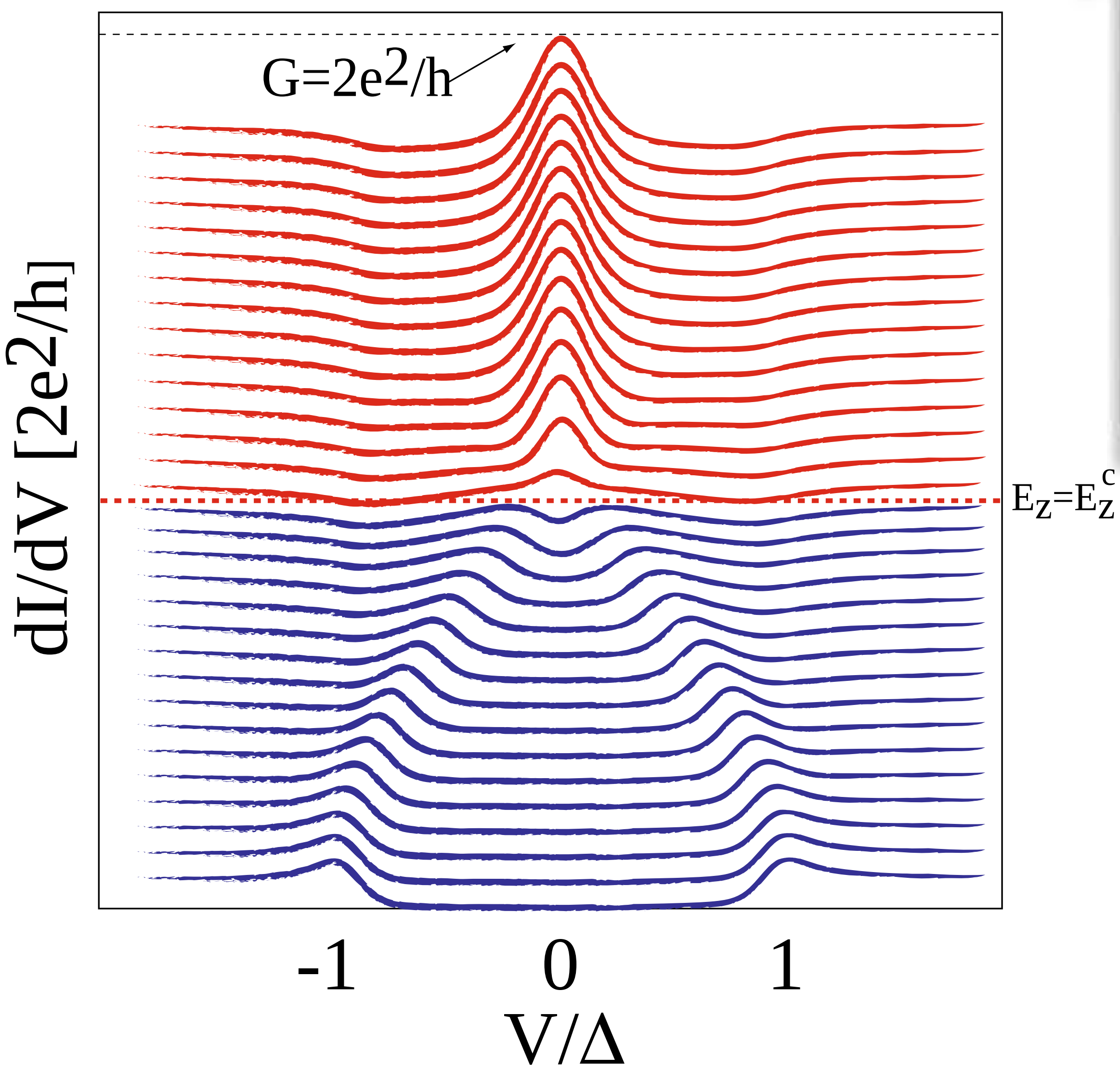
Topologically-trivial regime (BTK formula): For a single spinful channel, this formula describes all the possible values of Andreev conductance, from tunneling to fully ballistic, depending on the transparency of the normal-superconductor interface

$G \sim 0$ Tunneling	$G = 4e^2/h$ Ballistic





Law, Lee and Ng, PRL 103, 237001 (2009)



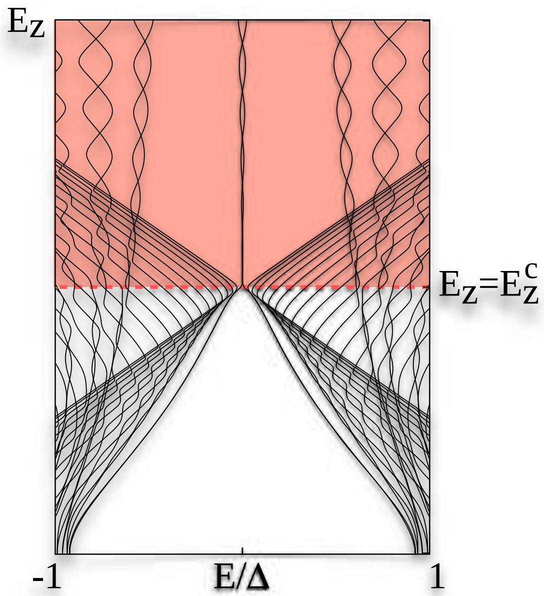
Resonant Andreev reflection in the NS interface: owing to the presence of the Majorana bound state there is a peak at V=0. Particle-hole symmetry implies unitary transport

$$G = \frac{2e^2}{h}$$

Recipe for Majoranas:

- Make a Normal-Superconductor junction with In As or InSb nanowires coupled to, say, Niobium contacts.
- Gate the system to reach low density.
- Increase magnetic field along the wire direction.
- **Look for zero bias peaks in conductance that emerge after a gap closing (transition from trivial to non-trivial phase) as B field increases.**

HOW TO MEASURE MAJORANAS IN NANOWIRES? ANDREEV REFLECTION IN NORMAL-SUPERCONDUCTOR JUNCTIONS



Nonlinear conductance (finite voltage)

$$I = e\int \frac{d\omega}{h}\frac{\Gamma_e\Gamma_h}{\omega^2+(\Gamma_e+\Gamma_h)^2/4}[f(\omega-eV)-f(\omega+eV)]$$

Perfect Electron-hole symmetry

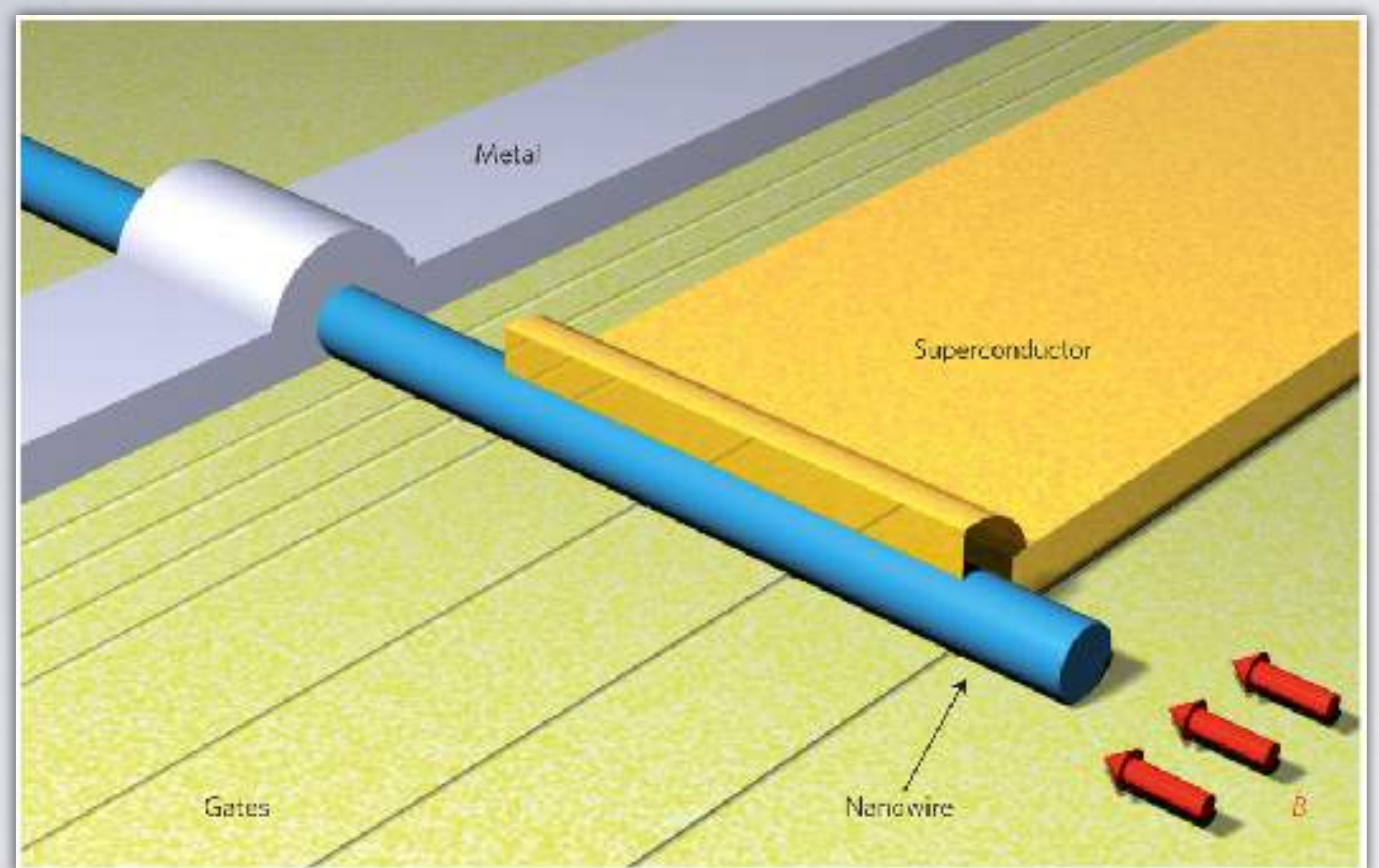
$$\Gamma_e=\Gamma_h=\Gamma$$

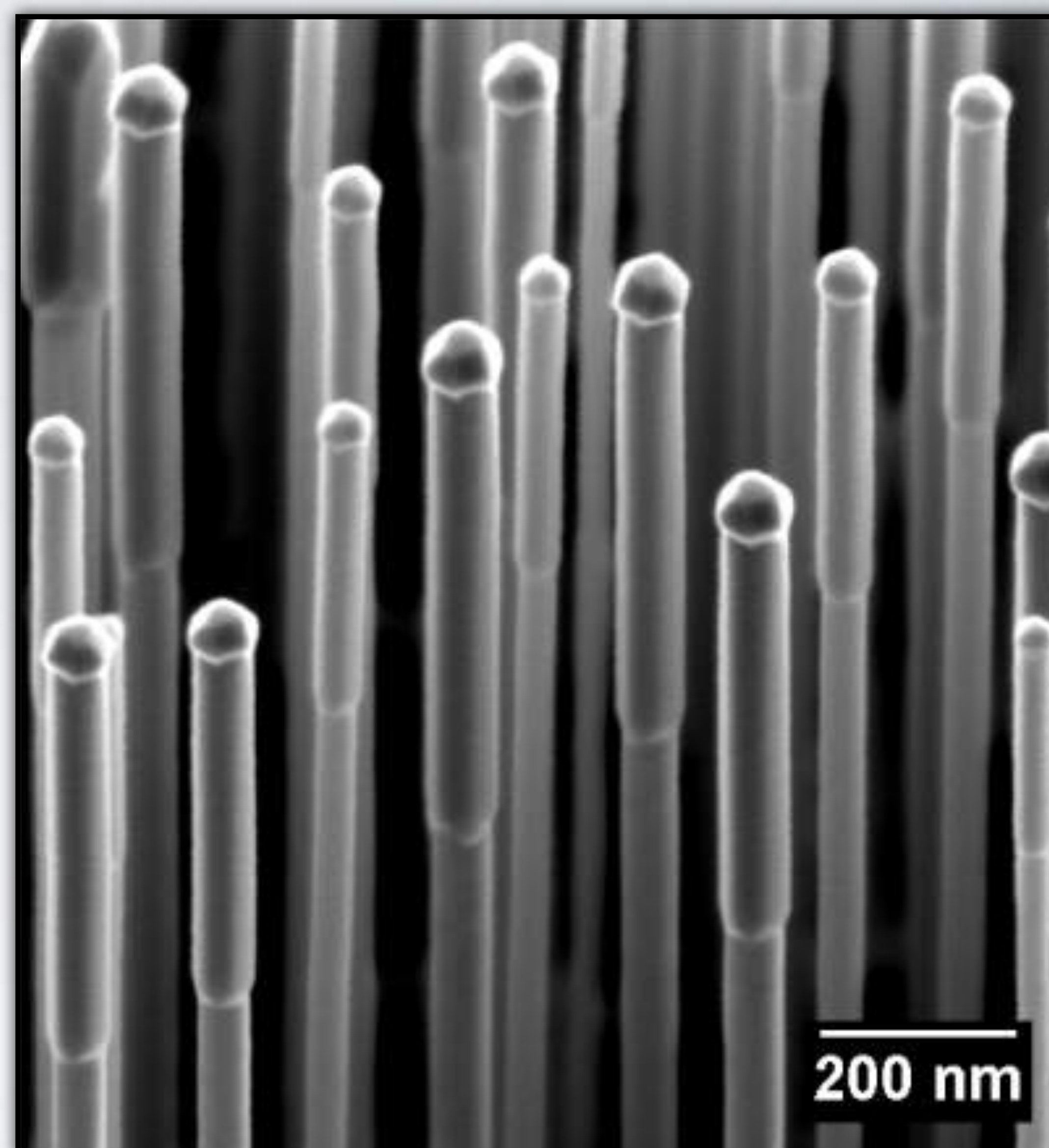
$$\frac{dI}{dV}=\frac{2e^2}{h}\frac{\Gamma^2}{eV^2+\Gamma^2}$$

Zero bias anomaly with quantized conductance at zero temperature

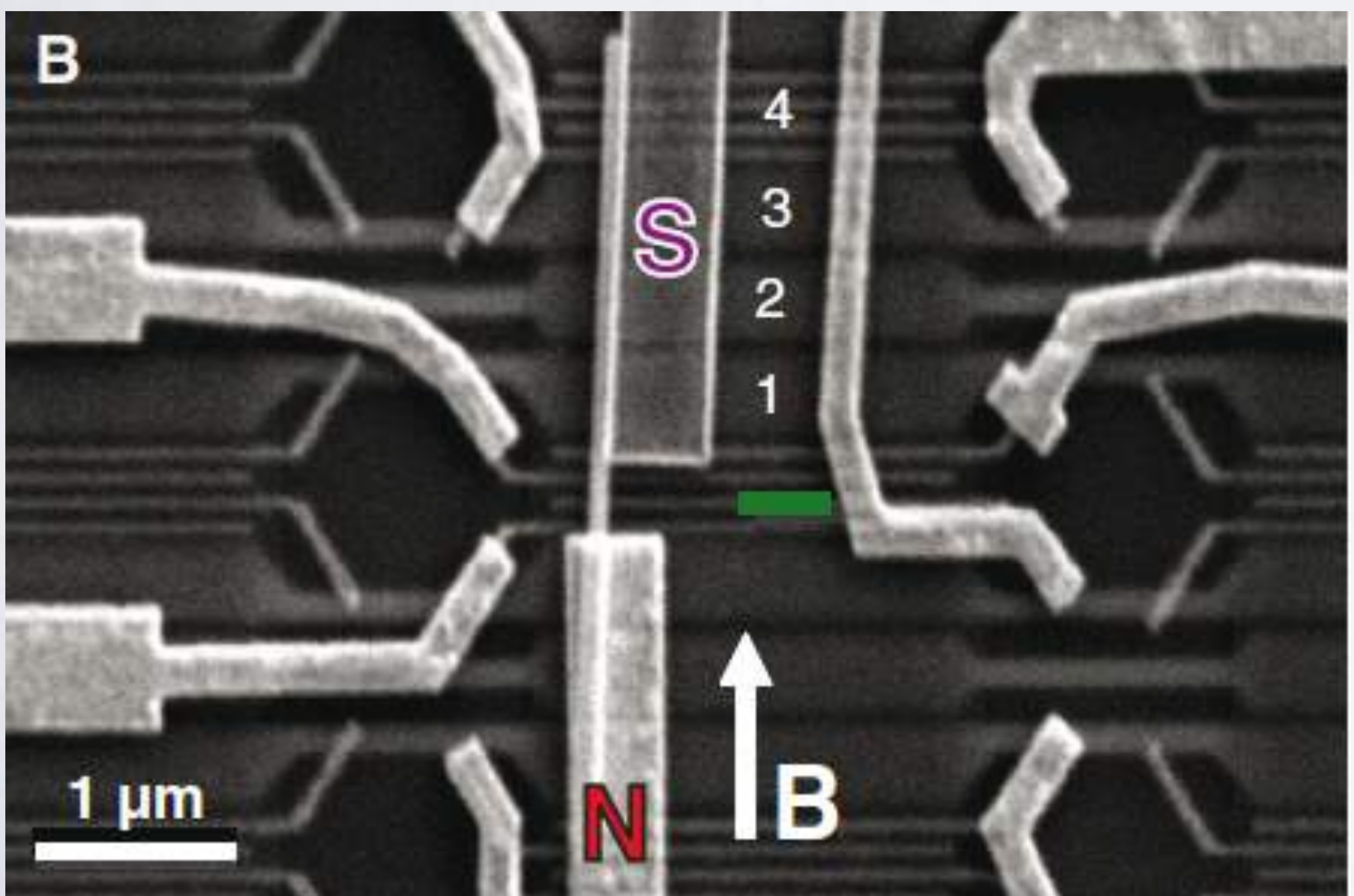
Finite temperature

$$\frac{dI}{dV}=\frac{2e^2}{h}\int d\omega\frac{\Gamma^2}{\omega^2+\Gamma^2}\frac{1}{4k_BTcosh^2((\omega-eV)/2k_BT)}$$



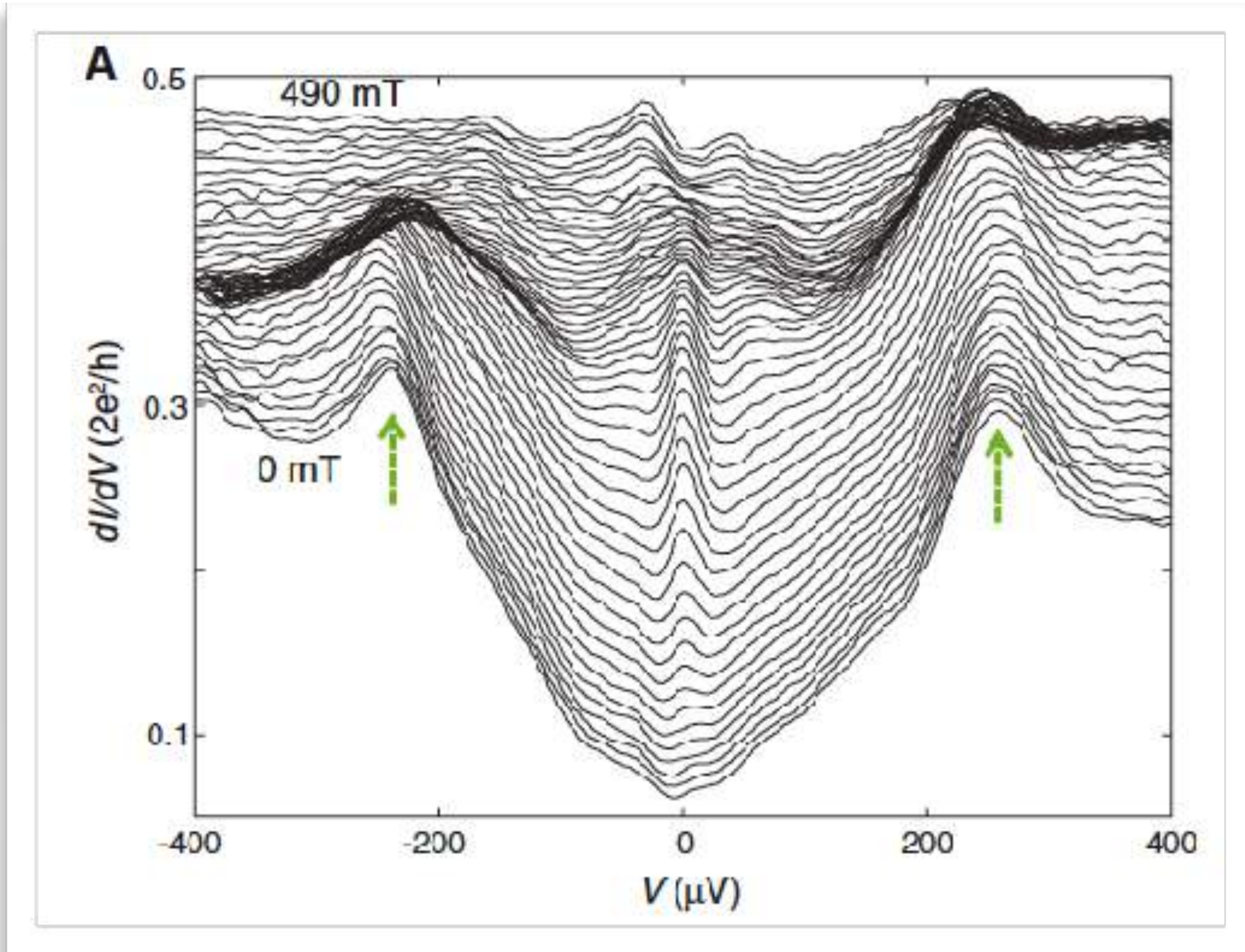


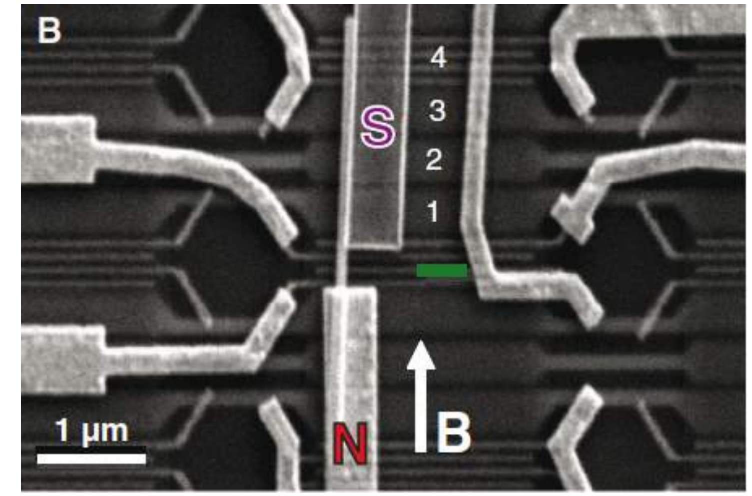
InSb nanowires (Erik Bakker's Eindhoven-Delft)

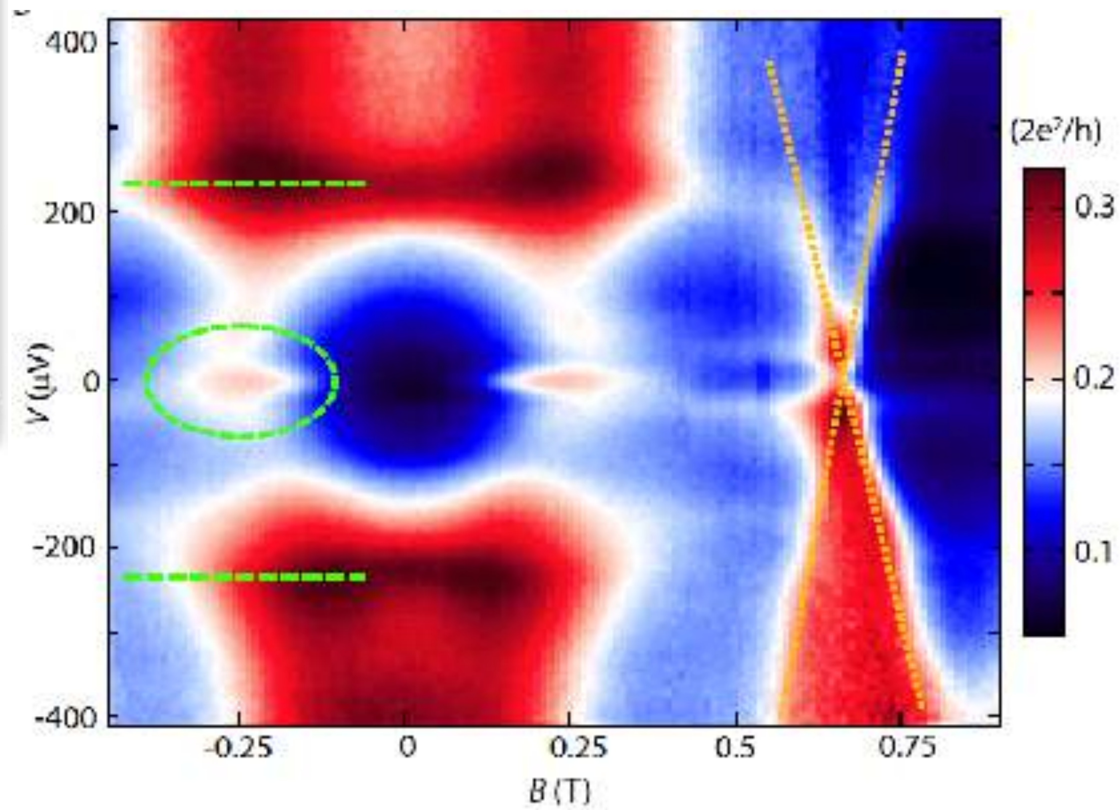


Majorana device
(Leo Kouwenhoven -Delft)

ZERO-BIAS ANOMALY EXPERIMENTS I: DELFT







Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

12 April 2012 / Page 1 / 10.1126/science.1222360

V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1]†

[1]Kavli Institute of Nanoscience, Delft University of Technology, 2600 GA Delft, Netherlands.
[2]Department of Applied Physics, Eindhoven University of Technology, 5600 MB Eindhoven, Netherlands.

Since 2012 there has been considerable debate of whether other physical scenarios giving ZBAs can mimic Majoranas:

- Disorder? Liu et al, Phys. Rev. Lett. 109, 267002 (2012).
- Weak antilocalization? Pikulin et al, New J. Phys. 14 125011
- Kondo? Lee et al, Phys. Rev. Lett. 109, 186802 (2012); Finck et al, Phys. Rev. Lett. 110, 126406 (2013).
- 0.7 anomalies? Churchill et al, Phys. Rev. B 87, 241401(R) (2013). Estrada Saldaña et al (Science Advances, 2019).
- Andreev bound states? Lee et al, Nature Nano 9, 79 (2014)

THESE QUESTIONS CAN BE BROADLY CLASSIFIED IN TWO MORE GENERAL GROUPS:

1) ROLE OF DISORDER, SOFT INDUCED GAPS, ETC.

II) WHAT IS THE ROLE OF ANDREEV LEVELS IN "MAJORANA" NANOWIRES? IS QUANTUM DOT PHYSICS RELEVANT? SMOOTH CONFINEMENT?

GREAT VARIETY OF SAMPLE GEOMETRIES, CONTACTS, GATE CONFIGURATIONS…VERY DIFFERENT FROM THE IDEAL LUTCHYN AND OREG MODELS:
THE DEVIL IS IN THE DETAILS!