



# Alberto Cortijo



### VIII Edition ICMM School

## Lecturers

Ramón Aguado  
Leni Bascones  
Maria José Calderón  
Alberto Cortijo

October, 6-10, 2025

Addressed to PhD candidates, master students and postdoctoral researchers, theorists and experimentalists

**More info & registration**  
<https://wp.icmm.csic.es/emergence/>



Register early to secure a spot.  
There is no registration fee.

Quantum  
spin liquids

nonmagnetic



Paramagnets

Diamagnets

Materials

magnetic

(some sort of magnetic  
order)

colinear

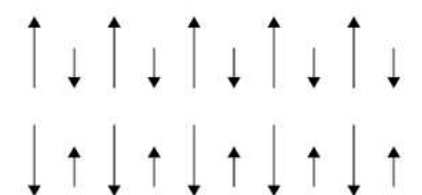


Incommensurate

commensurate

Not Crystal symmetry compensated

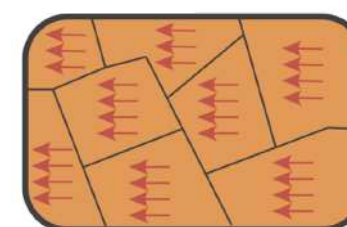
Ferrimagnetism



$$M \neq 0$$

$$N = M_a - M_b \neq 0$$

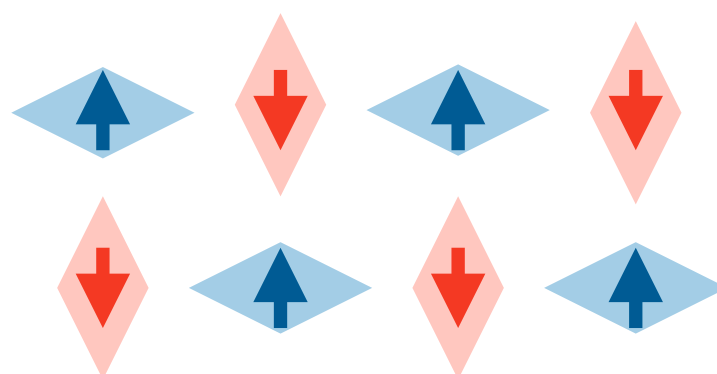
Ferromagnetism



$$M = M_a + M_b \neq 0$$

$$N = 0$$

Altermagnetism

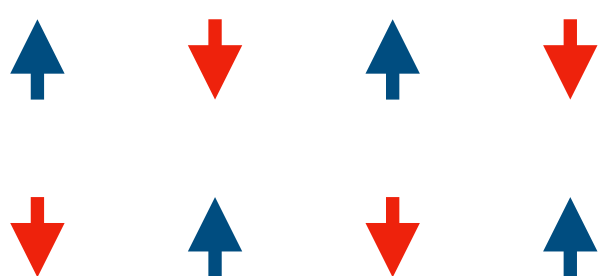


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Crystal symmetry compensated

Antiferromagnetism

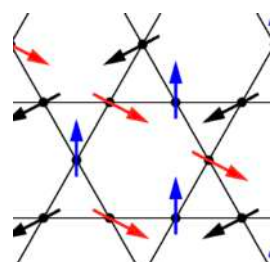


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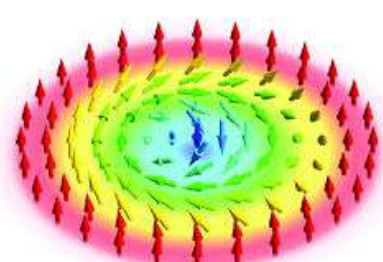
noncolinear

coplanar



P-wave magnets

noncoplanar



Phys. Rev. X 12, 040002

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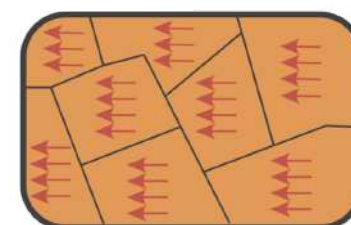
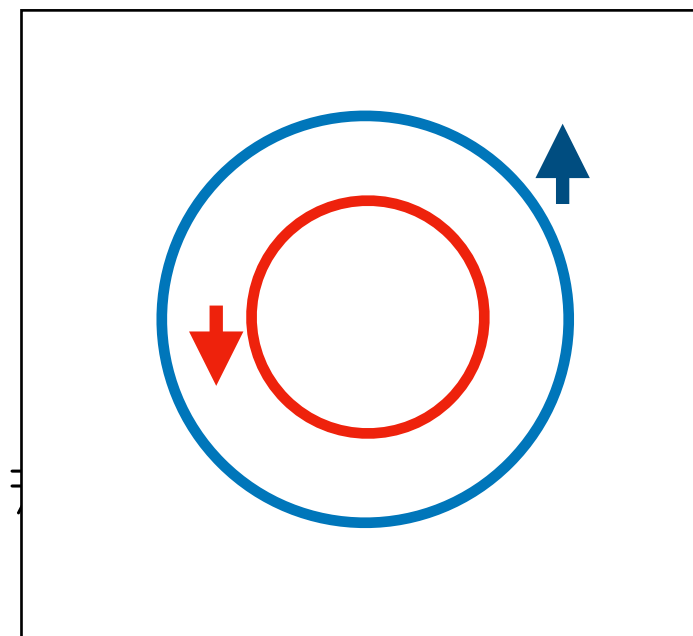
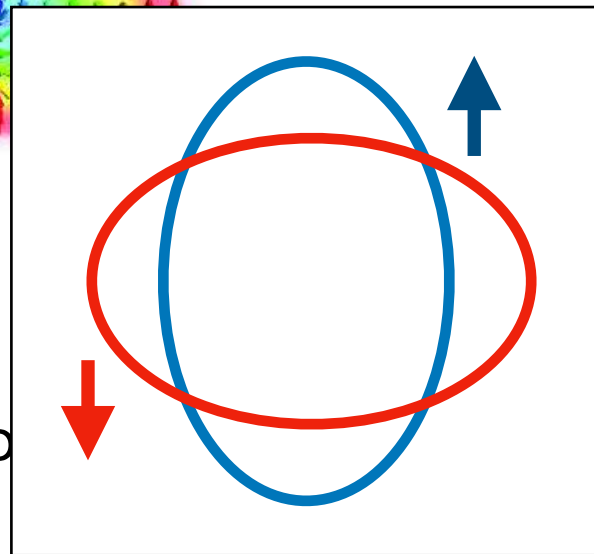
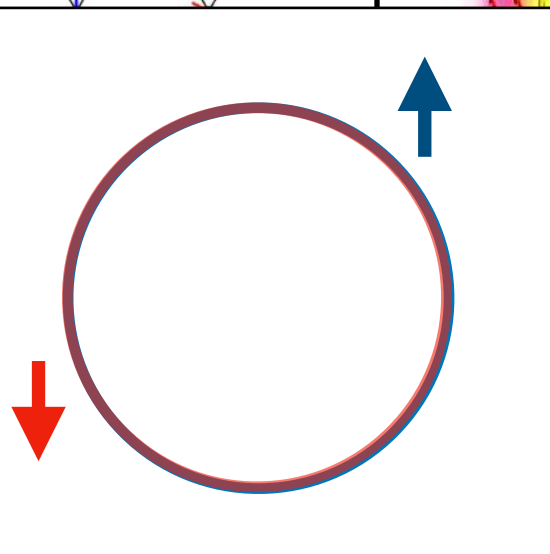
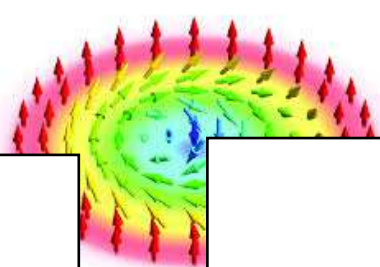
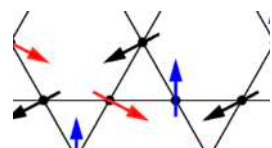
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Altermagnetism

Ferromagnetism



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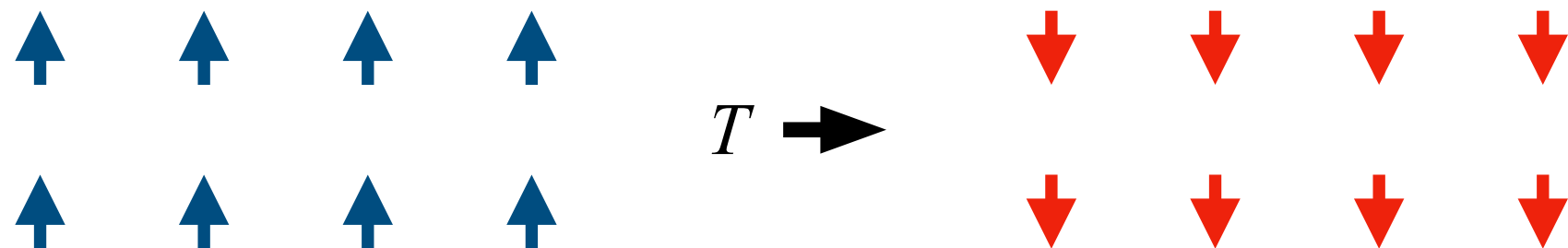
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What does symmetry compensated means?

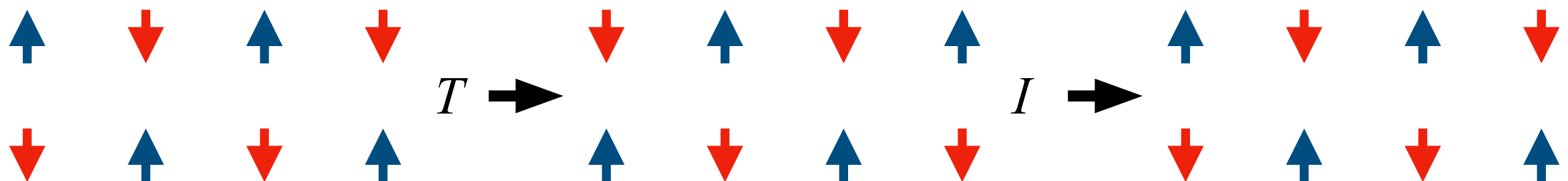
A way to counteract the breakdown of TRS breakdown by lattice symmetries.

FerroMagnets



If the lattice is inversion invariant:  $T \cdot I$  is not a symmetry.

AntiFerroMagnets

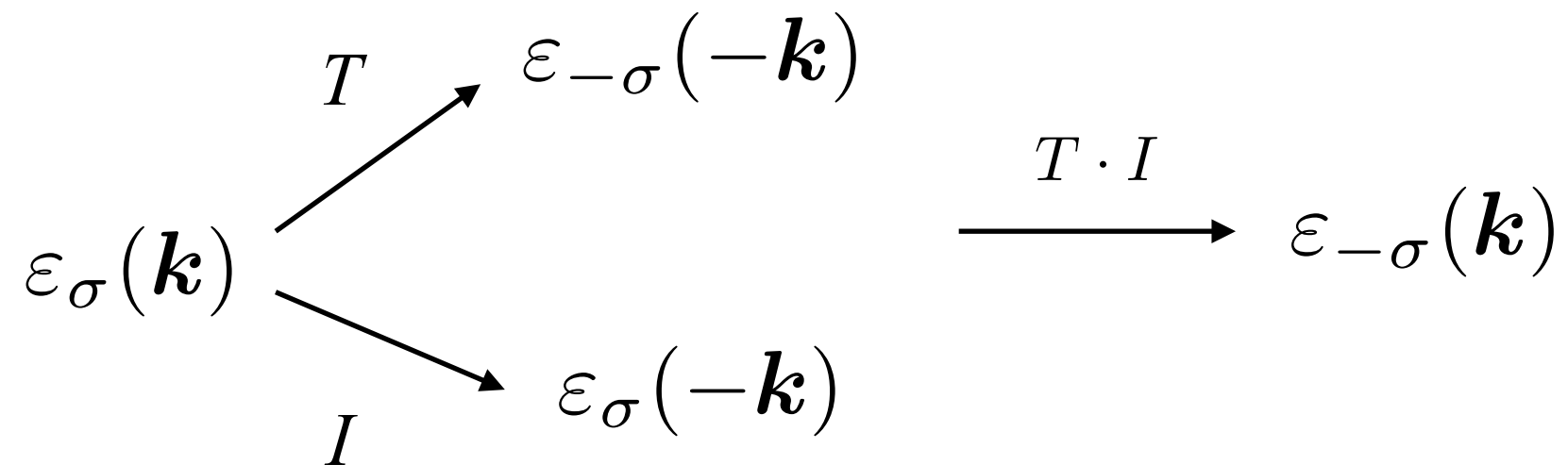


If the lattice is not inversion invariant:  $T \cdot I$  is a symmetry.

Importantly, here the system might or might not be inversion invariant.  
It is the combined operation what is a symmetry

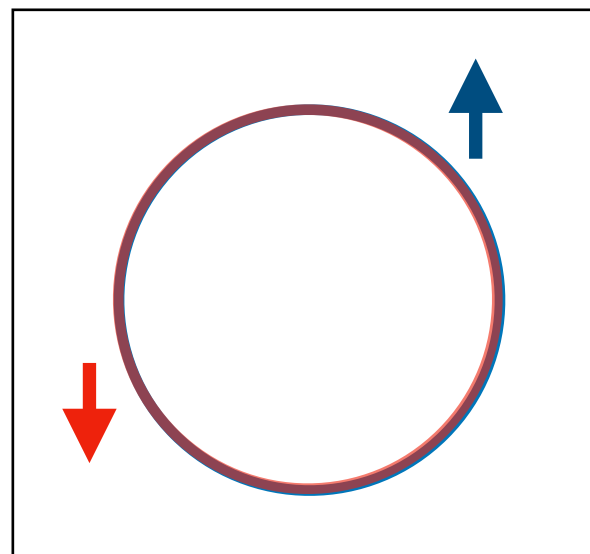
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$T \cdot I$  is a symmetry:  $\varepsilon_{-\sigma}(\mathbf{k}) = \varepsilon_{\sigma}(\mathbf{k})$

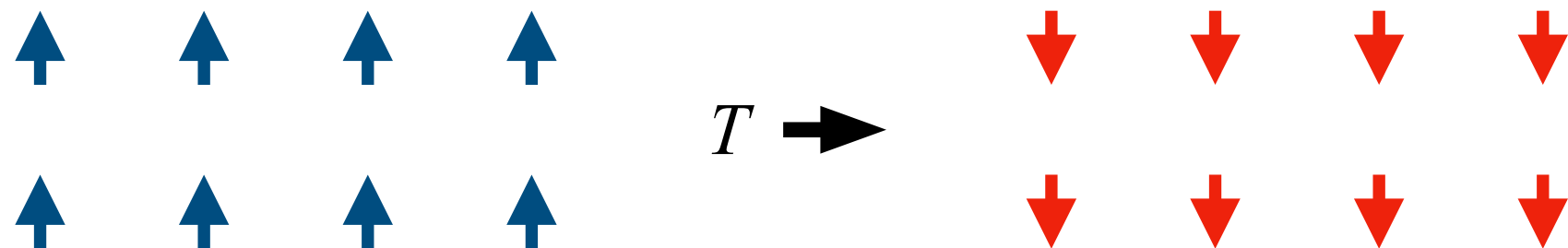
Spin split bands are degenerate



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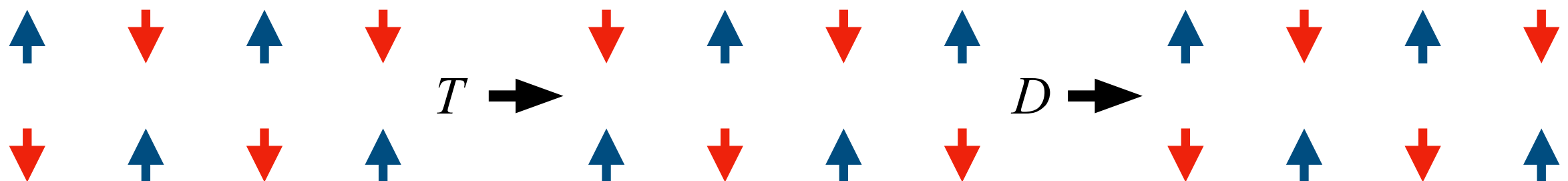
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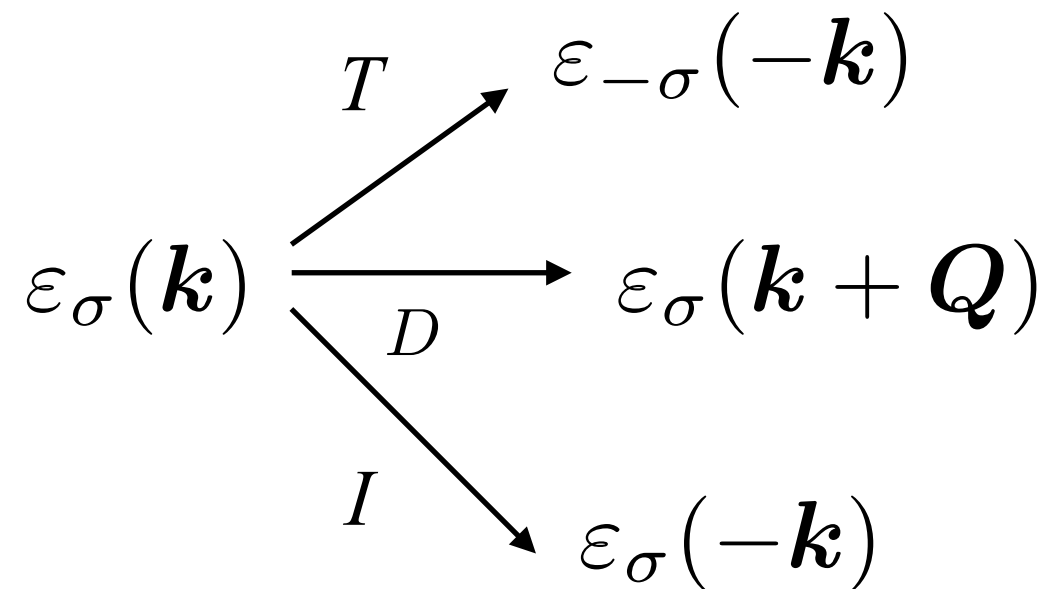


If the lattice is inversion invariant:  $(T \cdot D) \cdot I$  is a symmetry.

Importantly, here the system might or might not be inversion invariant.  
It is the combined operation what is a symmetry

What does symmetry compensated means?

If the lattice is inversion invariant:  $T \cdot I$  is a symmetry.



$$T \cdot D$$

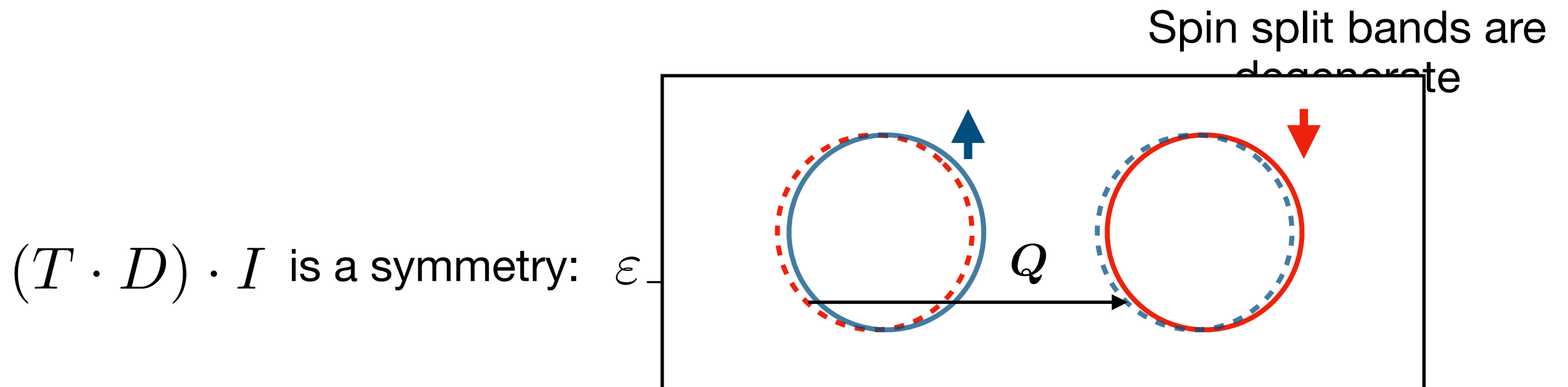
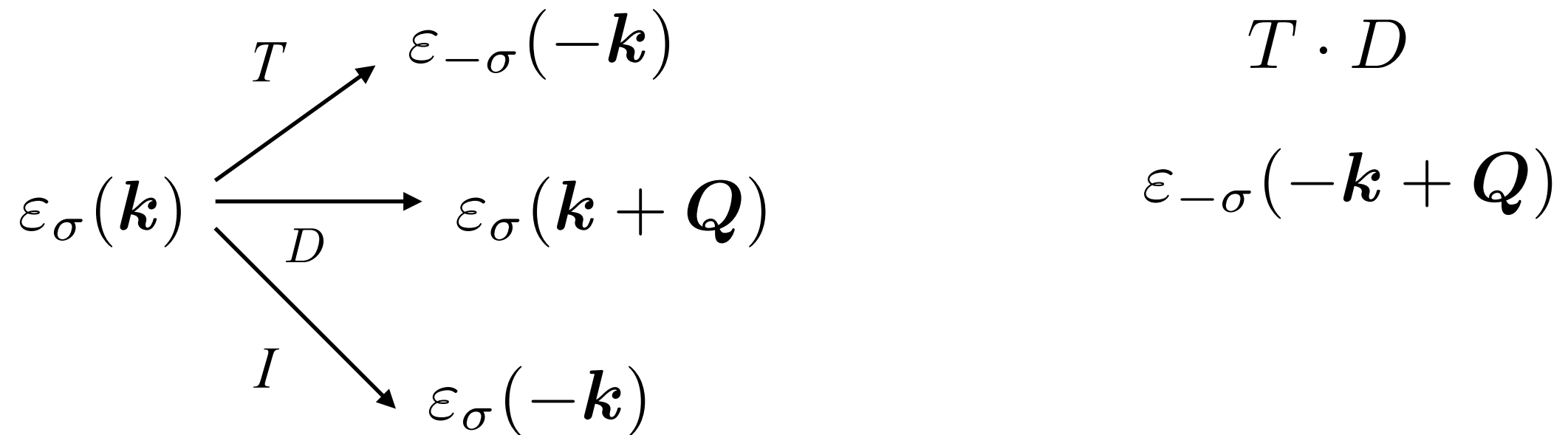
$$\varepsilon_{-\sigma}(-\mathbf{k} + \mathbf{Q})$$

Spin split bands are degenerate

$$(T \cdot D) \cdot I \text{ is a symmetry: } \varepsilon_{-\sigma}(\mathbf{k} + \mathbf{Q}) = \varepsilon_{\sigma}(\mathbf{k})$$

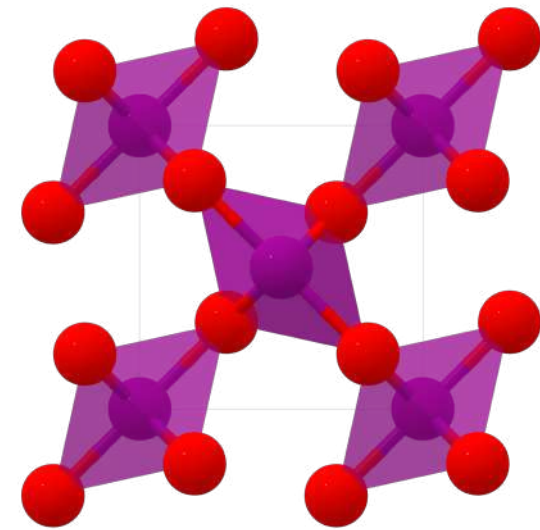
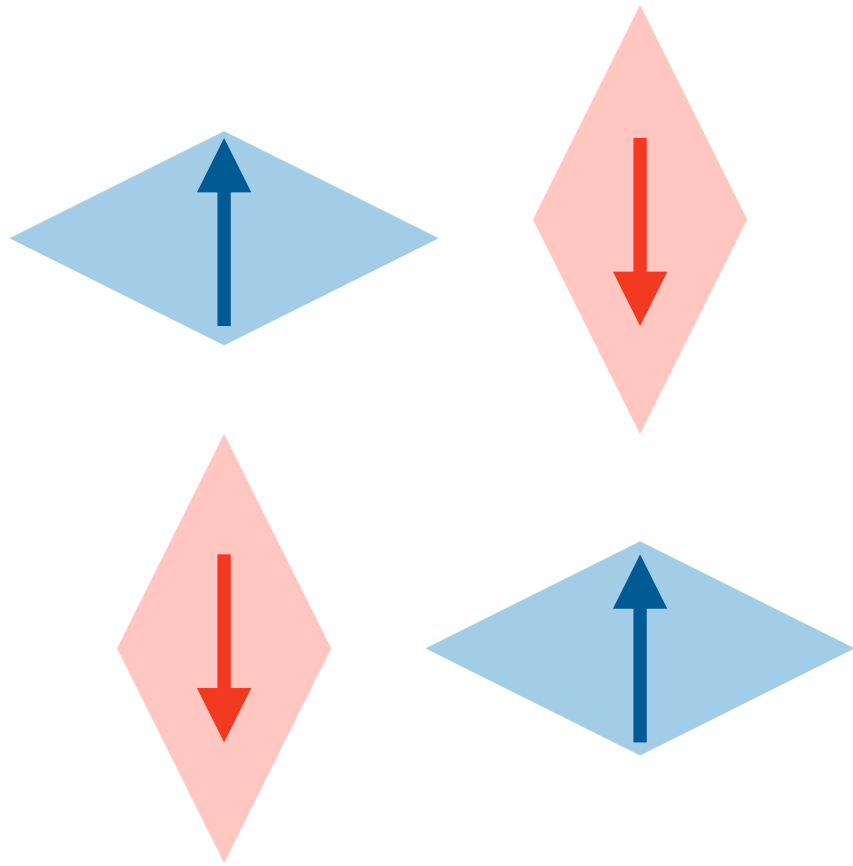
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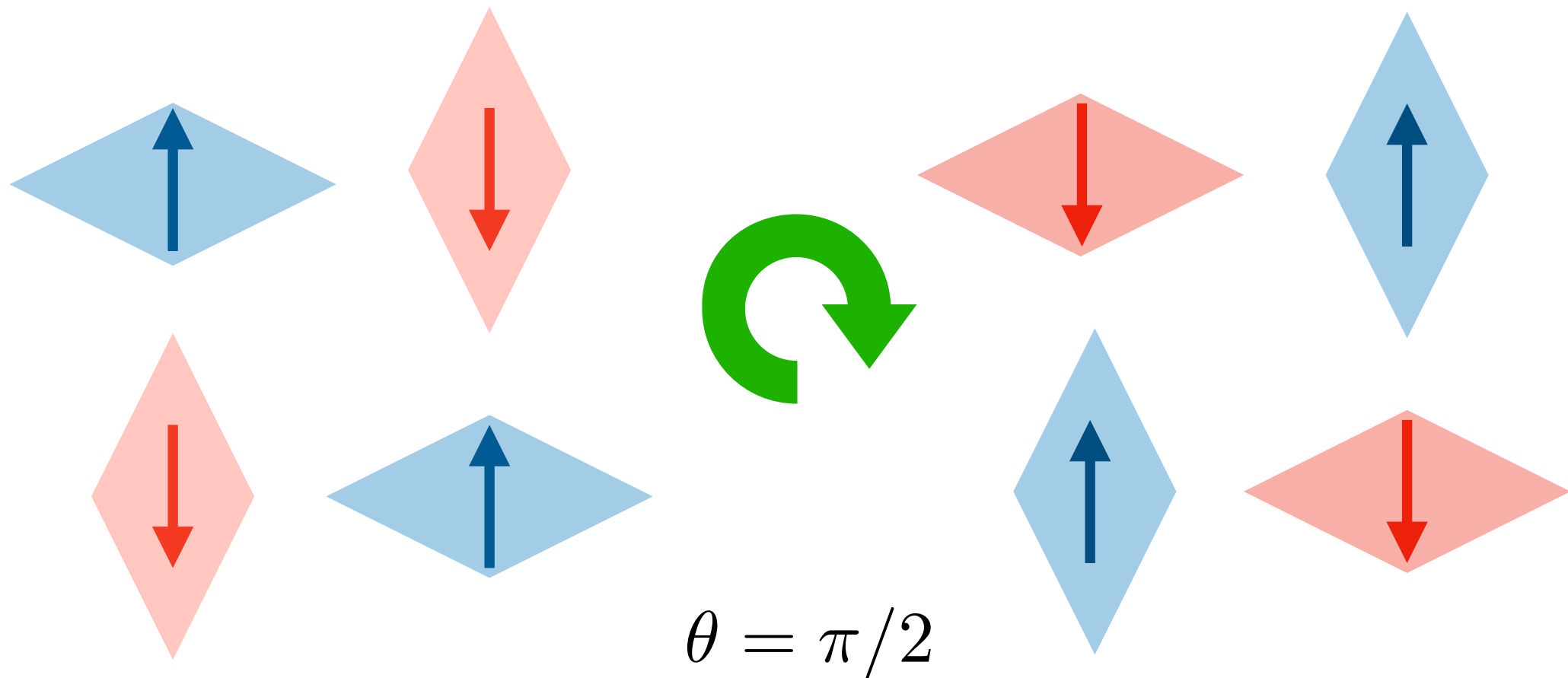
Now, in altermagnets, the previous symmetries do not apply:



$MnO_2$

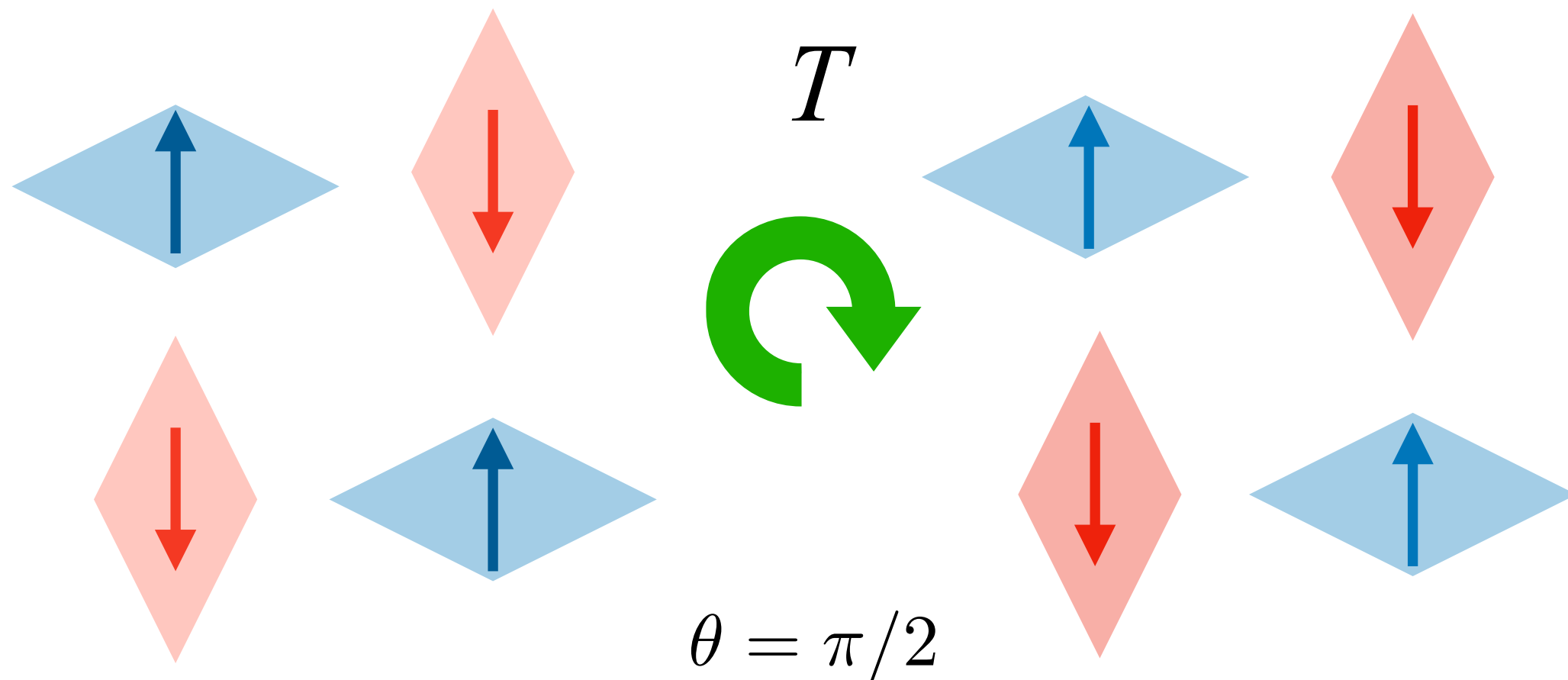
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What does symmetry compensated means?

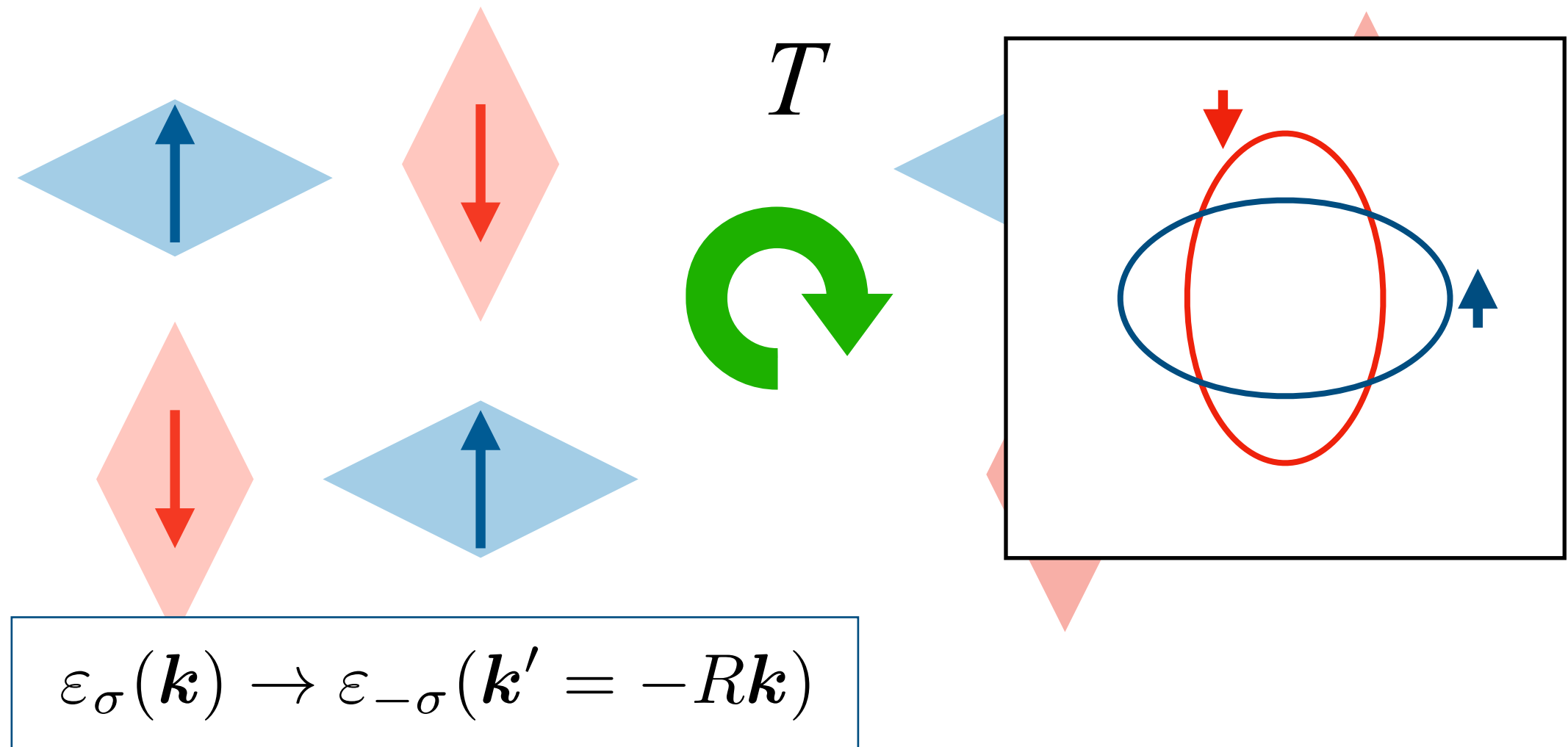
Now, in altermagnets, the previous symmetries do not apply:



In this example, the system is invariant under  $T$  and rotation of  $\pi/2$

# What does symmetry compensated means?

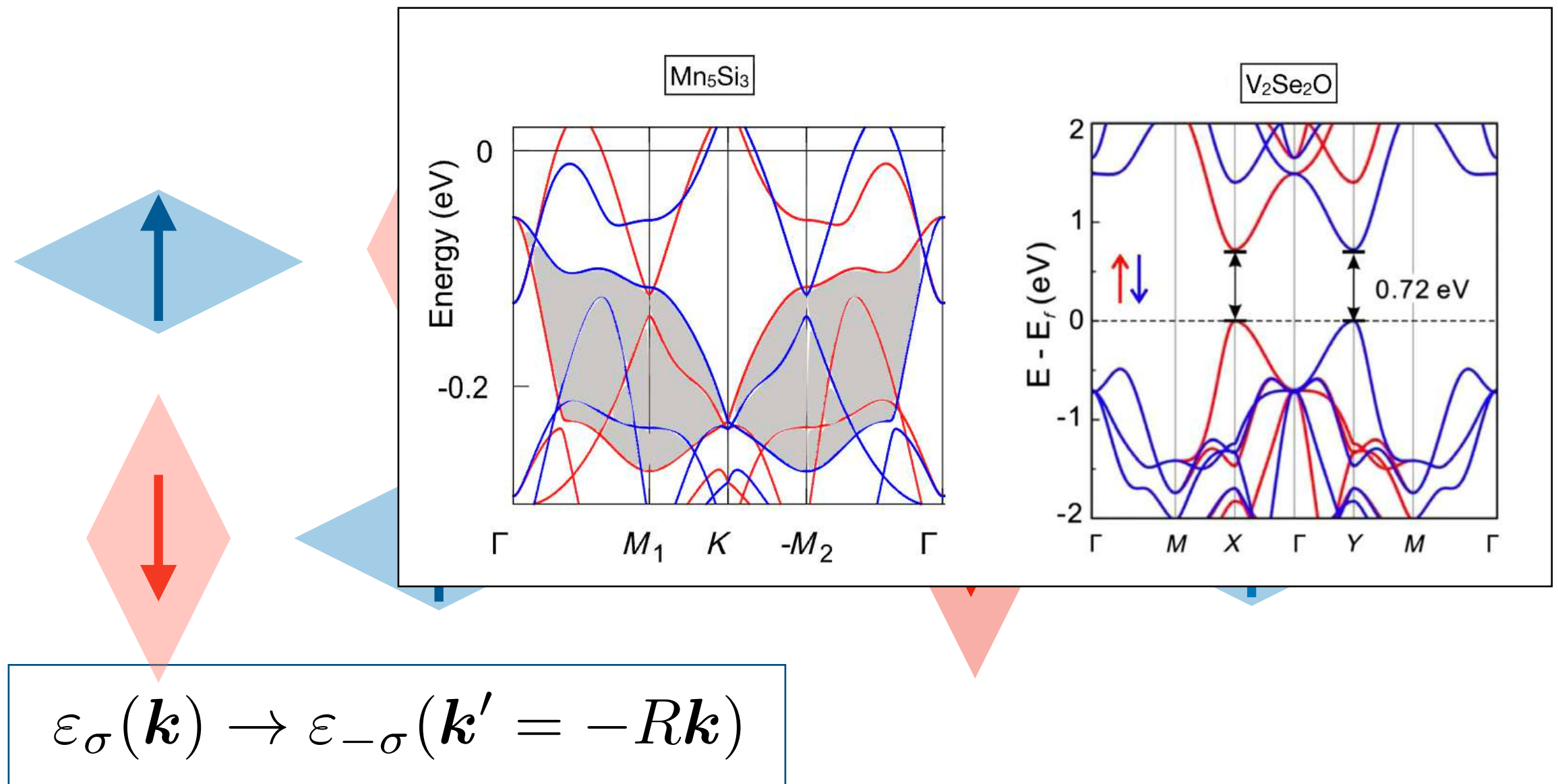
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Kramers degeneracy (Zeeman splitting) is lifted although the magnetization is zero

# What does symmetry compensated means?

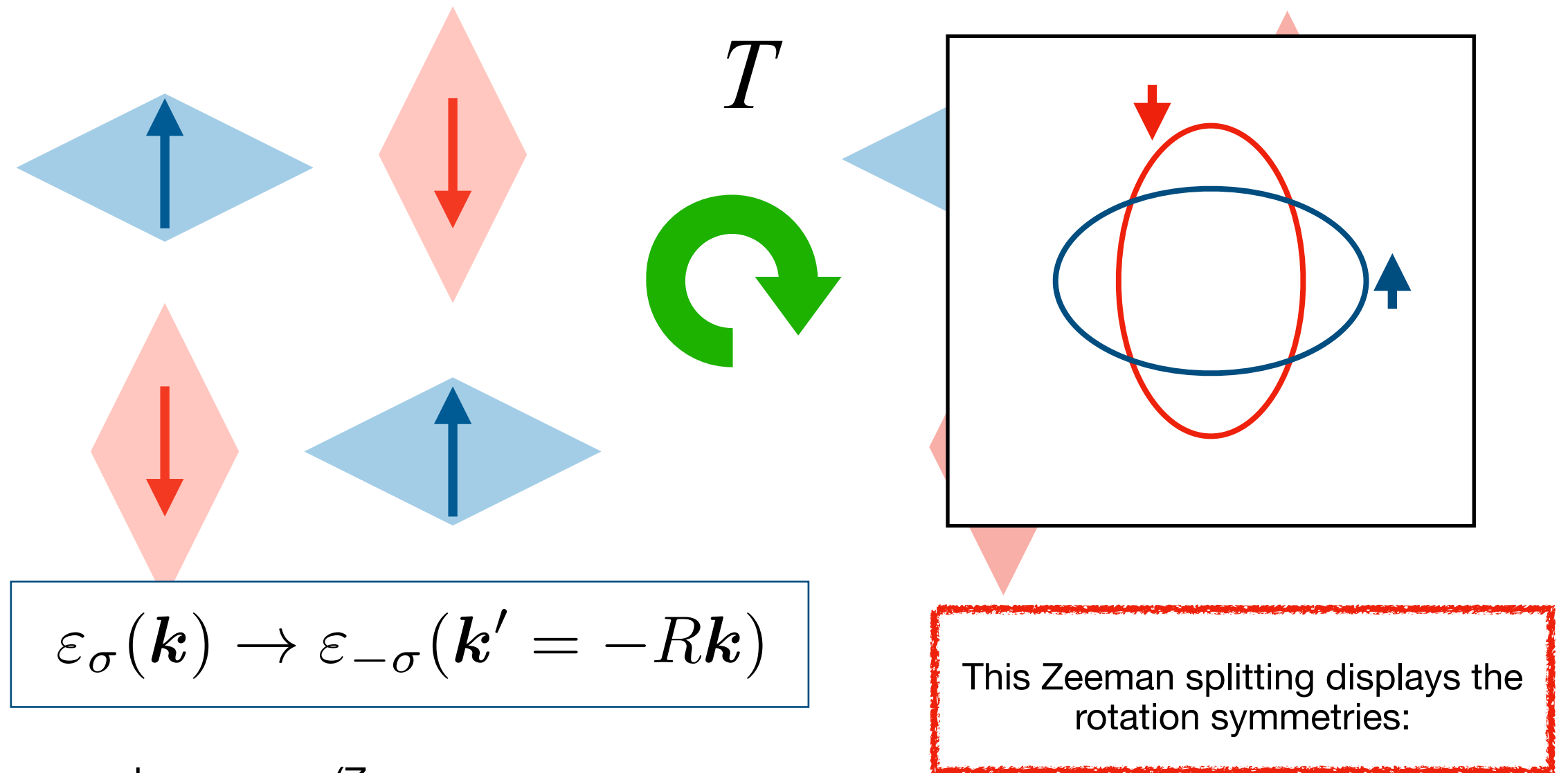
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# What does symmetry compensated means?

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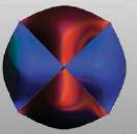
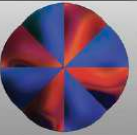
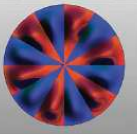
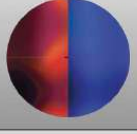
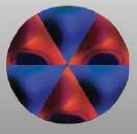
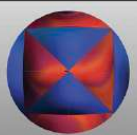


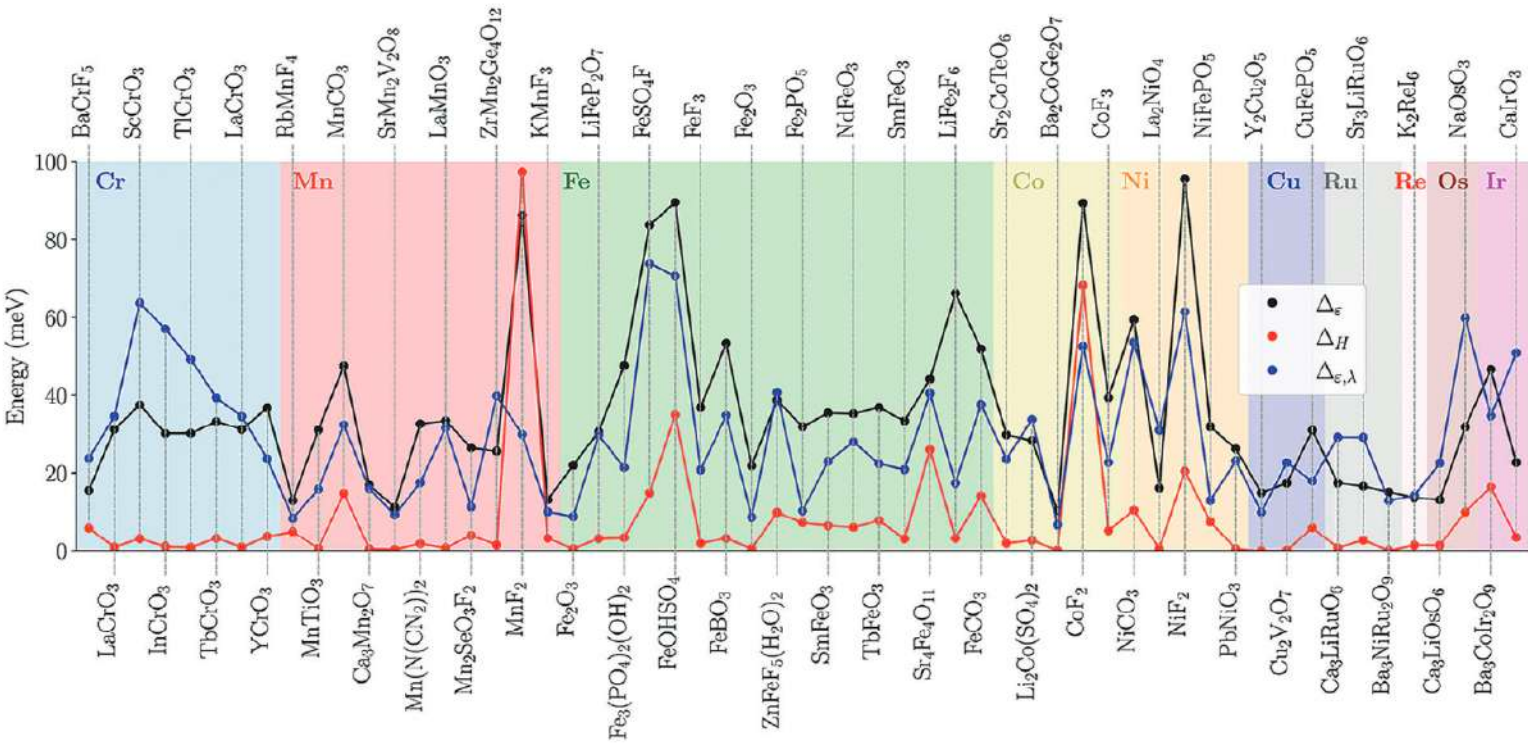
Kramers degeneracy (Zeeman splitting) is lifted although the magnetization is zero

$$\Delta\varepsilon(\mathbf{k}) = \varepsilon_{\uparrow}(\mathbf{k}) - \varepsilon_{\downarrow}(\mathbf{k}) \sim \sin k_1 \sin k_2$$

# Symmetries in altermagnets

| SG ( $P$ )      | Wyckoff ( $S$ )    | $\Gamma_N$ | Spin splitting ( $f_{\Gamma_N}(\mathbf{k})$ ) |
|-----------------|--------------------|------------|-----------------------------------------------|
| 11 ( $C_{2h}$ ) | 2a-2d ( $C_i$ )    | $B_g$      | $\alpha k_y k_x + \beta k_y k_z$              |
| 12 ( $C_{2h}$ ) | 4e, 4f ( $C_i$ )   | $B_g$      | $\alpha k_y k_x + \beta k_y k_z$              |
| 13 ( $C_{2h}$ ) | 2a-2d ( $C_i$ )    | $B_g$      | $\alpha k_y k_x + \beta k_y k_z$              |
| 14 ( $C_{2h}$ ) | 2a-2d ( $C_i$ )    | $B_g$      | $\alpha k_y k_x + \beta k_y k_z$              |
| 15 ( $C_{2h}$ ) | 2a-2d ( $C_i$ )    | $B_g$      | $\alpha k_y k_x + \beta k_y k_z$              |
| 49 ( $D_{2h}$ ) | 2a-2d ( $C_{2h}$ ) | $B_{1g}$   | $k_x k_y$                                     |
| 51 ( $D_{2h}$ ) | 2a-2d ( $C_{2h}$ ) | $B_{2g}$   | $k_x k_z$                                     |
| 53 ( $D_{2h}$ ) | 2a-2d ( $C_{2h}$ ) | $B_{3g}$   | $k_y k_z$                                     |
| 55 ( $D_{2h}$ ) | 2a-2d ( $C_{2h}$ ) | $B_{1g}$   | $k_x k_y$                                     |
| 58 ( $D_{2h}$ ) | 2a-2d ( $C_{2h}$ ) | $B_{1g}$   | $k_x k_y$                                     |
| 63 ( $D_{2h}$ ) | 4a,4b ( $C_{2h}$ ) | $B_{3g}$   | $k_y k_z$                                     |
| 64 ( $D_{2h}$ ) | 4a,4b ( $C_{2h}$ ) | $B_{3g}$   | $k_y k_z$                                     |
| 65 ( $D_{2h}$ ) | 4e,4f ( $C_{2h}$ ) | $B_{1g}$   | $k_x k_z$                                     |
| 66 ( $D_{2h}$ ) | 4c-4f ( $C_{2h}$ ) | $B_{1g}$   | $k_x k_y$                                     |
| 67 ( $D_{2h}$ ) | 4c-4f ( $C_{2h}$ ) | $B_{3g}$   | $k_x k_y$                                     |
| 72 ( $D_{2h}$ ) | 4c,4d ( $C_{2h}$ ) | $B_{1g}$   | $k_x k_y$                                     |
| 74 ( $D_{2h}$ ) | 4a,4b ( $C_{2h}$ ) | $B_{3g}$   | $k_x k_y$                                     |
| 74 ( $D_{2h}$ ) | 4c,4d ( $C_{2h}$ ) | $B_{2g}$   | $k_x k_y$                                     |

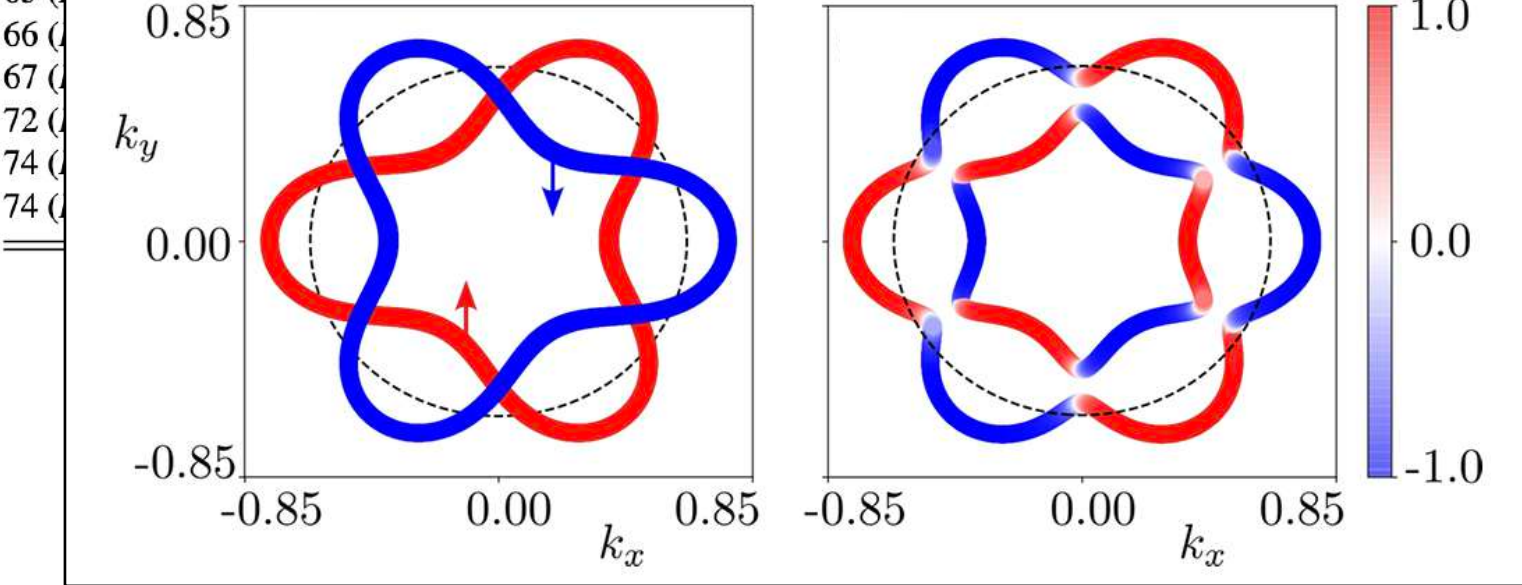
| Spin-momentum locking                                                                                                |                  |                                                                                                       | G           | $R_s^{\text{III}}$     | H          | A        | Candidate                                                            |
|----------------------------------------------------------------------------------------------------------------------|------------------|-------------------------------------------------------------------------------------------------------|-------------|------------------------|------------|----------|----------------------------------------------------------------------|
| Planar<br><br>( $k_x, k_y, k_z$ ) | P-2<br>$d$ -wave | <br>( $k_x, k_y$ ) | $mmm$       | $^2m^2m^1m$ (8)        | $2/m$      | $C_{2x}$ | La <sub>2</sub> CuO <sub>4</sub> , FeSb <sub>2</sub>                 |
|                                                                                                                      |                  |                                                                                                       | $4/m$       | $^24/1m$ (8)           |            | $C_{4z}$ | KRu <sub>4</sub> O <sub>8</sub>                                      |
|                                                                                                                      | P-4<br>$g$ -wave |                    | $4/mmm$     | $^24/1m^2m^1m$ (16)    | $mmm$      | $C_{4z}$ | RuO <sub>2</sub> ,<br>MnO <sub>2</sub> , MnF <sub>2</sub>            |
|                                                                                                                      |                  |                                                                                                       |             | $^14/1m^2m^2m$ (16)    | $4/m$      | $C_{2x}$ | KMnF <sub>3</sub>                                                    |
|                                                                                                                      | P-6<br>$i$ -wave |                    | $6/mmm$     | $^16/1m^2m^2m$ (24)    | $6/m$      | $C_{21}$ |                                                                      |
| Bulk<br>                         | B-2<br>$d$ -wave |                    | $2/m$       | $^22/2m$ (4)           | $\bar{7}$  | $C_{2z}$ | CuF <sub>2</sub>                                                     |
|                                                                                                                      | B-4<br>$g$ -wave |                    | $\bar{3}m$  | $^1\bar{3}^2m$ (12)    | $\bar{3}$  | $C_{21}$ | CoF <sub>3</sub> , FeF <sub>3</sub> , Fe <sub>2</sub> O <sub>3</sub> |
|                                                                                                                      |                  |                                                                                                       | $6/m$       | $^26/2m$ (12)          |            | $C_{6z}$ |                                                                      |
|                                                                                                                      |                  |                                                                                                       | $6/mmm$     | $^26/2m^2m^1m$ (24)    | $\bar{3}m$ | $C_{6z}$ | CrSb, MnTe, VNb <sub>3</sub> S <sub>6</sub>                          |
|                                                                                                                      | B-6<br>$i$ -wave |                  | $m\bar{3}m$ | $^1m^1\bar{3}^2m$ (48) | $m\bar{3}$ | $C_{4z}$ |                                                                      |

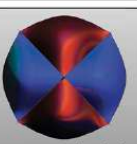
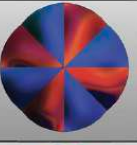
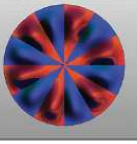
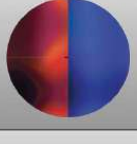
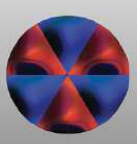
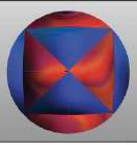


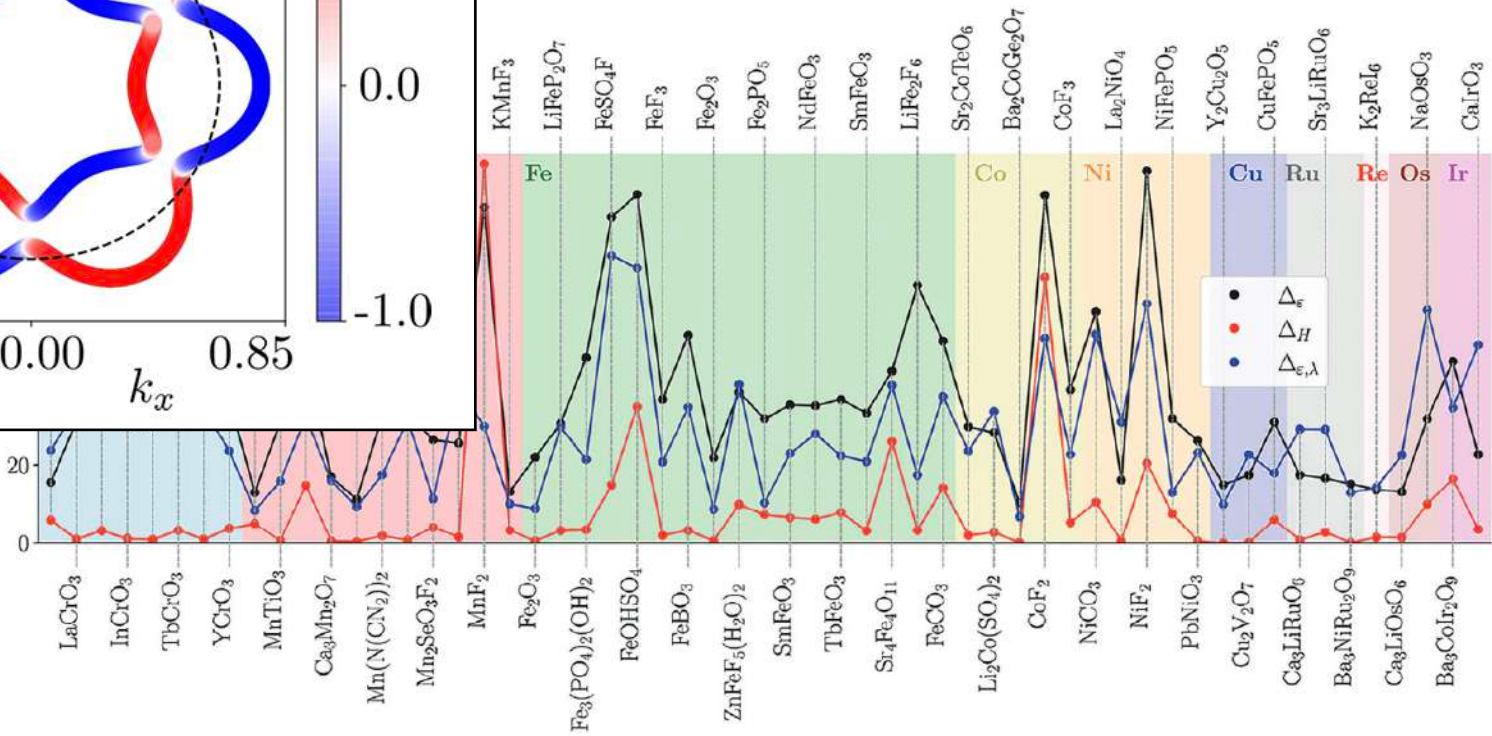
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|                 |                     |          |           |
|-----------------|---------------------|----------|-----------|
| 49 ( $D_{2h}$ ) | 2a-2d ( $C_{2h}$ )  | $B_{1g}$ | $k_x k_y$ |
| 51 ( $D_{2h}$ ) | 2a-2d ( $C_{2h}$ )  | $B_{2g}$ | $k_x k_z$ |
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| Spin-momentum locking                                                                                           |               |                                                                                                       | G           | $R_s^{III}$          | H          | A        | Candidate                                                            |
|-----------------------------------------------------------------------------------------------------------------|---------------|-------------------------------------------------------------------------------------------------------|-------------|----------------------|------------|----------|----------------------------------------------------------------------|
| Planar<br><br>( $k_x, k_y$ ) | P-2<br>d-wave | <br>( $k_x, k_y$ ) | mmm         | $2m^2m^1m$ (8)       | 2/m        | $C_{2x}$ | La <sub>2</sub> CuO <sub>4</sub> , FeSb <sub>2</sub>                 |
|                                                                                                                 |               |                                                                                                       | 4/m         | $24/1m$ (8)          |            | $C_{4z}$ | KRu <sub>4</sub> O <sub>8</sub>                                      |
|                                                                                                                 | P-4<br>g-wave |                    | 4/mmm       | $24/1m^2m^1m$ (16)   | mmm        | $C_{4z}$ | RuO <sub>2</sub> ,<br>MnO <sub>2</sub> , MnF <sub>2</sub>            |
|                                                                                                                 |               |                                                                                                       |             | $14/1m^2m^2m$ (16)   | 4/m        | $C_{2x}$ | KMnF <sub>3</sub>                                                    |
|                                                                                                                 | P-6<br>i-wave |                    | 6/mmm       | $16/1m^2m^2m$ (24)   | 6/m        | $C_{21}$ |                                                                      |
| Bulk<br>                    | B-2<br>d-wave |                    | 2/m         | $2^2/2m$ (4)         | $\bar{7}$  | $C_{2z}$ | CuF <sub>2</sub>                                                     |
|                                                                                                                 | B-4<br>g-wave |                    | $\bar{3}m$  | $1\bar{3}^2m$ (12)   | $\bar{3}$  | $C_{21}$ | CoF <sub>3</sub> , FeF <sub>3</sub> , Fe <sub>2</sub> O <sub>3</sub> |
|                                                                                                                 |               |                                                                                                       | 6/m         | $26/2m$ (12)         |            | $C_{6z}$ |                                                                      |
|                                                                                                                 |               |                                                                                                       | 6/mmm       | $26/2m^2m^1m$ (24)   | $\bar{3}m$ | $C_{6z}$ | CrSb, MnTe, VNb <sub>3</sub> S <sub>6</sub>                          |
|                                                                                                                 | B-6<br>i-wave |                  | $m\bar{3}m$ | $1m1\bar{3}^2m$ (48) | $m\bar{3}$ | $C_{4z}$ |                                                                      |



Ok, which properties do alter magnets display?

|                                                                 | FM              | AM   | AF   |                |
|-----------------------------------------------------------------|-----------------|------|------|----------------|
| Nonrelativistic net magnetization                               | nonzero or zero | zero | zero |                |
| Nonrelativistic Kramers spin degeneracy                         | no              | no   | yes  |                |
| Anomalous Hall                                                  | yes             | yes  | no   | Allowed by SOC |
| Magnetooptics                                                   | yes             | yes  | no   |                |
| Nonrelativistic spin-polarized current                          | yes             | yes  | no   |                |
| Suppression of Andreev reflection: diffuse contact              | yes             | no   | no   |                |
| Suppression of Andreev reflection: ballistic contact            | yes             | yes  | no   |                |
| Supports singlet superconductivity                              | no              | no   | yes  |                |
| Supports locally (in k-space) unitary triplet superconductivity | no              | no   | yes  |                |
| Supports k-averaged unitary triplet superconductivity           | no              | yes  | yes  |                |
| Giant or tunneling magnetoresistance and spin-transfer torque   | yes             | yes  | no   |                |

In essence, any transport/optical/device properties related to the presence of a Zeeman splitting

## Ok, which properties do alter magnets display?

|                                                                 | FM              | AM   | AF   |                |
|-----------------------------------------------------------------|-----------------|------|------|----------------|
| Nonrelativistic net magnetization                               | nonzero or zero | zero | zero |                |
| Nonrelativistic Kramers spin degeneracy                         | no              | no   | yes  |                |
| Anomalous Hall                                                  | yes             | yes  | no   | Allowed by SOC |
| Magnetooptics                                                   | yes             | yes  | no   |                |
| Nonrelativistic spin-polarized current                          | yes             | yes  | no   |                |
| Suppression of Andreev reflection: diffuse contact              | yes             | no   | no   |                |
| Suppression of Andreev reflection: ballistic contact            | yes             | yes  | no   |                |
| Supports singlet superconductivity                              | no              | no   | yes  |                |
| Supports locally (in k-space) unitary triplet superconductivity | no              | no   | yes  |                |
| Supports k-averaged unitary triplet superconductivity           | no              | yes  | yes  |                |
| Giant or tunneling magnetoresistance and spin-transfer torque   | yes             | yes  | no   |                |

This Zeeman splitting has different physical origin than Spin-Orbit Coupling, and it averages to zero

Ok, which properties do alter magnets display?

| Spin-momentum locking         |        |               |             |                        |            |          |                                                                      | AF             |  |
|-------------------------------|--------|---------------|-------------|------------------------|------------|----------|----------------------------------------------------------------------|----------------|--|
|                               |        |               | G           | $R_s^{III}$            | H          | A        | Candidate                                                            |                |  |
| Nonrelativistic net mag       | Planar | P-2<br>d-wave | mmm         | $^2m^2m^1m$ (8)        | 2/m        | $C_{2x}$ | La <sub>2</sub> CuO <sub>4</sub> , FeSb <sub>2</sub>                 | zero           |  |
|                               |        |               | 4/m         | $^24/1m$ (8)           |            | $C_{4z}$ | KRu <sub>4</sub> O <sub>8</sub>                                      |                |  |
|                               |        | P-4<br>g-wave | 4/mmm       | $^24/1m^2m^1m$ (16)    | mmm        | $C_{4z}$ | RuO <sub>2</sub> ,<br>MnO <sub>2</sub> , MnF <sub>2</sub>            | yes            |  |
|                               |        |               |             | $^14/1m^2m^2m$ (16)    | 4/m        | $C_{2x}$ | KMnF <sub>3</sub>                                                    | no             |  |
| Nonrelativistic Kramers       | Bulk   | P-6<br>f-wave | 6/mmm       | $^16/1m^2m^2m$ (24)    | 6/m        | $C_{21}$ |                                                                      | Allowed by SOC |  |
| Anomalous Hall                |        |               |             |                        |            |          |                                                                      | no             |  |
| Magnetooptics                 |        |               |             |                        |            |          |                                                                      | no             |  |
| Nonrelativistic spin-pol      | Bulk   | B-2<br>d-wave | 2/m         | $^22/2m$ (4)           | $\bar{1}$  | $C_{2z}$ | CuF <sub>2</sub>                                                     | no             |  |
| Suppression of Andreev        |        |               |             |                        |            |          |                                                                      | no             |  |
| Suppression of Andreev        |        | B-4<br>g-wave | $\bar{3}m$  | $^1\bar{3}^2m$ (12)    | $\bar{3}$  | $C_{21}$ | CoF <sub>3</sub> , FeF <sub>3</sub> , Fe <sub>2</sub> O <sub>3</sub> | yes            |  |
| Supports singlet super        |        |               |             |                        |            |          |                                                                      | yes            |  |
| Supports locally (in k-sp     |        | B-6<br>f-wave | 6/mmm       | $^26/2m$ (12)          | $\bar{3}m$ | $C_{6z}$ | CrSb, MnTe, VNb <sub>3</sub> S <sub>6</sub>                          | yes            |  |
| Supports k-averaged u         |        |               |             |                        |            |          |                                                                      | yes            |  |
| Giant or tunneling mag torque | Bulk   | B-6<br>f-wave | $m\bar{3}m$ | $^1m^1\bar{3}^2m$ (48) | $m\bar{3}$ | $C_{4z}$ |                                                                      | no             |  |
|                               |        |               |             |                        |            |          |                                                                      |                |  |

This Zeeman splitting has different physical origin than Spin-Orbit Coupling, and it averages to zero

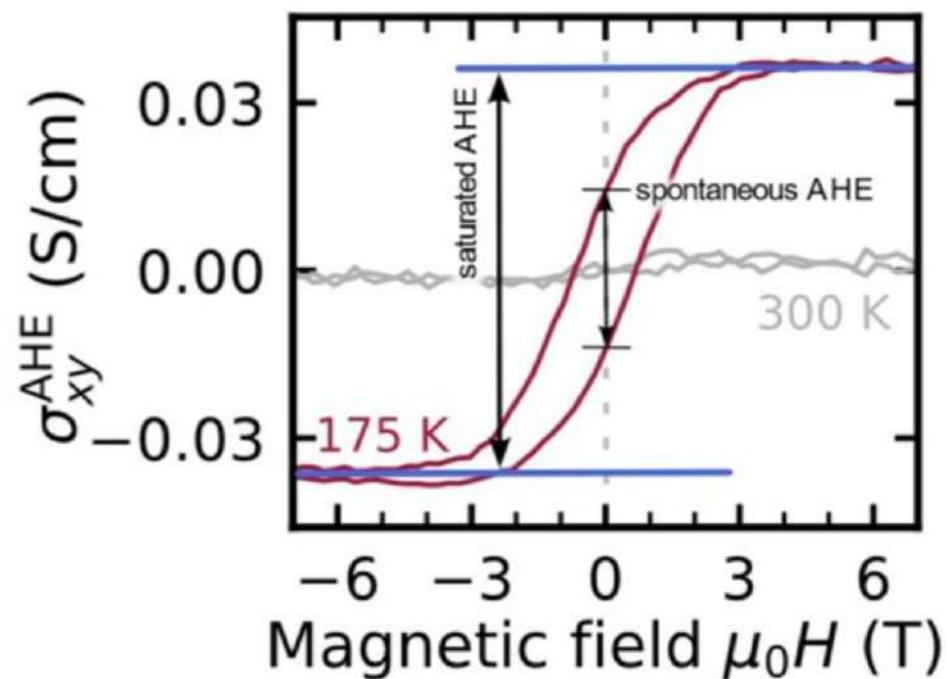
# Anomalous Hall effect in ferromagnets and Altermagnets



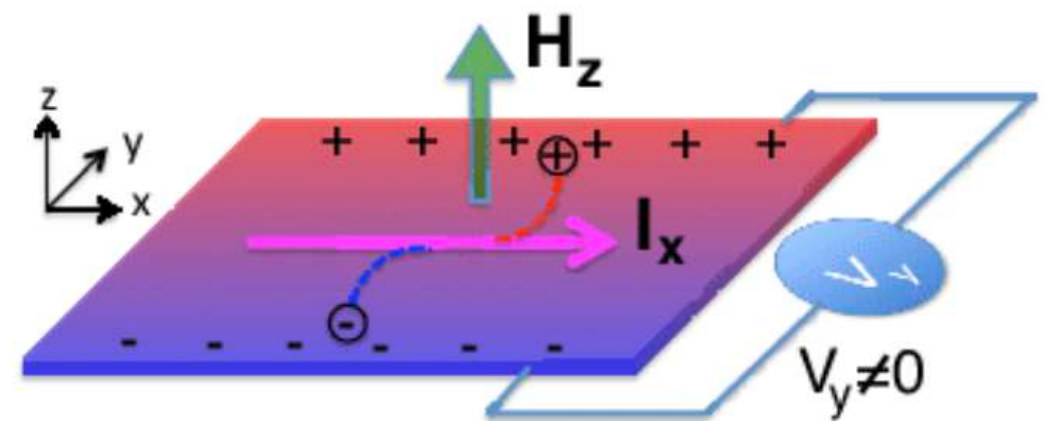
Edwin H Hall (1855-1938)

The anomalous Hall effect is the appearance of a transverse current to an applied electric field, and to the magnetization

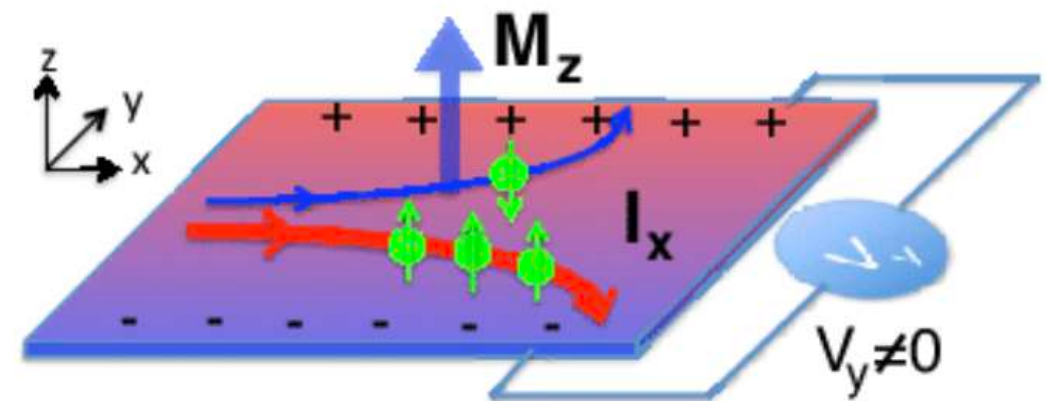
$$\sigma_{12} \sim \sigma_H B + \sigma^{AHE} M$$



MnTe



Hall effect



Anomalous Hall effect

# Anomalous Hall effect in ferromagnets and Altermagnets

Something important to keep in mind is that, in order to get a nonzero AHE, we need finite SOC terms, both for FM and altermagnets

$$H_{FM} = -2t(\cos k_1 \cos k_2)\sigma_0 + M\sigma_3$$

$$H_{FM} = -2t(\cos k_1 \cos k_2)\sigma_0 + t_J(\sin k_1 \sin k_2)\sigma_3$$



$$H_{FM} = t(k_1^2 + k_2^2)\sigma_0 + M\sigma_3$$

$$H_{AM} = t(k_1^2 + k_2^2)\sigma_0 + t_J k_1 k_2 \sigma_3$$

# Anomalous Hall effect in ferromagnets and Altermagnets

Something important to keep in mind is that, in order to get a nonzero AHE, we need finite SOC terms, both for FM and altermagnets

$$\begin{aligned} H_{FM} &= t(k_1^2 + k_2^2)\sigma_0 + M\sigma_3 \\ H_{AM} &= t(k_1^2 + k_2^2)\sigma_0 + t_J k_1 k_2 \sigma_3 \end{aligned} \quad \rightarrow \quad H = d_0(\mathbf{k})\sigma_0 + d_3(\mathbf{k})\sigma_3$$
$$d_1(\mathbf{k}) = d_2(\mathbf{k}) = 0$$

$$\Omega_3^\pm = \pm \frac{1}{2d^3} \varepsilon_{abc} d_a \partial_1 d_b \partial_2 d_c = 0$$

$$\sigma^{AHE} = \sum_{\pm} \frac{e^2}{h} \int \frac{d^2 k}{(2\pi)^2} f_F^{p\pm}(\mathbf{k}) \Omega_3^\pm(\mathbf{k}) = 0$$

# Anomalous Hall effect in ferromagnets and Altermagnets

Something important to keep in mind is that, in order to get a nonzero AHE, we need finite SOC terms, both for FM and altermagnets

$$\begin{aligned}
 H_{FM} &= t(k_1^2 + k_2^2)\sigma_0 + M\sigma_3 \\
 H_{AM} &= t(k_1^2 + k_2^2)\sigma_0 + t_J k_1 k_2 \sigma_3
 \end{aligned}
 \rightarrow
 \begin{aligned}
 H &= d_0(\mathbf{k})\sigma_0 + d_3(\mathbf{k})\sigma_3 \\
 d_1(\mathbf{k}) &= d_2(\mathbf{k}) = 0
 \end{aligned}$$

$$\Omega_3^\pm = \pm \frac{1}{2d^3} \varepsilon_{abc} d_a \partial_1 d_b \partial_2 d_c = 0 \qquad \sigma^{AHE} = \sum_{\pm} \frac{e^2}{h} \int \frac{d^2 k}{(2\pi)^2} f_F^{p\pm}(\mathbf{k}) \Omega_3^\pm(\mathbf{k}) = 0$$

$$H_{FM} = t(k_1^2 + k_2^2)\sigma_0 + M\sigma_3 + \underbrace{\lambda k_1 \sigma_2 - \lambda k_2 \sigma_1}_{\lambda(\mathbf{k} \times \boldsymbol{\sigma})}$$

SOC is needed  $\rightarrow$

$$H_{FM} = t(k_1^2 + k_2^2)\sigma_0 + t_J k_1 k_2 \sigma_3 + \underbrace{\lambda k_1 \sigma_2 - \lambda k_2 \sigma_1}_{\lambda(\mathbf{k} \times \boldsymbol{\sigma})}$$

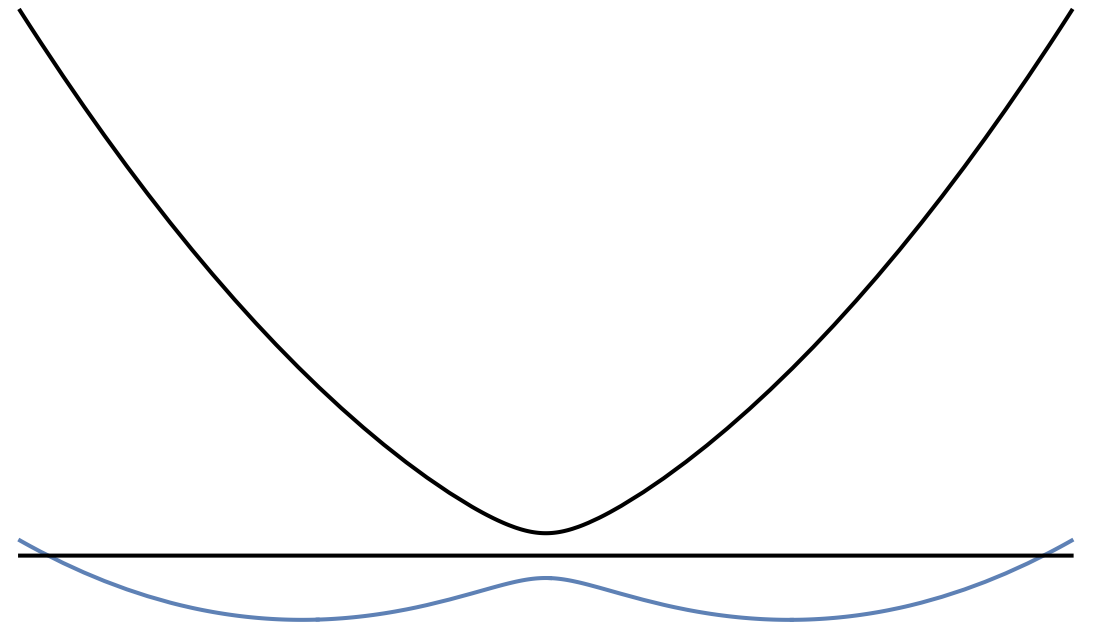
# Anomalous Hall effect in ferromagnets and Altermagnets

$$\sigma^{AHE} = \sum_{\pm} \frac{e^2}{h} \int \frac{d^2 k}{(2\pi)^2} f_F^{\pm}(\mathbf{k}) \mathbf{\Omega}_3^{\pm}(\mathbf{k})$$

$$H_{FM} = t(k_1^2 + k_2^2)\sigma_0 + M\sigma_3 + \underbrace{\lambda k_1\sigma_2 - \lambda k_2\sigma_1}_{\lambda(\mathbf{k} \times \boldsymbol{\sigma})}$$

$$\mathbf{\Omega}_3^{\pm} = -\frac{1}{2} \frac{M\lambda^2}{(M^2 + \lambda^2 k^2)^{\frac{3}{2}}}$$

$$\sigma^{AHE} = -\frac{e^2}{h} \text{sign}(M) + \frac{e^2}{h} \frac{M\lambda^2}{(k_{\mu}^2 + M^2)}$$



$$tk^2 - \sqrt{\lambda^2 k^2 + M^2} = \mu$$

# Anomalous Hall effect in ferromagnets and Altermagnets

$$\sigma^{AHE} = \sum_{\pm} \frac{e^2}{h} \int \frac{d^2 k}{(2\pi)^2} f_F^{\pm}(\mathbf{k}) \Omega_3^{\pm}(\mathbf{k})$$

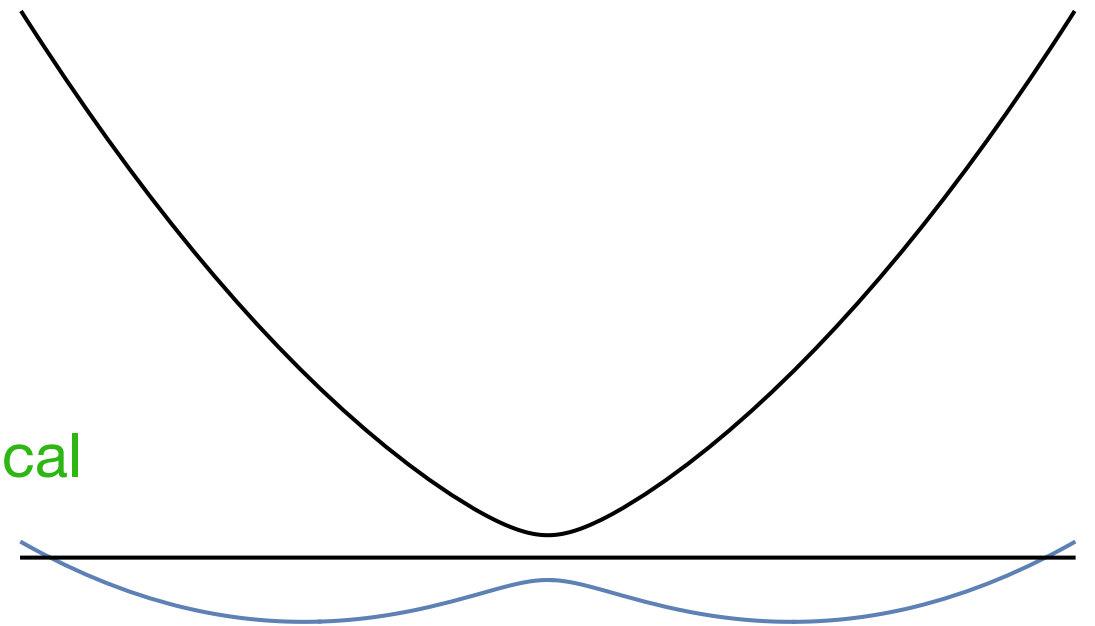
$$H_{FM} = t(k_1^2 + k_2^2)\sigma_0 + M\sigma_3 + \underbrace{\lambda k_1\sigma_2 - \lambda k_2\sigma_1}_{\lambda(\mathbf{k} \times \boldsymbol{\sigma})}$$

$$\Omega_3^{\pm} = -\frac{1}{2} \frac{M\lambda^2}{(M^2 + \lambda^2 k^2)^{\frac{3}{2}}}$$

$$\sigma^{AHE} = -\frac{e^2}{h} \text{sign}(M) + \frac{e^2}{h} \frac{M\lambda^2}{(k_{\mu}^2 + M^2)}$$

Topological

Non topological



$$tk^2 - \sqrt{\lambda^2 k^2 + M^2} = \mu$$

# Anomalous Hall effect in ferromagnets and Altermagnets

$$\sigma^{AHE} = \sum_{\pm} \frac{e^2}{h} \int \frac{d^2k}{(2\pi)^2} f_F^{\pm}(\mathbf{k}) \Omega_3^{\pm}(\mathbf{k})$$

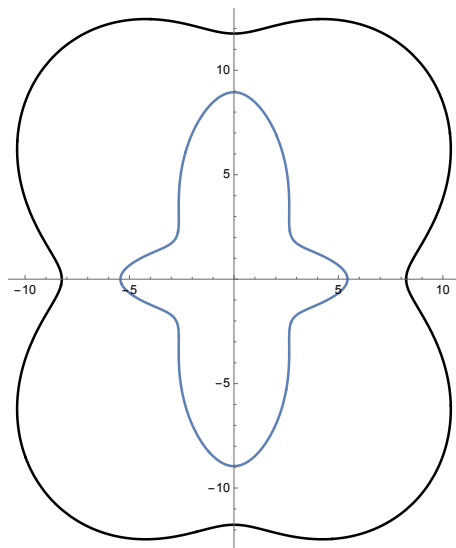
$$H = tk^2\sigma_0 + t_J(k_1k_2 + \varepsilon_{12})\sigma_3 + \underbrace{\lambda(k_1\sigma_2 - k_2\sigma_1)}_{\lambda(\mathbf{k} \times \boldsymbol{\sigma})}$$

$$\Omega_3^{\pm} = -\frac{1}{2} \frac{\lambda^2 t_J k^2 \sin 2\theta}{(t_J^2 k^4 \sin^2 2\theta + \lambda^2 k^2)^{\frac{3}{2}}}$$

$$\mu = tk^2 - \sqrt{\lambda^2 k^2 + t_J^2 (k^2 \sin 2\theta + \varepsilon_{12})^2}$$

$$\sigma^{AHE} \sim \frac{e^2}{h} \varepsilon_{12} t_J f(\lambda, \mu, t)$$

Strain is needed!!



$$\varepsilon_{12} \rightarrow -\varepsilon_{12} \quad \text{under } C_4$$

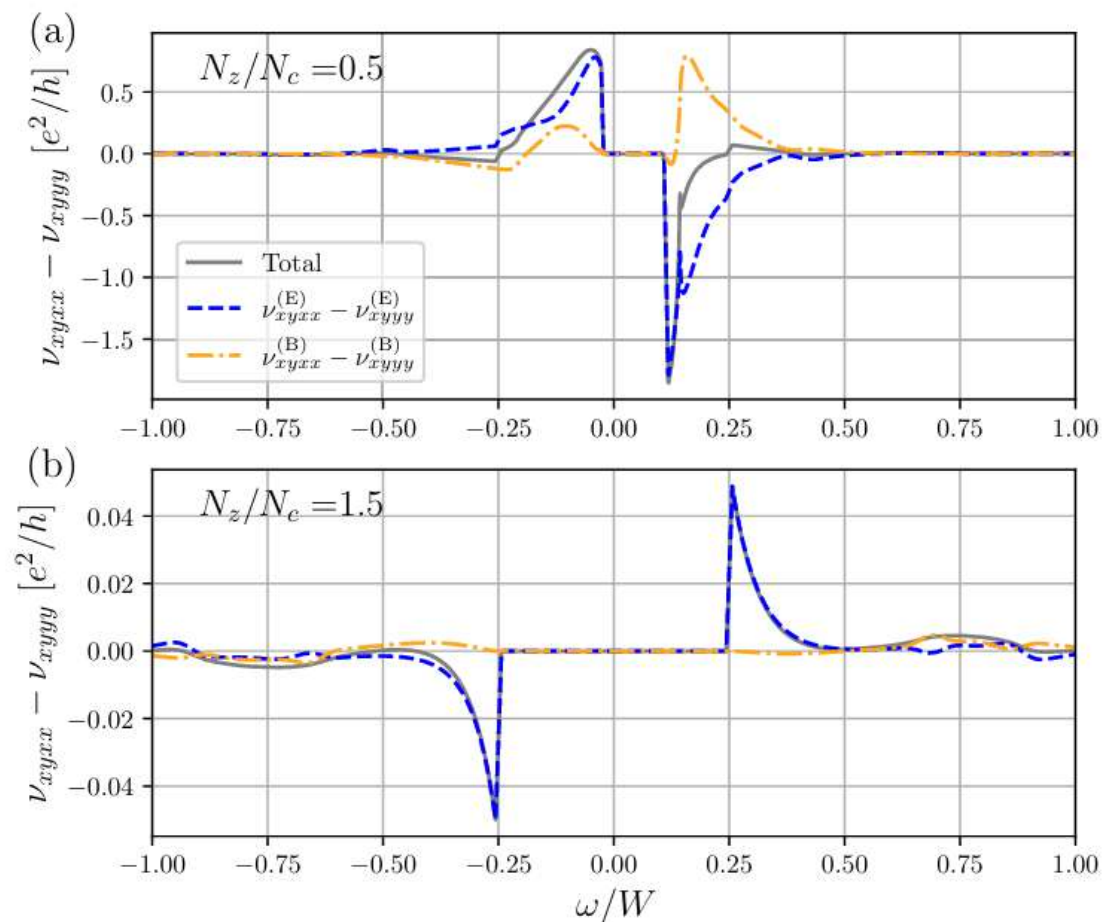
$$t_J \sigma_3 \rightarrow -t_J \sigma_3 \quad \text{under } T$$

$$\varepsilon_{12} t_J \sigma_3 \rightarrow \varepsilon_{12} t_J \sigma_3$$

$$\text{under } C_4 \cdot T$$

Piezomagnetic effect

# Anomalous Hall effect in ferromagnets and Altermagnets



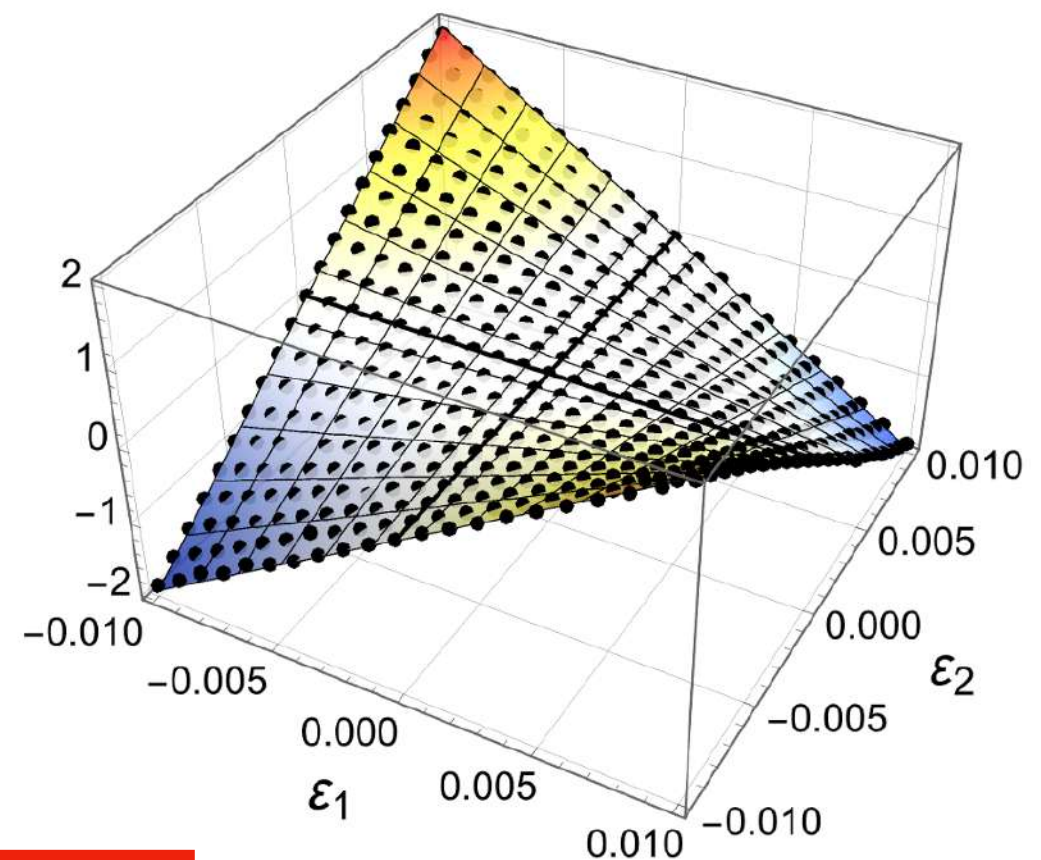
arXiv > cond-mat > arXiv:2506.06257

Condensed Matter > Materials Science

[Submitted on 6 Jun 2025 (v1), last revised 15 Jul 2025 (this version, v2)]

## Tuning of altermagnetism by strain

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### Elasto-Hall conductivity and the anomalous Hall effect in altermagnets

[Keigo Takahashi](#)<sup>1</sup>, [Charles R. W. Steward](#)<sup>2</sup>, [Masao Ogata](#)<sup>1,3</sup>, [Rafael M. Fernandes](#)<sup>4,5</sup>, and [Jörg Schmalian](#)<sup>2,6</sup>

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Piezomagnetic effect

Strain is needed!!