Emergence of Quantum Phases in Novel Materials

VIII Edition ICMM School

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EXCELENCIA SEVERO OCHOA

BIBLIOGRAPHY

"Introduction to Many-body physics" Piers Coleman (2015)

"Introduction to superconductivity" Tinkham (1973)

SPC history:

"Superconductivity: a very short introduction" S. Blundell (2009)

Collection of old reviews

- Conventional SPC "Superconductivity" Edited by Parks. 1968.
- Conventional and unconventional SPC "Superconductivity" 2008 (Fe SPC not included)

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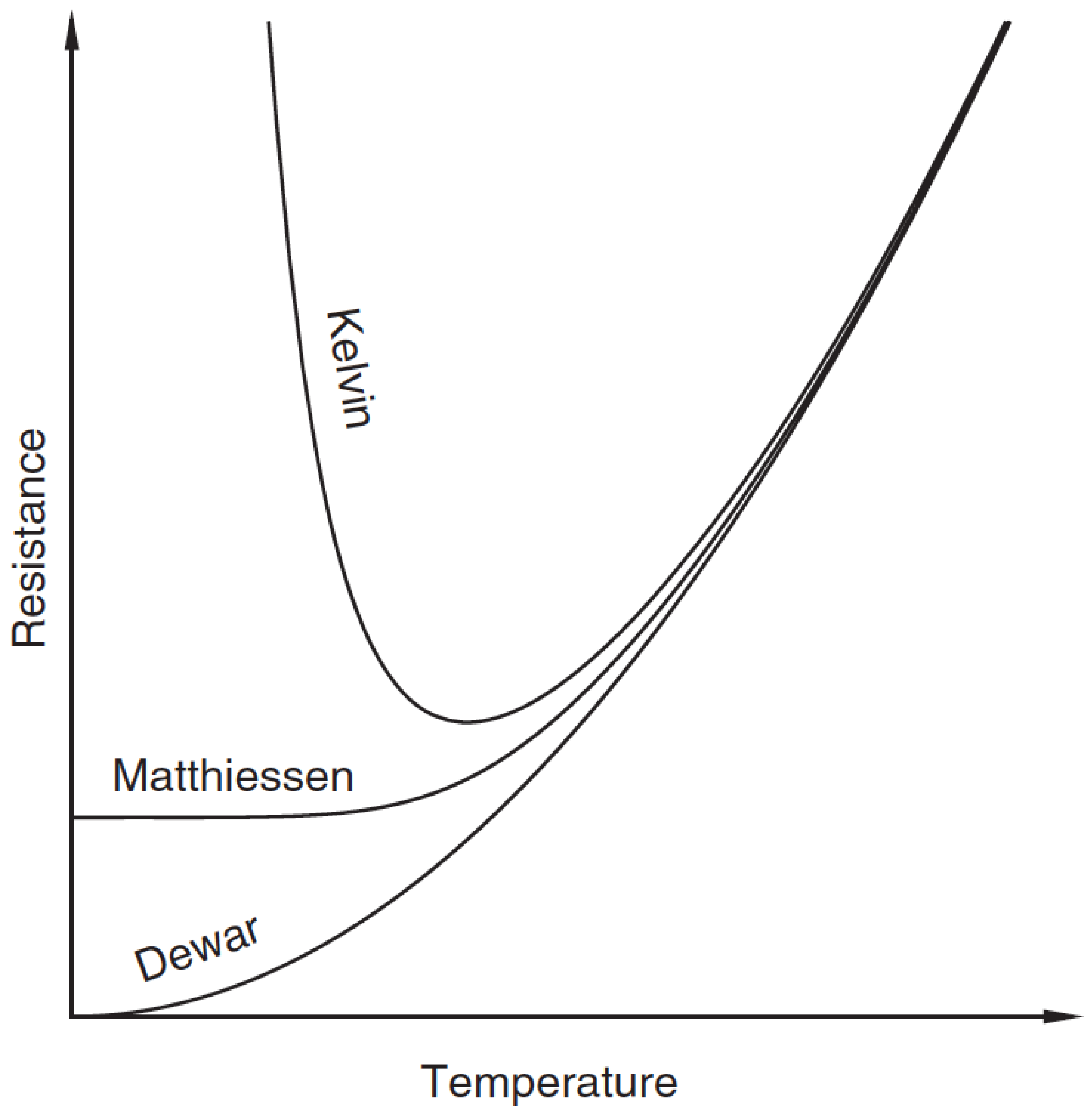
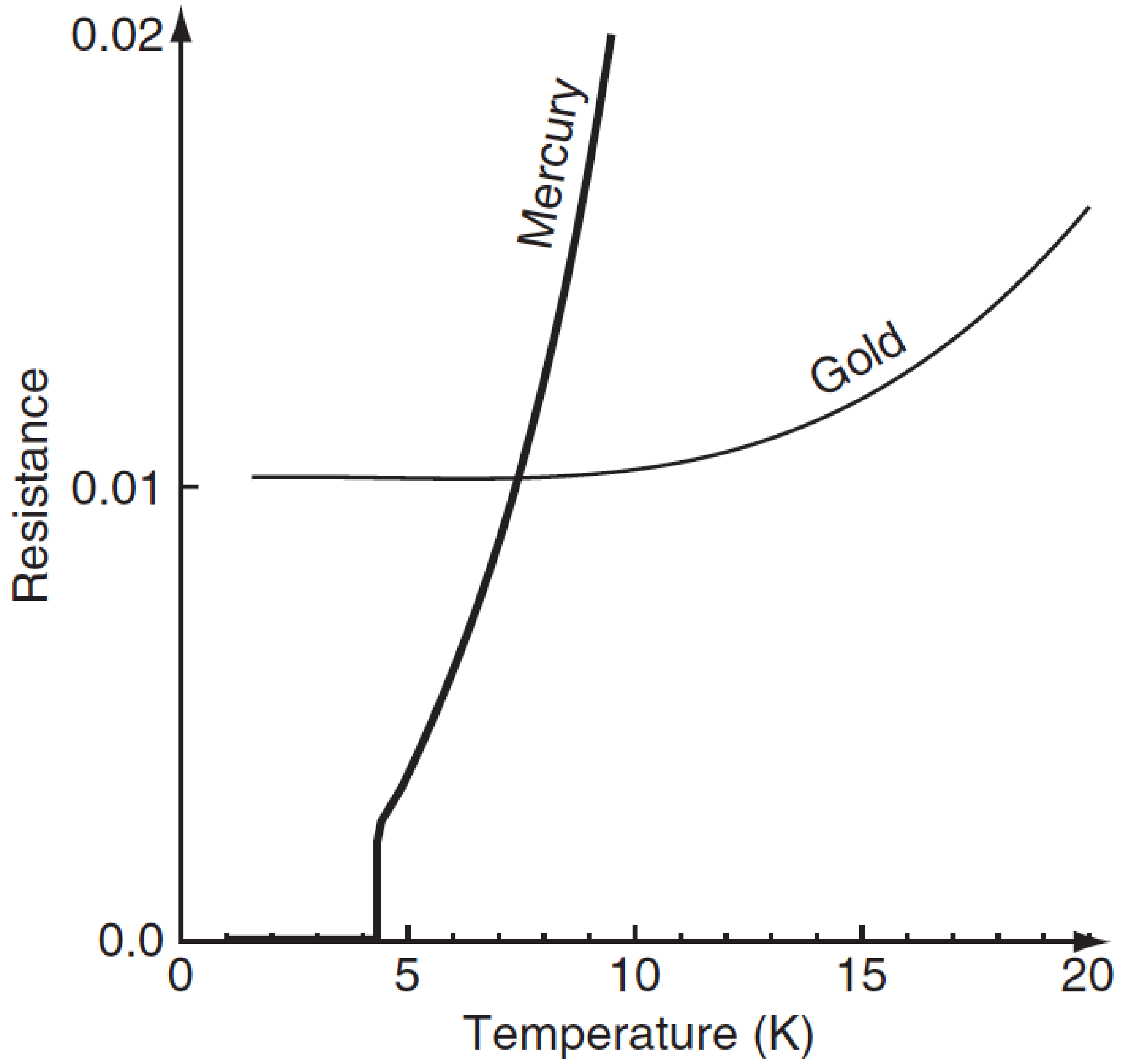
THE DISCOVERY (1911)

NOBEL PRIZE 1913

Resistivity in metals at low T at the turn of the 20th century





Heike Kamerlingh Onnes liquefied He: first to measure low T resistivity

S. Blundell “Superconductivity: a very short introduction”

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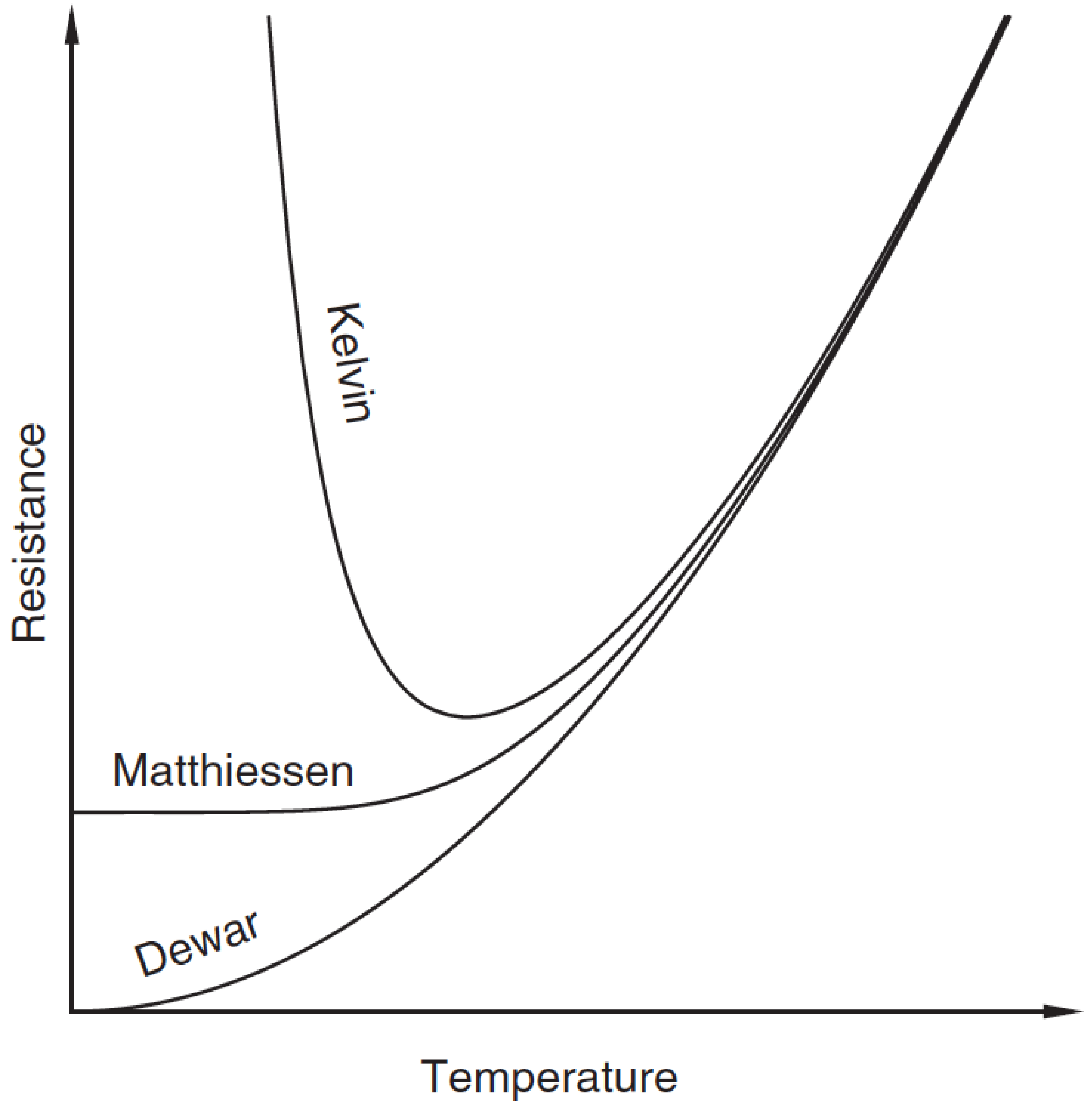

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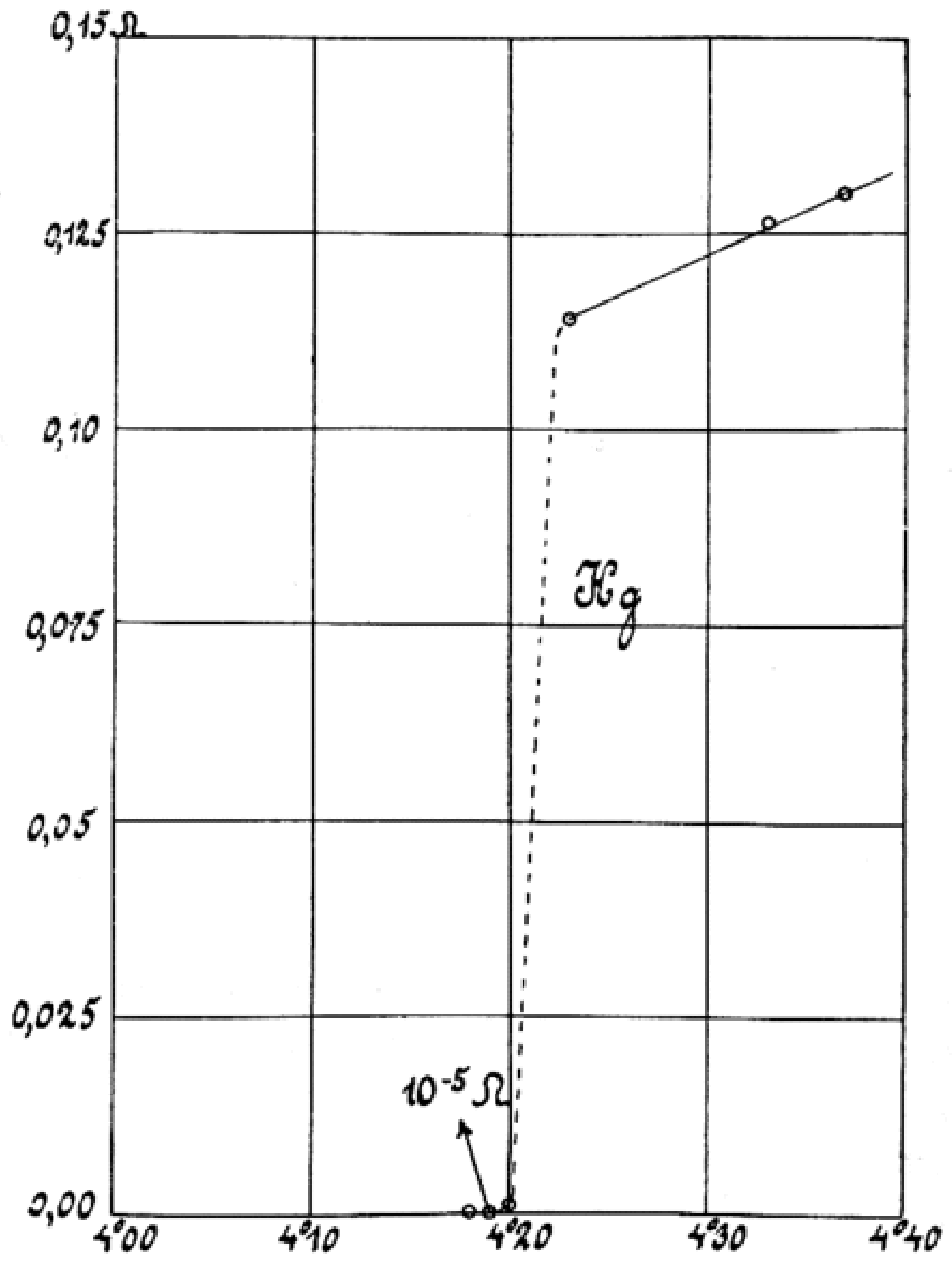

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Heike Kamerlingh Onnes liquefied He: first to measure low T resistivity

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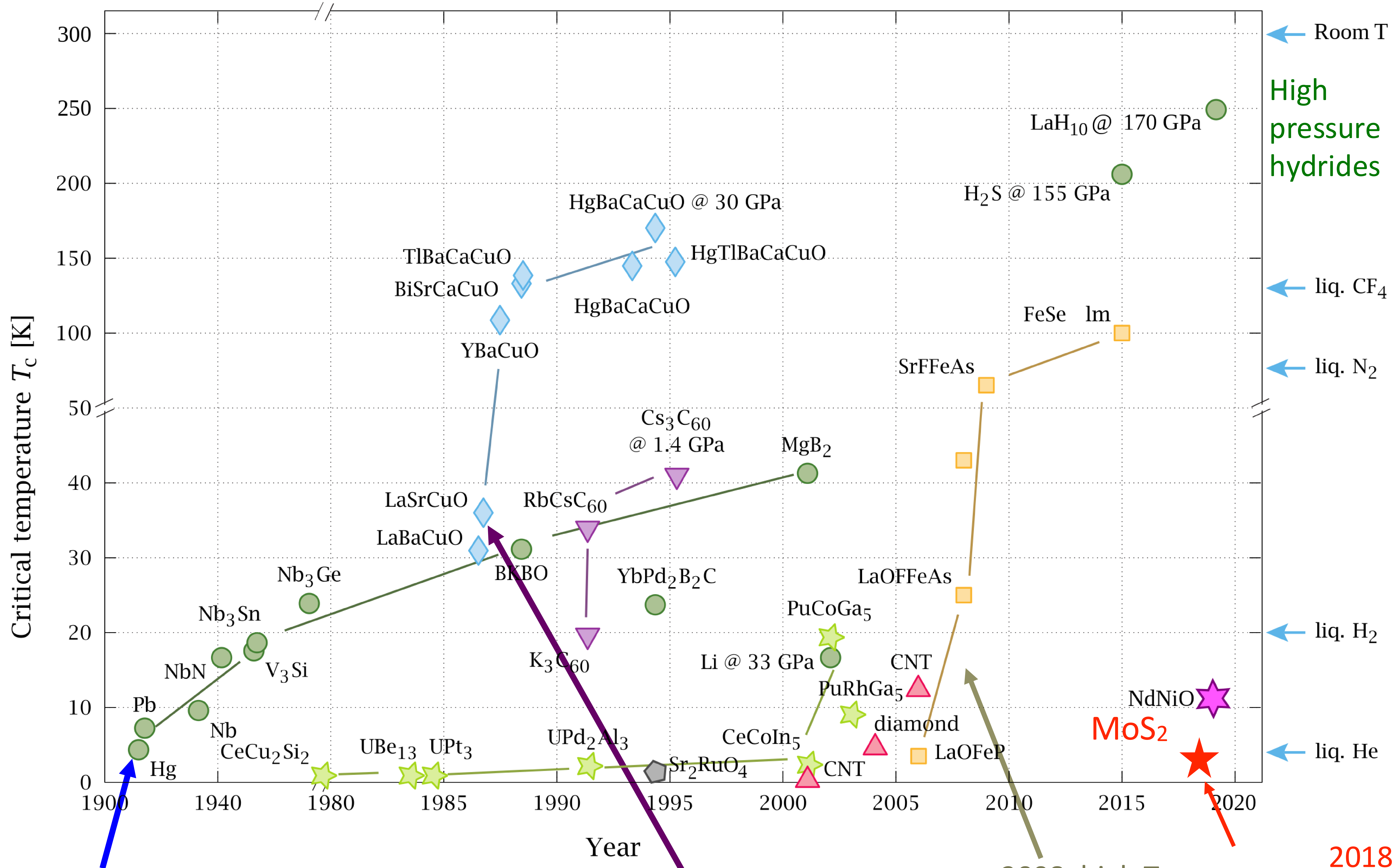
https://web.physics.wustl.edu/jss/NewPeriodicTable.pdf

ambient pressure superconductor: T_c(K) / T_c^{max}(K) / P(GPa)

high pressure superconductor: T_c^{max}(K) / P(GPa)

H																	He
Li 0.0004 14 30	Be 0.026											B 11 250	C	N	O 0.6 100	F	Ne
Na	Mg											Al 1.14	Si 8.2 15.2	P 13 30	S 17.3 190	Cl	Ar
K	Ca 29 217	Sc 19.6 106	Ti 0.39 3.35 56.0	V 5.38 16.5 120	Cr	Mn	Fe 2.1 21	Co	Ni	Cu	Zn 0.875	Ga 1.091 7 1.4	Ge 5.35 11.5	As 2.4 32	Se 8 150	Br 1.4 100	Kr
Rb 2.5 55	Sr 7 50	Y 19.5 115	Zr 0.546 11 30	Nb 9.50 9.9 10	Mo 0.92	Tc 7.77	Ru 0.51	Rh .00033	Pd	Ag	Cd 0.56	In 3.404	Sn 3.722 5.3 11.3	Sb 3.9 25	Te 7.5 35	I 1.2 25	Xe
Cs 1.3 12	Ba 5 18	insert La-Lu	Hf 0.12 8.6 62	Ta 4.483 4.5 43	W 0.012	Re 1.4	Os 0.655	Ir 0.14	Pt	Au	Hg-α 4.153	Tl 2.39	Pb 7.193	Bi .00053 8.5 9.1	Po	At	Rn
Fr	Ra	insert Ac-Lr	Rf	Ha													
		La-fcc 6.00 13 15	Ce 1.7 5	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb 4.8 180	Lu 12.4 174	
		Ac	Th 1.368	Pa 1.4	U 0.8(β) 2.4(α) 1.2	Np	Pu	Am 0.79 2.2 6	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

And many compounds!



1911
Discovery

1986 High T_c, Cuprates

2008, high T_c
Fe superc.

2018
TBG
+other multilayered graphene

2dim: Chiral and topological superconductivity

Adapted from wikipedia

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OUTLINE

- Superconductivity I
  - Properties (zero resistivity, Meissner effect)
  - Understanding (pairing, BCS, Ginzburg-Landau)
  - Electron-phonon interaction (conventional superconductivity)

- Superconductivity II (unconventional superconductivity)
  - What are the distinct issues.
  - What are (some of) the proposals.

- Mesoscopic and topological superconductivity —> Ramón Aguado

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A BIT OF (THE EARLY) HISTORY

1911: Discovery of zero resistance state (perfect conductor)

1933: Meissner effect (perfect diamagnetic)

1935: London Equations (phenomenological approach)

1937: Superfluidity in ^{4}He (a superconductor is a charged superfluid)

1950: Isotope effect (hint on the mechanism)

Electron-electron interaction mediated by phonons

Ginzburg-Landau

1953: Pippard's coherence length

1955: Idea of gap (Bardeen)

Bardeen-Pines effective interaction (phonons)

1956: Cooper pairs, Schrieffer wave-function

Abrikosov extension of Ginzburg-Landau (vortices)

1957: BCS

1959: Gor'kov derived Ginzburg-Landau theory from BCS

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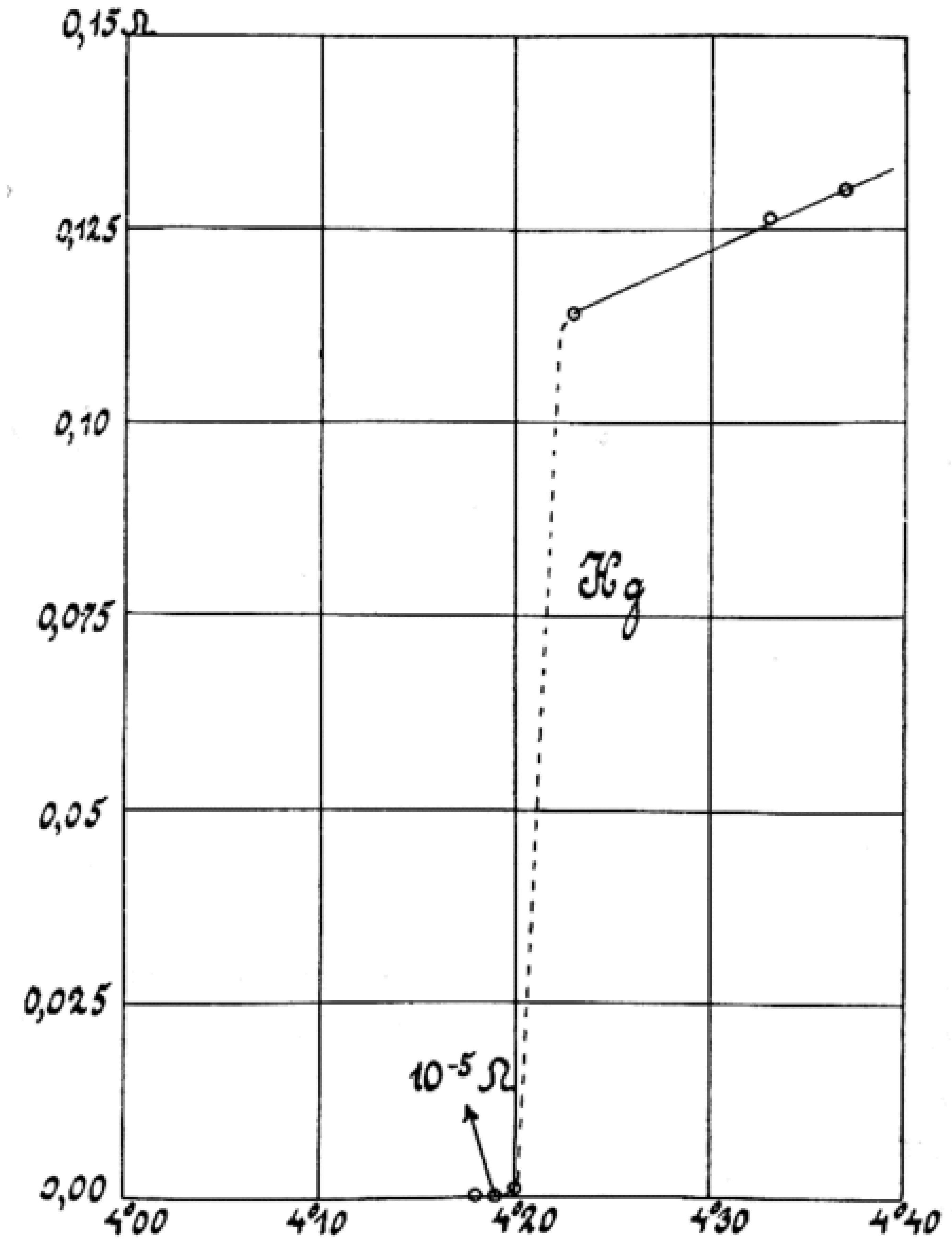
1911



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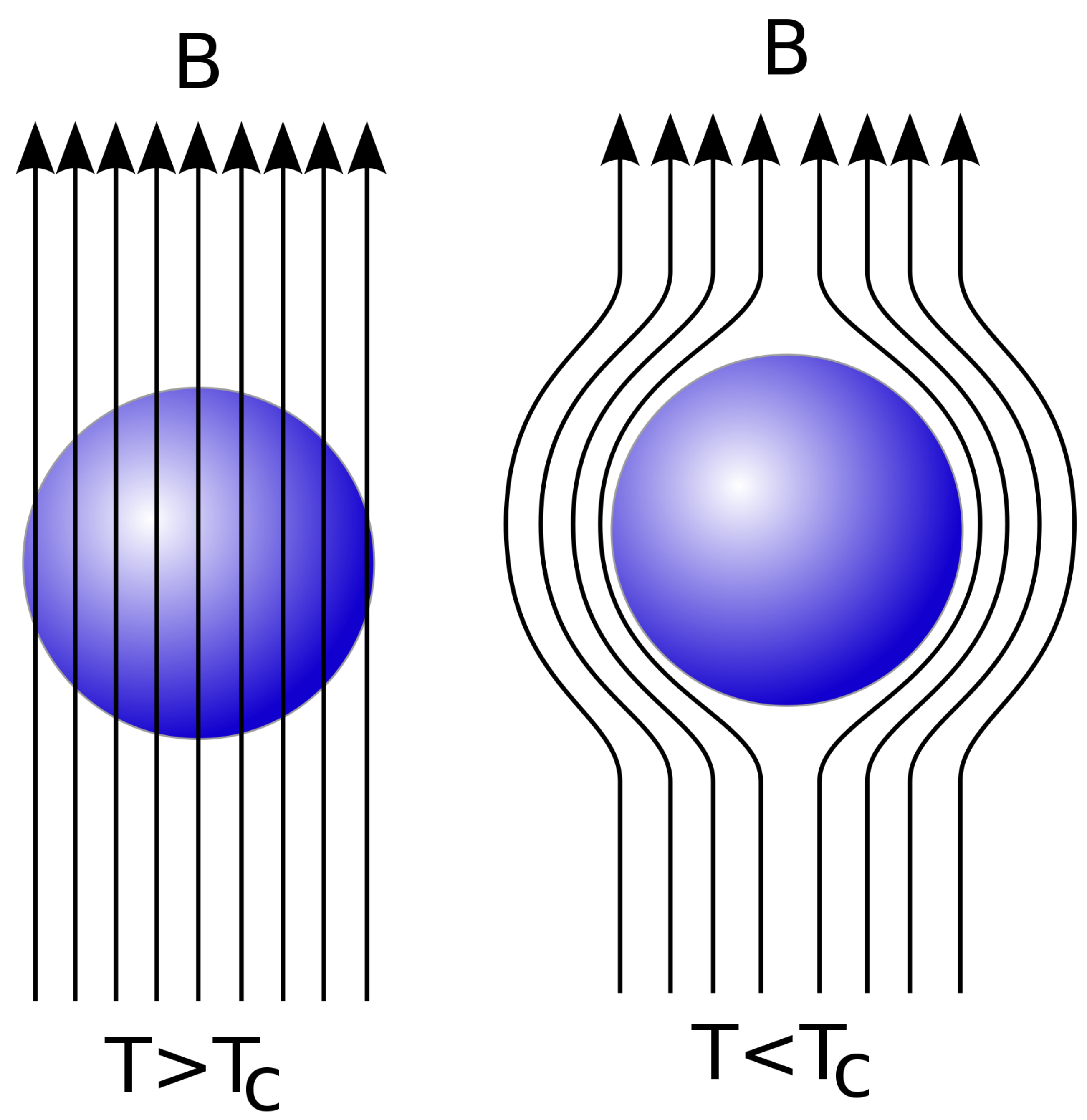
1933

The Meissner-Ochsenfeld effect

SUPERCONDUCTORS ARE PERFECT DIAMAGNETS



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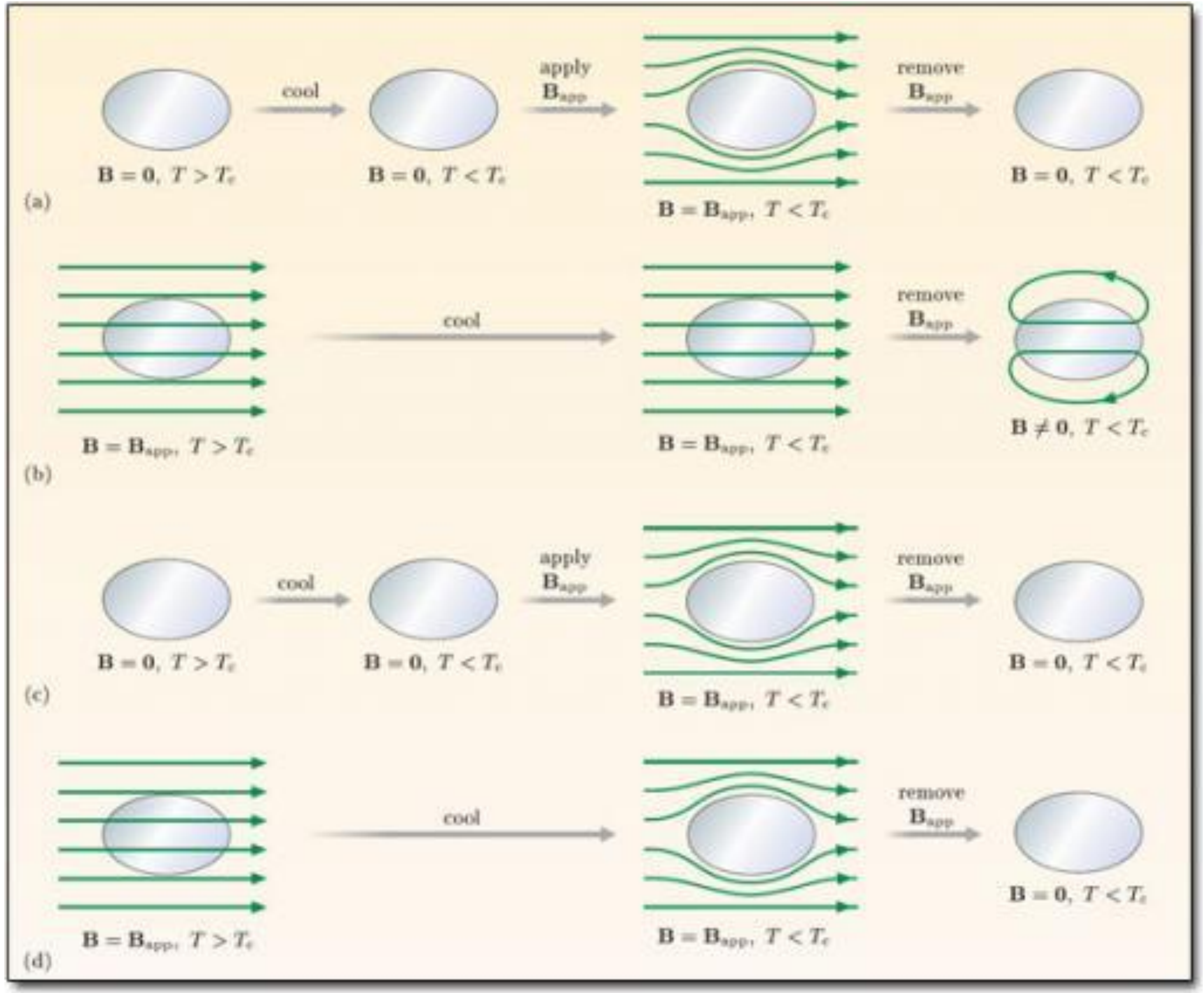
SUPERCONDUCTORS ≠ PERFECT CONDUCTORS

Perfect conductor

Super-conductor



Perfect conductors (R=0) do not allow change in magnetic field.

δB—> V; if R=0, I—>∞ by Ohm's law.

Supercurrents completely screen out the magnetic field.

www.open.edu: Superconductivity

Ohm’s law relates current and electric field through resistance.

It does not apply to superconductors.

Meissner effect implies a relation between current and magnetic field.

LONDON EQUATIONS

(phenomenological approach)

Long range order of the momentum vector:

A condensation in momentum space (<p>=0)

—> wave-function rigidity (macroscopic quantum coherence)

LONDON EQUATIONS

Tinkham's book, Coleman's book

$$\vec{p} = m\vec{v} + e\vec{A}/c$$

<p>=0

$$\vec{J}_s = n_s e\langle \vec{v}_s \rangle = \frac{-n_s e^2 \vec{A}}{mc} = \frac{-\vec{A}}{\Lambda c}$$

$$\Lambda = \frac{m}{n_s e^2}$$

n_s density of superconducting e-

Time derivative:
PERFECT CONDUCTIVITY

$$\vec{E} = \frac{\partial}{\partial t}\left(\Lambda \vec{J}_s\right)$$

Curl: SUPERCURRENT SCREENS MAGNETIC FIELD

$$\vec{h} = -c\nabla \times \left(\Lambda \vec{J}_s\right)$$

<u>Accelerative super current</u> also arises from Drude model w/o scattering

$$m\frac{d\vec{v}}{dt} = e\vec{E} - \cancel{\frac{m\vec{v}}{\tau}}$$

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LONDON EQUATIONS

Curl: SUPERCURRENT SCREENS MAGNETIC FIELD

$$\vec{h} = -c\nabla \times \left(\Lambda \vec{J}_s\right)$$

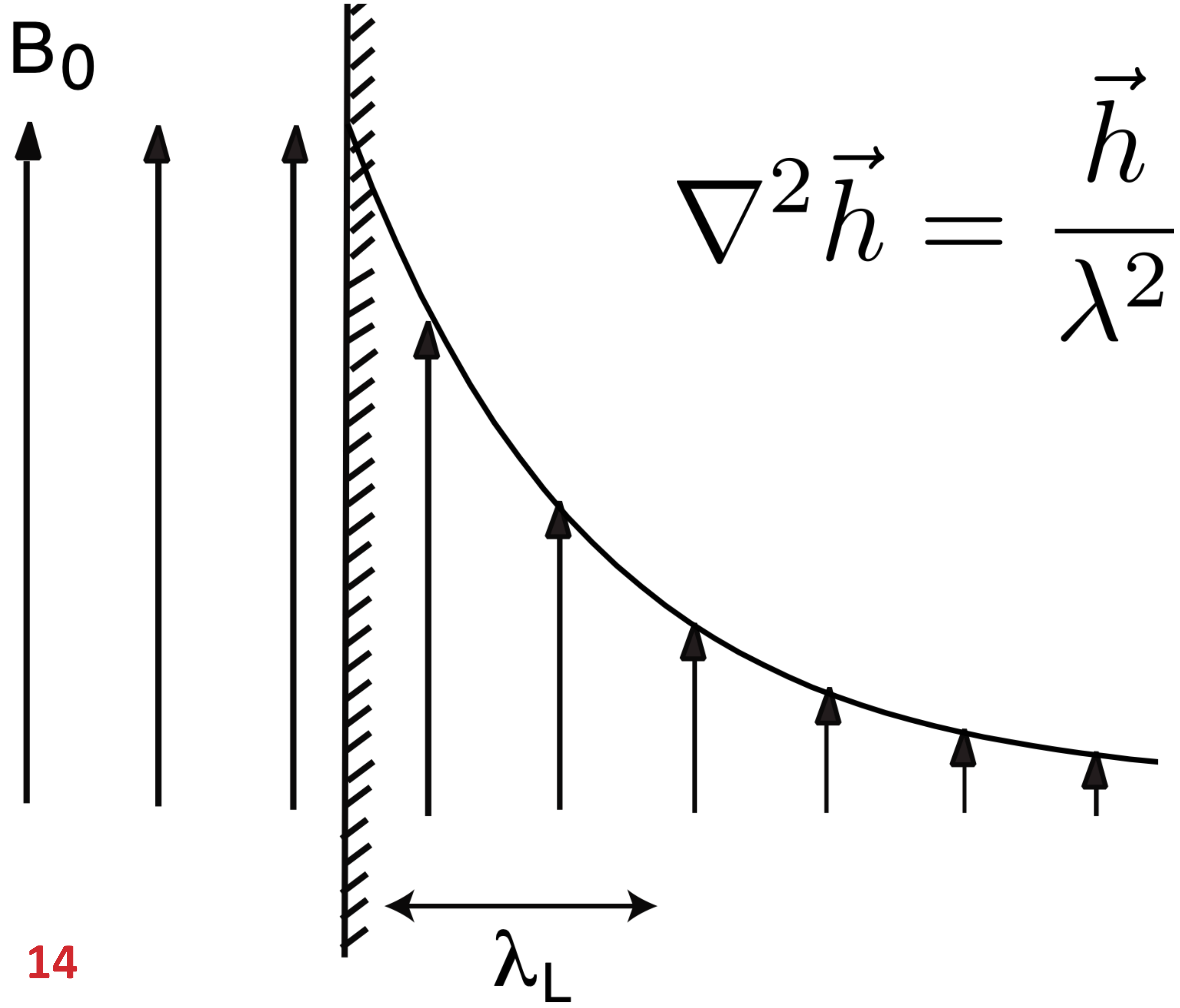
+MAXWELL EQUATION

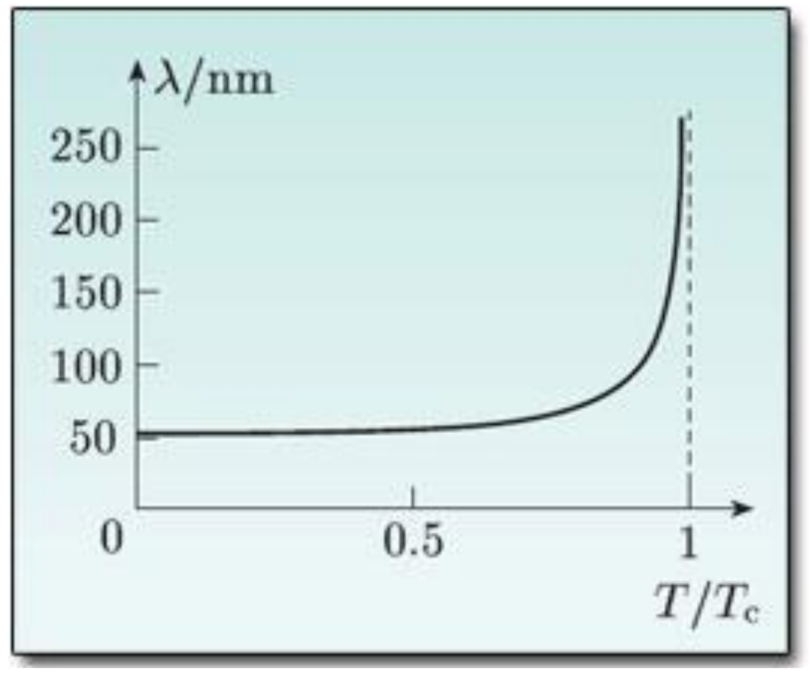
$$\nabla \times \vec{h} = 4\pi \vec{J}_s / c$$

$$\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} = -\nabla^2 \vec{B}$$

$$\nabla^2 \vec{h} = \frac{\vec{h}}{\lambda^2}$$

$$\vec{h}(r) = \vec{h}(0) e^{-r/\lambda}$$





LONDON PENETRATION LENGTH

$$\lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}$$

How efficiently is the magnetic field screened.

Tinkham's book, Coleman's book

PIPPARD'S COHERENCE LENGTH

From the uncertainty principle:

Only e^- within $k_B T_c$ of E_F participate in superconductivity

$$\Delta p \approx \frac{k_B T_c}{v_F} \qquad \Delta x \gtrsim \frac{\hbar}{\Delta p} \approx \frac{\hbar v_F}{k_B T_c} \sim \xi_0$$

From a non-local generalization of the London equations

$$\xi_0 = 0.15 \frac{\hbar v_F}{k_B T_c}$$

ξ_0: wave-function size

Two characteristic length scales:

- penetration length λ
- coherence length ξ_0

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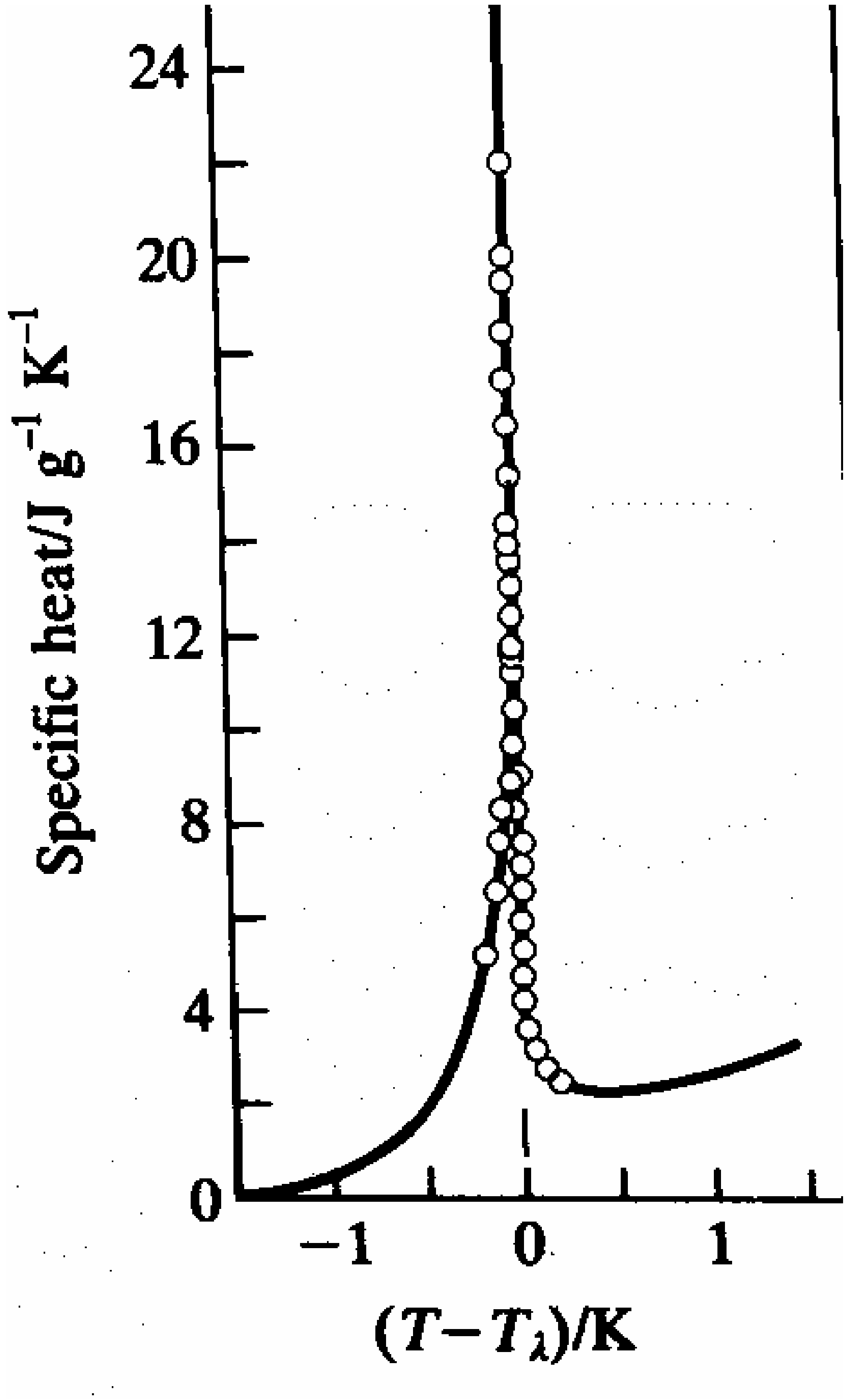
^{4}HE SUPERFLUIDITY (1937)

^{4}He liquifies at 4.2K (He I: normal quantum liquid)

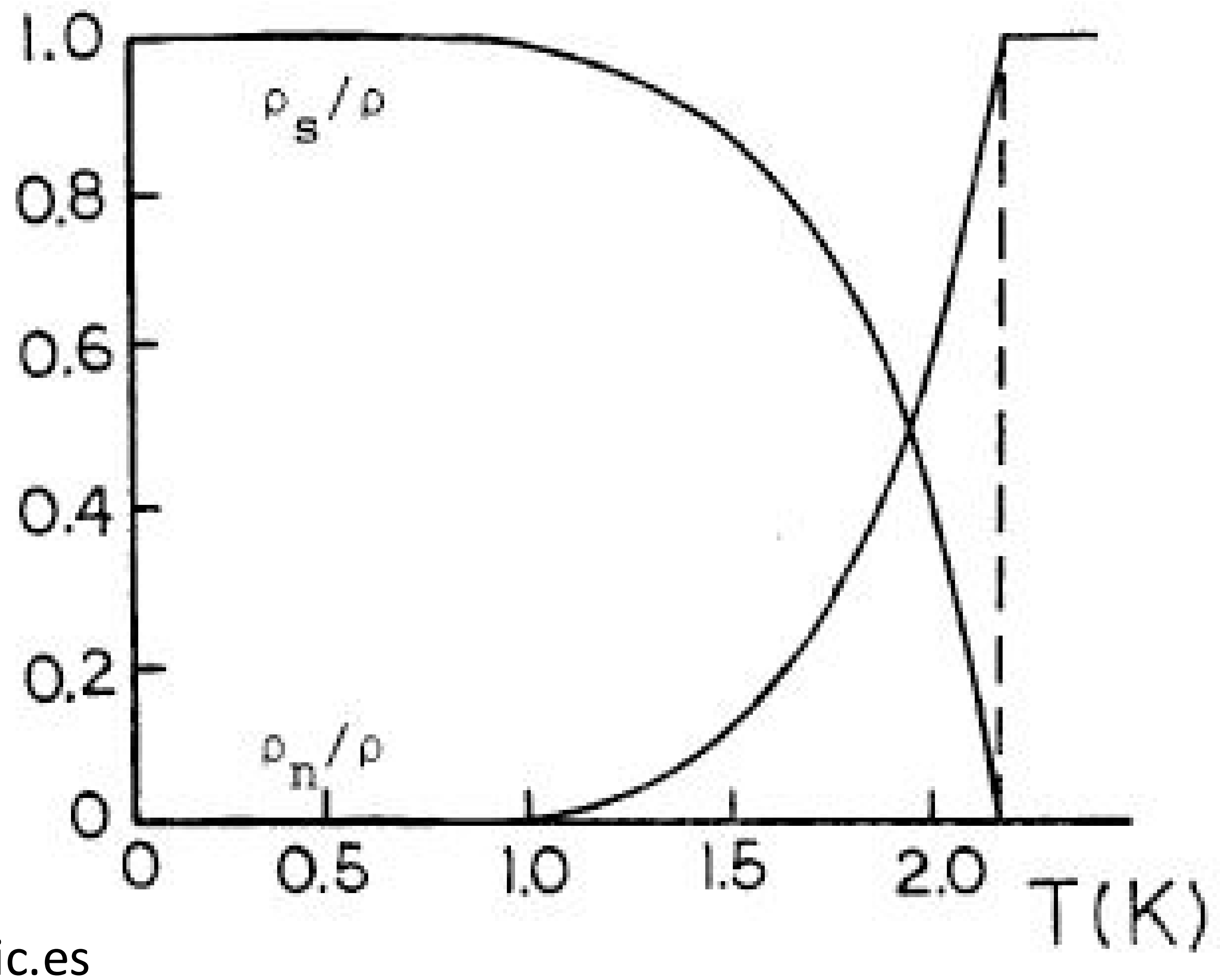
At 2.17K, transition to superfluid (He II). λ transition.



- Superfluidity →Viscosity=0.
- A Bose Einstein condensation with interactions

Two fluid model

- normal fluid
- superfluid



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SUPERFLUIDITY & SUPERCONDUCTIVITY

A superconductor is a charged superfluid.

Note: What is condensing in the superconductor??
Electrons are fermions, not bosons!!
Before 1957, a microscopic theory was still missing.

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LANDAU TRANSITION THEORY U(1)

Macroscopic theory in terms of order parameters.

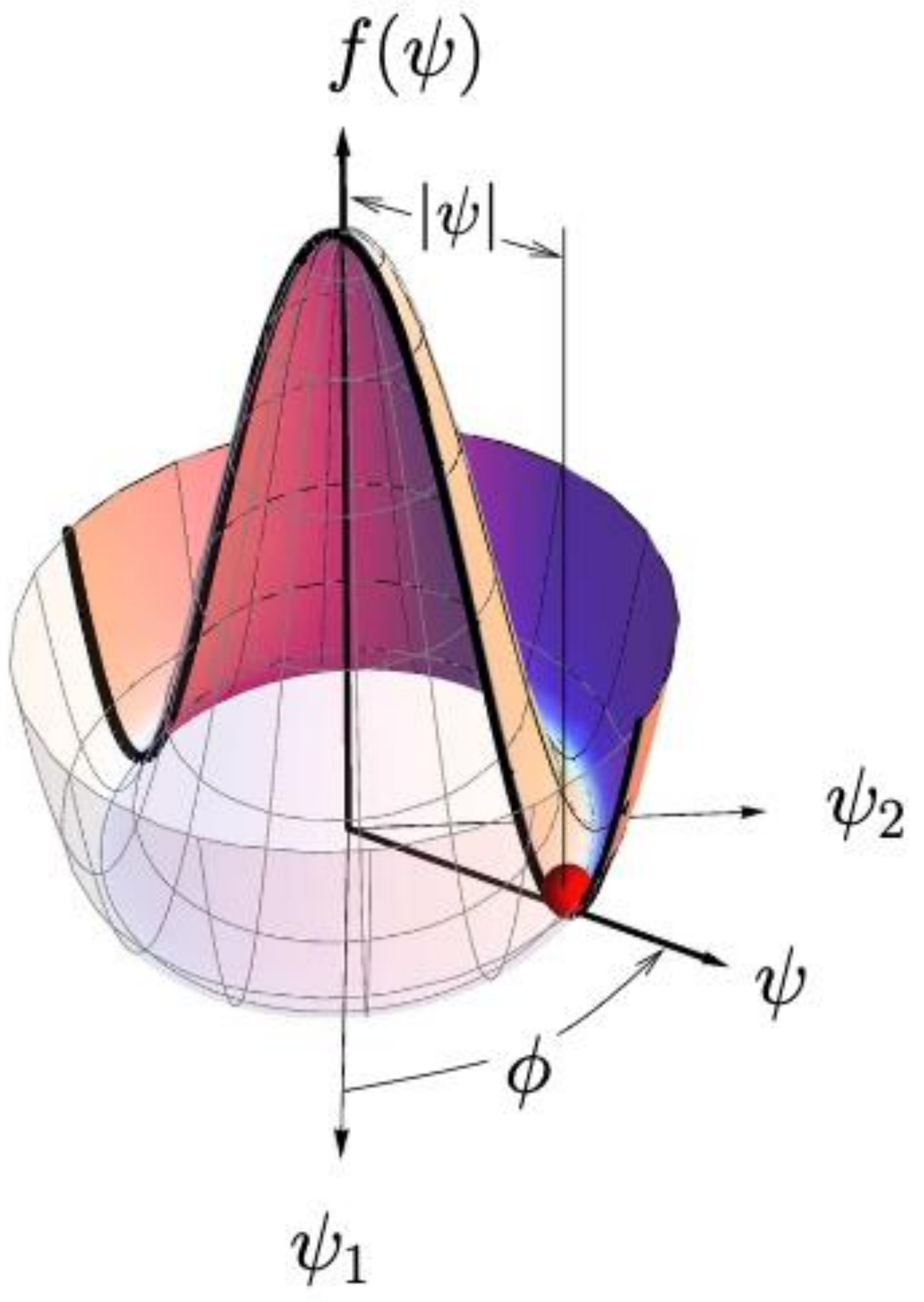
- Valid close to the critical point.
- Valid for lengths larger than the coherence length.

$$f[\Psi] = r(\Psi^*\Psi) + \frac{u}{2}(\Psi^*\Psi)^2$$

$$\Psi \equiv \Psi_1 + i\Psi_2 \equiv |\Psi| e^{i\phi}$$

For $T<T_c$

$$\Psi = \sqrt{\frac{|r|}{u}} e^{i\phi}$$



A well-defined phase breaks the U(1) continuous symmetry (spontaneous symmetry breaking)

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GINZBURG-LANDAU

(allows for inhomogeneous order parameters, to describe rigidity)

New gradient terms. For a superfluid (no charge):

Kinetic energy

$$f_{GL}[\Psi, \nabla\Psi] = \frac{\hbar^2}{2m}|\nabla\Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4$$

London's phenomenological wavefunction is the complex order parameter.

$$\Psi(x) = |\Psi(x)|e^{i\phi(x)} \qquad n_s = |\Psi|^2$$

The rigidity of the order parameter is in the gradient term

$$\nabla\Psi = (\nabla|\Psi| + i|\Psi|\nabla\phi)e^{i\phi}$$

$$f_{GL} = \frac{\hbar^2}{2m}|\Psi|^2(\nabla\phi)^2 + [\frac{\hbar^2}{2m}(\nabla|\Psi|)^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4]$$

Phase stiffness

Coleman's book

GINZBURG-LANDAU

(allows for inhomogeneous order parameters)

Coleman's book

$$f_{GL} = \frac{\hbar^2}{2m}|\Psi|^2(\nabla\phi)^2 + [\frac{\hbar^2}{2m}(\nabla|\Psi|)^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4]$$

r=a(T−Tc)

Correlation length (length scale of the amplitude fluctuations).

$$\Psi(r) = \Psi(0) + \delta\Psi(r) \Rightarrow f[\delta\Psi] = \frac{\hbar}{2m}(\nabla\delta\Psi)^2 + 2r(\delta\Psi)^2 \Rightarrow \delta\Psi(r) \sim e^{-r/\xi}$$

$$\xi(T) = \xi(0)\left|1 - \frac{T}{T_c}\right|^{-1/2}; \xi(0) = \sqrt{\frac{\hbar^2}{2maT_c}}$$

$\xi(0)$ is the Pippard's coherence length.

Short ξ: order parameter varies rapidly in space

Long ξ: smooth variation

GINZBURG-LANDAU

Amplitude (Ψ) fluctuations are confined to scales shorter than ξ. For longer lengths those terms are constant and only remain the fluctuations in the phase (ϕ).

$$f_{GL} = \frac{\rho_\phi}{2}(\nabla\phi)^2 + \text{constant}$$

A twist in the phase is an increase in the kinetic energy associated with a superfluid velocity

Calculate the current

$$\vec{J} = -i\frac{\hbar}{2m}\left(\psi^*\vec{\nabla}\psi - \vec{\nabla}\psi^*\psi\right)$$

Using:

$$\psi(x) = \sqrt{n_s(x)}e^{i\phi(x)}$$

$$\vec{J}_s = n_s\frac{\hbar}{m}\nabla\phi$$

Superfluid velocity

GINZBURG LANDAU

(now with charge —> include interaction with electromagnetic field)

$$F[\Psi, \mathbf{A}] = \int d^d x \left[\frac{\hbar^2}{2M} |(\nabla - \frac{ie^*}{\hbar}\mathbf{A})\Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4 + \frac{(\nabla \times \mathbf{A})^2}{2\mu_0} \right]$$

The free energy is gauge invariant (any change in the phase of the order parameter can be compensated by a gauge transformation)

$$\psi(x) \to \psi(x) e^{i\alpha(x)} \quad \mathbf{A} \to \mathbf{A} + \frac{\hbar}{e^*}\nabla\alpha$$

The electromagnetic part of the Free energy has an associated length: the London penetration length λ.

(Note e*. e*=2e only after BCS theory)

GINZBURG LANDAU (now with charge)

$$\psi(x) \to \psi(x)e^{i\alpha(x)} \quad \mathbf{A} \to \mathbf{A} + \frac{\hbar}{e^*}\nabla\alpha$$

The electromagnetic field absorbs the phase gradient
and the Goldstone mode gets a mass

ANDERSON-HIGGS MECHANISM

(see section 12.6.2 of Coleman's book for a demonstration)

(in a neutral superfluid you have Goldstone —massless— modes)

GINZBURG LANDAU (now with charge)

$$F[\Psi, \mathbf{A}] = \int d^d x \left[\frac{\hbar^2}{2M} |(\nabla - \frac{ie^*}{\hbar}\mathbf{A})\Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4 + \frac{(\nabla \times \mathbf{A})^2}{2\mu_0} \right]$$

Now the current is:

$$\mathbf{J}(x) = \frac{e^*\hbar}{M}|\Psi|^2 \nabla\phi - \frac{e^{*2}}{M}|\psi|^2\mathbf{A} = e^* n_s \frac{\hbar}{M}\left(\vec{\nabla}\phi - \frac{e\,*}{\hbar}\mathbf{A}\right) = e^* n_s \mathbf{v}_s$$

An external vector potential produces superflow

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JOSEPHSON EFFECTS (predicted 1962; measured 1963; applied in SQUID 1964; Nobel Prize 1973)

DC Josephson Effect: A zero voltage supercurrent should flow between two superconducting

$$I_s = I_c \sin\Delta\phi$$

Critical current at the junction

Difference in the phase of the Ginzburg-Landau wave-functions

—> 2025 Nobel Prize
Macroscopic quantum tunnelling and energy quantisation in an electric circuit.

AC Josephson Effect:
An applied voltage leads to an oscillatory current

$$\frac{d(\Delta\phi)}{dt} = \frac{2eV}{\hbar}$$

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GINZBURG LANDAU (now with charge)

$$F[\Psi, \mathbf{A}] = \int d^d x \left[\frac{\hbar^2}{2M} |(\nabla - \frac{ie^*}{\hbar}\mathbf{A})\Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4 + \frac{(\nabla \times \mathbf{A})^2}{2\mu_0} \right]$$

The length scale for A: London penetration length λ.
The length scale for $(\nabla\psi)^2$ is the coherence length ξ.

Ginzburg Landau parameter: λ/ξ.

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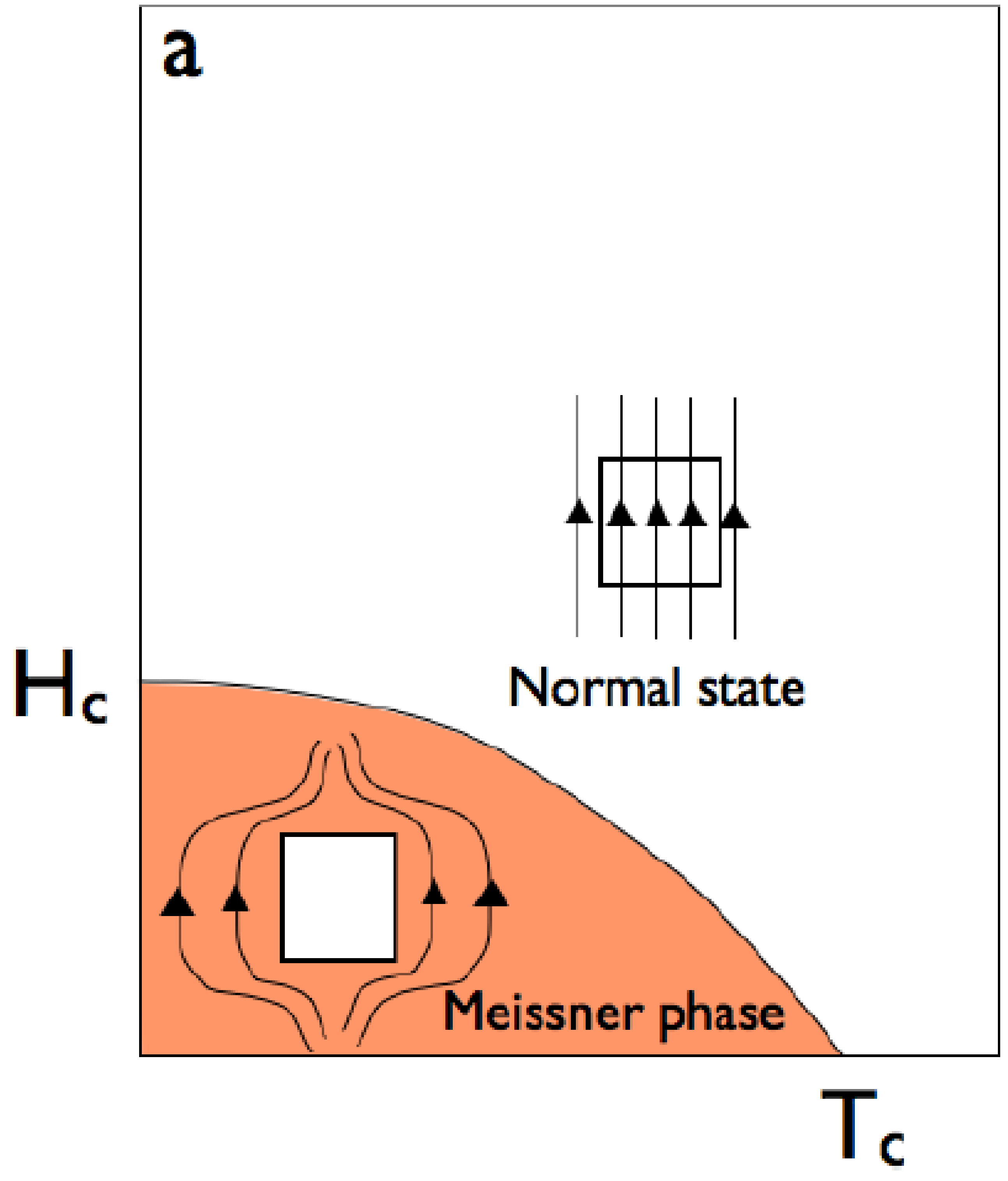
CRITICAL FIELD H_C

There is a maximum value of magnetic field H_c that can be applied before superconductivity is destroyed. There is also a critical current I_c.

First superconductors had a low H_C. Bad news to build magnets with superconducting cables.

Some superconductors (alloys) didn't exclude the magnetic field completely and had much larger critical fields (1930's).



Coleman's book

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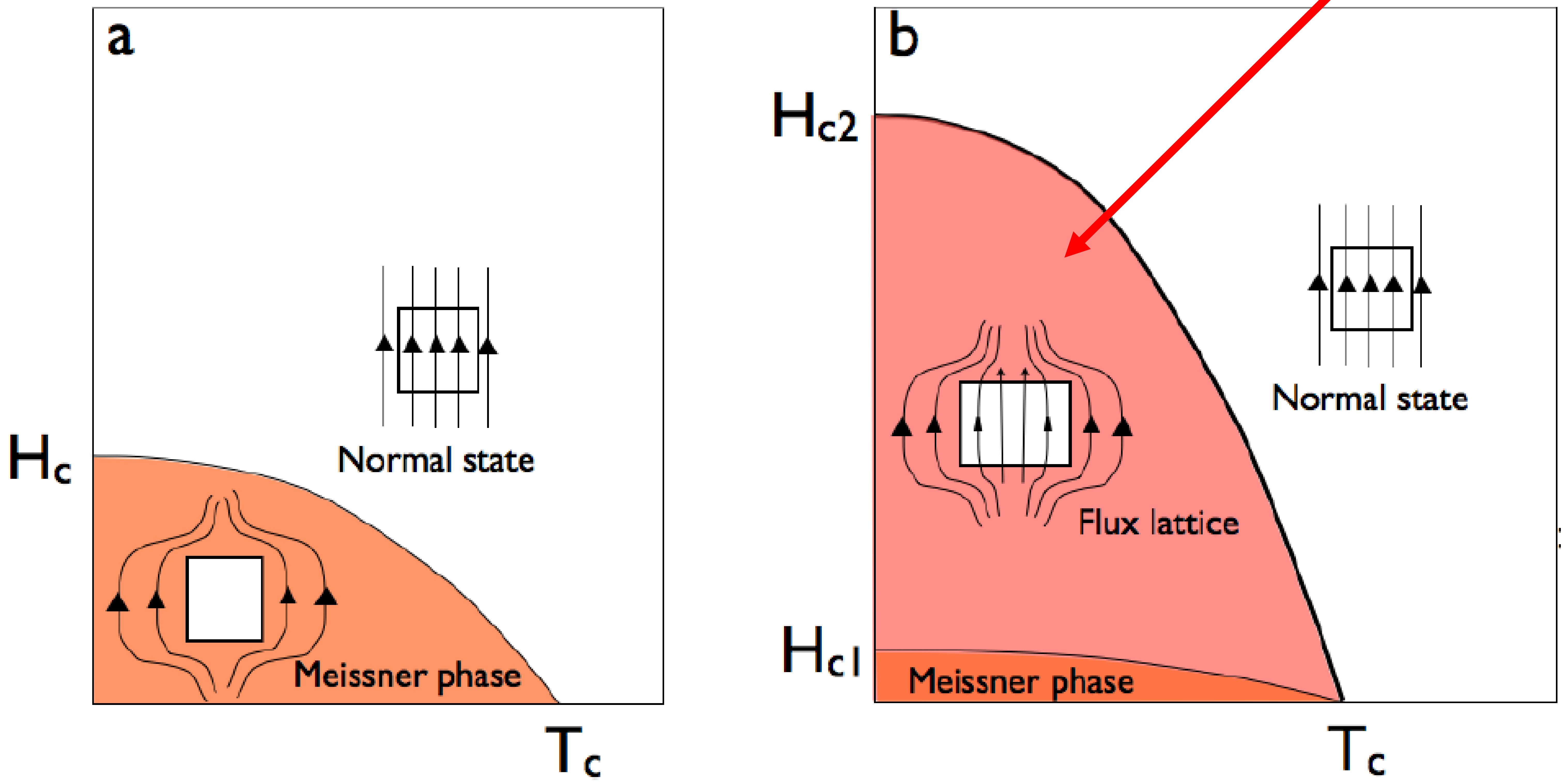
CRITICAL FIELD H_C

Coleman's book

Mixed state
or Shubnikov phase



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TYPE I/TYPE II SUPERCONDUCTORS

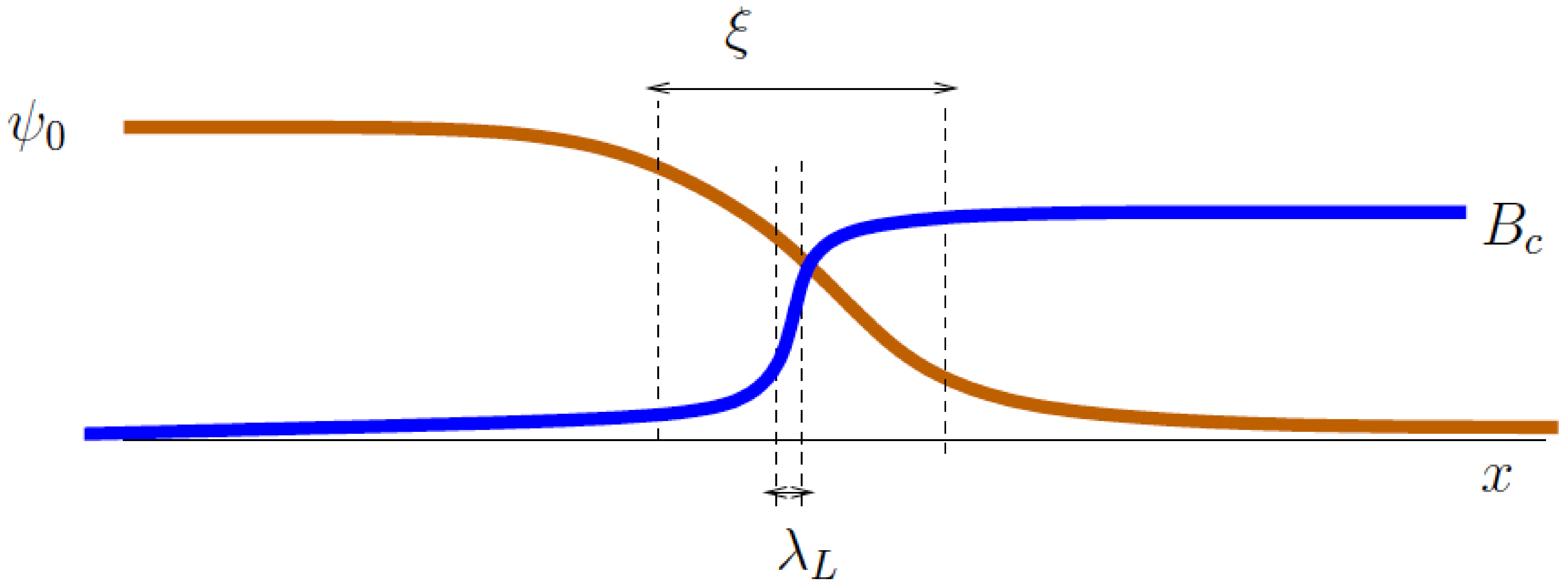
The firstly discovered superconductors were all Type I (λ<<ξ).

London penetration length

$$\lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}$$

Correlation length

$$\xi(T) = \xi(0)\left|1 - \frac{T}{T_c}\right|^{-1/2} ;\ \xi(0) = \sqrt{\frac{\hbar^2}{2maT_c}}$$



TYPE I/TYPE II SUPERCONDUCTORS

Coleman's book

The first superconductors were all Type I (λ<<ξ).

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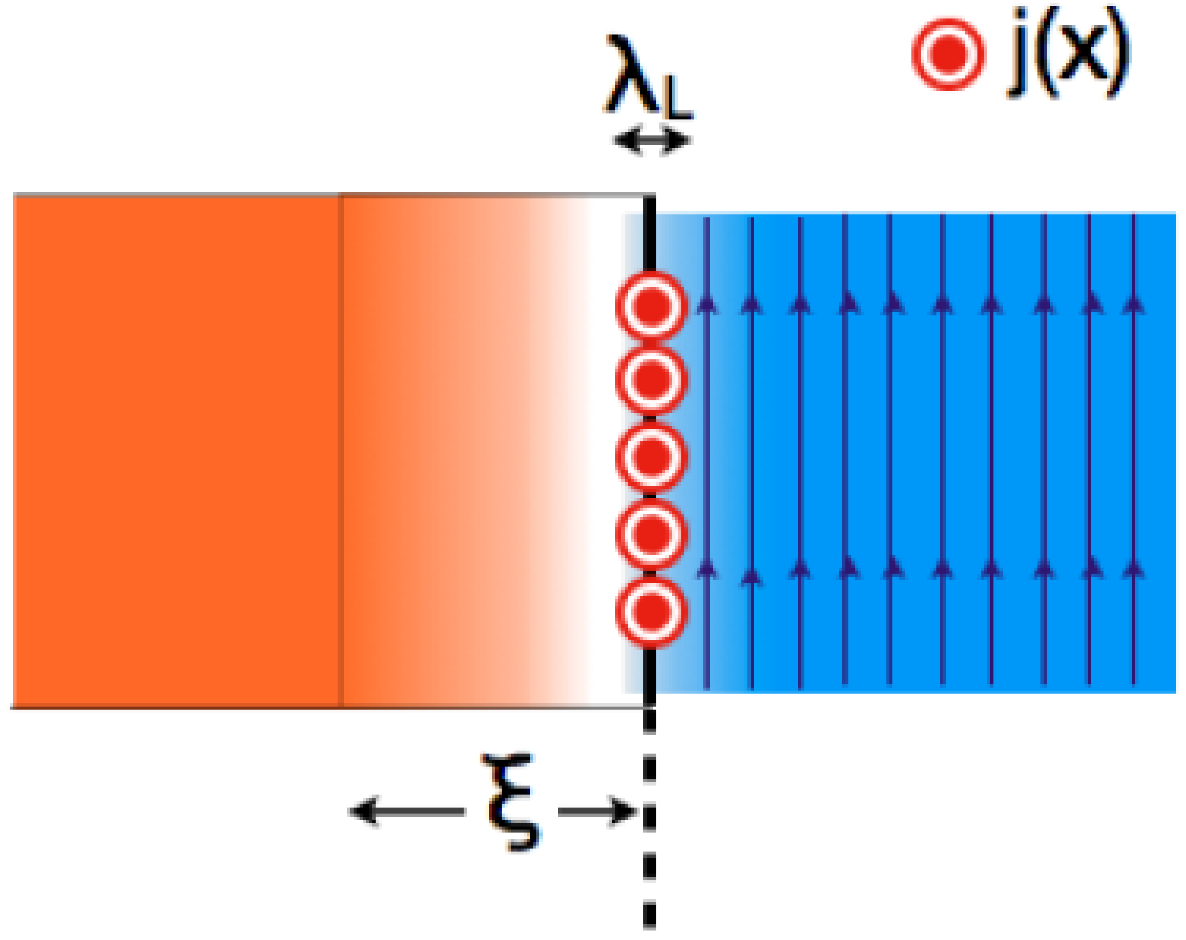
The correlation length

$$\xi(T) = \xi(0)\left|1-\frac{T}{T_c}\right|^{-1/2} ; \xi(0) = \sqrt{\frac{\hbar^2}{2maT_c}}$$

London and Ginzburg-Landau considered λ<ξ.
The supercurrent that screens the applied magnetic field occurs in a thin layer.



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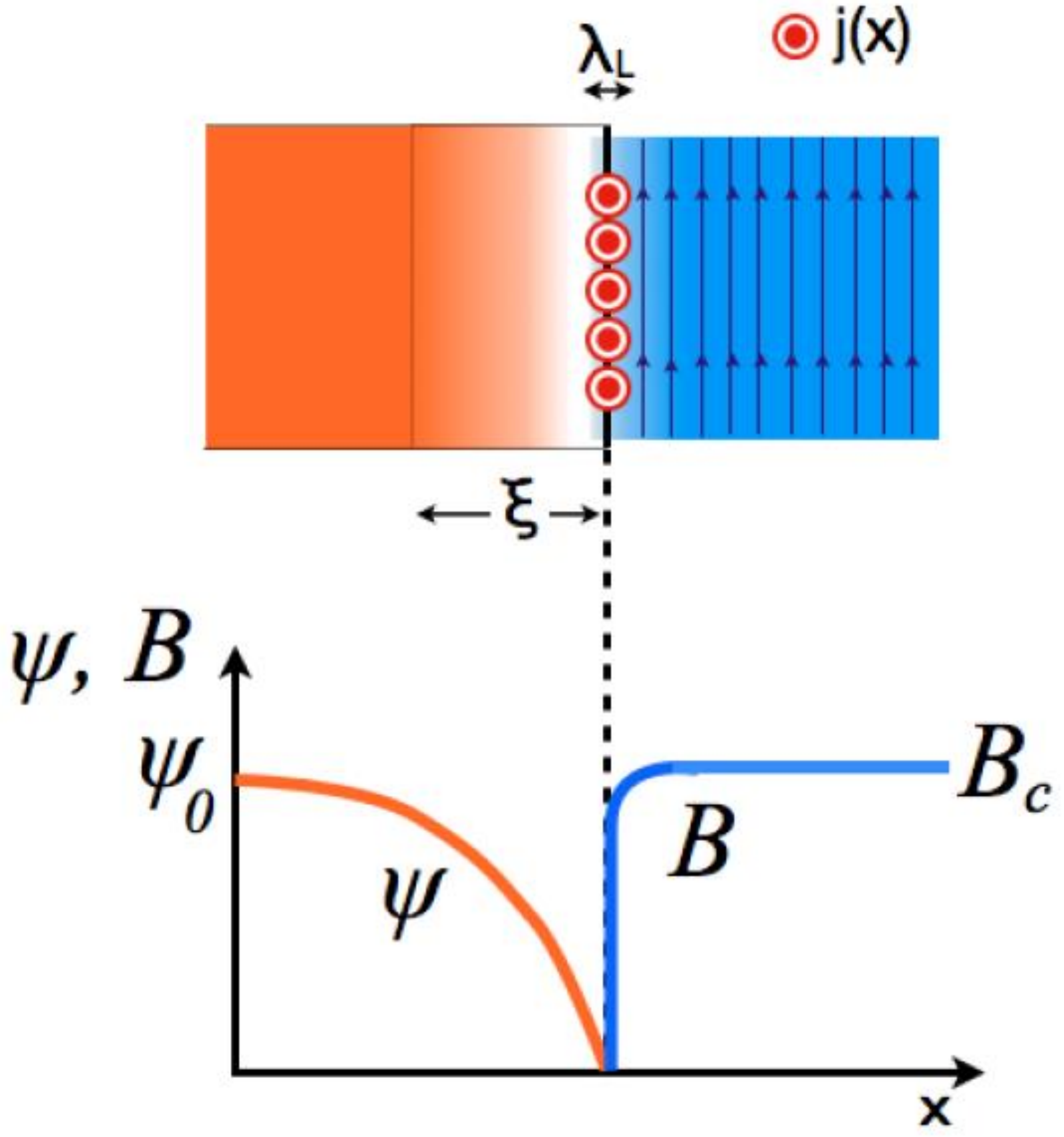
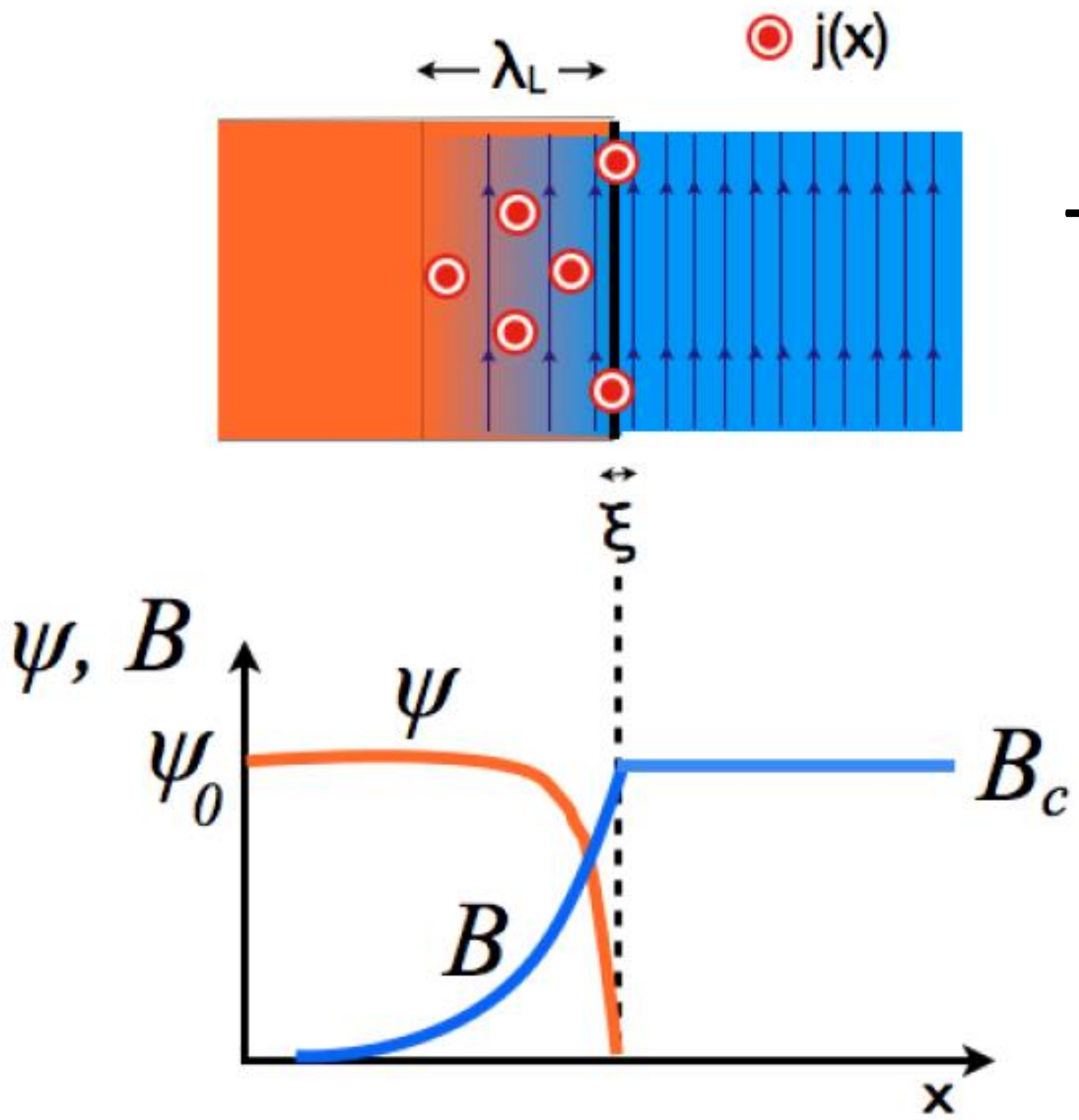
TYPE I/TYPE II SUPERCONDUCTORS

Type I





Type II

Modifying the wave-function costs a lot of energy. Not much energy gain by allowing H in. N/S interfaces are costly. At H_c there is a first order transition into the normal state.

Modifying the wave-function doesn't cost too much. You gain energy by allowing the magnetic field in. N/S interfaces are favoured. Two critical fields. **Vortices**. (Abrikosov)

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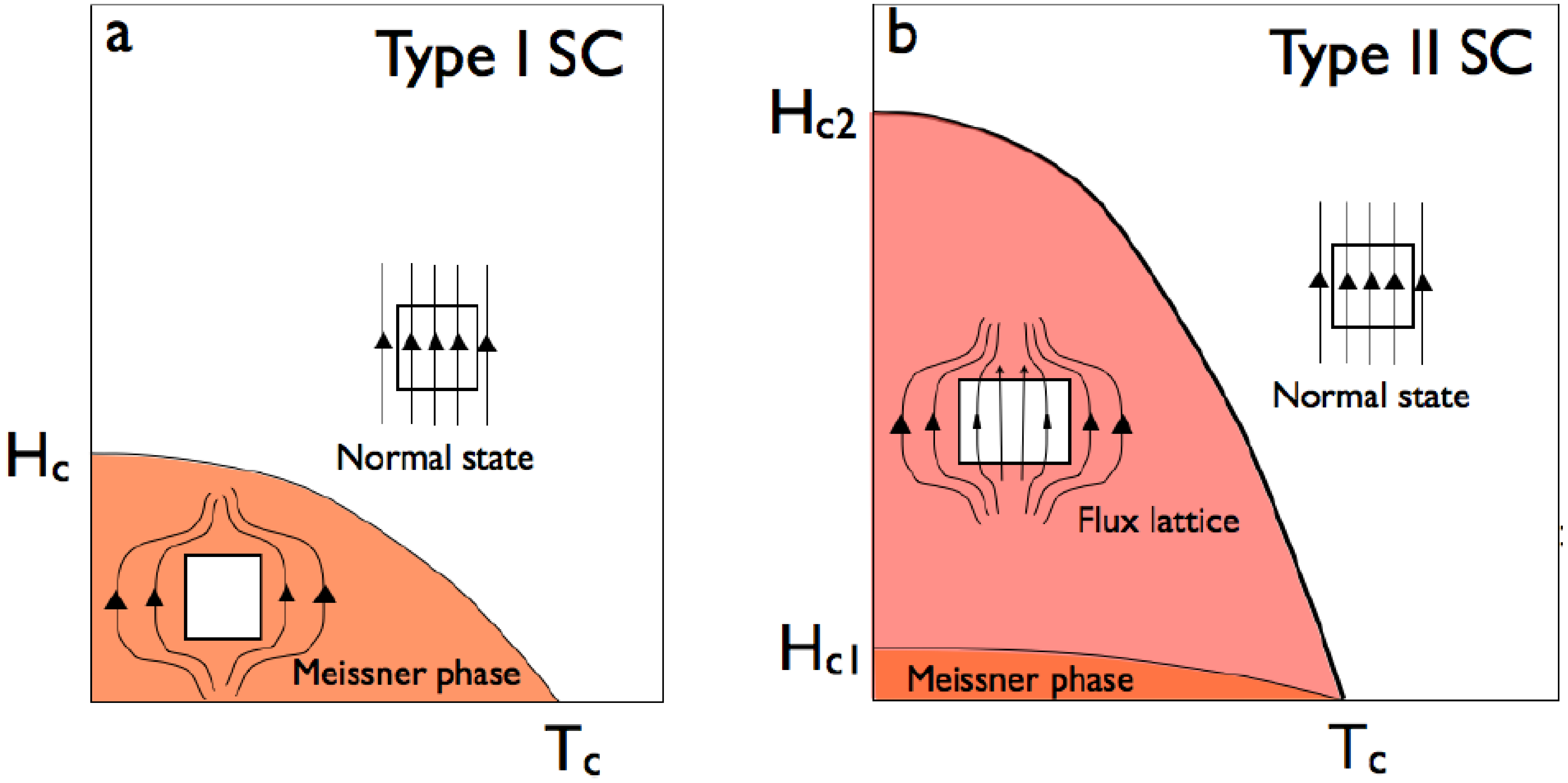
CRITICAL FIELD H_C

$$\frac{\lambda_L}{\xi} < \frac{1}{\sqrt{2}} \qquad\qquad \frac{\lambda_L}{\xi} > \frac{1}{\sqrt{2}}$$



$$B_{c2} = \frac{\phi_0}{2\pi\xi^2} \qquad B_{c1} = \frac{\phi_0}{2\pi\lambda^2} \qquad B_{c1}B_{c2} = B_c^2$$

Coleman's book

SOME EXAMPLES

http://www.qm.phy.cam.ac.uk/teaching/

Compound	T_c [K]	ξ [nm]	λ [nm]	$\kappa = \lambda/\xi$	
Al	1.18	1600	50	0.03	type-I
Pb	7.19	83	39	0.47	type-I
Nb	9.25	40	44	1.1	border
Nb_3Sn	18.2	3.6	124	34	type-II
$YBa_2Cu_3O_{7-\delta}$	90	(1.5)	(130)	(87)	type-II

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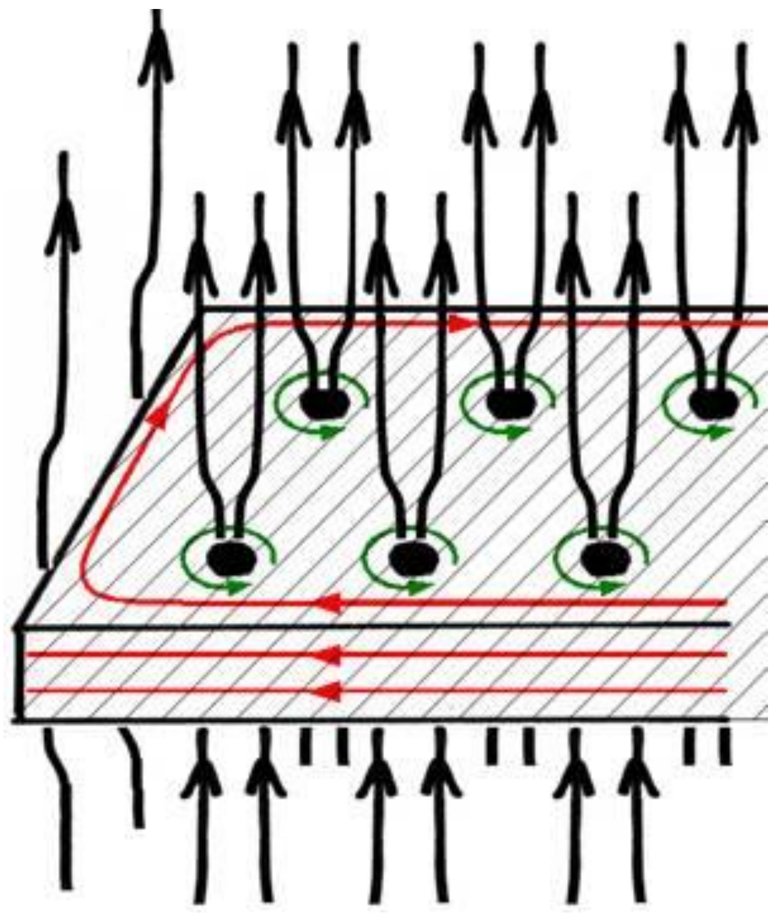
VORTICES

In type II superconductors, part of the field can go through the superconductor (the wave function can "adapt" to changes fast).
A supercurrent circulates around the magnetic flux to screen it: Vortices.

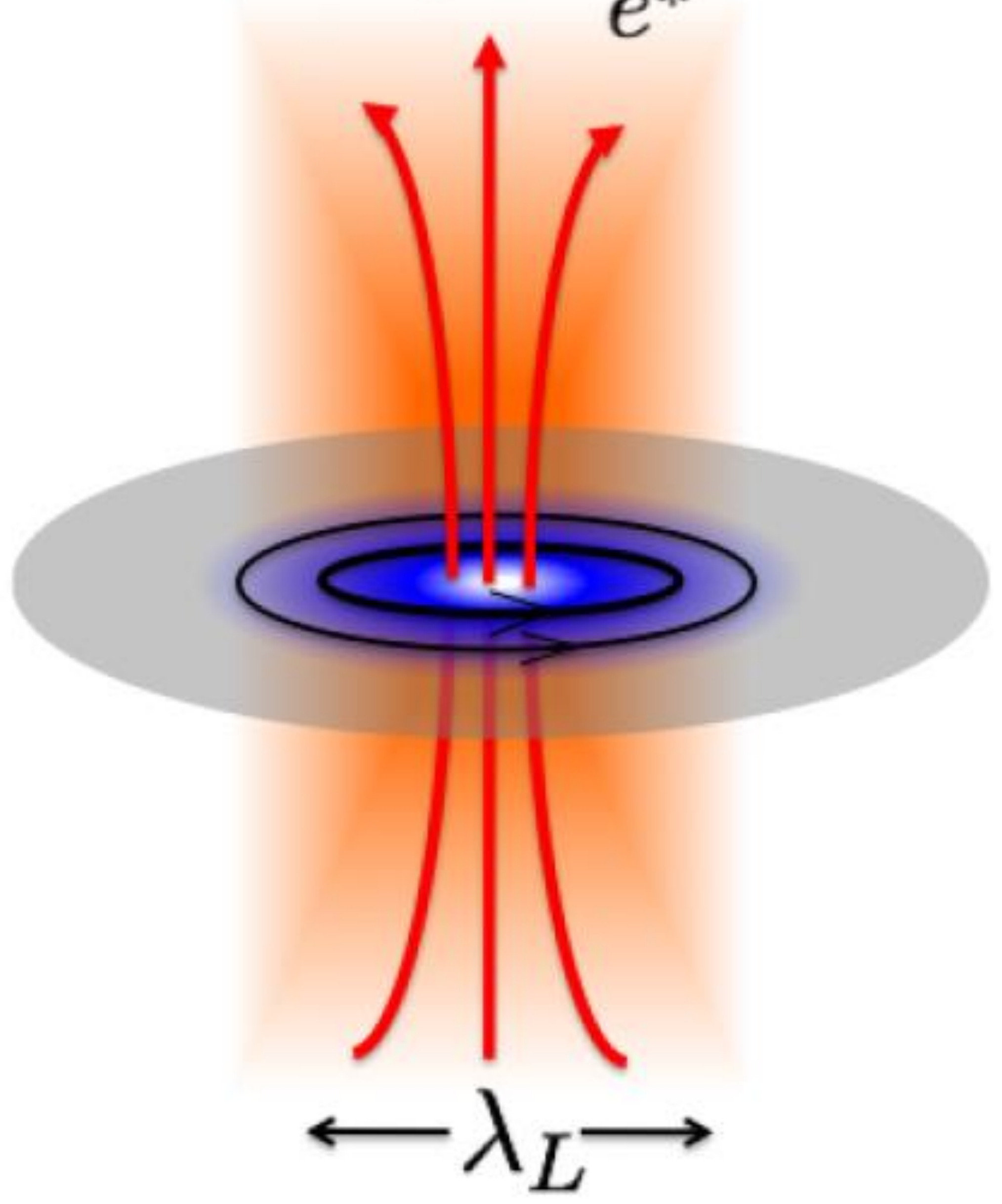


http://www.supraconductivite.fr/

Superconductivity is suppressed at the vortex core

The flux is quantized

$$\Phi_0 = \frac{h}{e^*}$$



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Coleman's book

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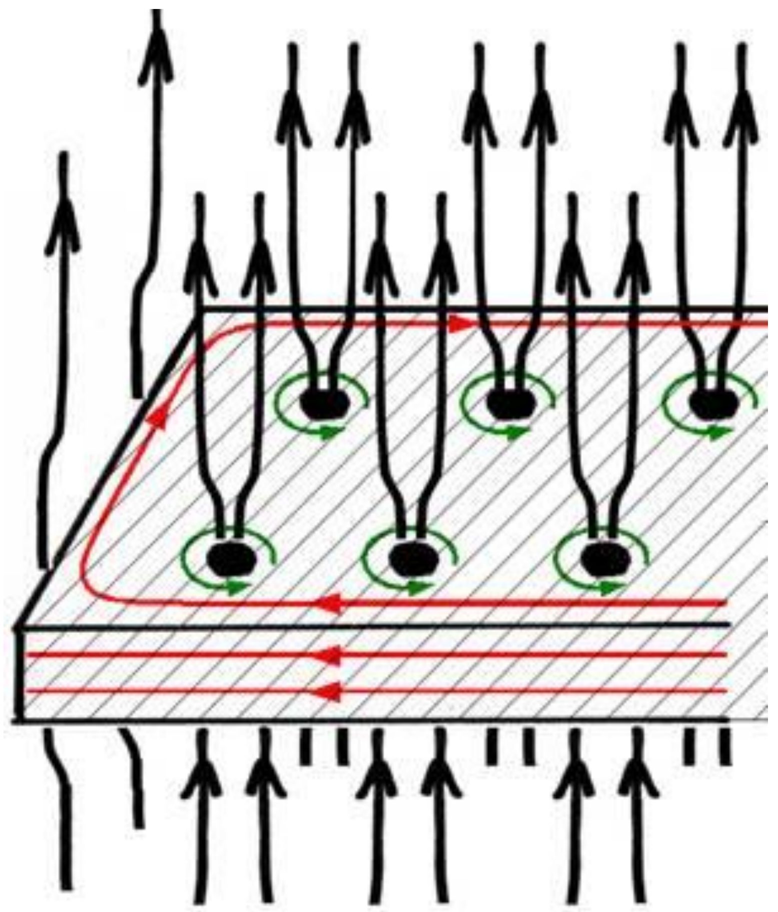

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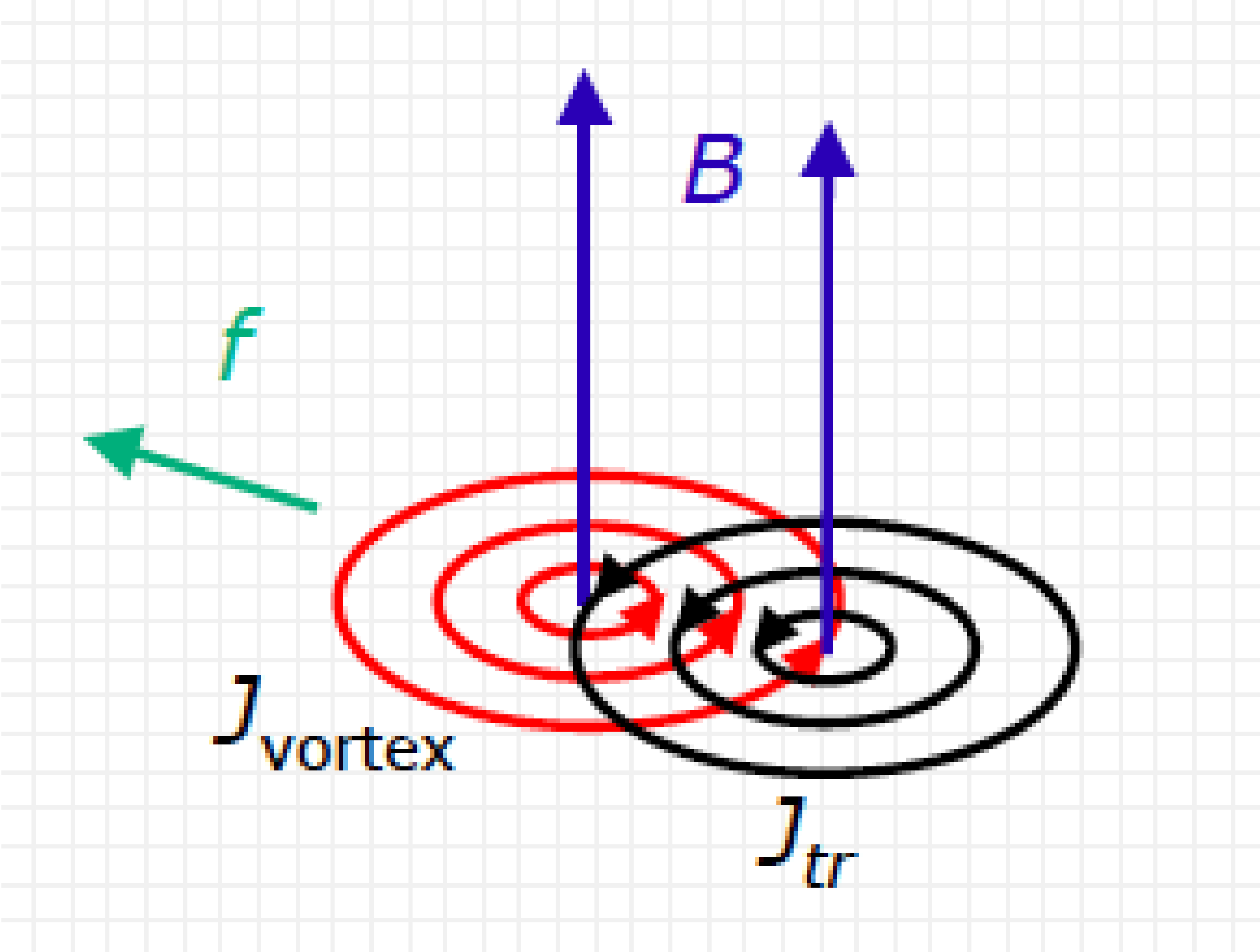

Vortices repel each other
(they have the same topological charge)

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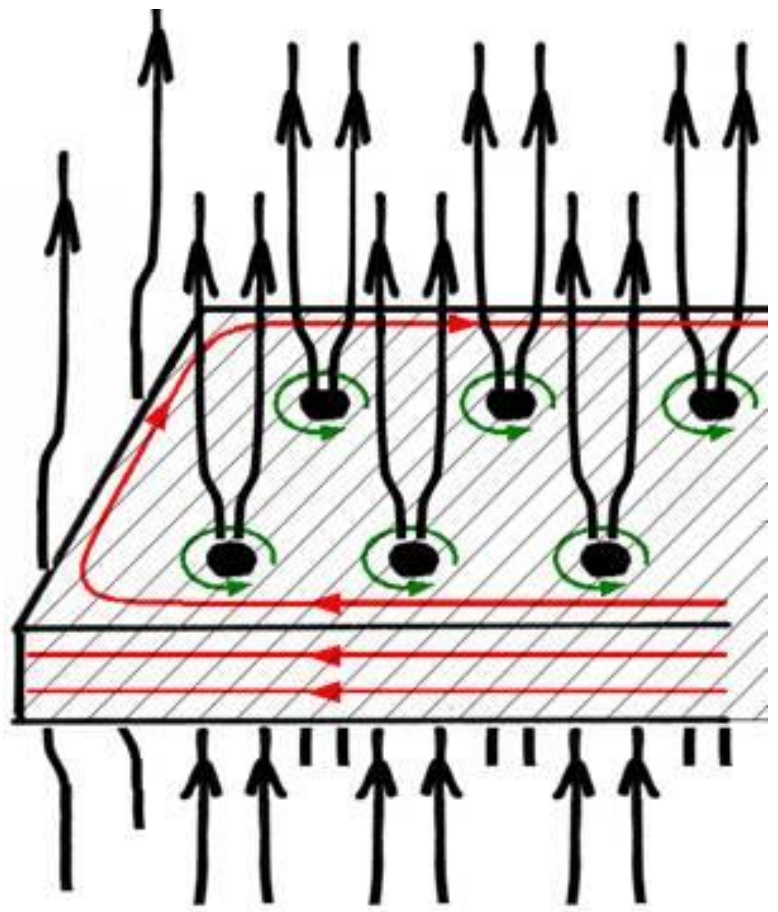
VORTICES

In type II superconductors, part of the field can go through the superconductor. A supercurrent circulates around the magnetic flux to screen it: Vortices.



http://www.supraconductivite.fr/

Pinning!!

www.icmm.csic.es/superconductividad

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PINNING

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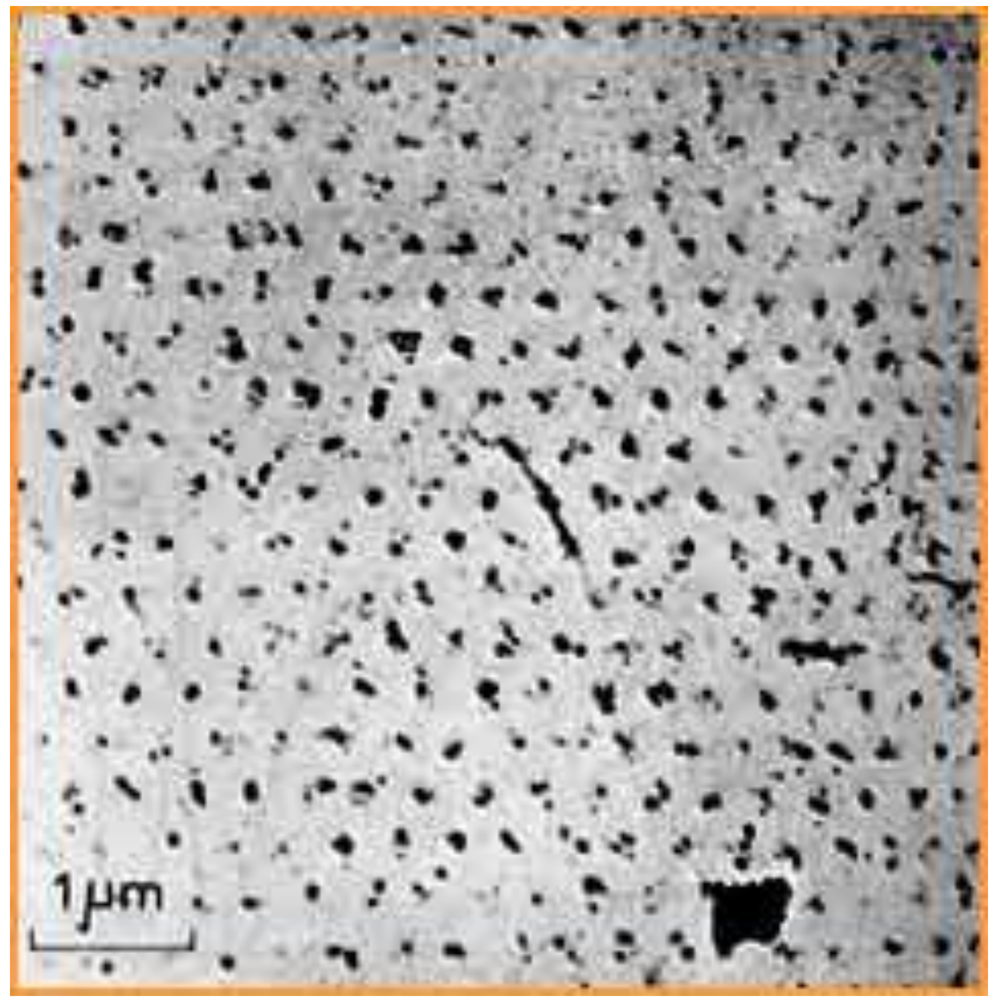
VORTICES

In type II superconductors, part of the field can go through the superconductor (the wave function can "adapt" to changes fast).
A supercurrent circulates around the magnetic flux to screen it: Vortices.

From London eqs: the magnetic flux is quantized. Vortices order in a regular array.

First observation of an array of vortices



Alloy of Pb with 4% In
Physics Letters 24A, 526 (1967)

Supercurrent exert forces on vortices leading to dissipation. Impurities pin the vortices so they cannot move.

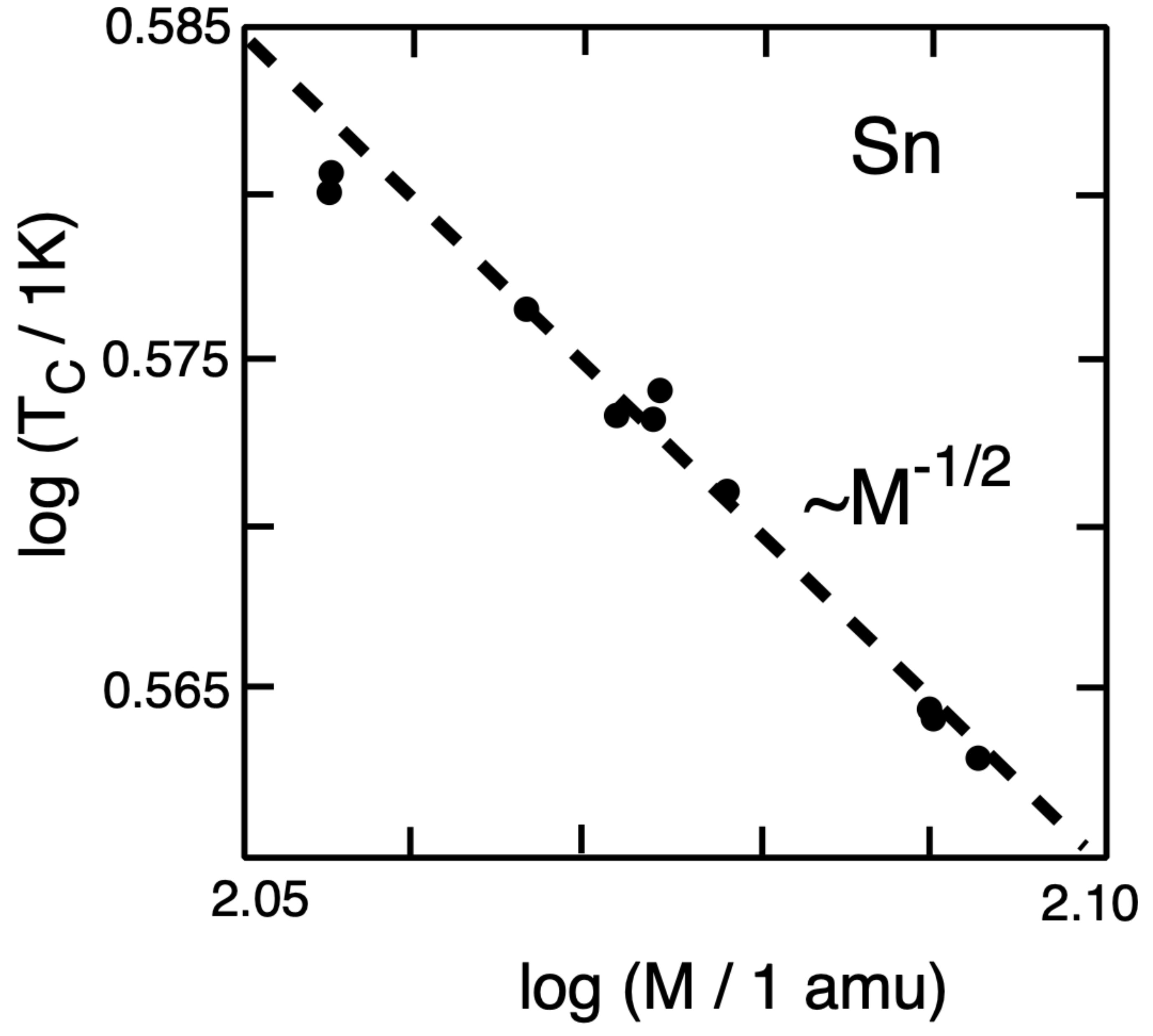
TOWARDS A MICROSCOPIC MODEL

ISOTOPE EFFECT

(C. Reynolds et al, E. Maxwell)

Evidence that superconductivity was related to electron-phonon interaction

$$T_c \propto \omega_D \approx \frac{1}{\sqrt{M}}$$



Independently, Fröhlich suggested that SPC occurs due to electron-lattice interaction. Also Bardeen, independently, as soon as he knew about the isotope effect.

$$V_{\mathrm{eff}}(\mathbf{k},\mathbf{k}') = -g^2_{\mathbf{k}-\mathbf{k}'}\frac{2\omega_{\mathbf{k}-\mathbf{k}'}}{\omega^2_{\mathbf{k}-\mathbf{k}'} - (\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'})^2}$$

Formulation of the electron-phonon interaction. Leads to attractive energy for low-energy transfer.

THE IDEA OF A GAP

Bardeen proposed that a gap in the spectrum would lead to the rigidity of the wave-function. Some experiments were showing a sign of a gap (activated specific heat).

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EFFECTIVE INTERACTION

Bardeen-Pines effective interaction:

screened Coulomb
+
retarded electron-phonon interaction

Bare Coulomb

$$V(r) = e^2/r \rightarrow V_q = 4\pi e^2/q^2$$

Screened Coulomb

$$V(r) = \frac{e^2 e^{-\kappa r}}{\epsilon r} \rightarrow V_q = \frac{4\pi e^2}{\epsilon(q^2 + \kappa^2)}$$

Coulomb is always repulsive

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EFFECTIVE INTERACTION

Electron-phonon interaction. Phonon of frequency ω_q.

$$V_q + |M_q|^2 \left(\frac{1}{\hbar\omega - \hbar\omega_q} + \frac{1}{-\hbar\omega - \hbar\omega_q} \right) = V_q + \frac{2|M_q|^2}{\hbar} \frac{\omega_q}{\omega^2 - \omega_q^2}$$

Change in E
for the e-

M_q: electron-phonon matrix element.
If electron-phonon interaction is too weak (good metals), no superconductivity arises

Modification of the dielectric screening by a moving particle in a lattice that can be deformed.

Overscreening: The effective electron-electron interaction can be negative (attractive)

Key issue: the dynamics of the phonons is much slower than the electrons'

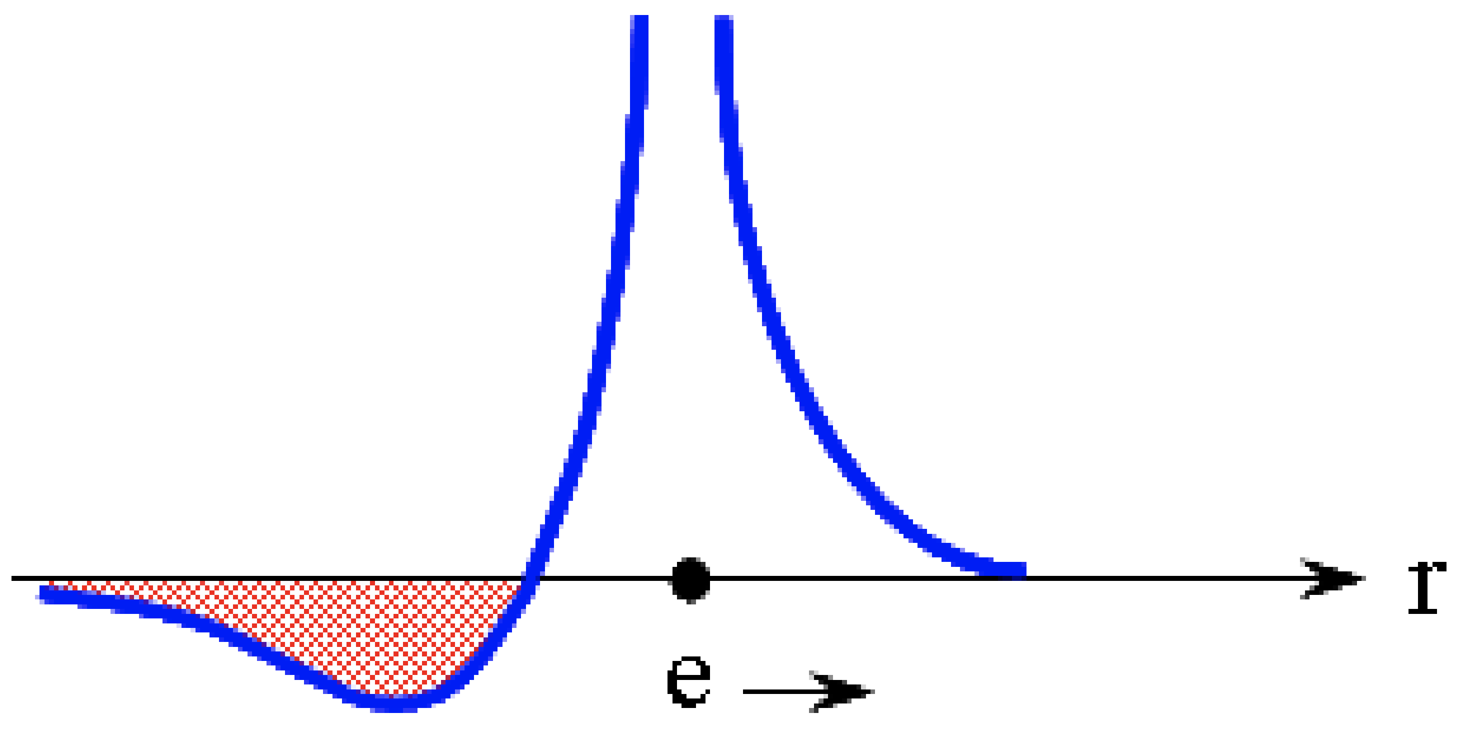
EFFECTIVE INTERACTION

Electron-phonon interaction. Phonon of frequency ω_q.

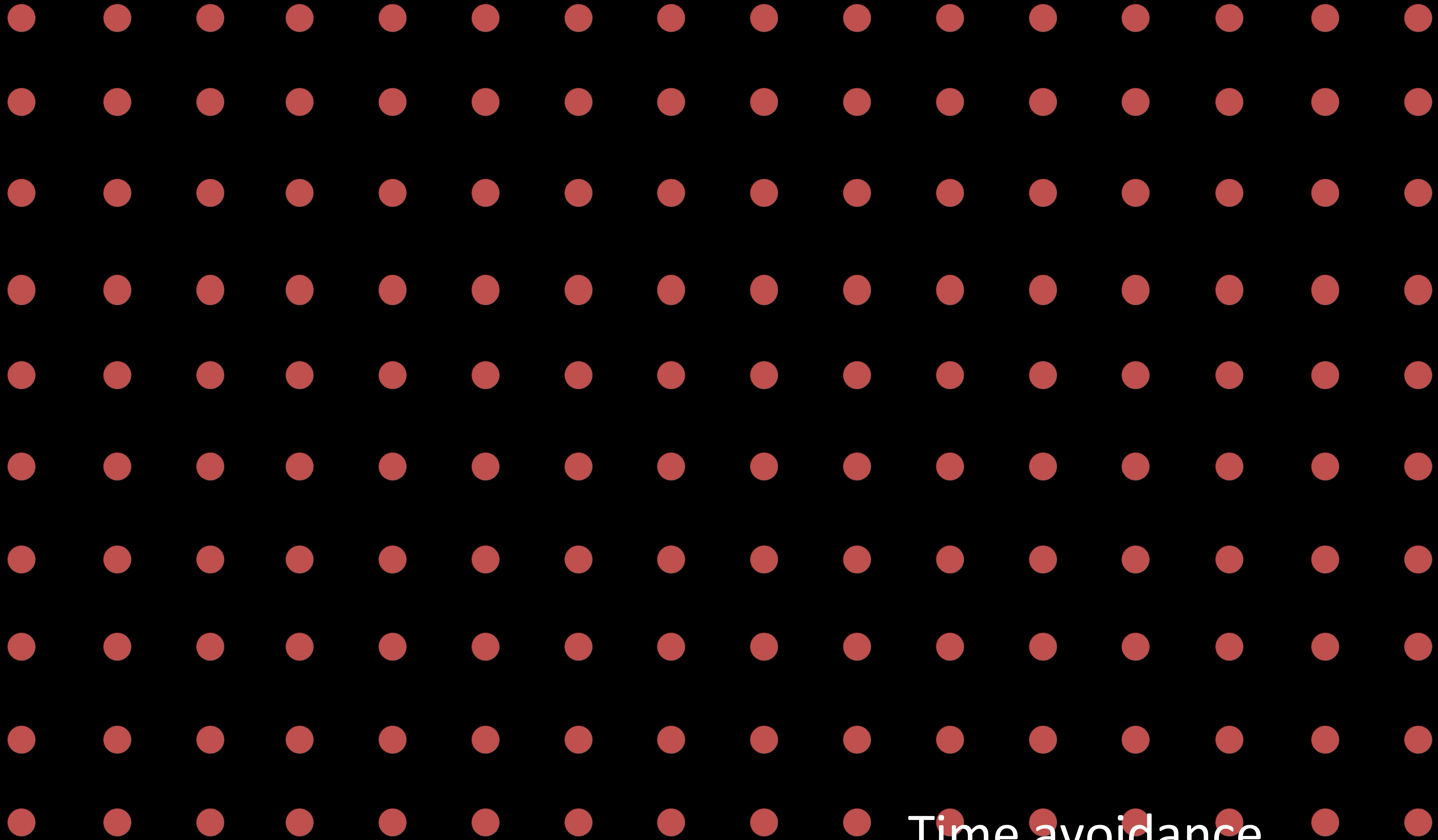
$$V_q + |M_q|^2 \left(\frac{1}{\hbar\omega - \hbar\omega_q} + \frac{1}{-\hbar\omega - \hbar\omega_q} \right) = V_q + \frac{2|M_q|^2}{\hbar} \frac{\omega_q}{\omega^2 - \omega_q^2}$$

http://www.qm.phy.cam.ac.uk/teaching/

Attraction if $\omega < \omega_q$



Moving Charge



Time avoidance

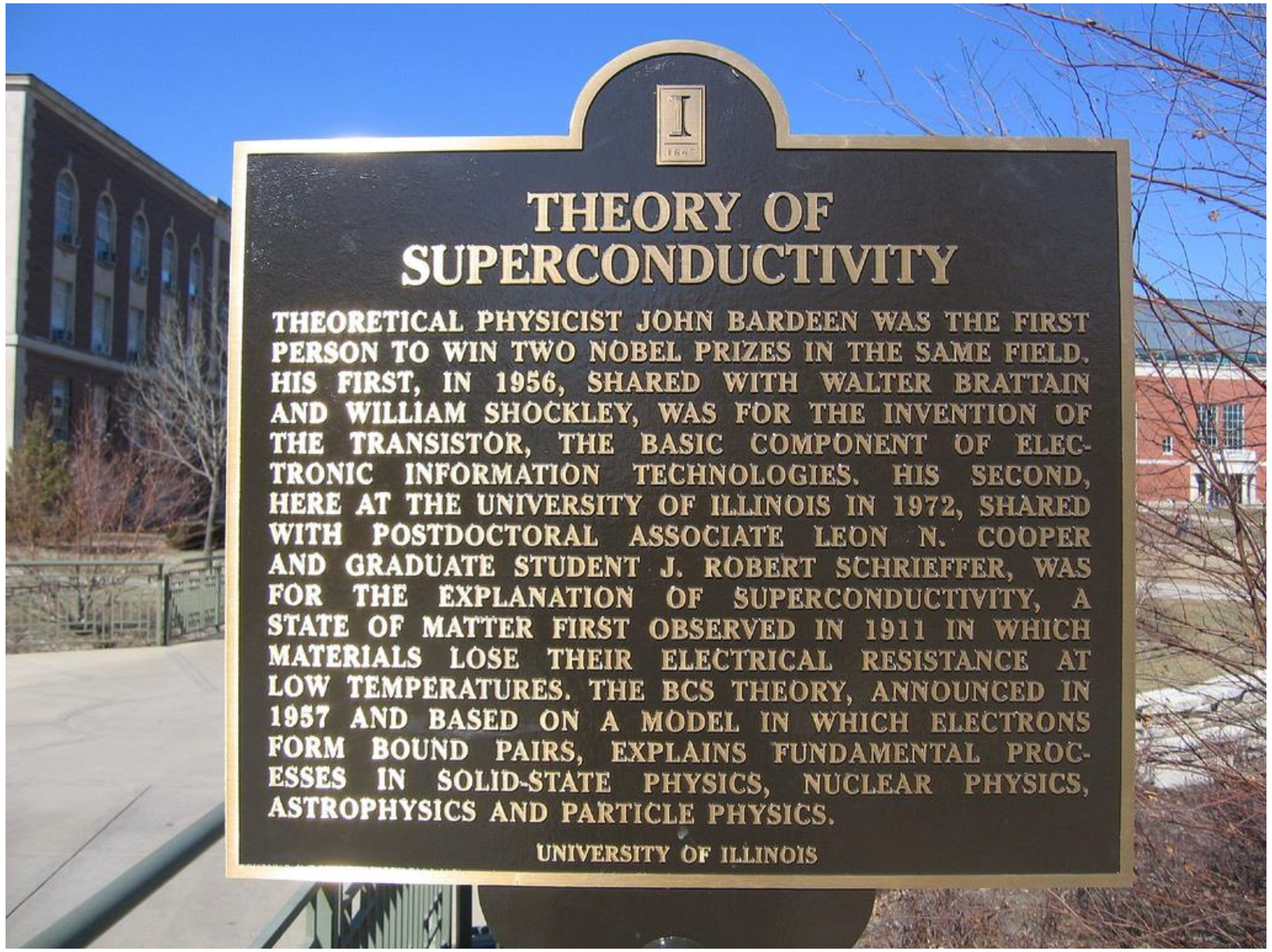
BCS theory (1957)

(Nobel prize 1972)

Bardeen-Cooper-Schrieffer

Interaction-pairs-wavefunction

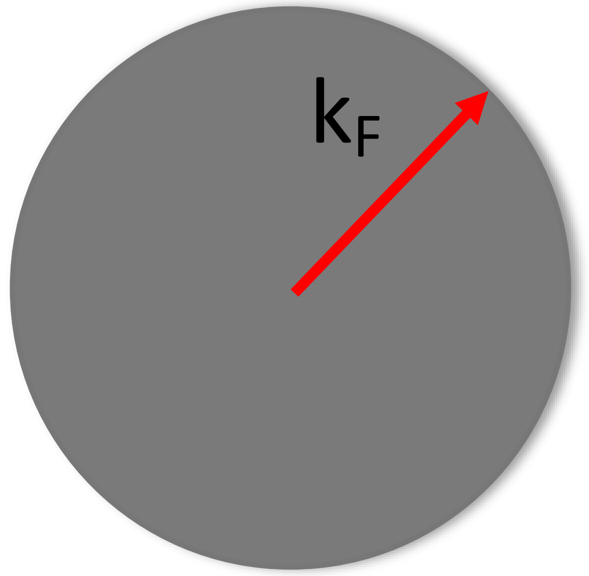
University of Illinois archives



Wikipedia

COOPER PAIRS (1956)

Arbitrarily weak attractions can bind pairs of electrons on the Fermi surface (free particles would need an energy minimum).



Add two electrons on top of the Fermi sea

$$|\Psi\rangle = \Lambda^\dagger |FS\rangle$$

Assume zero momentum and singlet pairing (expected lowest energy)

With pair creation operator
$$\Lambda^\dagger = \sum_{\mathbf{k}} \phi_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}$$

pair wavefunction
(s-wave symmetry for conventional superconductors)

COOPER PAIRS (1956)

Coleman's book

Basic Hamiltonian: Kinetic term plus effective interaction

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \hat{V}$$

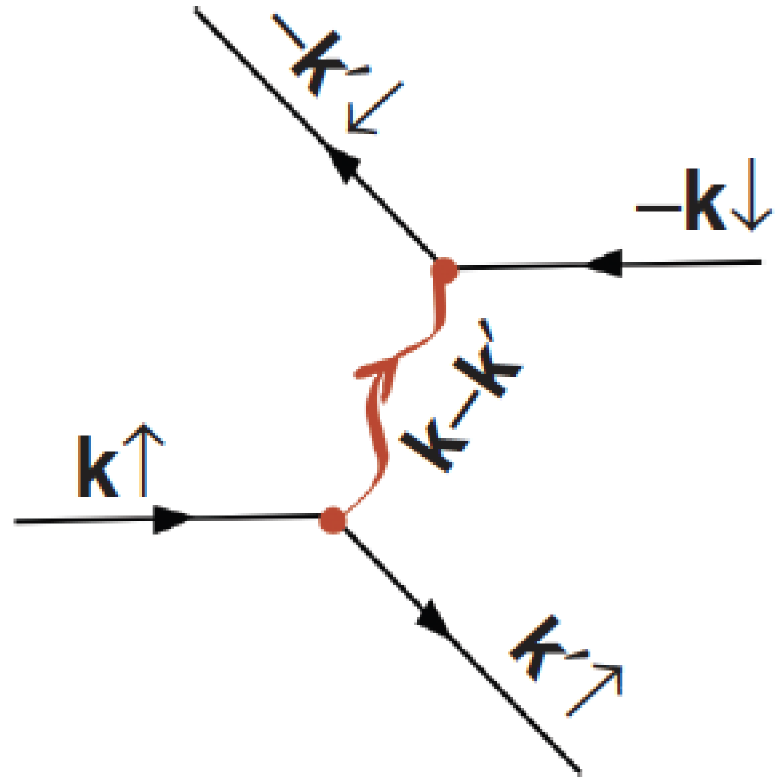
$$|\Psi\rangle = \Lambda^{\dagger}|FS\rangle \qquad \Lambda^{\dagger} = \sum_{\mathbf{k}} \phi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}$$

Define: $|\mathbf{k}_P\rangle \equiv c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}|FS\rangle$

$$H|\Psi\rangle = \sum_{|\mathbf{k}|>k_F} 2\epsilon_{\mathbf{k}} \phi_{\mathbf{k}} |\mathbf{k}_p\rangle + \sum_{|\mathbf{k}|,|\mathbf{k}'|>k_F} |\mathbf{k}_p\rangle\langle \mathbf{k}_p|\hat{V}|\mathbf{k}'_p\rangle \phi_{\mathbf{k}'}$$

For each k

$$E\phi_{\mathbf{k}} = 2\epsilon_{\mathbf{k}} \phi_{\mathbf{k}} + \sum_{|\mathbf{k}'|>k_F} \langle \mathbf{k}_p|\hat{V}|\mathbf{k}'_p\rangle \phi_{\mathbf{k}'}$$



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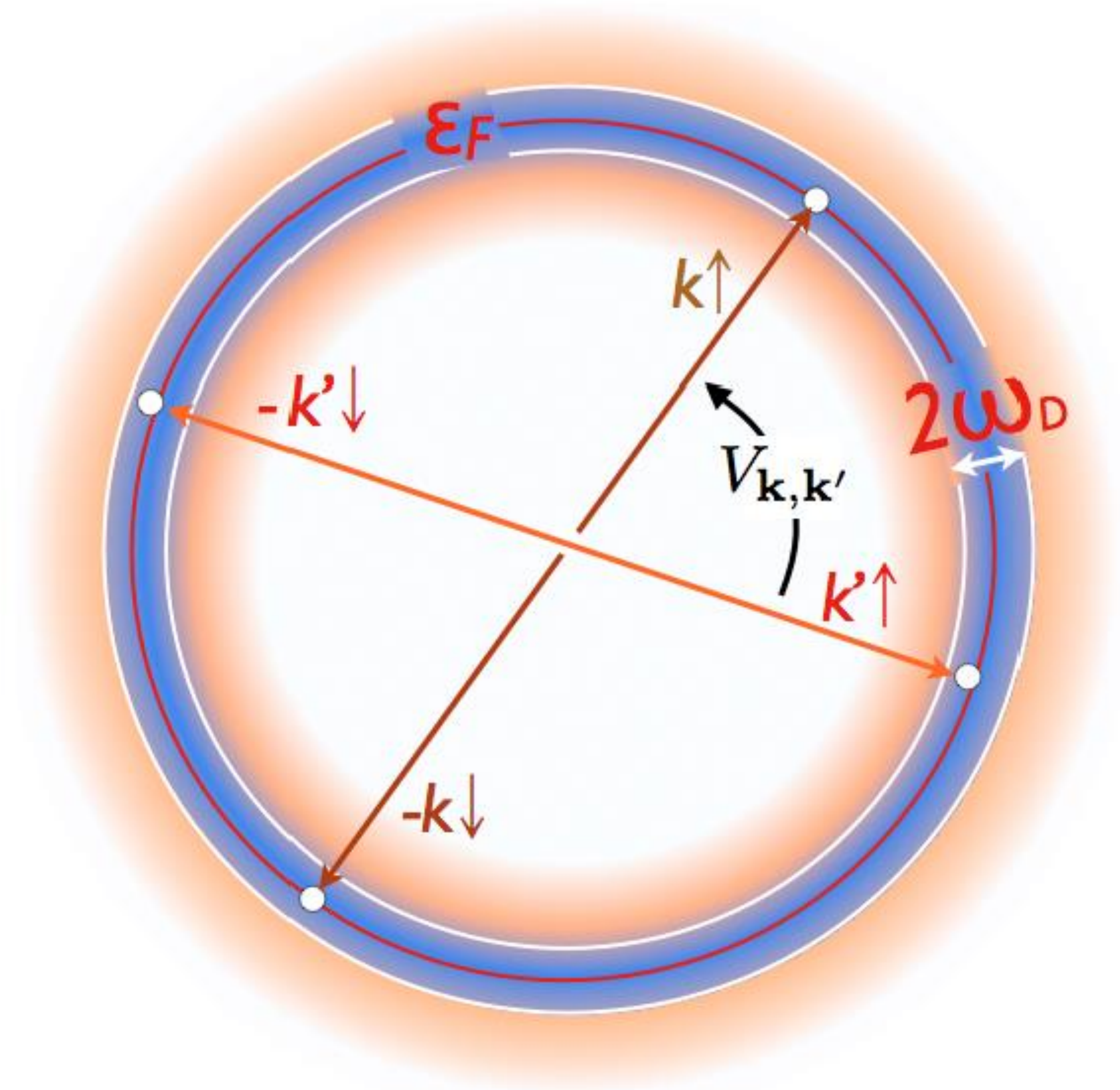
COOPER PAIRS (1956)

Coleman's book

Simplified model interaction $V_{kk'}$: only attractive within an energy ω_D (Debye frequency) of the Fermi energy

Debye frequency is the theoretical maximum frequency of phonons in a solid



$$V_{\mathbf{k},\mathbf{k}'} = \begin{cases} -g_0/V & (|\epsilon_{\mathbf{k}}|,\ |\epsilon_{\mathbf{k}'}| < \omega_D) \\ 0 & (\text{otherwise}). \end{cases}$$

COOPER PAIRS (1956)

$$E\phi_{\mathbf{k}} = 2\epsilon_{\mathbf{k}}\phi_{\mathbf{k}} + \sum_{|\mathbf{k}'|>k_F} \langle \mathbf{k}_p|\hat{V}|\mathbf{k}'_p\rangle\phi_{\mathbf{k}'}$$

$$V_{\mathbf{k},\mathbf{k}'} = \begin{cases} -g_0/V & (|\epsilon_{\mathbf{k}}|,\ |\epsilon_{\mathbf{k}'}| < \omega_D) \\ 0 & (\text{otherwise}). \end{cases}$$

$$(E - 2\epsilon_{\mathbf{k}})\phi_{\mathbf{k}} = -\frac{g_0}{V}\sum_{0<\epsilon_{\mathbf{k}'}<\omega_D}\phi_{\mathbf{k}'}$$

$$\phi_{\mathbf{k}} = -\frac{g_0/V}{E - 2\epsilon_{\mathbf{k}}}\sum_{0<\epsilon_{\mathbf{k}'}<\omega_D}\phi_{\mathbf{k}'} \;\cdots\cdots \qquad E = -2\omega_D e^{\frac{-2}{g_0 N(0)}}$$

Fermi sea is unstable to the formation of Cooper pairs

- Cooper pair is a bound state beneath the particle-hole continuum.
- Cooper pairs can condensate.

COOPER PAIRS (1956)

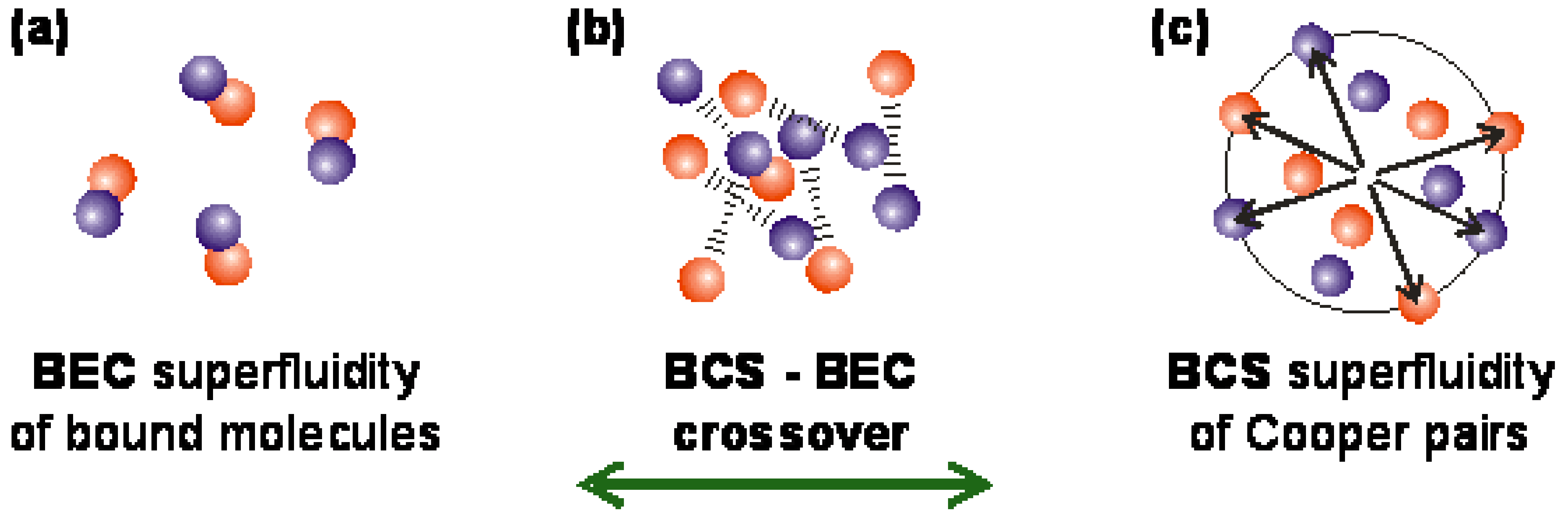
The degeneracy of the Fermi sea N(0) implies pairs form even for a very weak pairing interaction

$$E = -2\omega_D e^{\frac{-2}{g_0 N(0)}}$$

- The Cooper pair has a small smearing in k space

→ a large extension (coherence length) in real space (10^4 Å for Al).
- Pairs overlap.

http://www.qm.phy.cam.ac.uk/teaching/



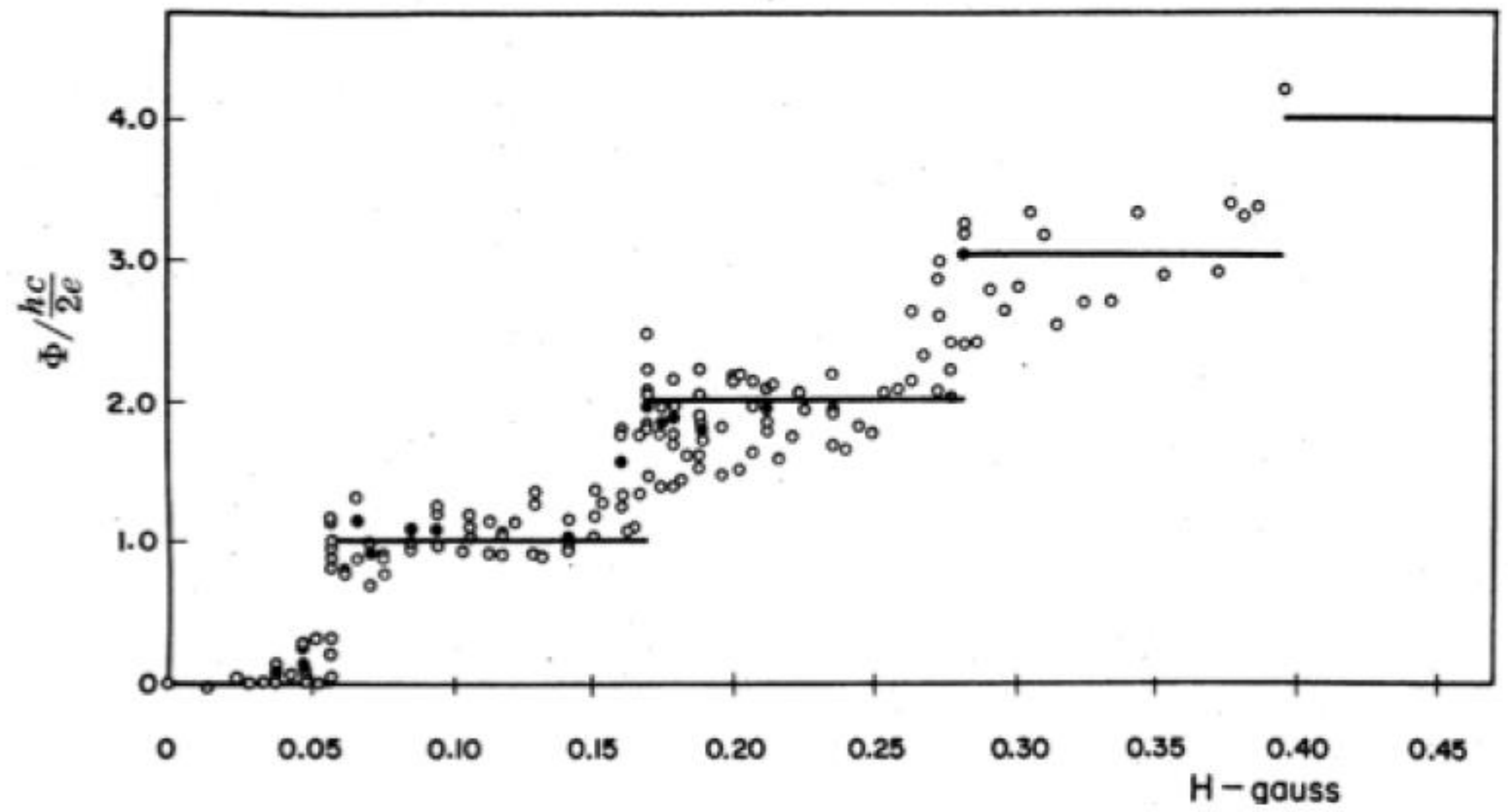
http://users.physics.harvard.edu/~greiner/fermicondensates.html

OBSERVATION OF COOPER PAIRS

The vortex magnetic flux is quantized (London, 1950)

$$\Phi_0 = \frac{h}{e^*} \equiv \frac{h}{2e}$$

Magnetic flux in a cylinder provides a proof for the charge of the Cooper pair.



Phys. Rev. Lett. 7, 43–46 (1961)

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SCHRIEFFER: BCS WAVE-FUNCTION

Coherent superposition of Cooper pair operators

$$|\Psi_{BCS}\rangle = \exp[\Lambda^\dagger]|0\rangle \qquad \Lambda^\dagger = \sum_{\mathbf{k}} \phi_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}$$

$$|\Psi_{BCS}\rangle = \prod_{\mathbf{k}} \exp[\phi_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}]|0\rangle = \prod_{\mathbf{k}} (1 + \phi_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow})|0\rangle$$

(all higher power orders are 0)

PAIRING HAMILTONIAN

Tinkham's book

Hamiltonian: Kinetic energy + pairing term

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

Mean field approximation

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$$

Mean field Hamiltonian

$$H = \sum_{k\sigma} \epsilon_k c^{\dagger}_{k\sigma} c_{k\sigma} - \sum_{k} \left(\Delta_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} + \Delta^{*}_{k} c_{-k\downarrow} c_{k\uparrow} \right)$$

PAIRING HAMILTONIAN

Mean field Hamiltonian

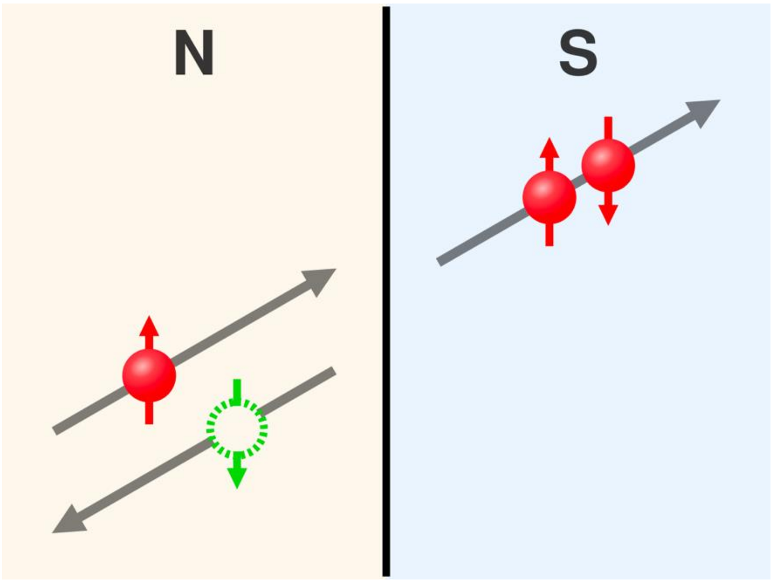
$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - \sum_k \left(\Delta_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \Delta^*_k c_{-k\downarrow} c_{k\uparrow} \right)$$

Pair creation: $e^- + e^- \rightarrow \text{pair}^{2-}$

By defining $c_{-\mathbf{k}\downarrow} = h^\dagger_{\mathbf{k}\downarrow}$ (hole creation operator)

This Hamiltonian also describes Andreev reflection:

$$e^- \rightarrow \text{pair}^{2-} + h^+$$



wikipedia

EXCITATION SPECTRUM

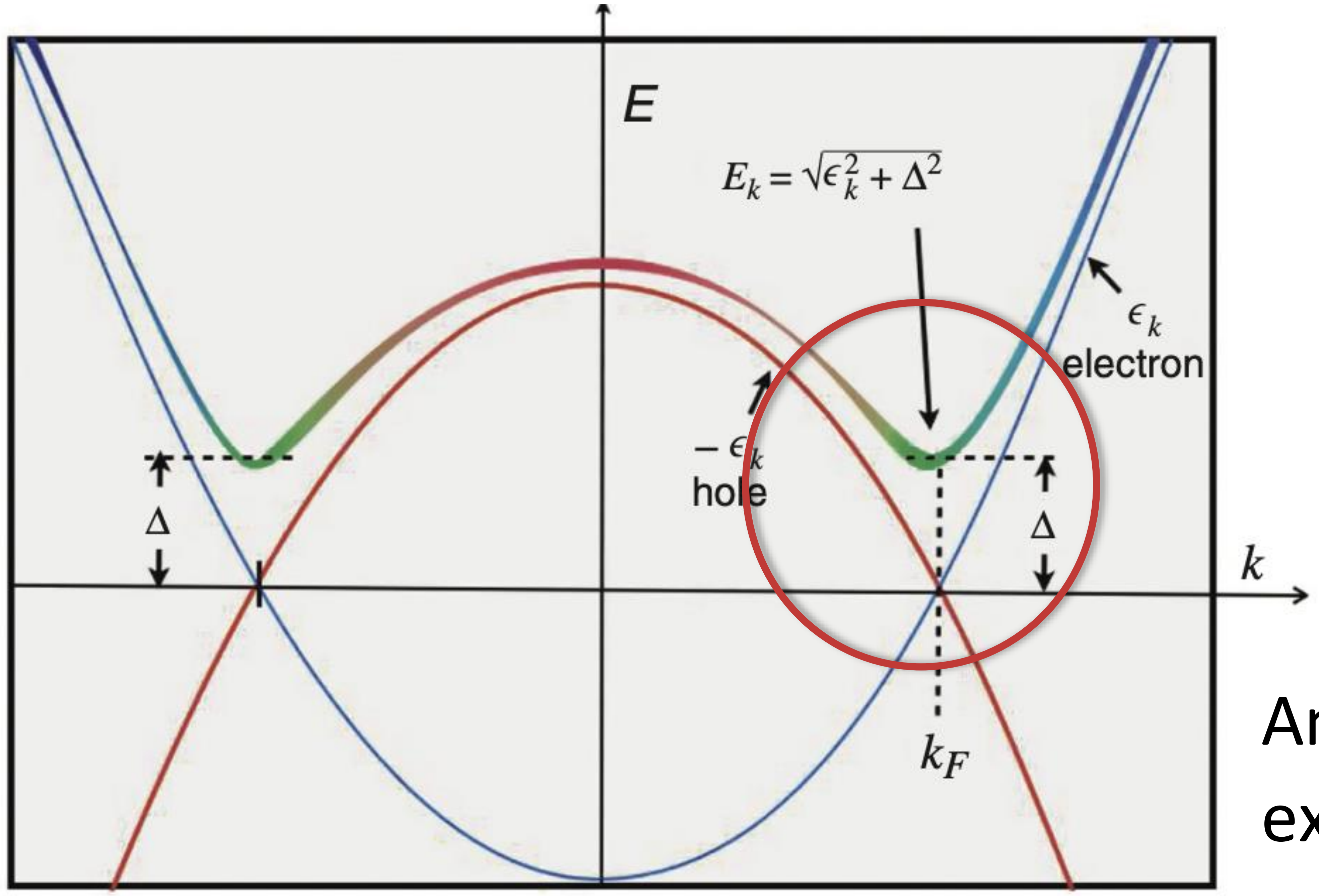
Tinkham's book, Coleman's book

Diagonalize (Bogoliubov transformation)

$$\alpha^\dagger_{k\uparrow} = c^\dagger_{k\uparrow} u_k + c_{-k\downarrow} v_k$$
$$\alpha_{-k\downarrow} = c_{-k\downarrow} u^*_k - c^\dagger_{k\uparrow} v^*_k$$

$$H = \sum_{k\sigma} E_k \alpha^\dagger_{k\sigma} \alpha_{k\sigma}$$

$$E_{\mathbf{k}} = \sqrt{\epsilon^2_{\mathbf{k}} + |\Delta_{\mathbf{k}}|^2}$$



The quasiparticles are linear combinations of electrons and holes.

Andreev scattering mixes electron and hole excitation spectra producing the gap Δ.

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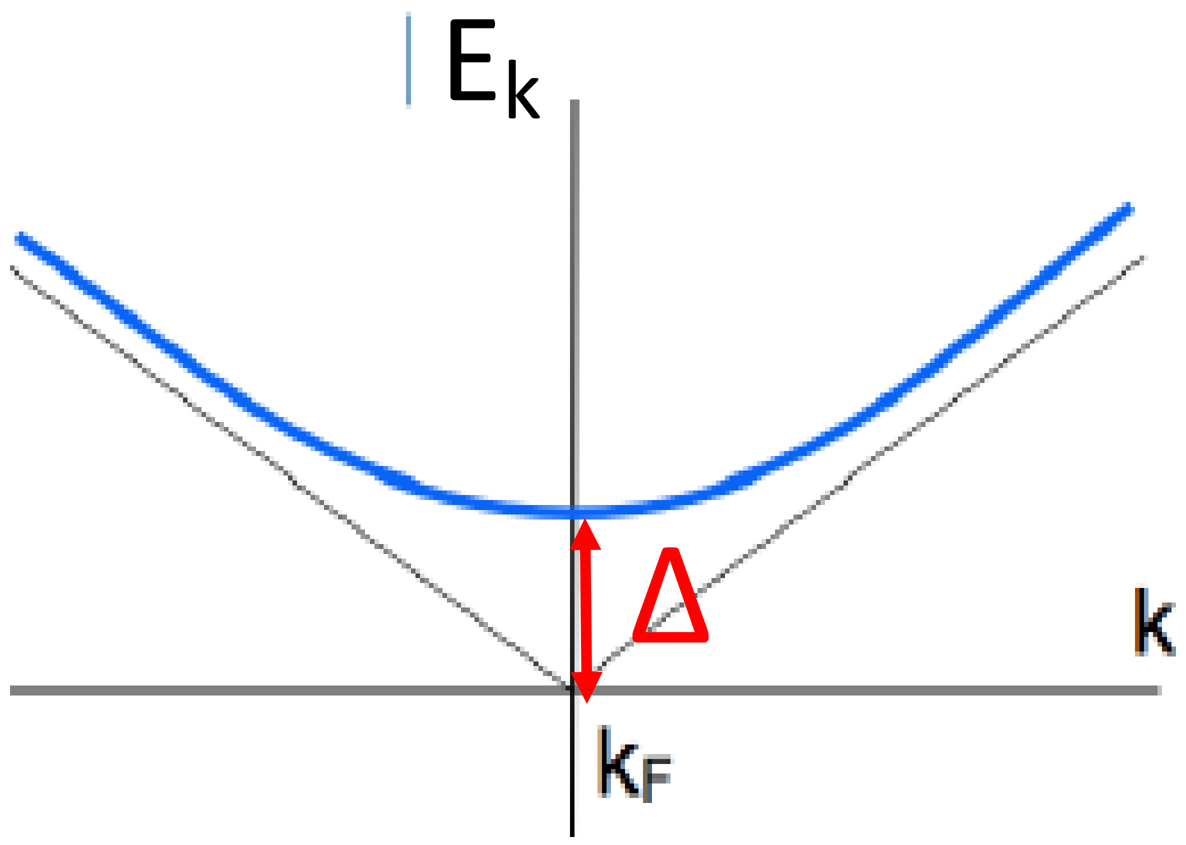
Coleman's book

Excitation energy of quasiparticles

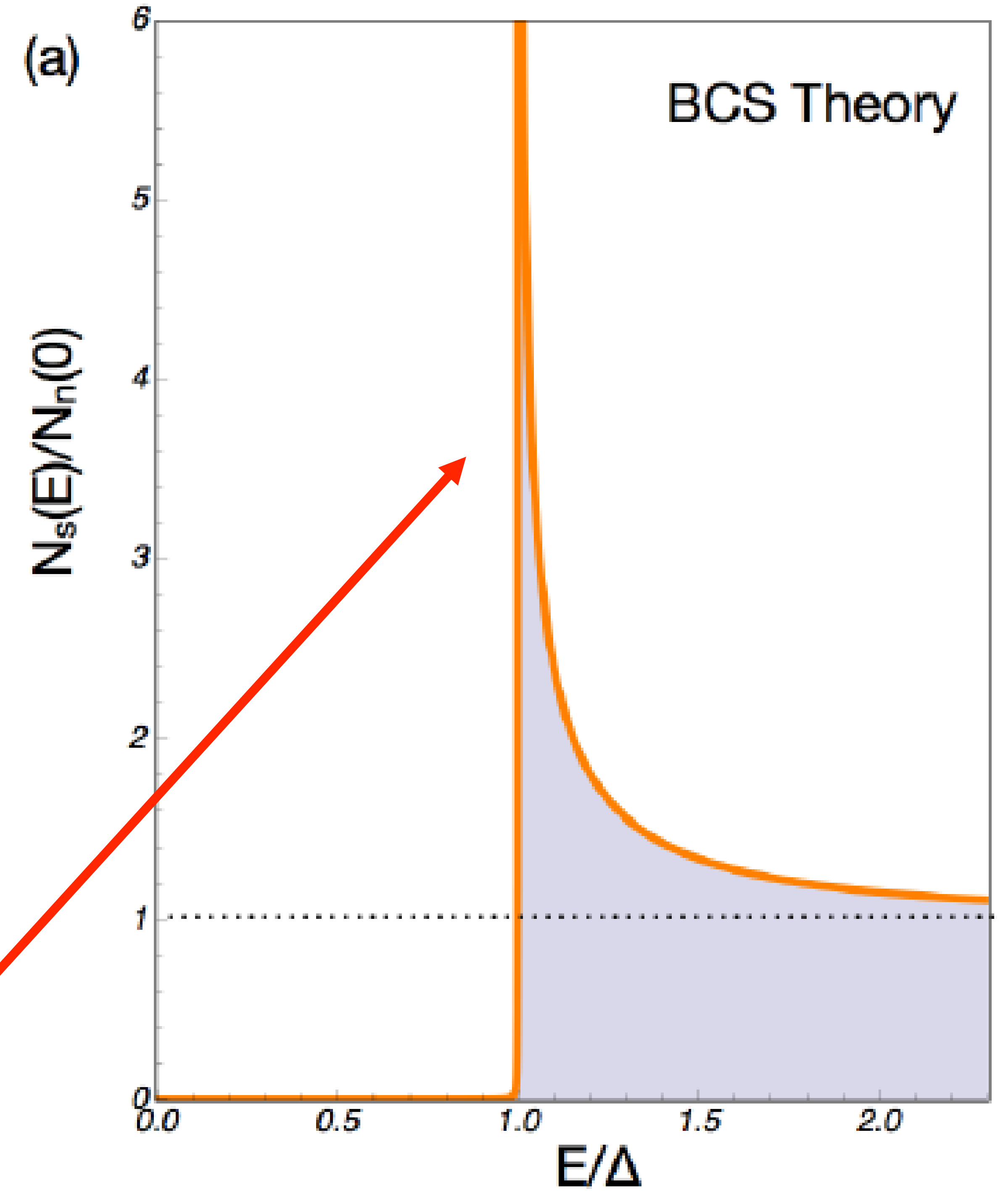
$$E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}|^2}$$



Density of Bogoliubov quasiparticles

$$N_s(E) = N_n(0)\frac{d|\epsilon_{\mathbf{k}}|}{dE_{\mathbf{k}}} =$$

$$= N_n(0)\left(\frac{E}{\sqrt{E^2 - |\Delta|^2}}\right)\theta(E - |\Delta|)$$



Coherence peak

BCS THEORY

General explicit equation for Δ (T=0)

$$\Delta_{\mathbf{k}} = -\frac{1}{2}\sum_{\mathbf{k}'}\frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}} V_{\mathbf{k},\mathbf{k}'}$$

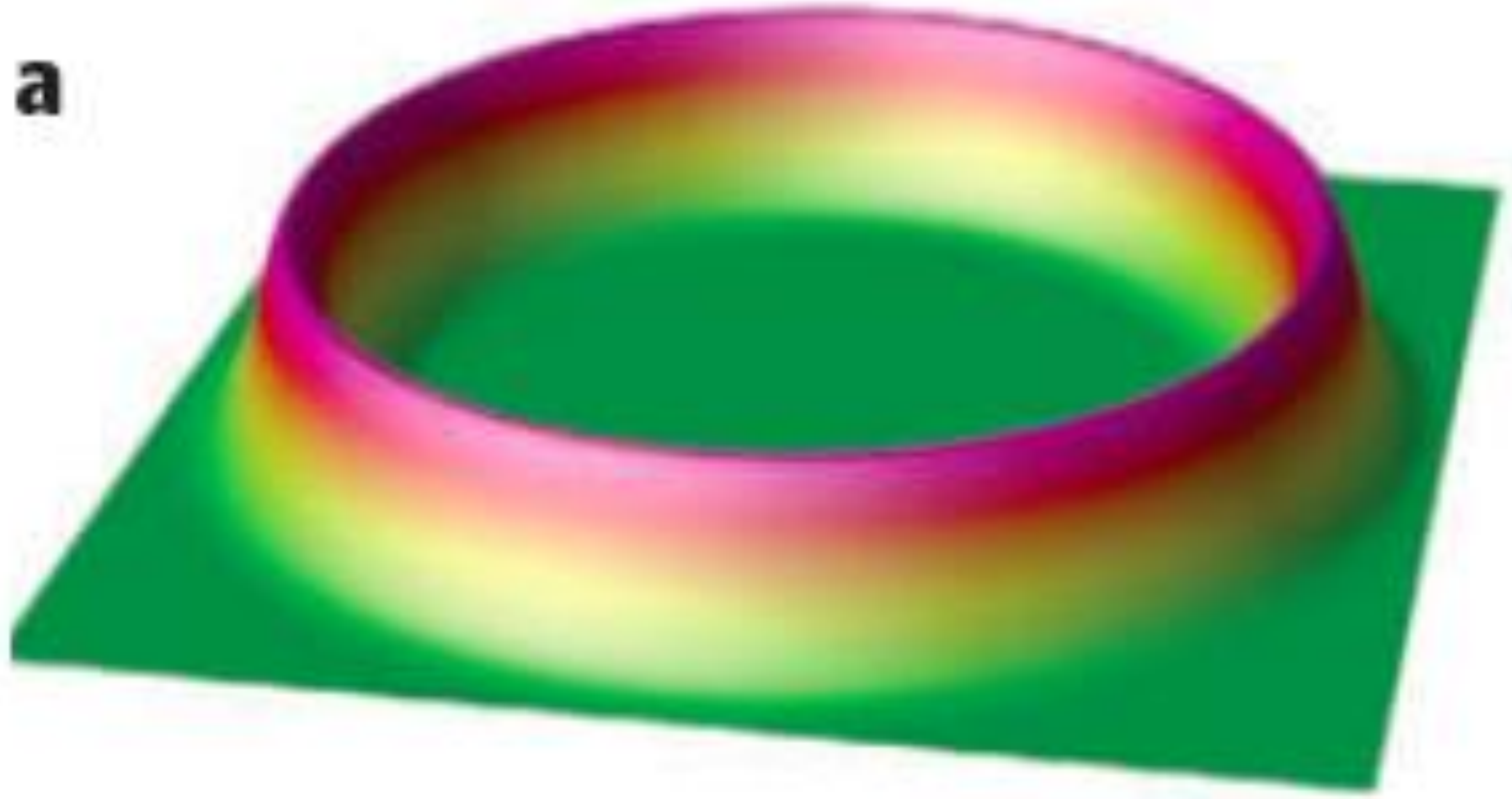
Within BCS $V_{kk'} = -V$ within $\hbar\omega_D$ of E_F

Then $\Delta_{kk'} = \Delta$ (isotropic)

$$\Delta \approx 2\hbar\omega_D e^{-1/N(0)V}$$

Isotropic pairing function: s-wave

a



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BCS THEORY

Tinkham's book

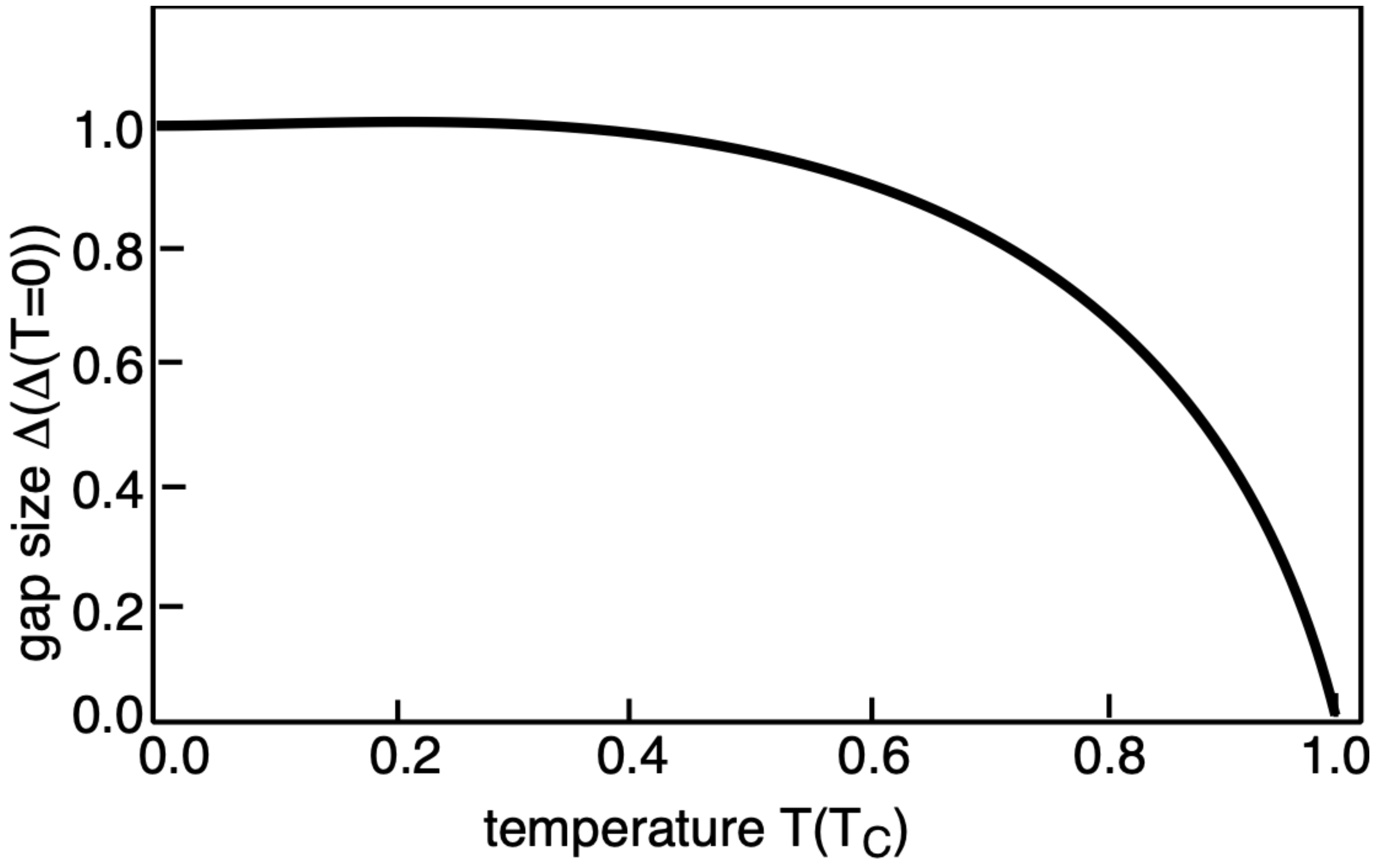
For finite T

$$\Delta_{\mathbf{k}} = -\frac{1}{2}\sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}} \tanh\left(\frac{\beta}{2}\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}\right)$$

For weak coupling superconductors

$$\frac{\Delta(T)}{\Delta(0)} \sim 1.76\left(1 - \frac{T}{T_c}\right)^{1/2}$$

Universal Δ(T) curve
in the weak coupling limit



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MIGDAL-ELIASHBERG
(INCLUSION OF RETARDATION EFFECTS)

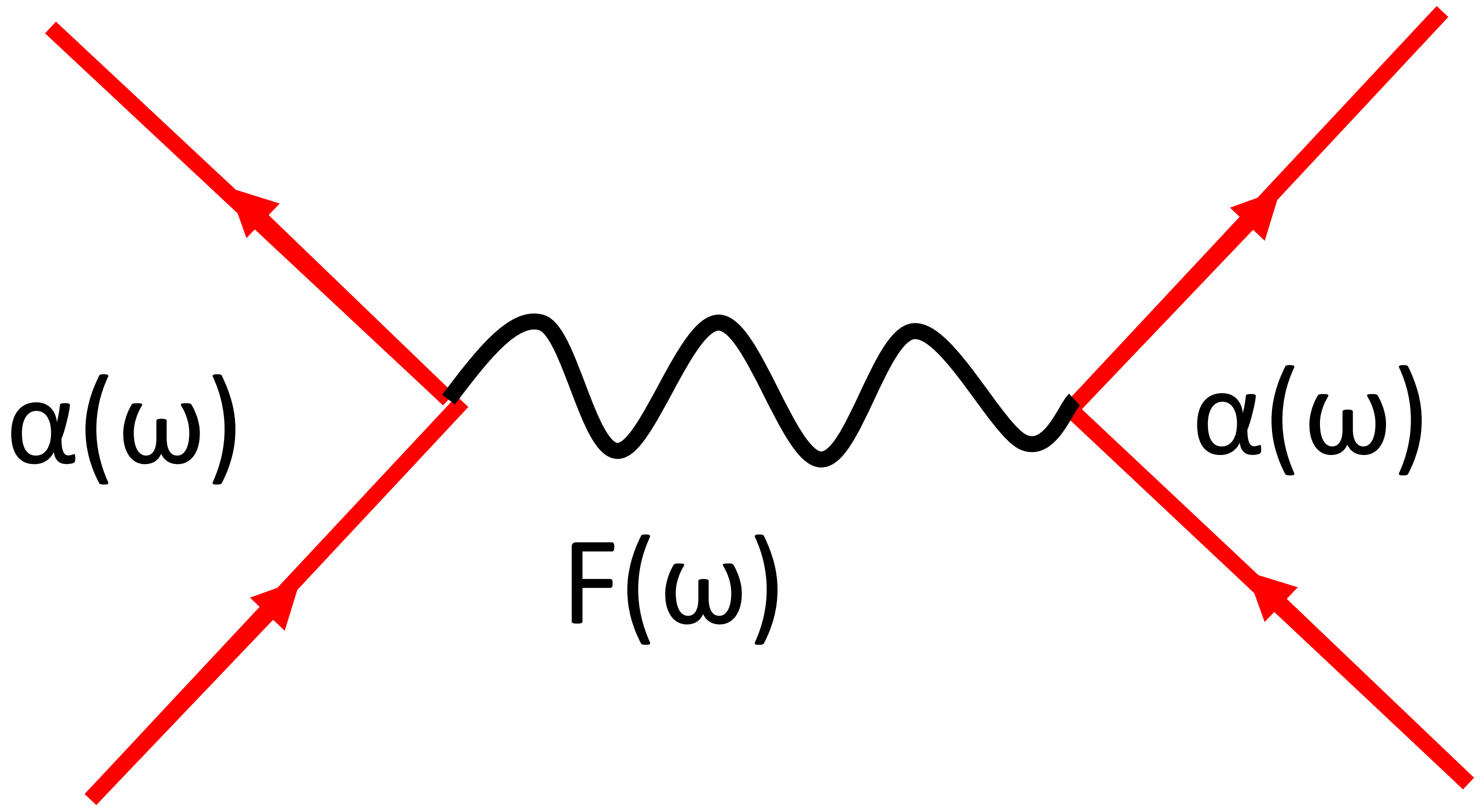
BCS theory included retardation effects in a minimal way by introducing an energy cut-off: the Debye frequency.

Eliashberg treated the electron-phonon interaction including correctly the retardation effects.

$$T_c = F(\alpha^2 F(\omega), \mu^*)$$

Electron-phonon spectral function

Coulomb pseudopotential



Migdal (1957): vertex corrections $O(m/M)^{1/2}$ can be dropped (equivalent to adiabaticity condition)

MIGDAL-ELIASHBERG

(INCLUSION OF RETARDATION EFFECTS)

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Eliashberg treated the electron-phonon interaction including correctly the retardation effects.

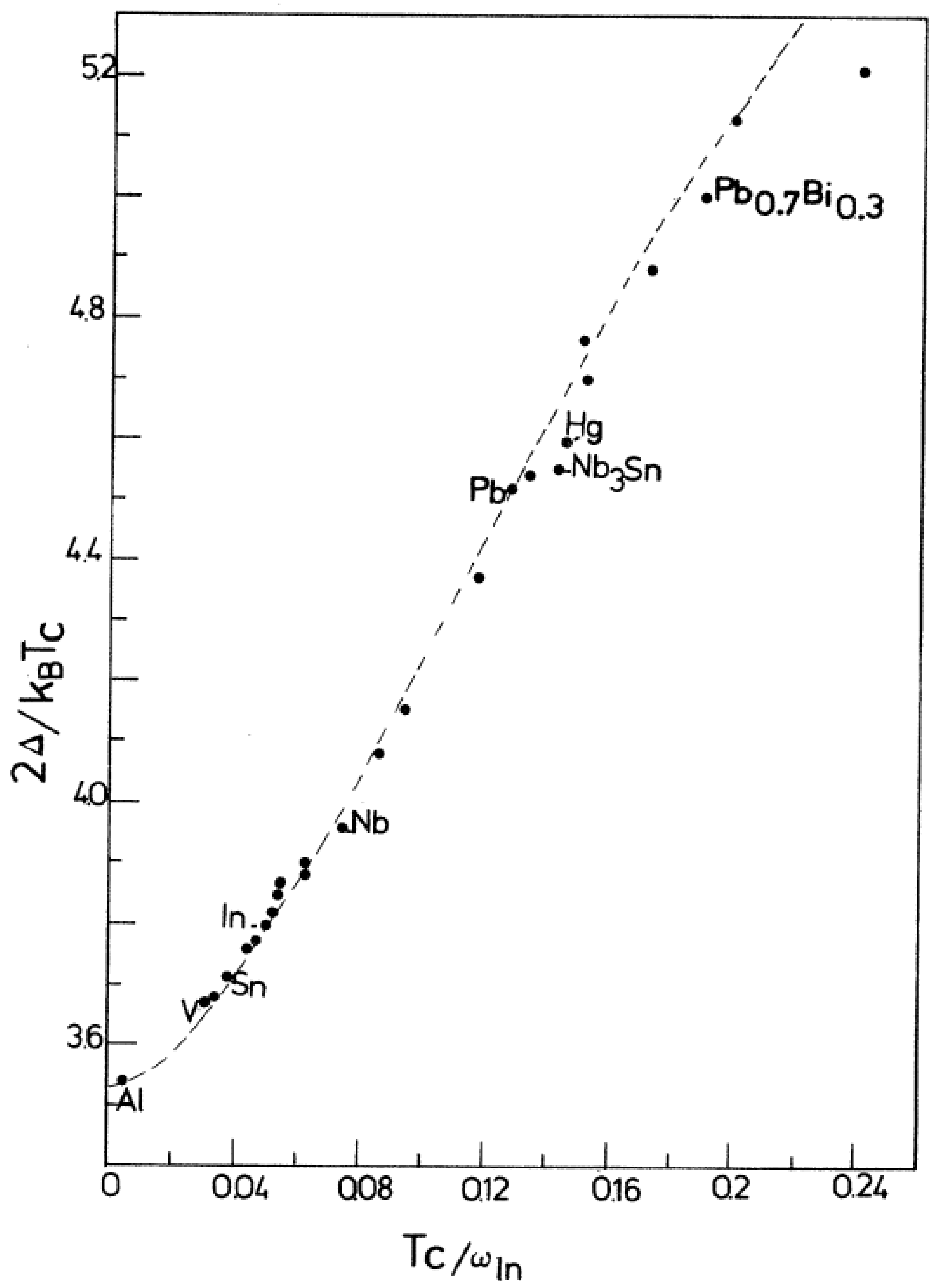
$$T_c = F(\alpha^2 F(\omega), \mu^*) \qquad \Delta = G(\alpha^2 F(\omega), \mu^*)$$

$$\frac{2\Delta}{k_B T_c} = 3.53 \left[1 + 12.5 \left(\frac{T_c}{\omega_{ln}} \right)^2 \log \left(\frac{\omega_{ln}}{2T_c} \right) \right]$$

Coupling strength

PRB 29, 184 (1984)

MIGDAL-ELIASHBERG



PRB 29, 184 (1984)

$$\frac{2\Delta(0)}{k_BT_c} = 3.53$$

MAX T_C FOR PHONON MEDIATED SUPERCONDUCTIVITY

McMillan PR 167, 331

$$T_c \propto \langle \omega_{ph} \rangle \exp[-1/(\lambda - \mu^*)]$$

Input: phonon spectrum

With the electron phonon coupling constant

$$\lambda = 2\int_0^\infty \frac{\alpha^2 F(\omega)}{\omega}$$

TABLE VIII. The predicted maximum superconducting transition temperature for four classes of materials found from the observed T_c and λ and Fig. 11, together with the observed T_c.

Pure metals

Alloys

Metal	T_c (°K)	λ	T_c^{max} (°K)	Observed maximum T_c	Material
Pb	7.2	1.3	9.2	8.8	Pb–Bi
Nb	9.2	0.82	22	10.8	Zr–Nb
V_3Si	17	0.82	40		
Nb_3Sn			28	20	Nb_3Al–Nb_3Ge

Note:

MgB_2 Tc=40K

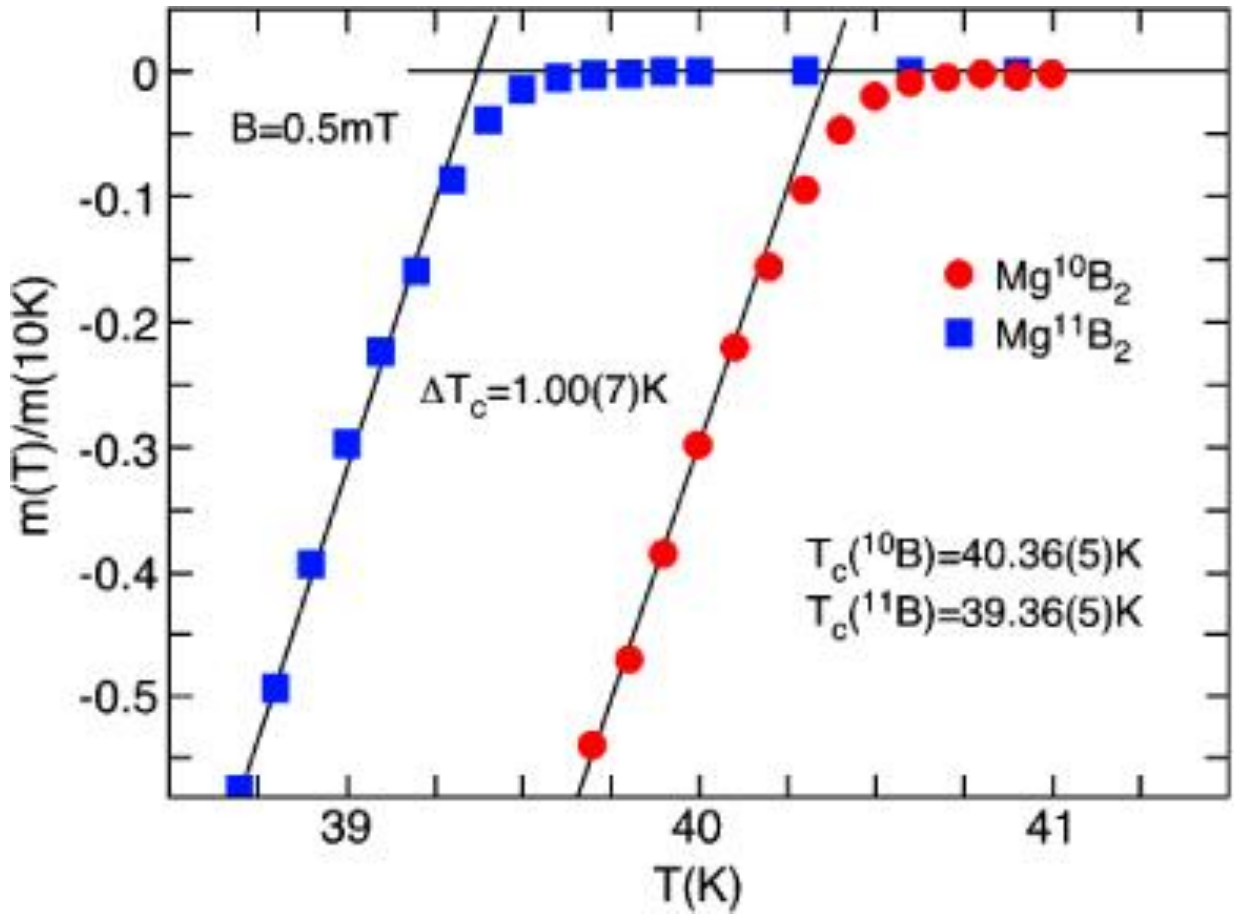
Canfield and Crabtree, Physics Today 2003.

MgB_2 (2001)

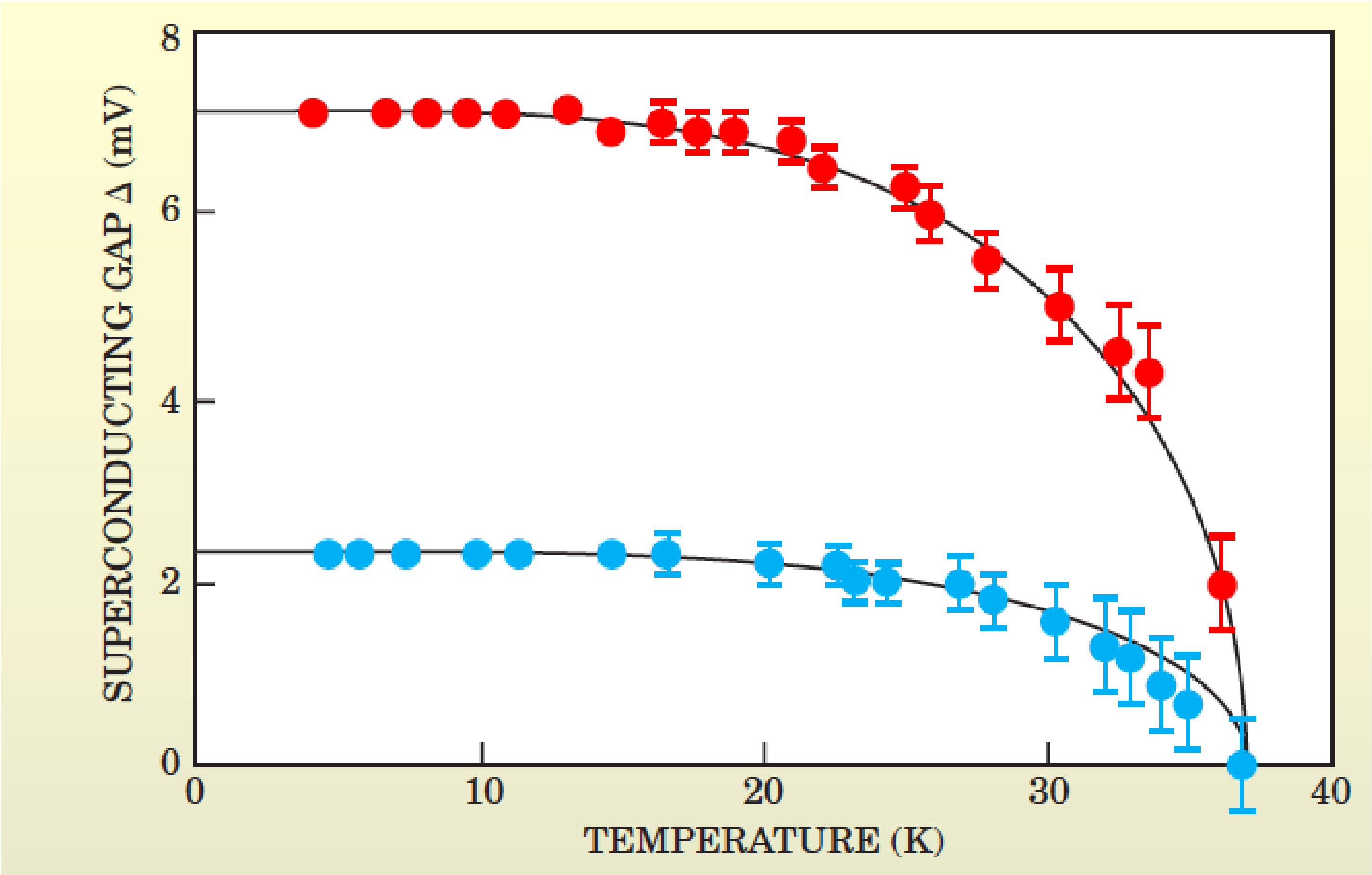
T_c=39 K. $\boxed{\text{MULTIBAND SUPERCONDUCTOR}}$

Conventional SPC.
2 differentiated bands.
One very strong phonon (in the B plane) coupled to the conduction electrons.

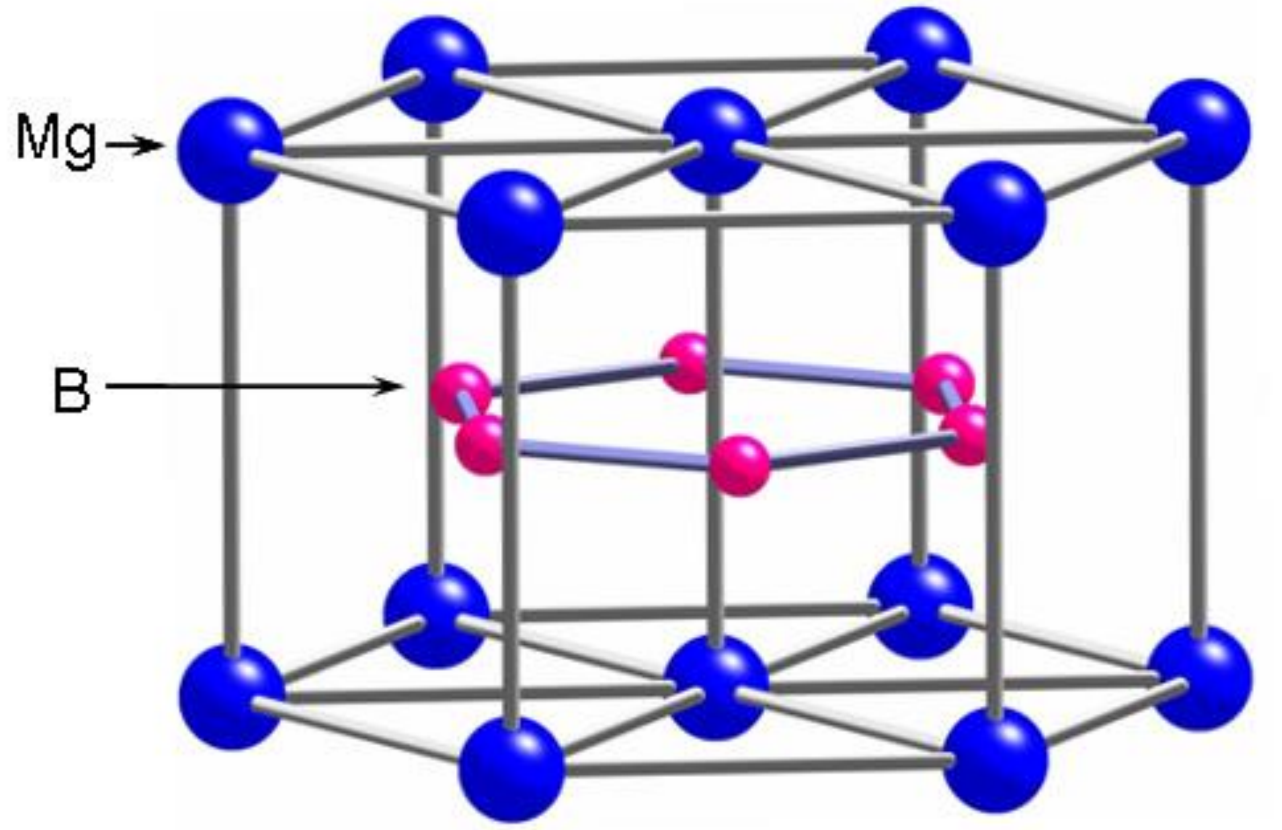
Isotope effect



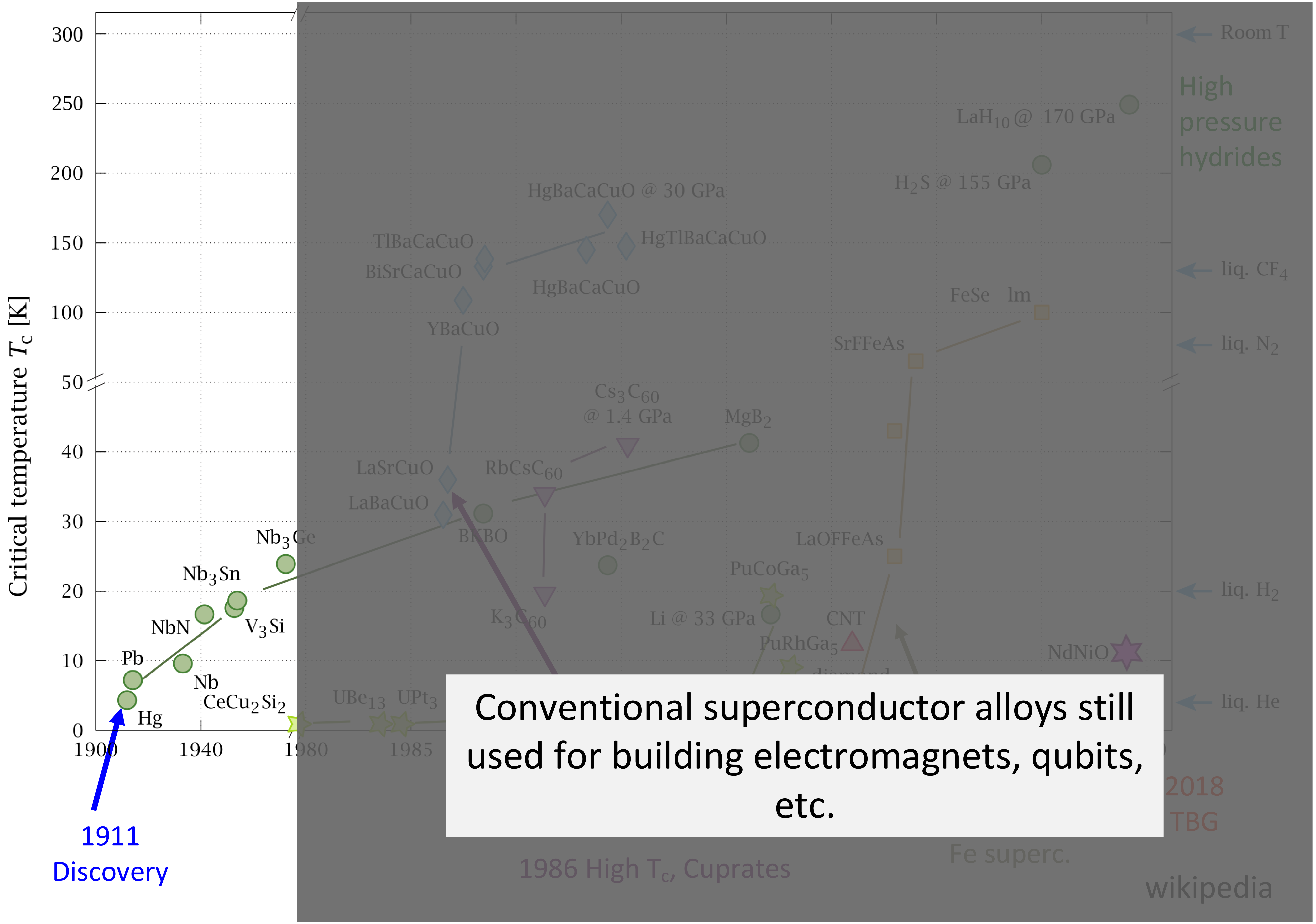
JPCM 24 233201



2 energy gaps with different electron-phonon strengths



DOI: 10.5772/17117

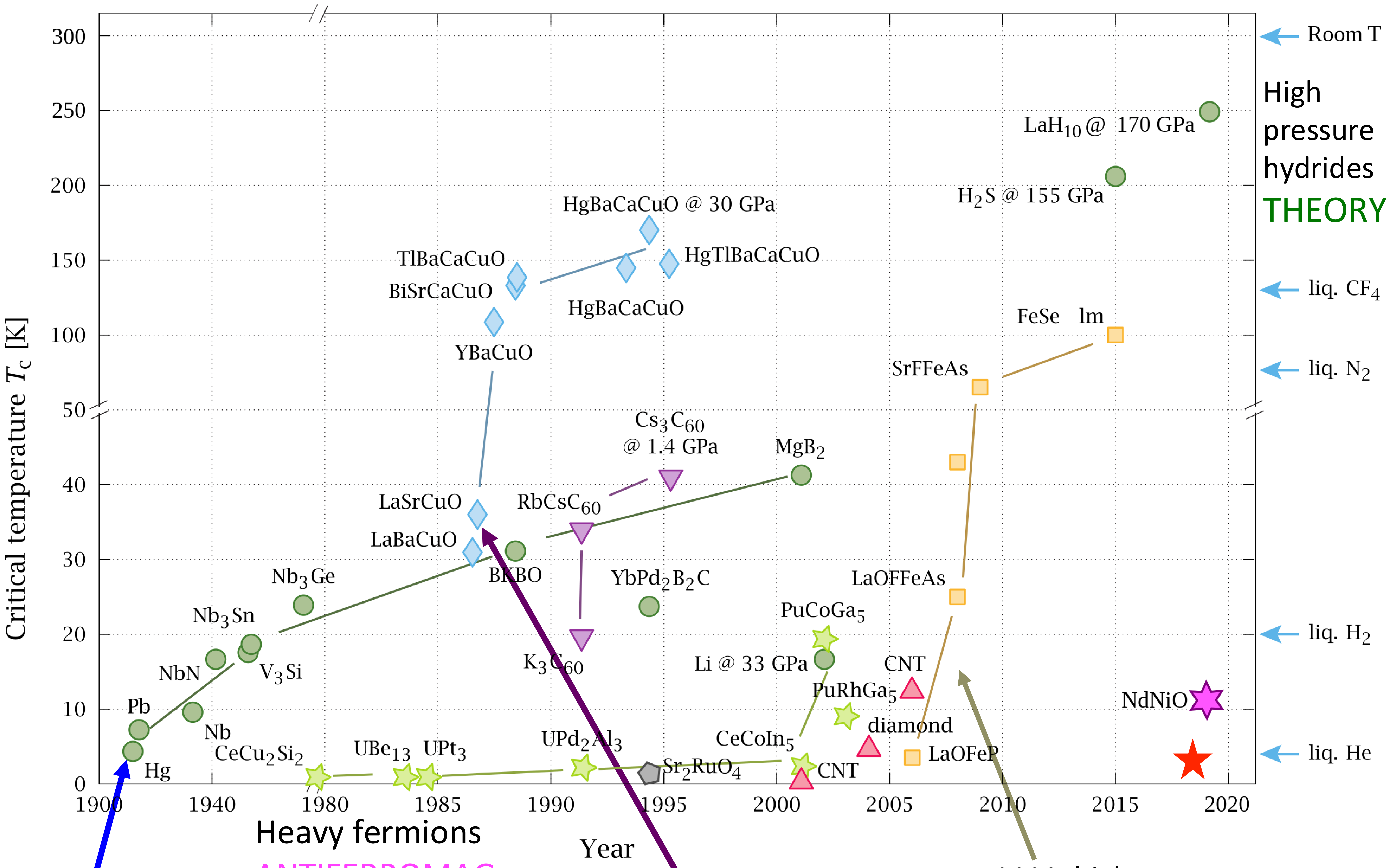


Conventional superconductor alloys still used for building electromagnets, qubits, etc.

Mazin, Nature 464, 183 (2010)

MATTHIAS'S RULES (1950's)

- High symmetry (cubic best) is good
- High density of electronic states is good.
- Stay away from oxygen
- Stay away from magnetism (PM impurities destroy superconductivity)
- Stay away from insulators
- Stay away from theorists 🤔



High pressure hydrides
THEORY

Heavy fermions
ANTIFERROMAG.

1911
Discovery

1986 High T_c, Cuprates
OXIDE, ANTIFERROMAGNET
PARENT COMPOUND: INSULATOR

2008, high T_c
Fe superc
ANTIFERRO

wikipedia

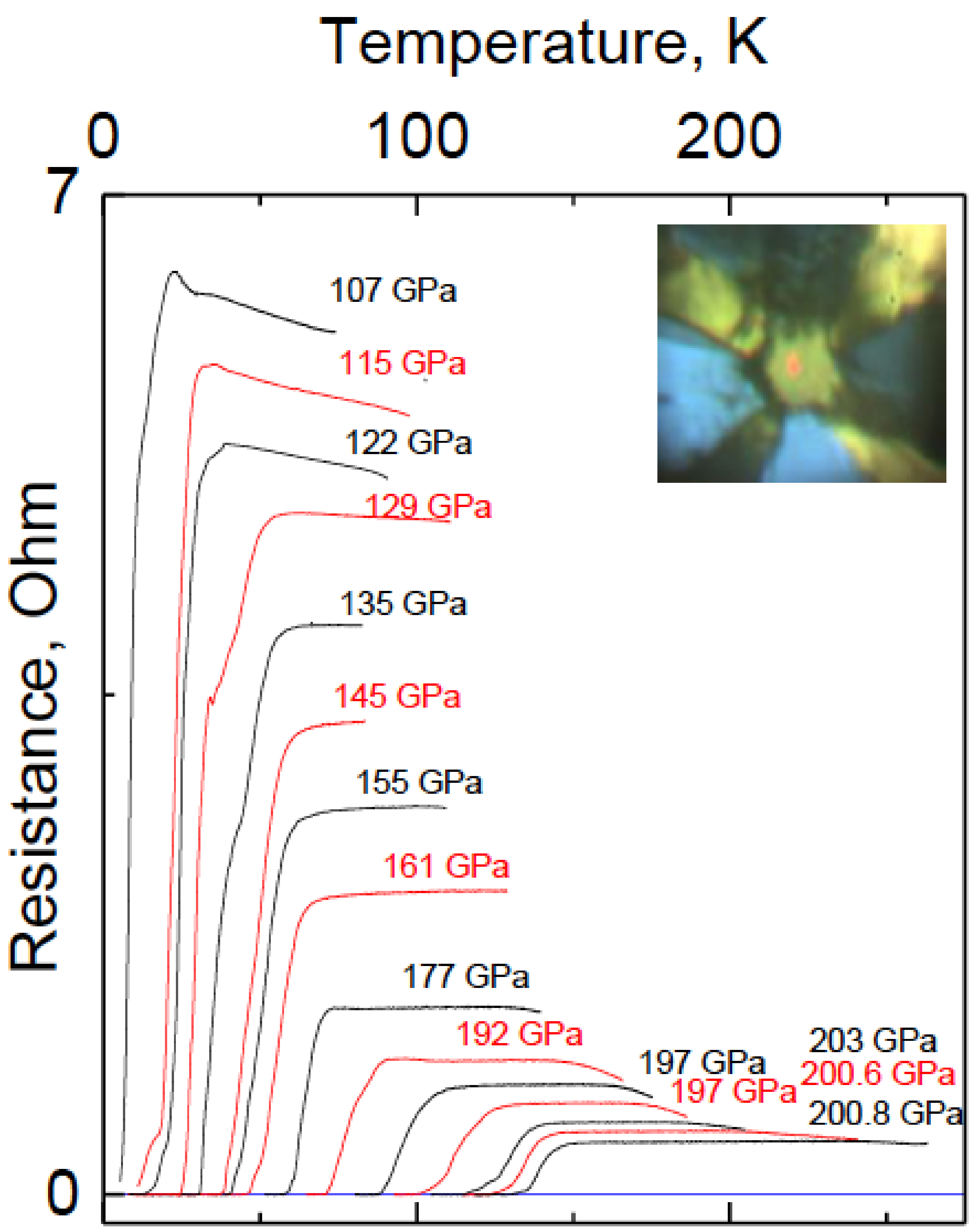
RECORD T_C FOR ELECTRON-PHONON MEDIATED SUPERCONDUCTIVITY

H_2S T_c=203 K (at 200 GPa); Nature 525, 73 (2015)
LaH_{10} T_C=250 K at 170 GPa; Nature 569, 528 (2019)

H_2S leads to H_3S under high pressures.

Hydrogen atoms are light → high phonon frequencies expected. Large Tc predicted
Ashcroft PRL 21, 1748 (1968); 21, 1748 (2004)

Role of anharmonic phonons.
Errea et al PRL 114, 157004



Very controversial topic at the moment.

Recent measurement of Meissner effect under high pressure in CeH_9: Nature 627, 73 (2024)

OUTLINE

- **Superconductivity**
  - Properties (zero resistivity, Meissner effect)
  - Understanding (pairing, BCS, Ginzburg-Landau)
  - Electron-phonon interaction (conventional superconductivity)
- **Unconventional superconductivity (unsolved)**
  - What are the distinct issues.
  - What are (some of) the proposals.

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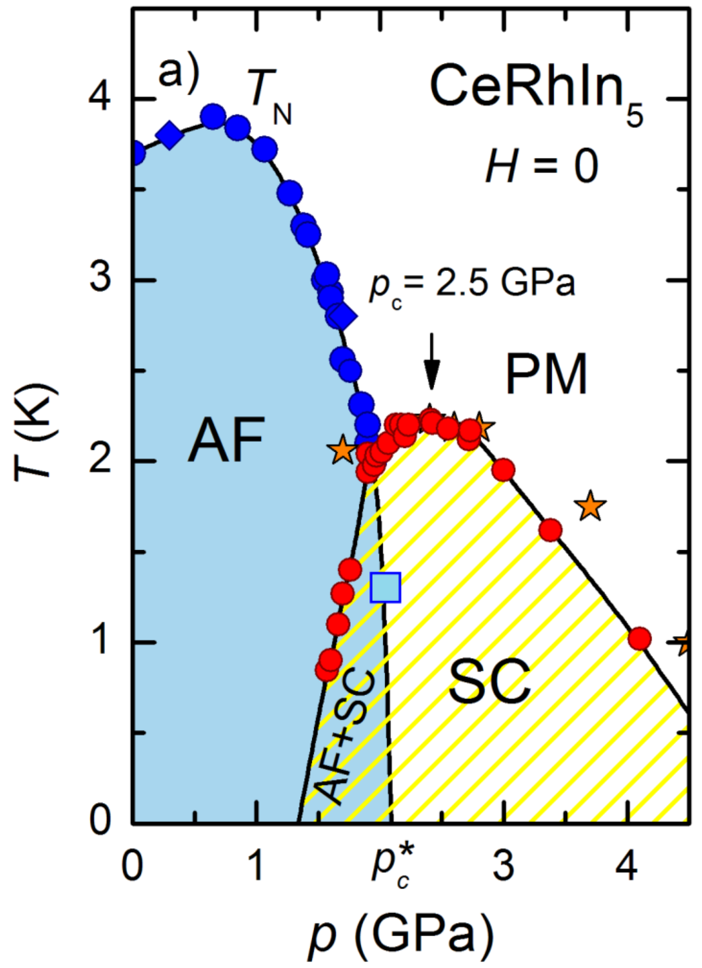
MATTHIAS'S RULES ?

Theory predicted superconductivity in hydrogen compounds (Ashcroft) and semiconductors ($SrTiO_3$, Cohen).

AF supercond.

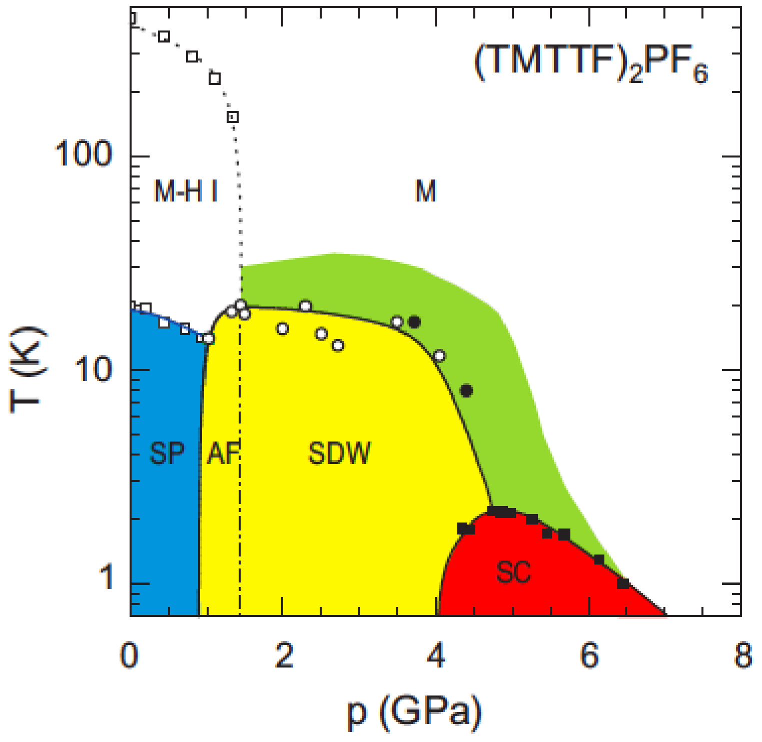
Heavy fermions



AF+SPC
Coexistence

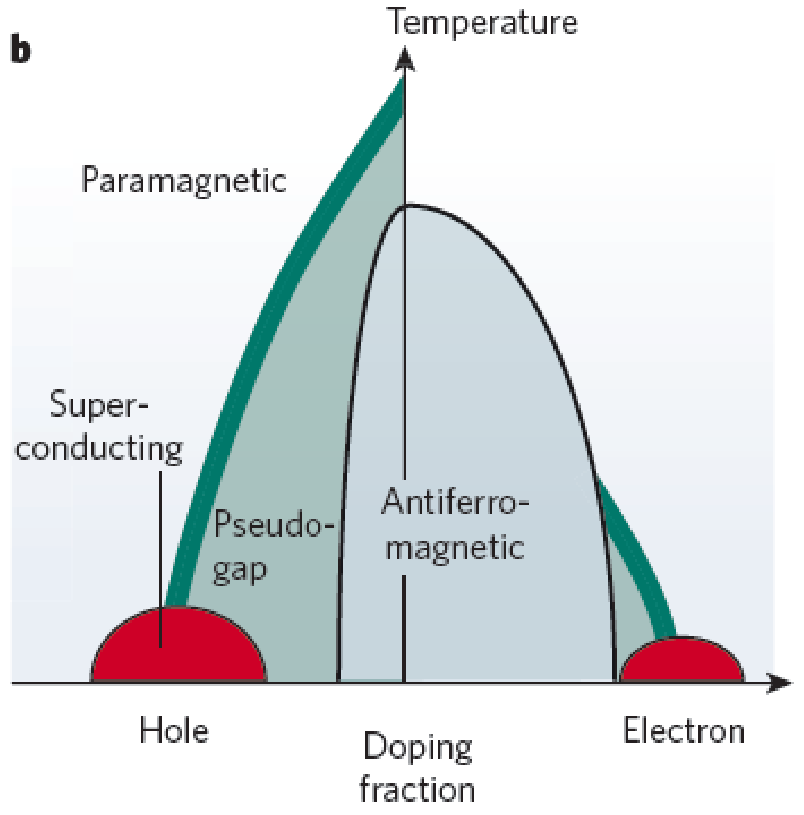
Oxides
Insulators
Magnetism

Organics

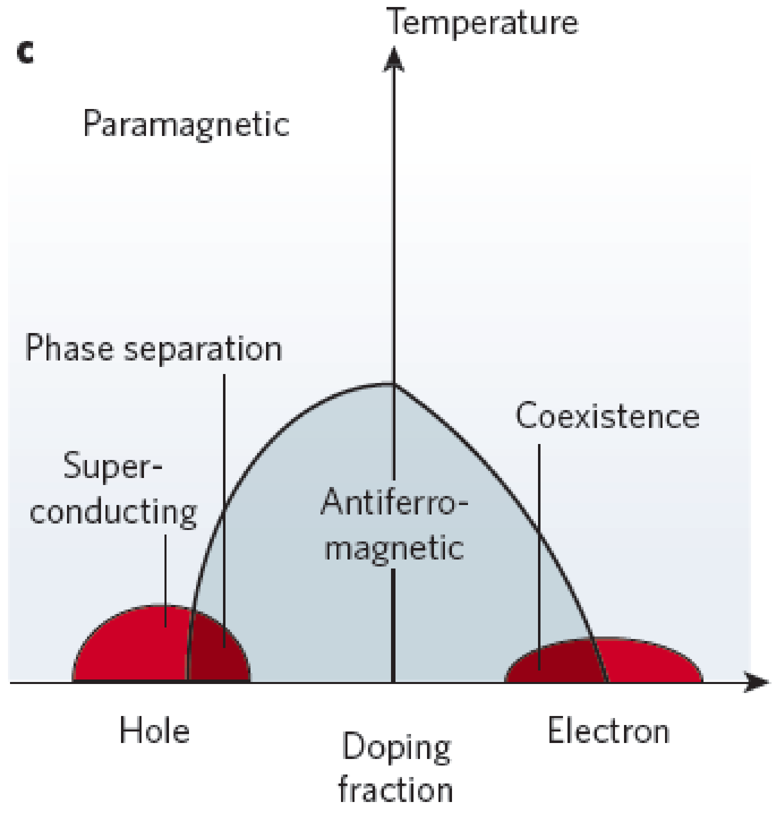


Eur. Phys. JB 21, 175

Cuprates (high Tc)

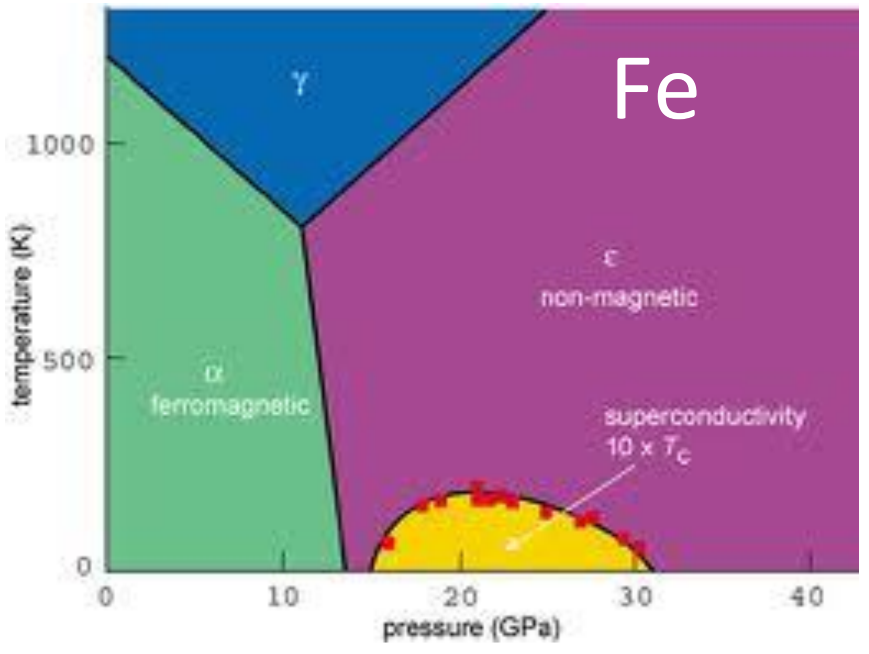


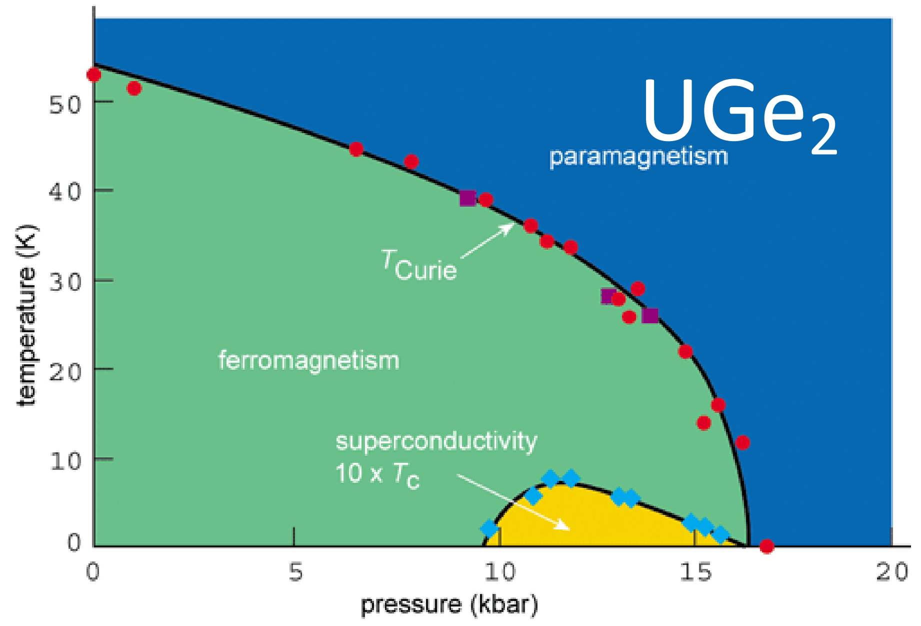
Fe-superconductors



Nature 464, 183 (2010)

FM superconductors





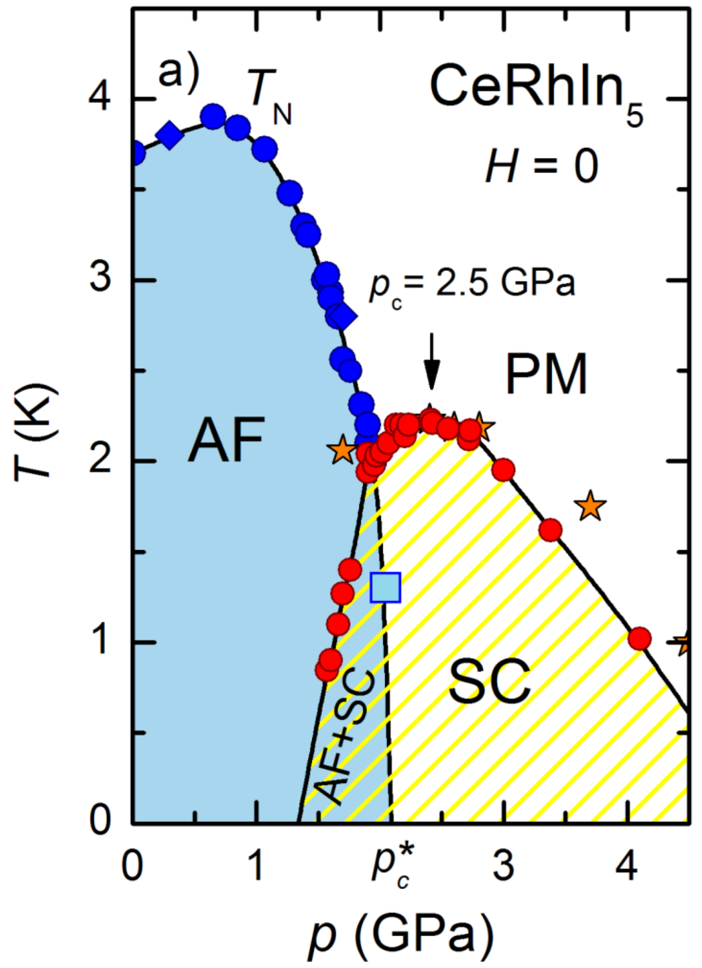
Physics World, Jan 2002

UNCONVENTIONAL SUPERCONDUCTORS

Not driven by conventional phonons (Tc too large for phonons).

AF supercond.

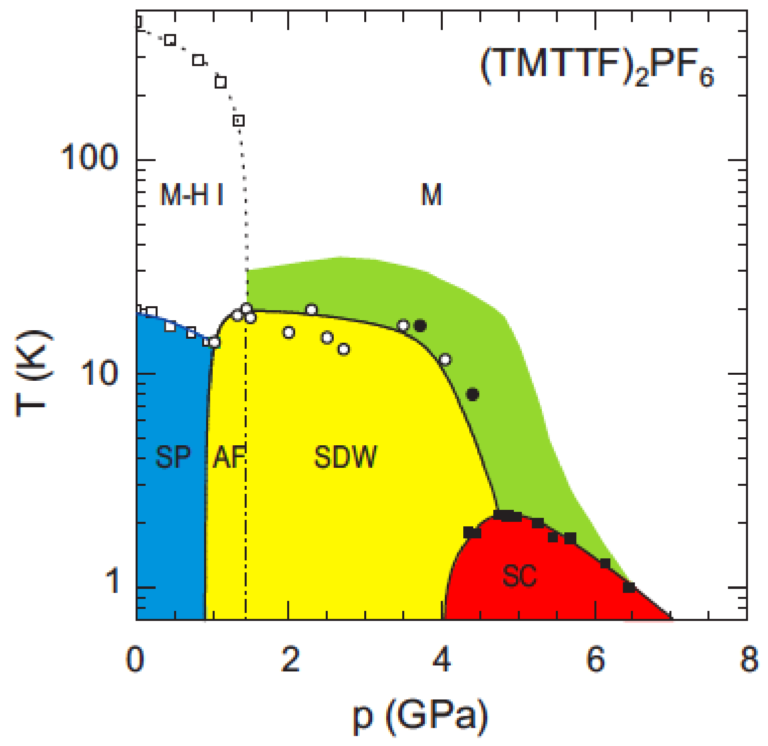
Heavy fermions



AF+SPC
Coexistence

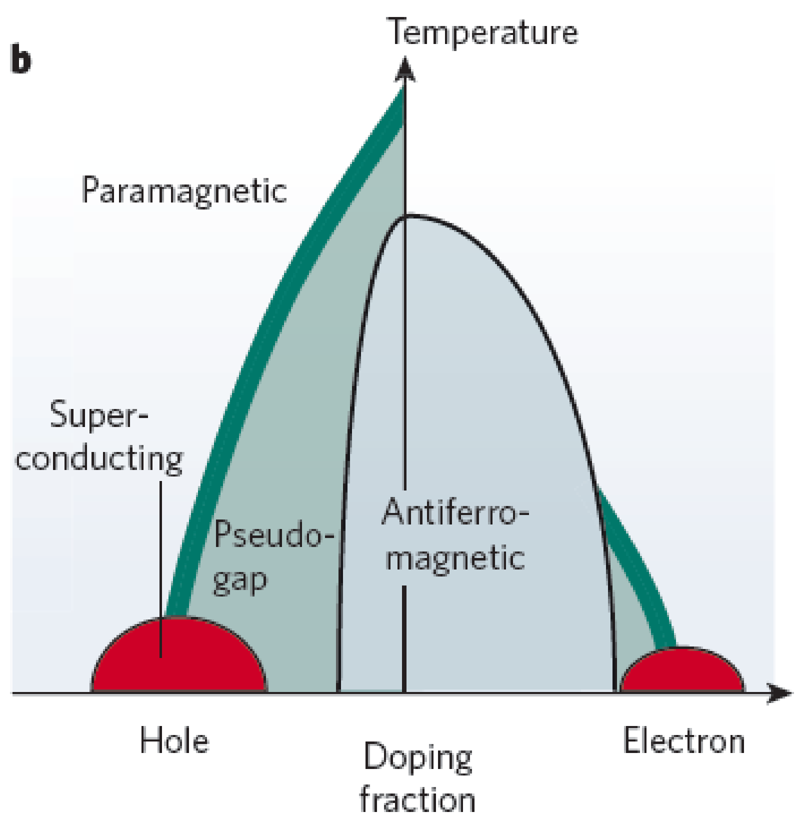
Oxides
Insulators
Magnetism

Organics

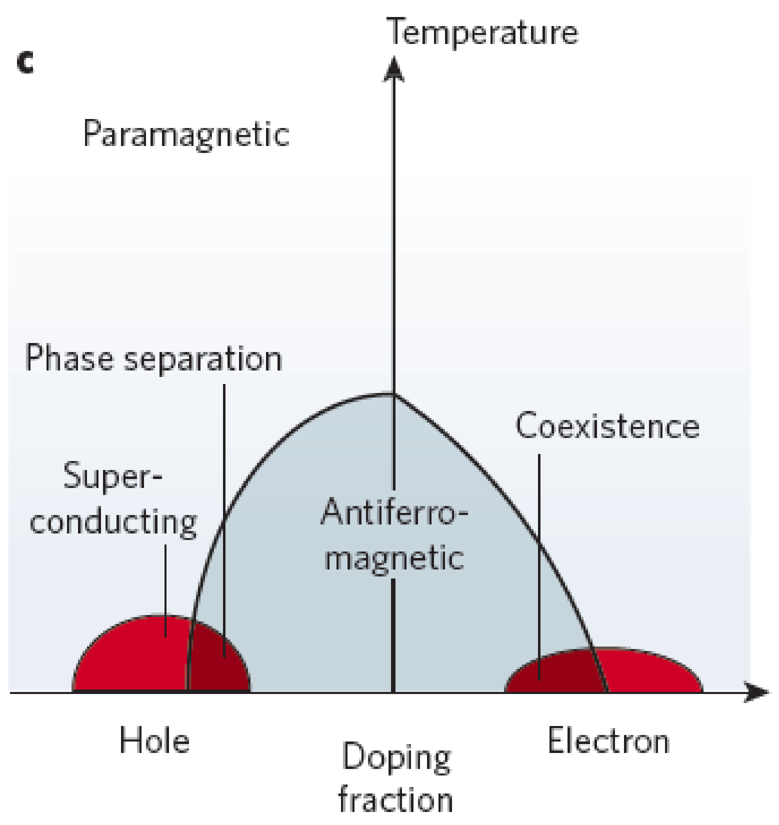


Eur. Phys. JB 21, 175

Cuprates
(high Tc)

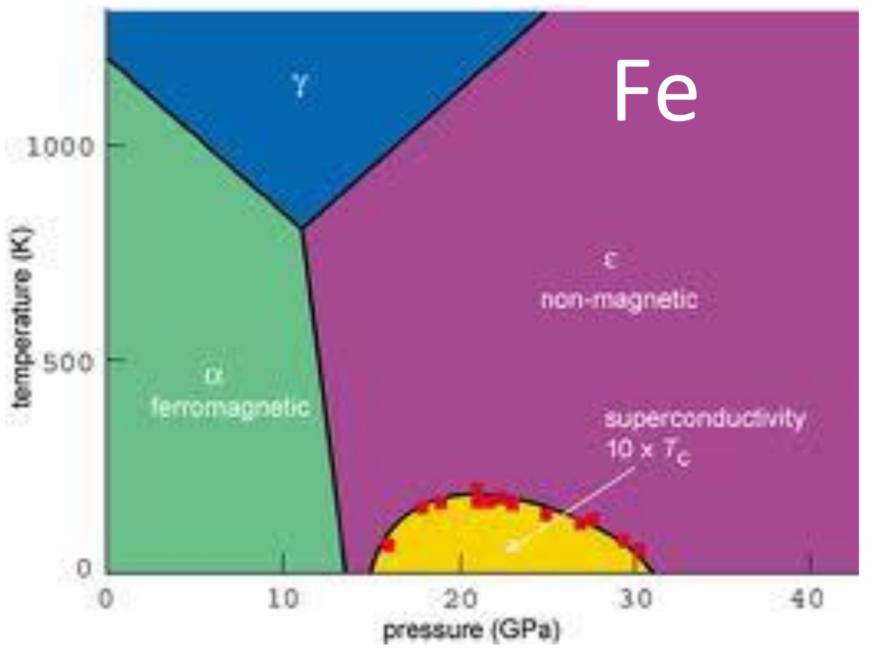


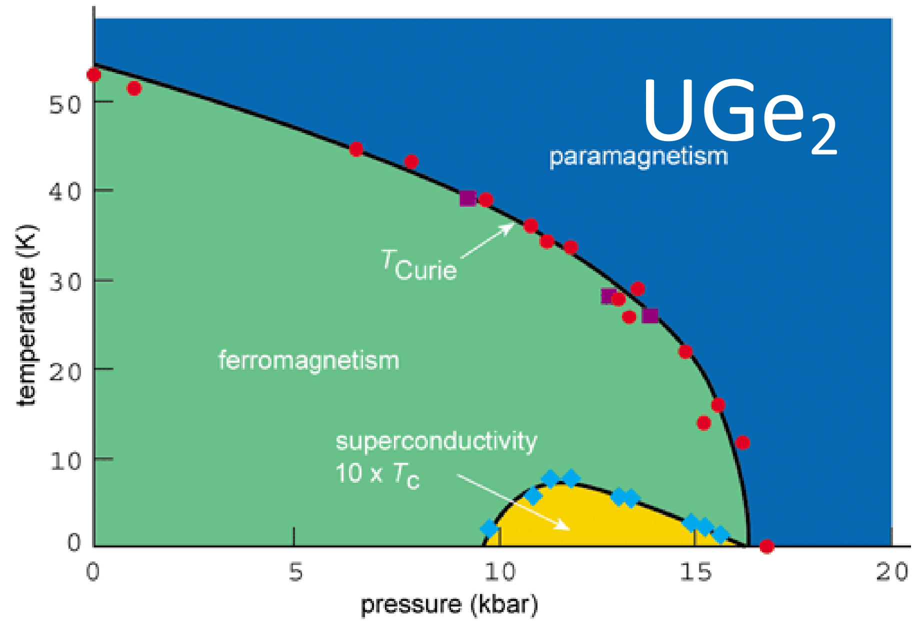
Fe-superconductors



Nature 464, 183 (2010)

FM superconductors





Physics World, Jan 2002

UNCONVENTIONAL SUPERCONDUCTORS

<u>The "normal" state is more complicated</u>

- Proximity or coexistence with magnetism
- Strong correlations.
  - Competing orders (stripes).
  - Is there a Fermi surface? Doped Mott insulator. Non-Fermi liquid behaviour. Pseudogap phases.
- Low dimensionality, anisotropies, nematicity.

UNCONVENTIONAL SUPERCONDUCTORS

<u>The superconducting state is different</u>

The pairing function Δ_k may

- Be non-isotropic (including nodes, sign changes)
- Have a finite orbital momentum (p- or d-wave symmetry)
- Be spin-triplet (p-wave symmetry)

- $\lambda >> \xi$ (type II) (conventional SPC can be type I or type II)
- Anisotropies

$$\frac{2\Delta}{k_B T_c} >> 3.53$$

(Too) many theories.
Two distinct approaches to the problem:

1. Stay within BCS (need a Fermi surface!) but a new pairing (glue) mechanism is needed (maybe spin fluctuations, orbital fluctuations…).

2. Start from the Mott state (no boson exchange required) and see how to gain energy from pairing
   - Resonating valence bonds
   - Kinetic energy driven
   - Quantum criticality…

Many theories.
Two distinct approaches to the problem:

1. Stay within BCS (need a Fermi surface!) but a new pairing (glue) mechanism is needed (maybe spin fluctuations, orbital fluctuations…).

   A: We know how to deal with it.
   D: Usually there is no Fermi surface!

Many theories.
Two distinct approaches to the problem:

2. Start from the Mott (correlated) state

- Resonating valence bonds
- Kinetic energy driven
- Quantum criticality…

A: It seems, in principle, more self-consistent.
D: We need to properly treat the Mott state first!!

Assume BCS is valid for non-conventional superconductors.

We need some attractive interaction (but no phonons!)
+Very strong electron-electron repulsive interaction.
Is there a way around it??

PAIRING SYMMETRY

$$\psi(\vec{r}_1 s_1, \vec{r}_2 s_2) = \varphi(\vec{r}_1, \vec{r}_2)\chi(s_1, s_2)$$

Spatial Spin

The pair wavefunction must be antisymmetric (Pauli exclusion principle)

Spin singlet →even parity orbital wave function s, d
Spin triplet → odd parity orbital wave-function p, f

Singlet SPC

s-wave: conventional spc
d-wave: cuprates
s-wave/d-wave: Fe scp

p-wave (Triplet SPC):

- Superfluidity in ^3He
- UPt_3, UTe_2
- Indications in some multilayered graphene systems.
- Proximity effects: s-wave SPC in proximity with a high SO semiconductor

Rev. Mod. Phys. 69, 645

SUPERFLUIDITY IN ^{3}HE (1972)

T_c=2.7 mK

Pairing cannot be mediated by the lattice. Nuclear forces are strongly repulsive in the core → no s-wave possible. Need of wavefunctions that vanish at r→0.

High angular momentum pairing (p-wave) was proposed as a way to overcome the short range repulsion (Pitaevskii 1959).

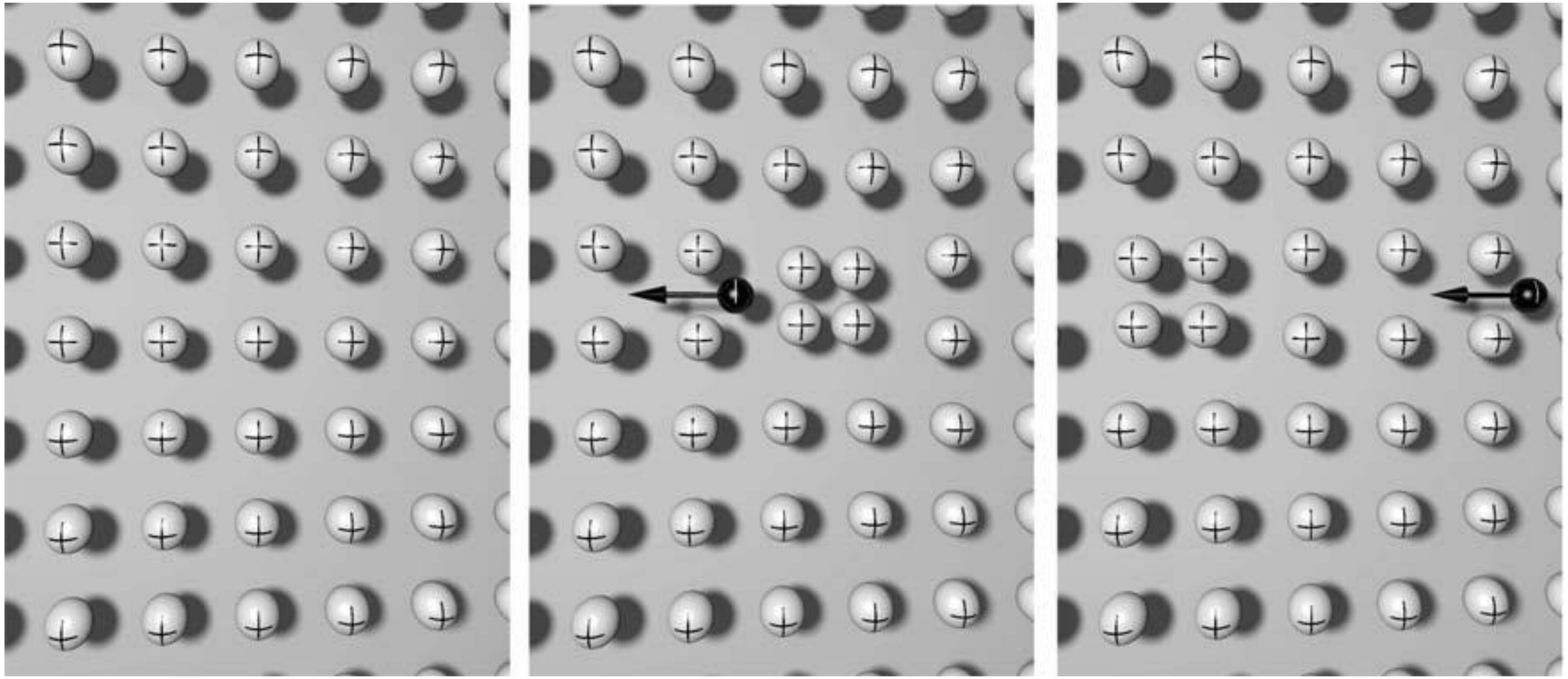
Glue: One possibility is mediation by ferromagnetic spin fluctuations; FM paramagnons (FM fluctuations suppress s-wave and enhance p-wave pairing).

Rev. Mod. Phys. 69, 645

SUPERFLUIDITY IN ^{3}HE

Attractive interactions by ferromagnetic fluctuations:

FM clouds are formed which attract the ^{3}He quasiparticles (something like magnetic polarons instead of lattice polarons)



Blundell's book

Lattice polarons

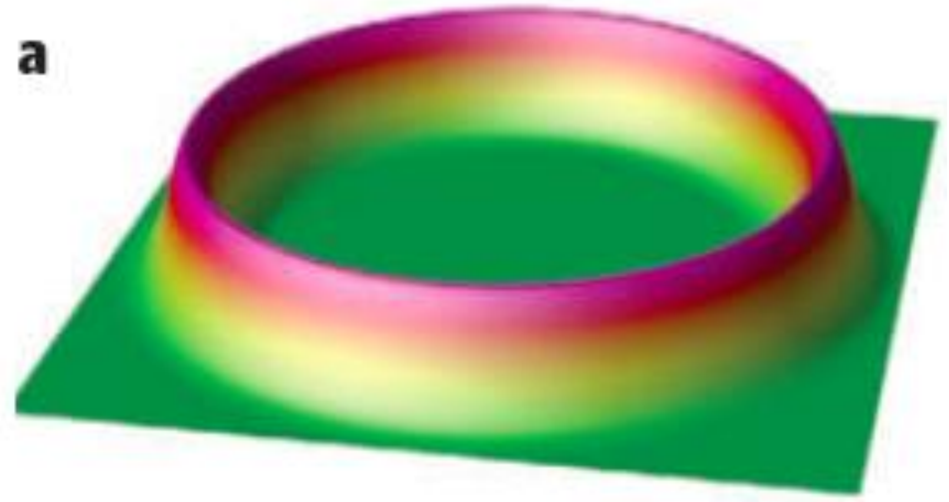
S-WAVE

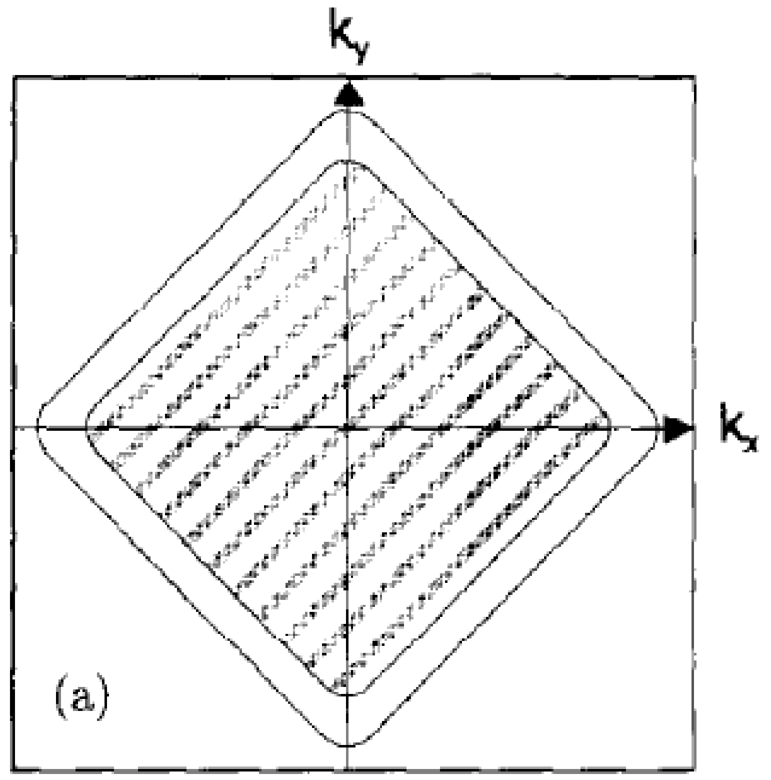
Gap equation from BCS
(T=0)

$$\boxed{\Delta_k = -\frac{1}{\Omega}\sum_{kk'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}}}$$

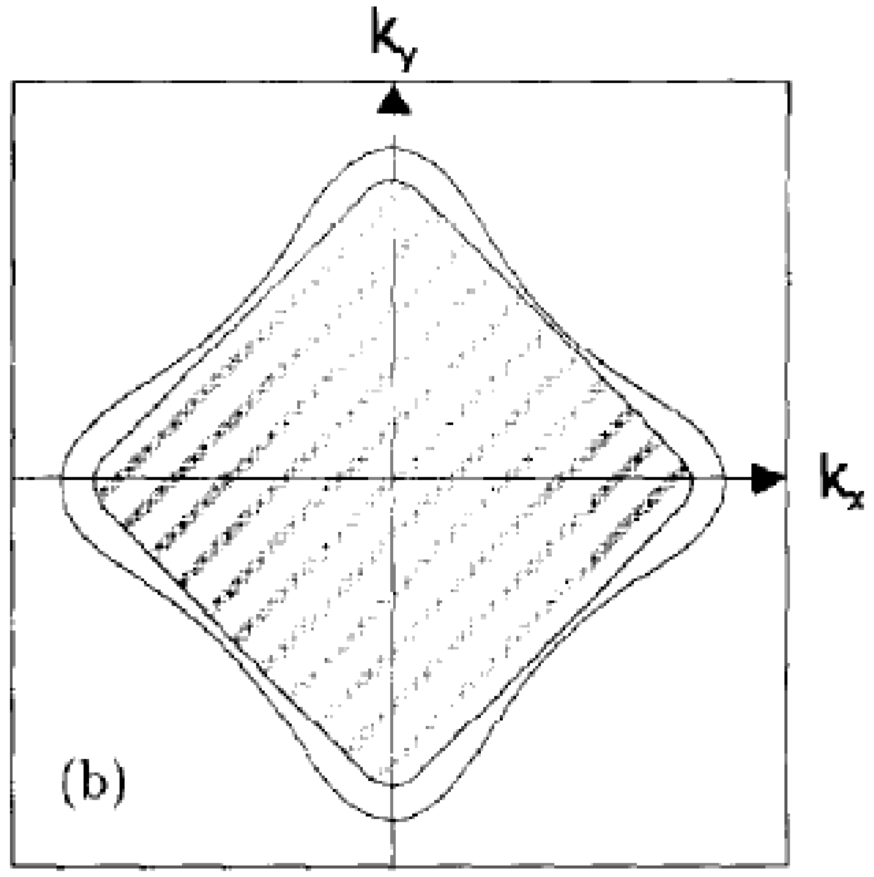
For $V_{kk'}$ constant and attractive: isotropic gap $\Delta_k = \Delta$

s wave gap
(spherical symmetry)





More generally, s-wave gap may be anisotropic with no sign changes.



Scalapino, Phys. Rep. 250,329 (1995)

D-WAVE

Gap equation from BCS
(T=0)

$$\boxed{\Delta_k = -\frac{1}{\Omega}\sum_{kk'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}}}$$

Repulsive $V_{kk'}$ $\Longleftrightarrow$ Anisotropic Δ_k with sign change!

$$\mathrm{sign}(\Delta_{\mathbf{k}}) = -\mathrm{sign}(V_{\mathbf{k},\mathbf{k}'})\mathrm{sign}(\Delta_{\mathbf{k}'})$$

An anisotropic pair potential leads to an anisotropic gap

$$V_{kk'} = -V_0 \gamma_k \gamma_{k'}$$

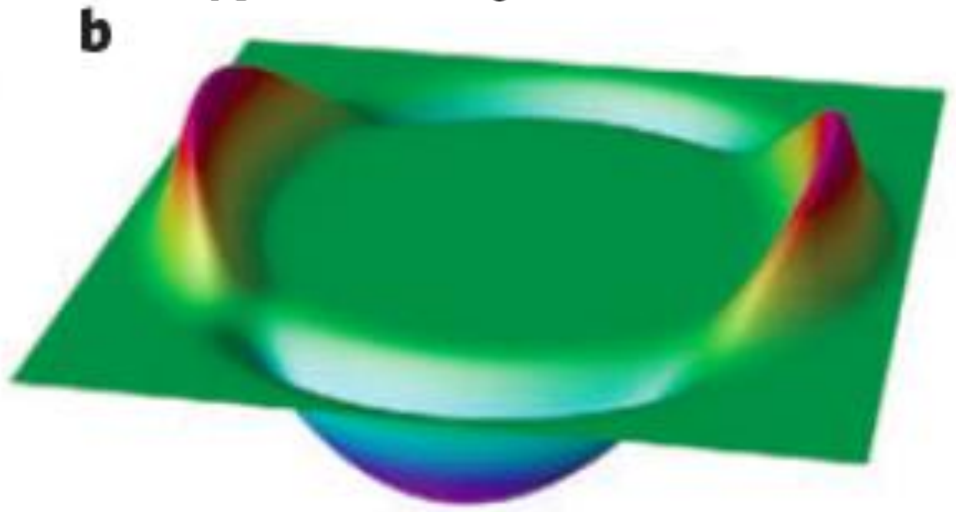
$$\Delta_k = \gamma_k \Delta_0$$

form factor

For instance, d-wave

$$\Delta_k = \Delta_0 \cos(2\phi)$$

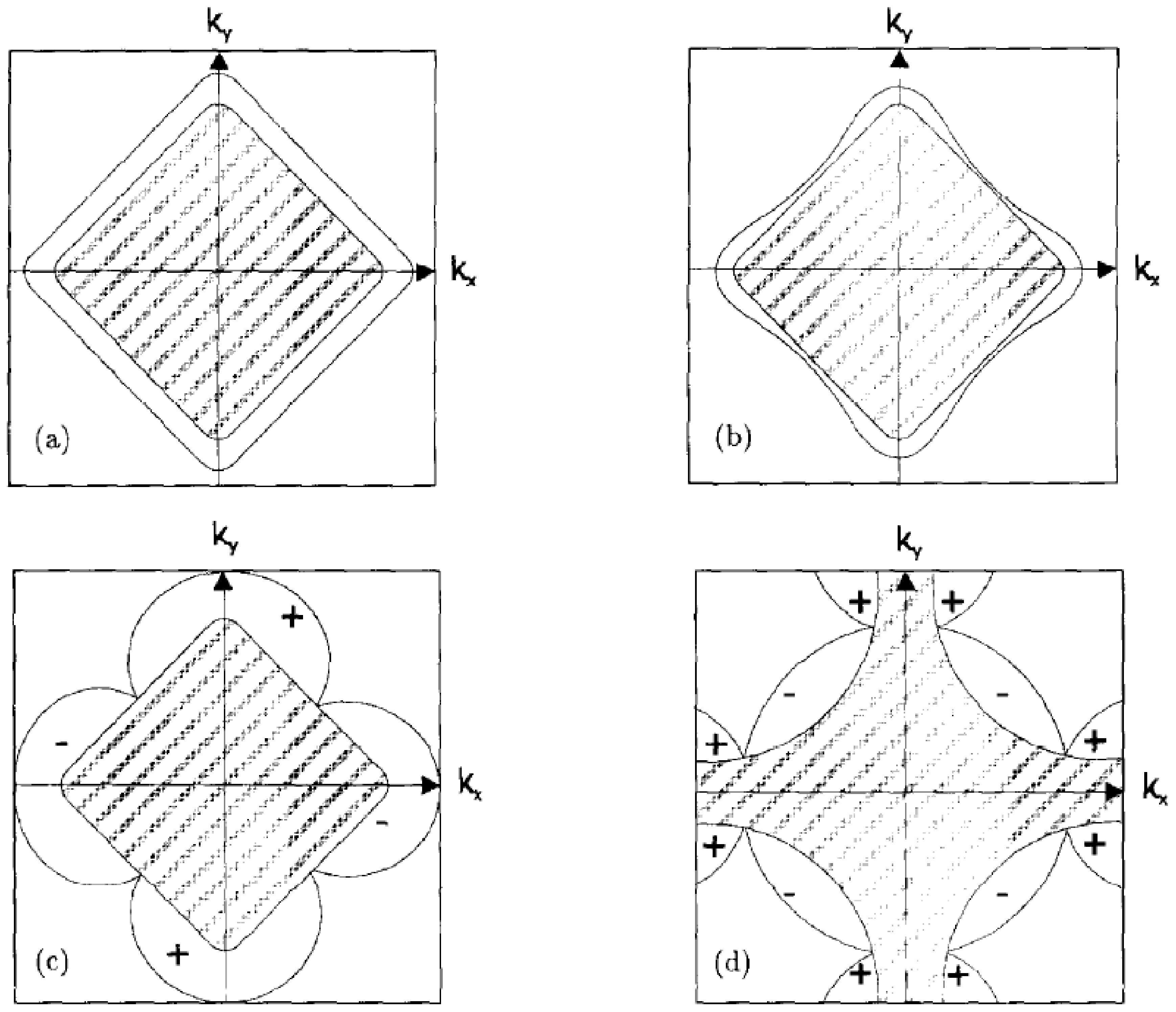
b



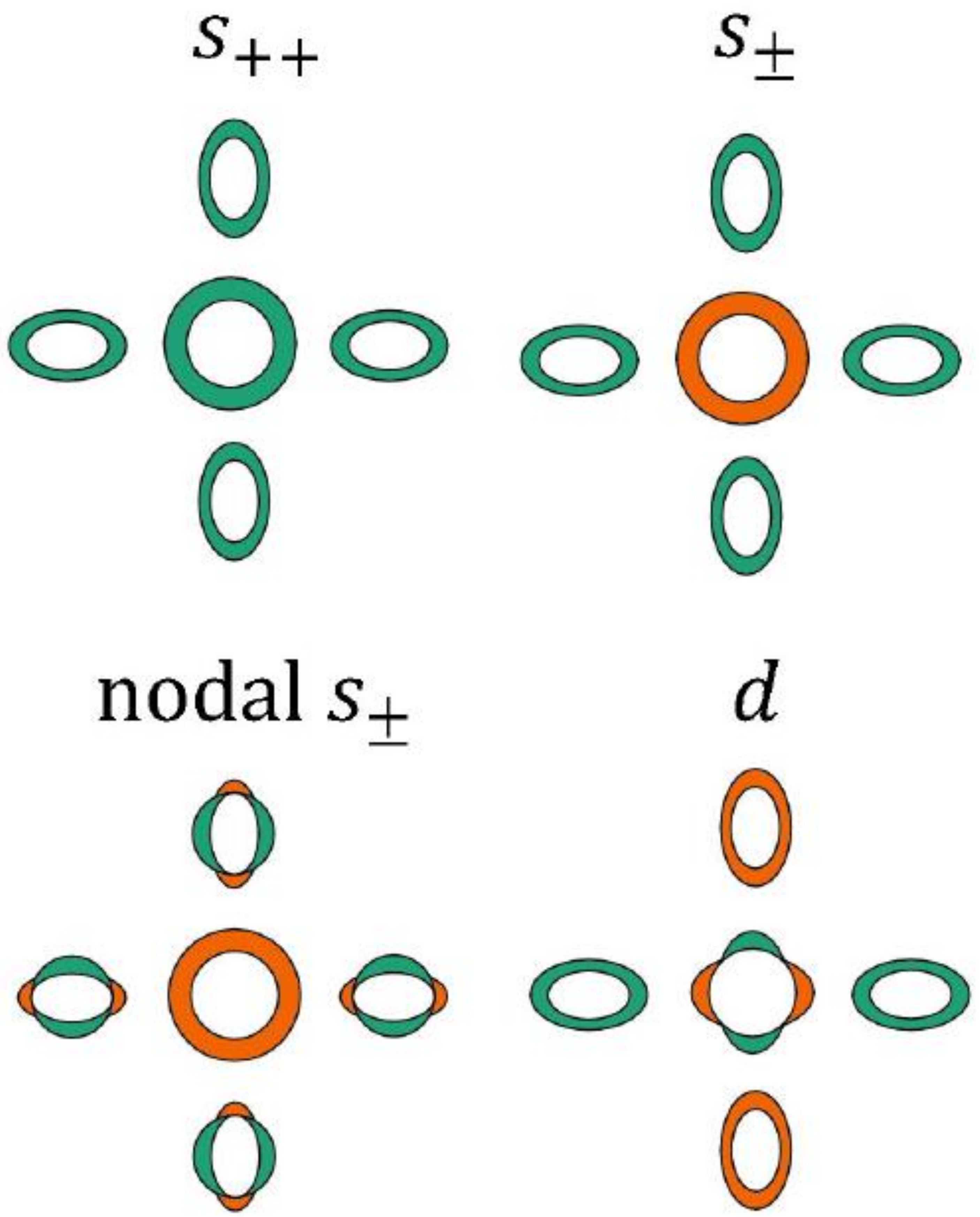
The gap has nodes and sign changes

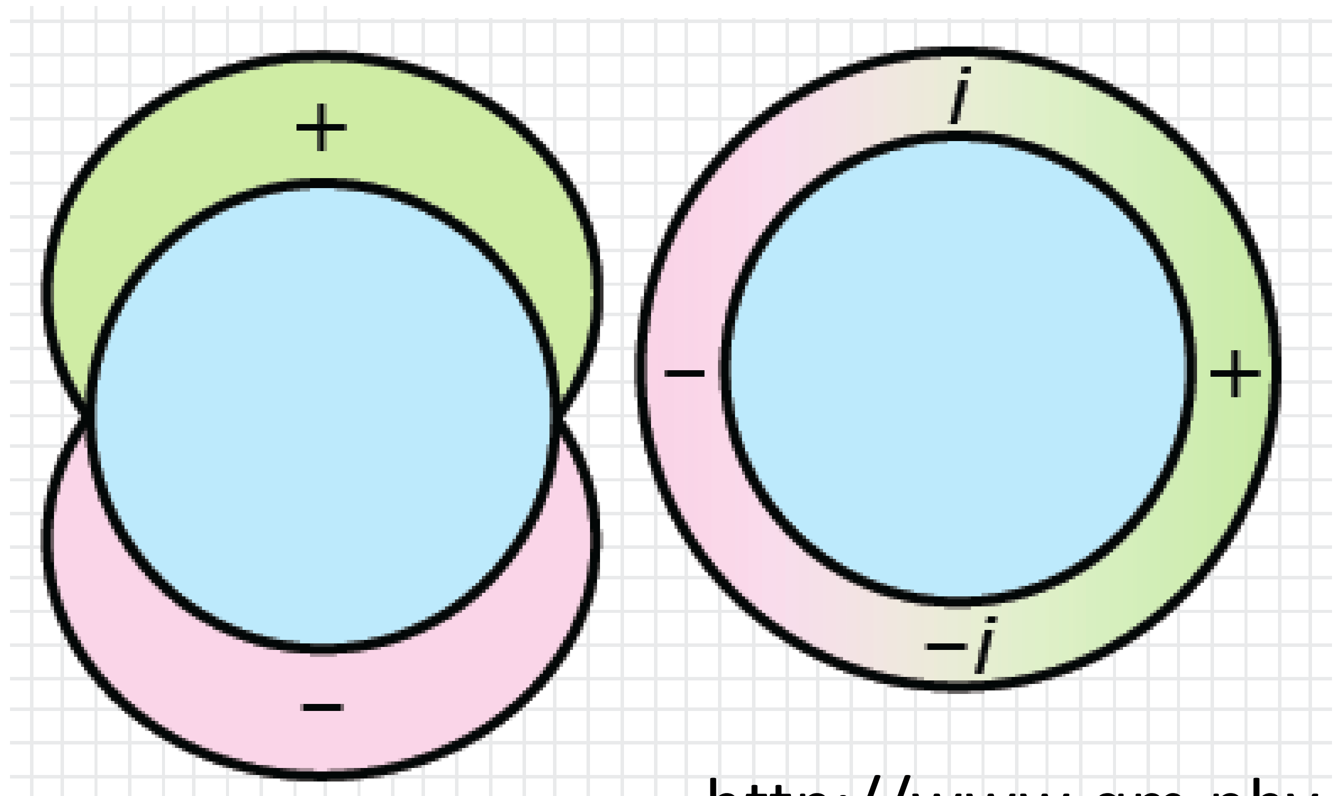
GAP SYMMETRIES…

Scalapino, Phys. Rep. 250,329 (1995)



Hirshfield et al. 1106.3712





http://www.qm.phy.cam.ac.uk/teaching/

Non-isotropic gaps lead to sensitivity of unconventional SPC to non-magnetic impurities

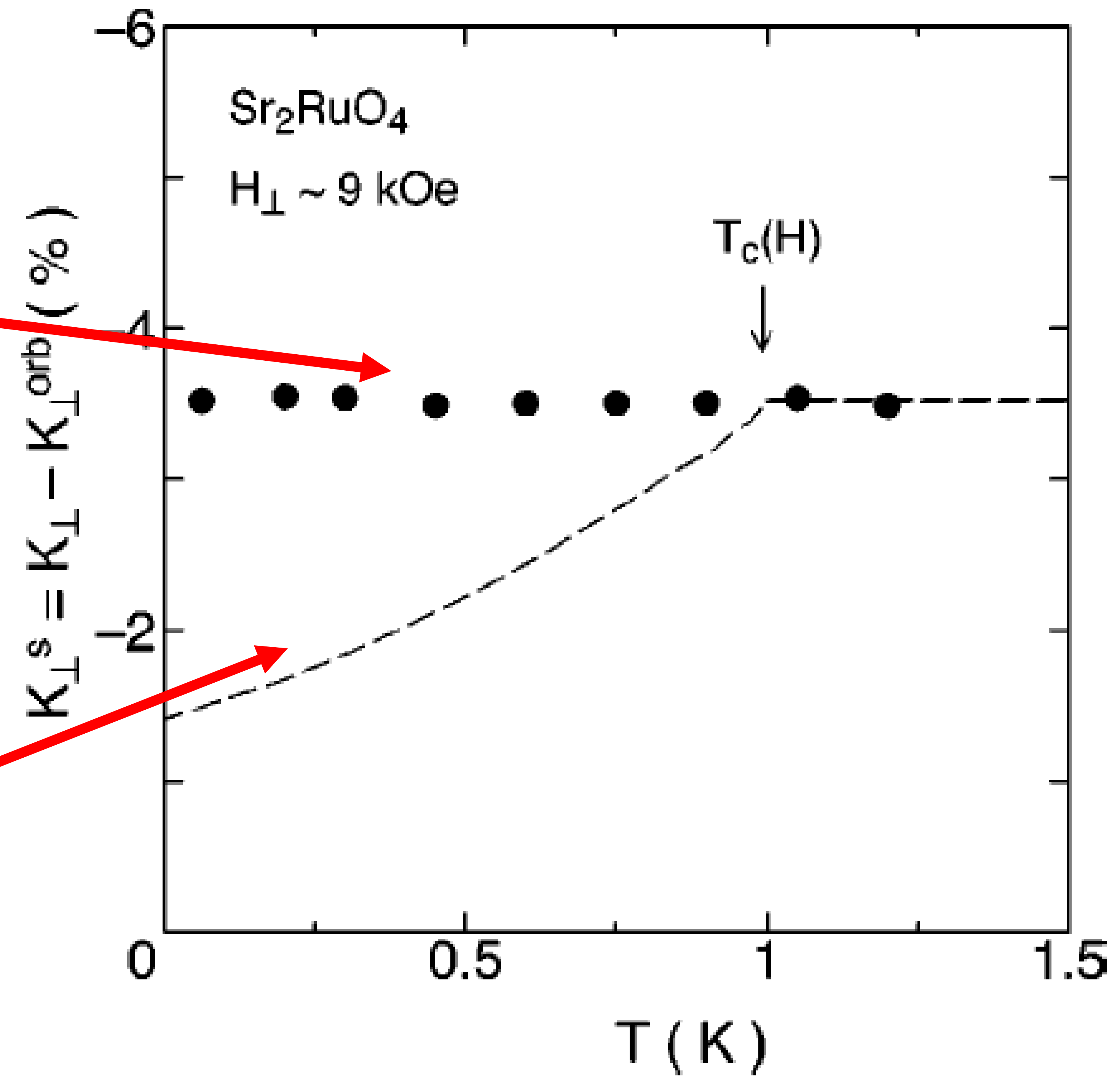
M.J. Calderón calderon@icmm.csic.es

SINGLET VERSUS TRIPLET

Can be distinguished with Knight shift experiments (nuclear magnetic resonance experiment)

triplet

Singlet
(spin quenching at low T)

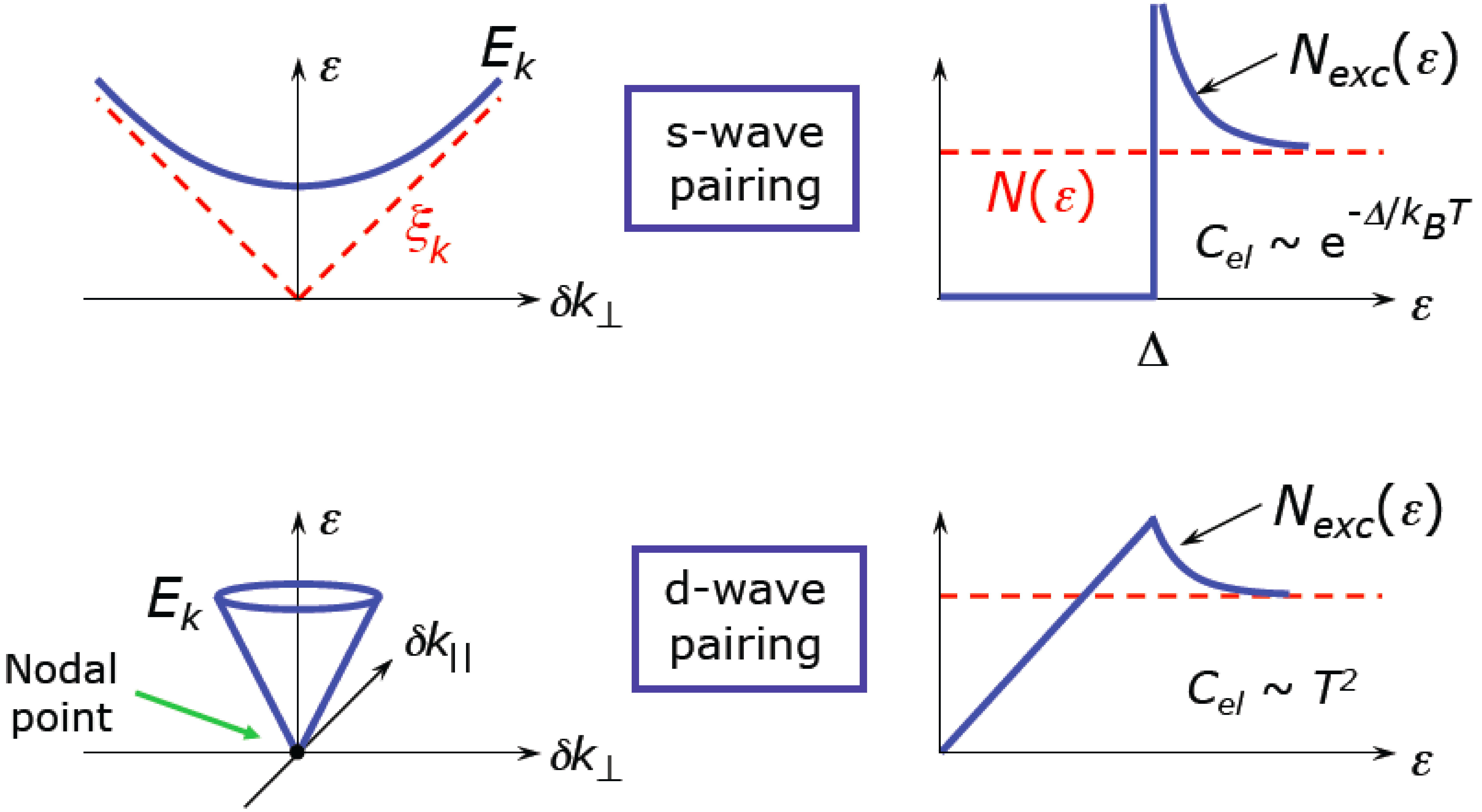


PRB 63, 060507 (2001)

(note Sr_2RuO_4 is currently known not to be a triplet superconductor)

NODES VERSUS NODELESS

http://www.qm.phy.cam.ac.uk/teaching/



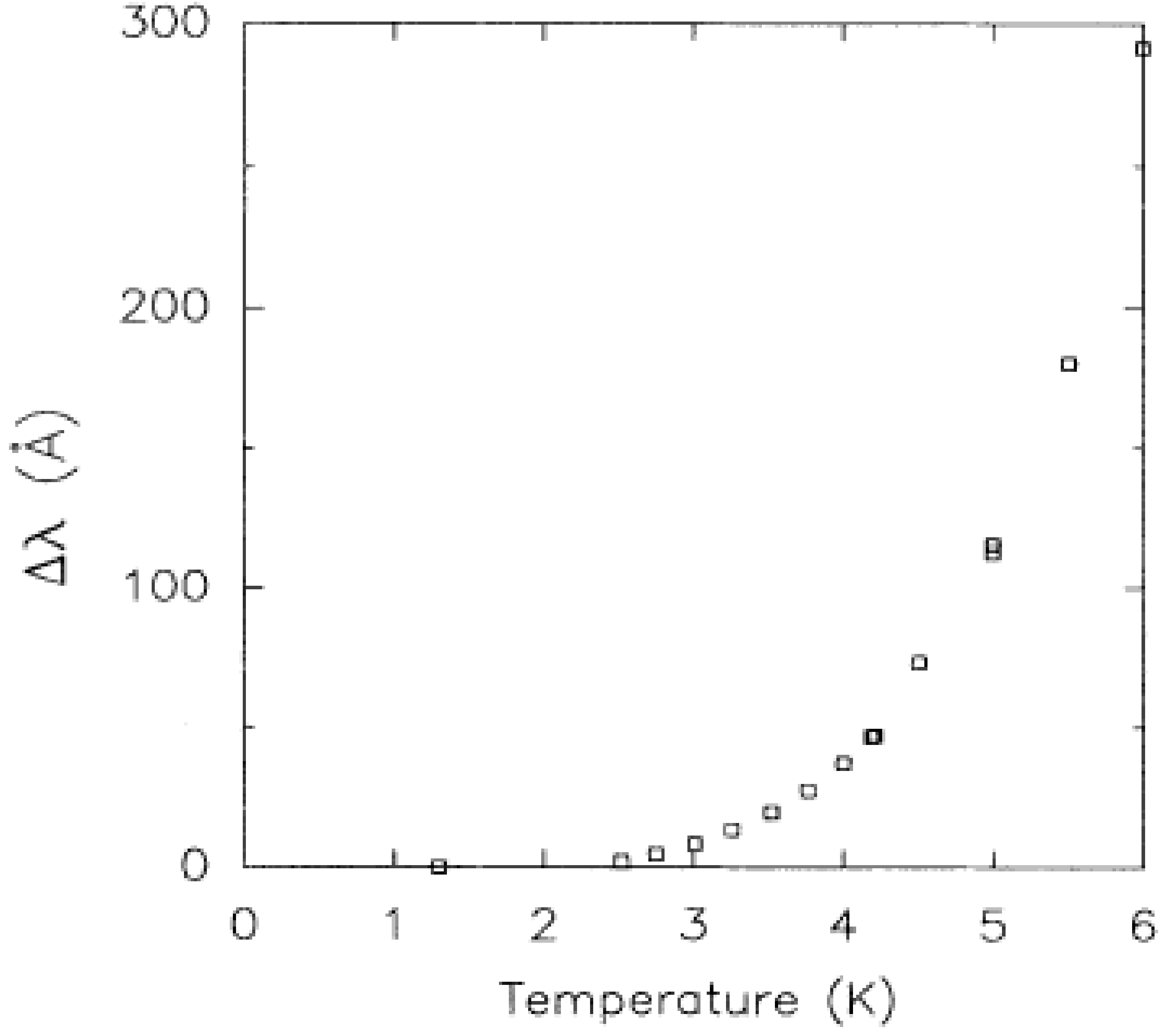
M.J. Calderón calderon@icmm.csic.es

Without nodes: activated behavior (λ, specific heat…)
With nodes: power law behaviors

Gap without nodes

London penetration length within BCS

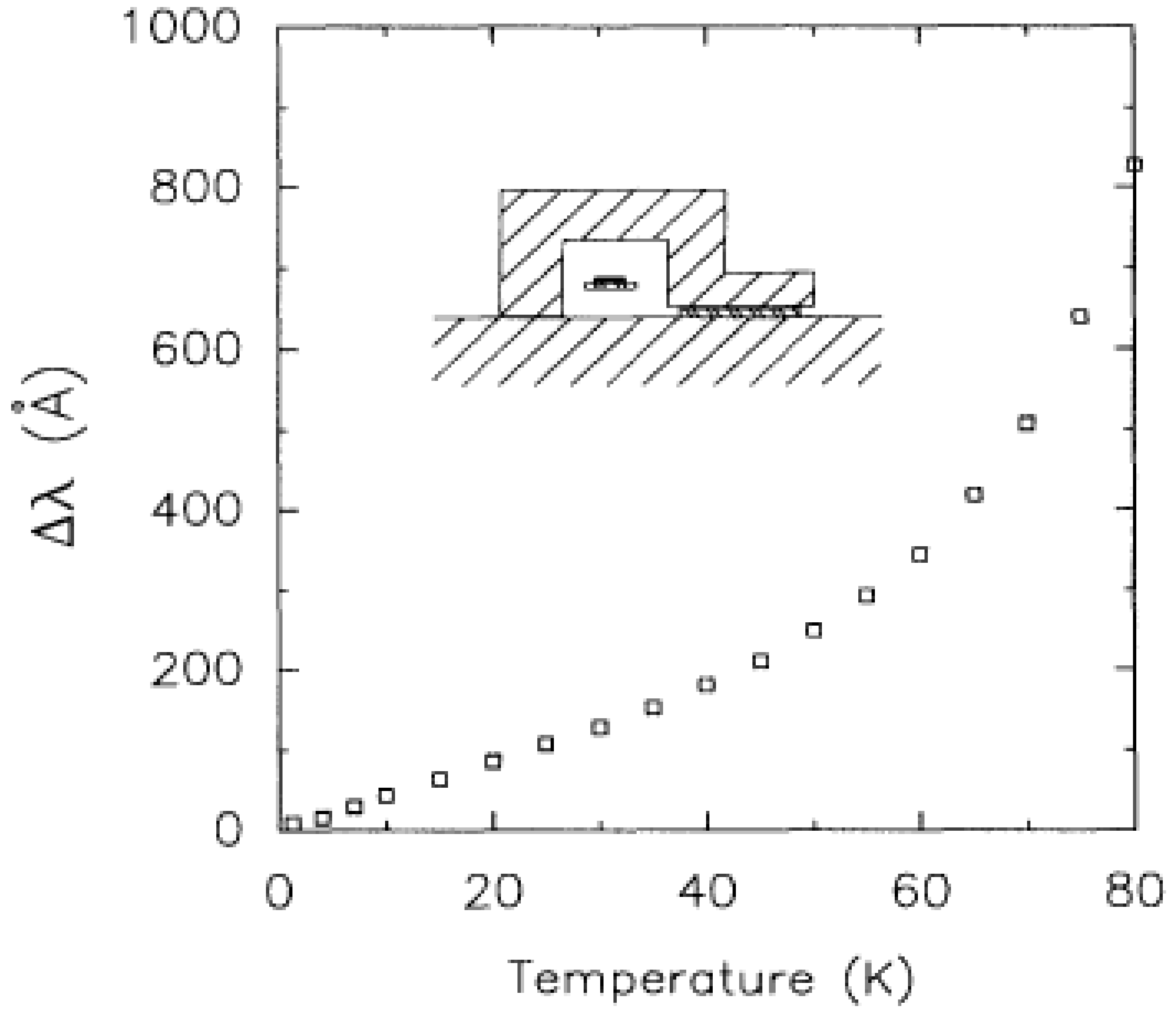
$$\frac{\Delta\lambda(T)}{\lambda(0)} \sim 3.33 \left(\frac{T_c}{T}\right)^{1/2} \exp(-1.76T_c/T)$$



$Pb_{0.95}Sn_{0.05}$ PRL 70, 3999 (1993)

Gap with nodes

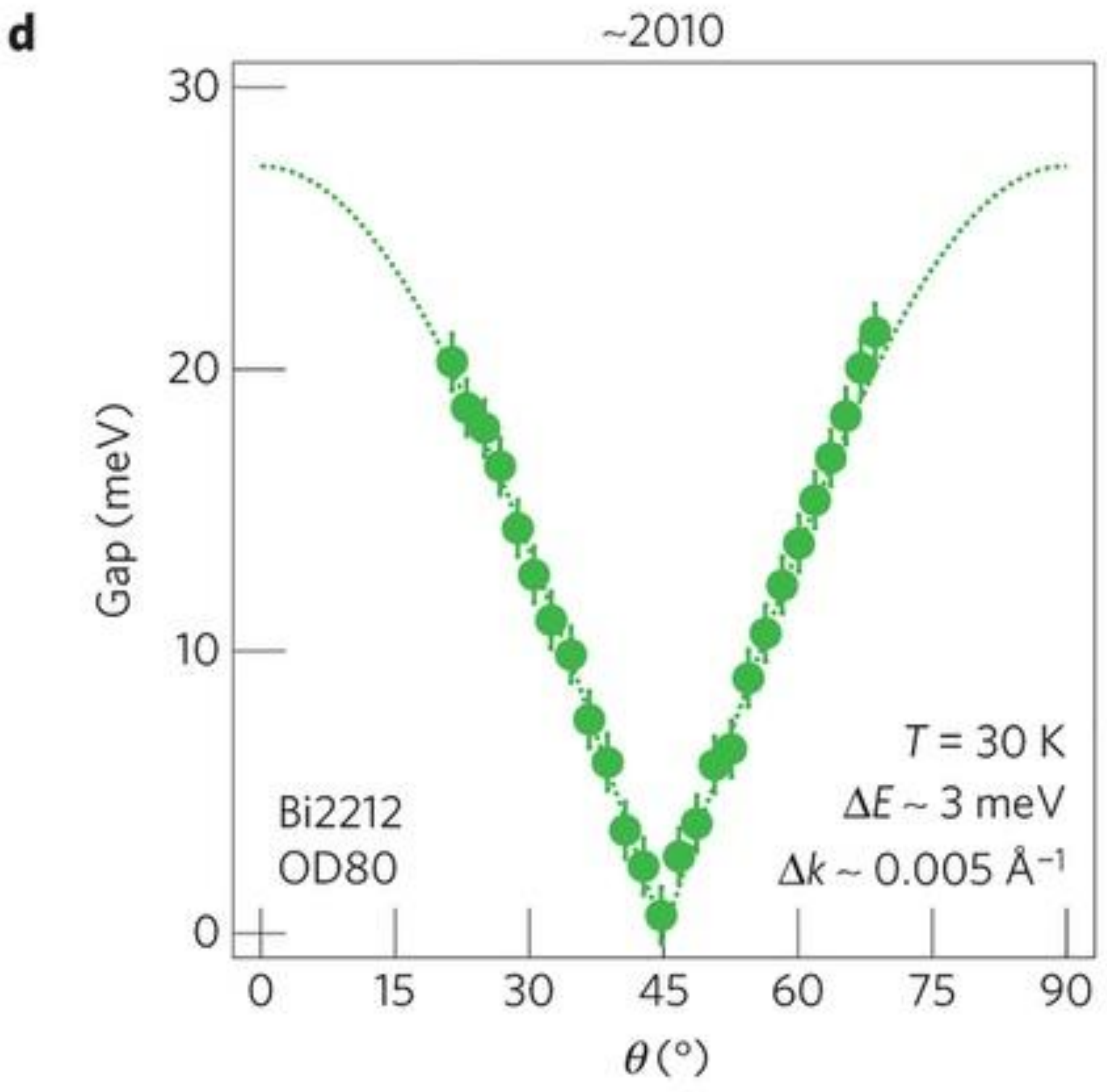
Power law dependencies



$YBa_2Cu_3O_{6.95}$

(no phase information)

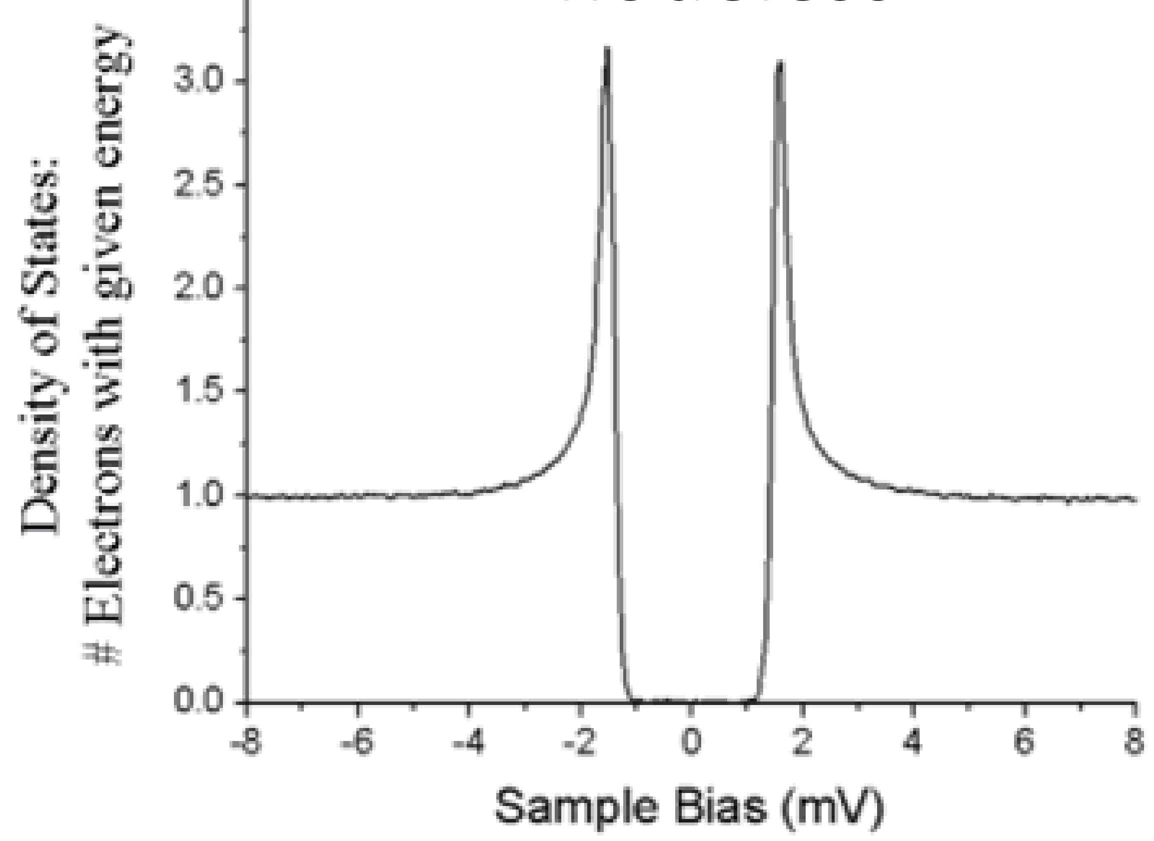
ARPES



Nature Physics 10, 483–495 (2014)

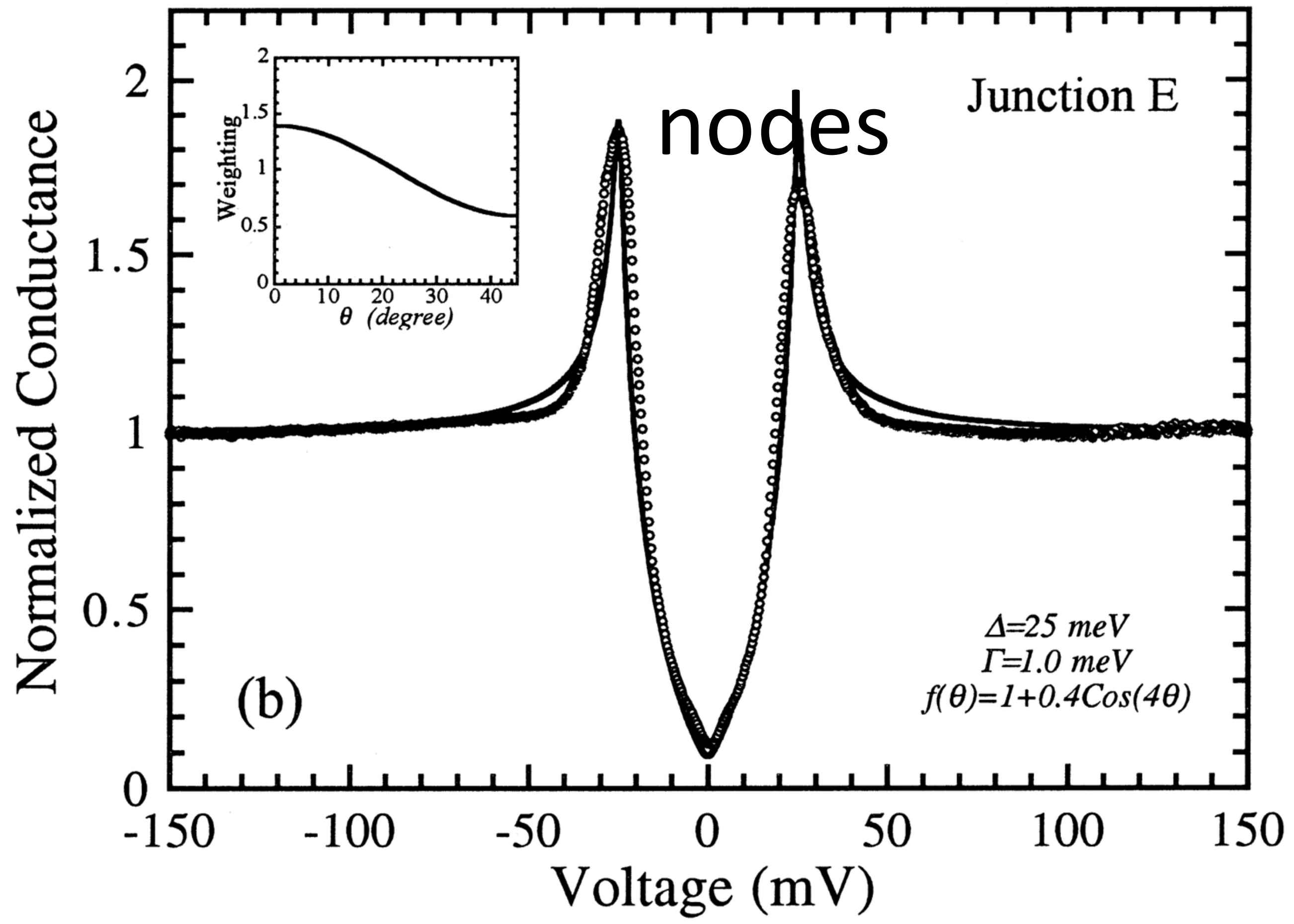
TUNNELING SPECTROSCOPY

nodeless



www.personal.psu.edu/ewh10

nodes



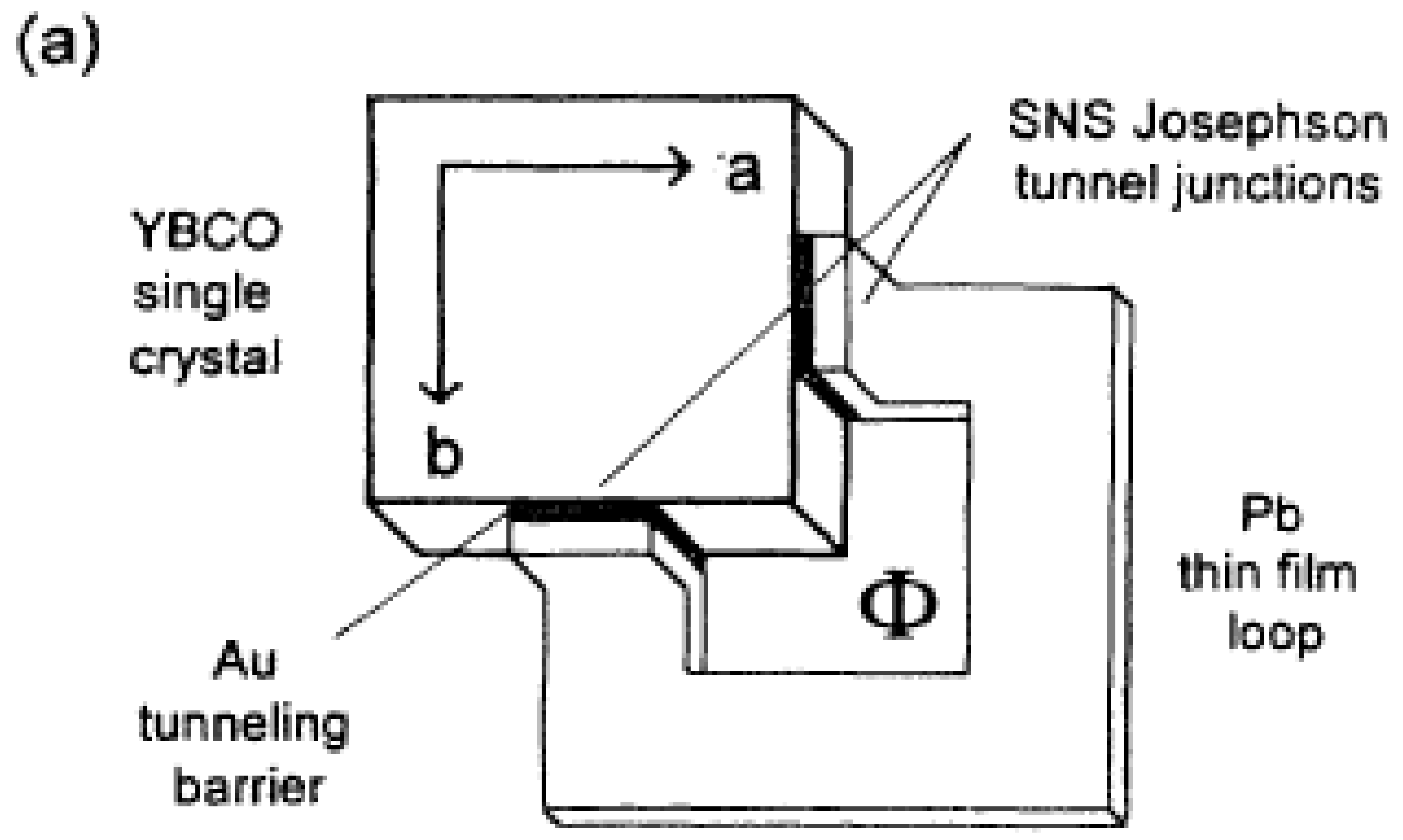
Physica C 320, 9

(no phase information)

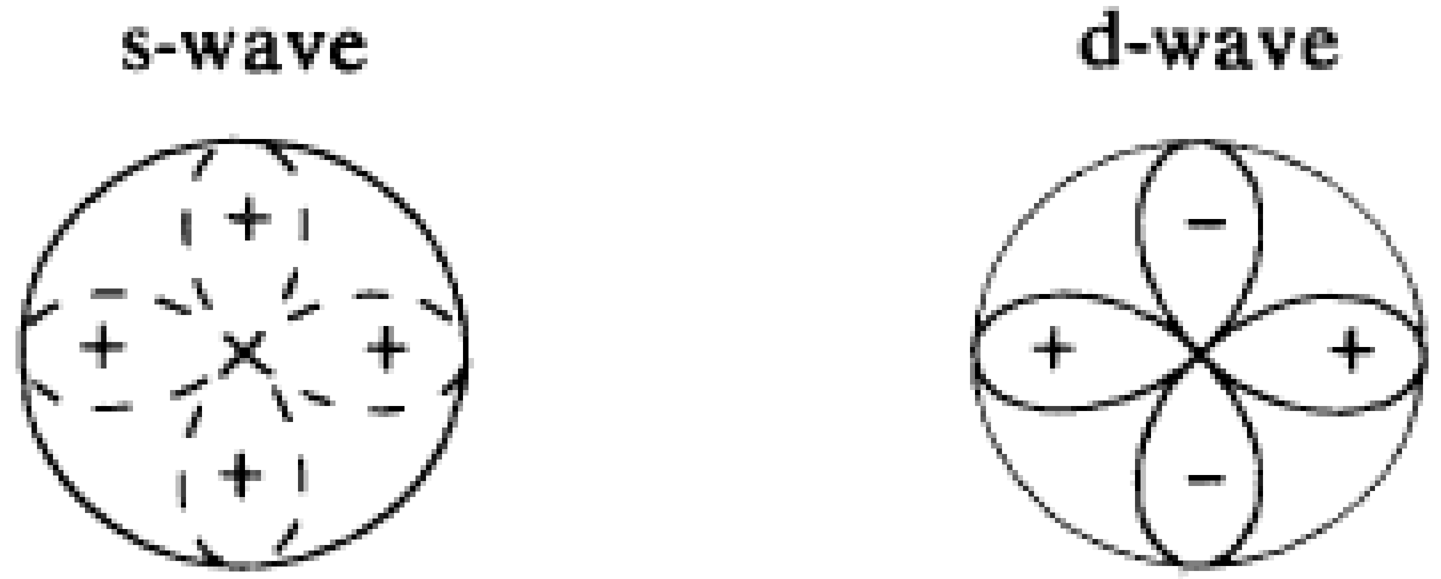
SENSITIVITY TO THE PHASE: JOSEPHSON EFFECT

$$I_s = I_c \sin\Delta\varphi$$

PRL 71, 2134 (1993)

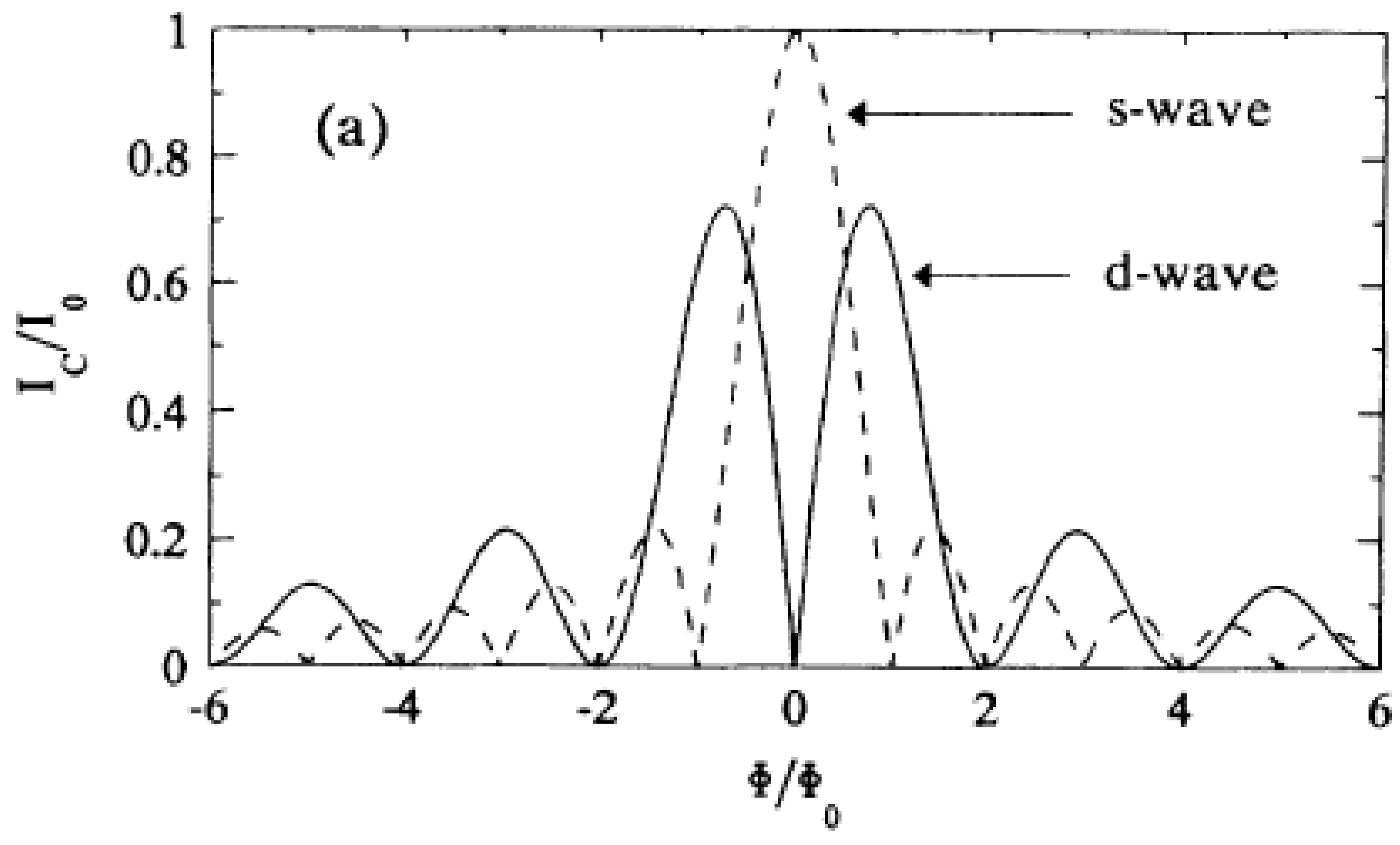


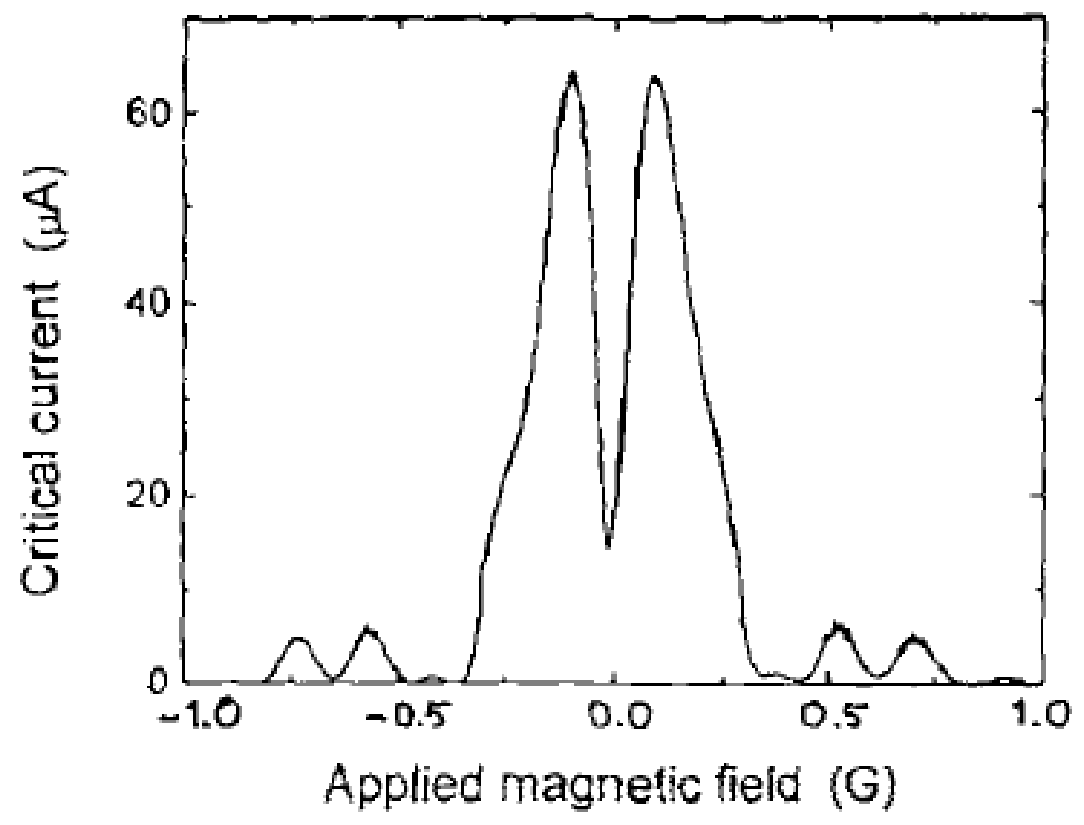
$$\phi_a - \phi_b + 2\pi\Phi/\Phi_0 + \delta_{ab} = 0$$



$\delta_{ab} = 0$ $\delta_{ab} = \pi$

Calculated critical current

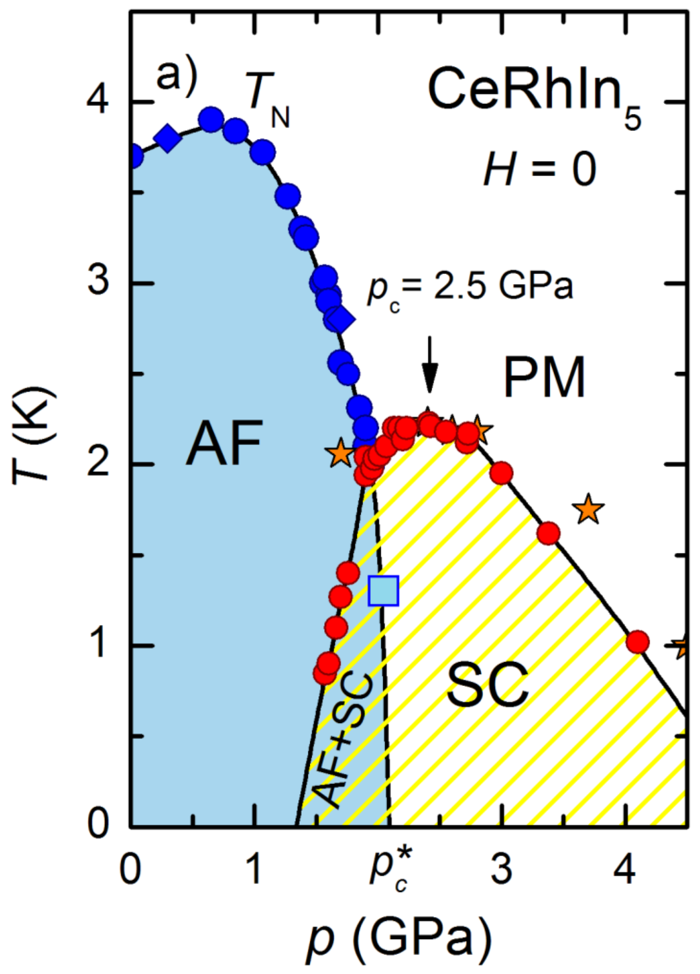




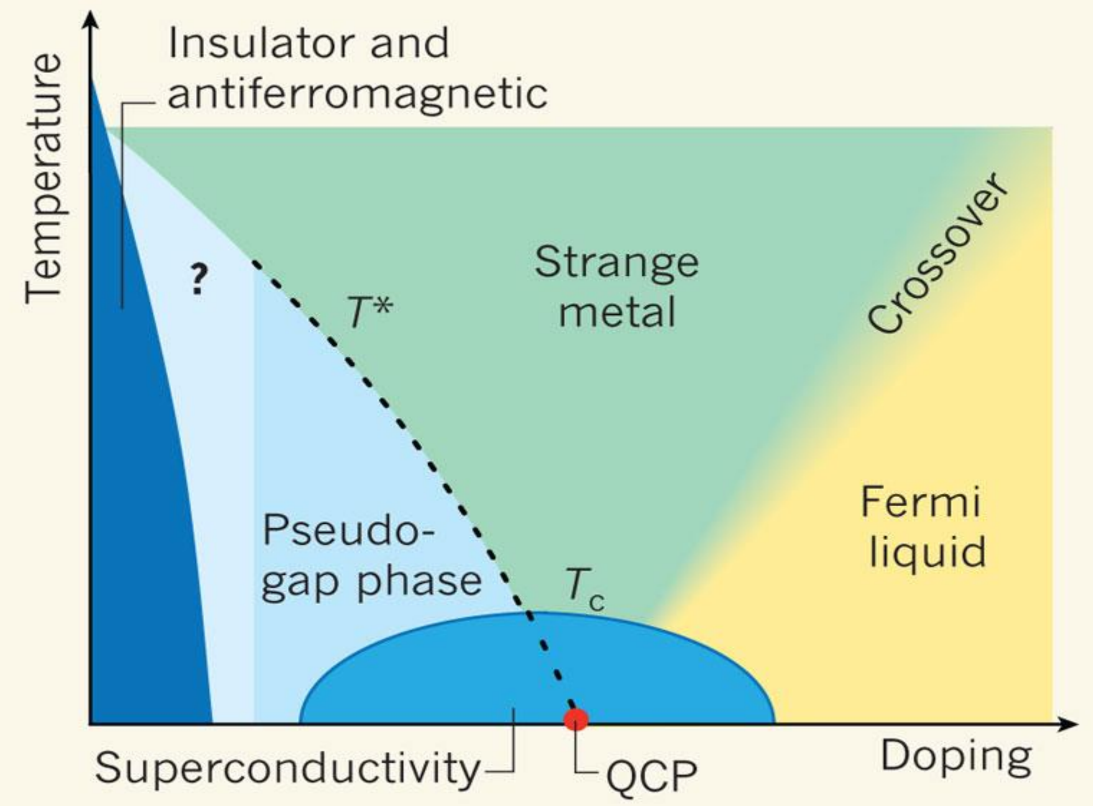
MATERIALS

SOME TYPICAL PHASE DIAGRAMS

Heavy fermions

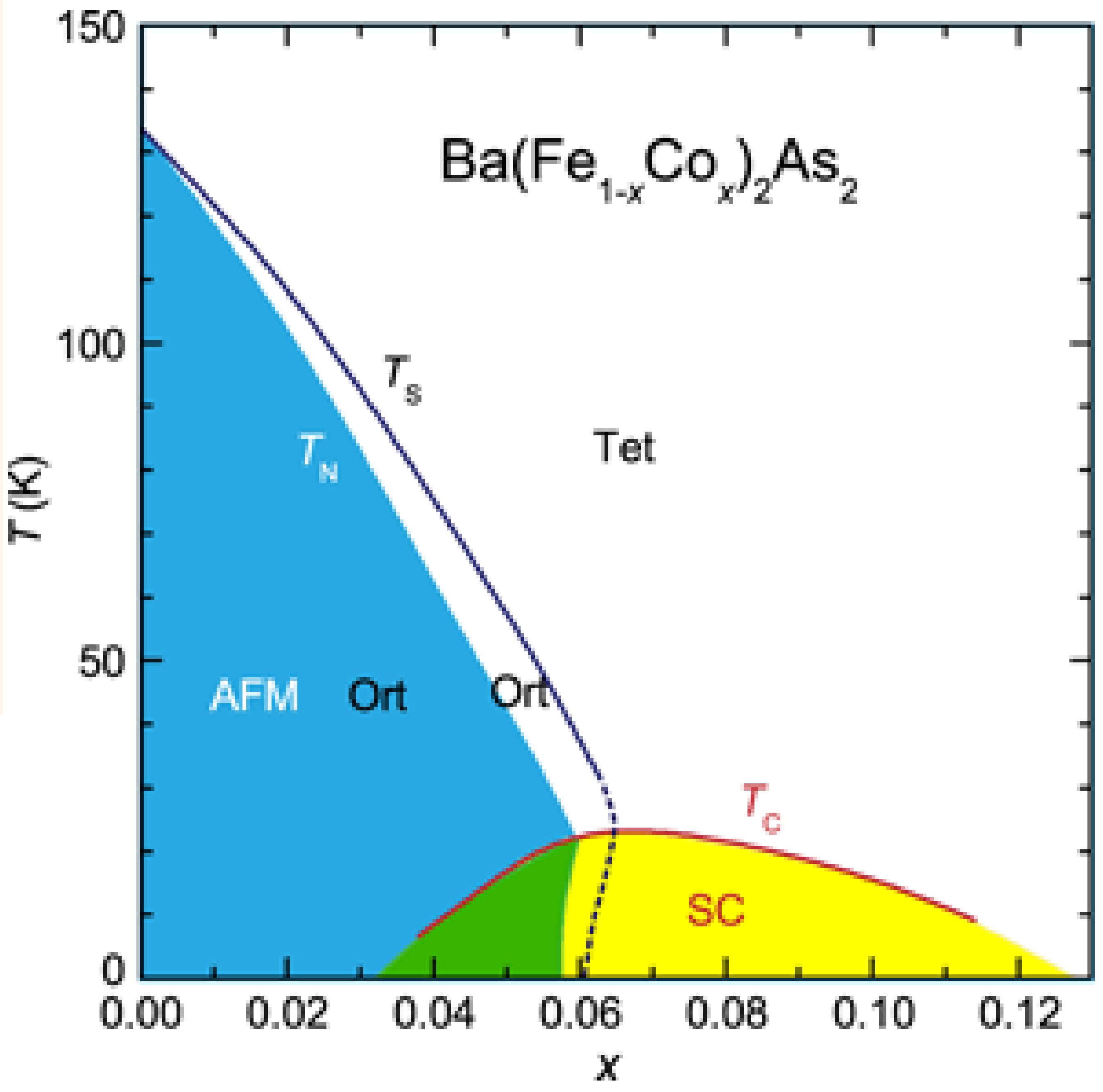


Cuprates



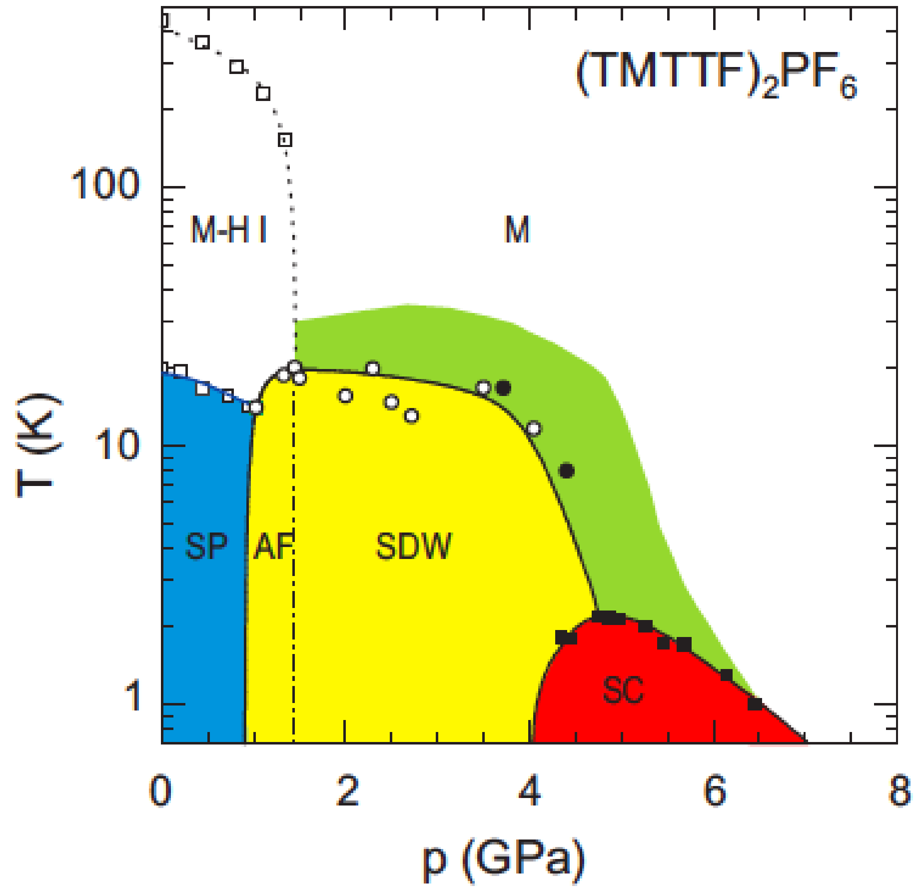
Nature 468, 184–185

Fe-superconductors



Nandi et al, PRL 104, 057006 (2010)

Organics



In common:
(AF)magnetic
phases

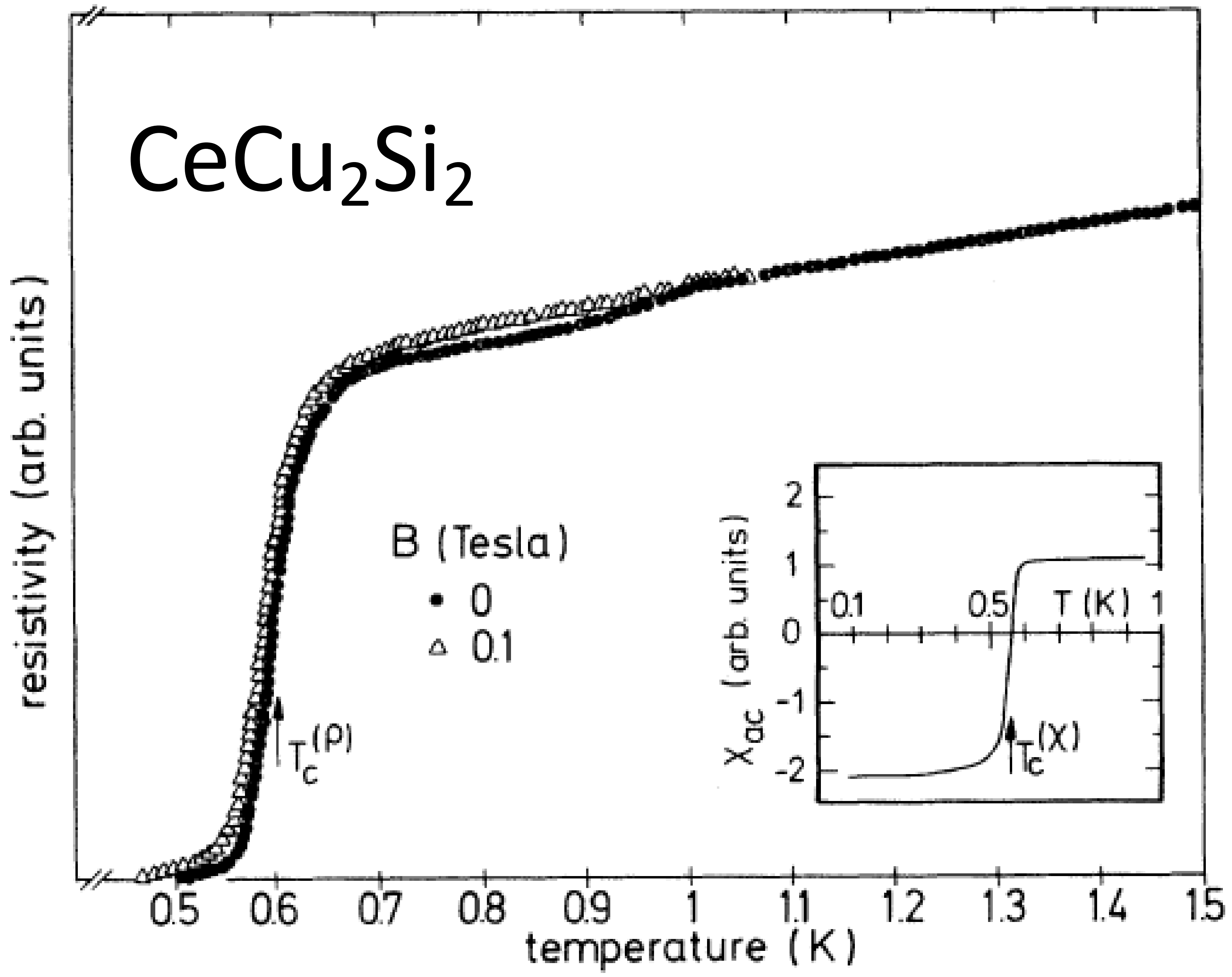
Eur. Phys. JB 21, 175

HEAVY FERMIONS (1979)

"Our experiments demonstrate for the first time that superconductivity can exist in a metal in which many-body interactions, probably magnetic in origin, have strongly renormalized the properties of the conduction-electron gas."

- Coexisting AF + SPC
- Reentrant SPC due to competition with Kondo
- Quantum criticality

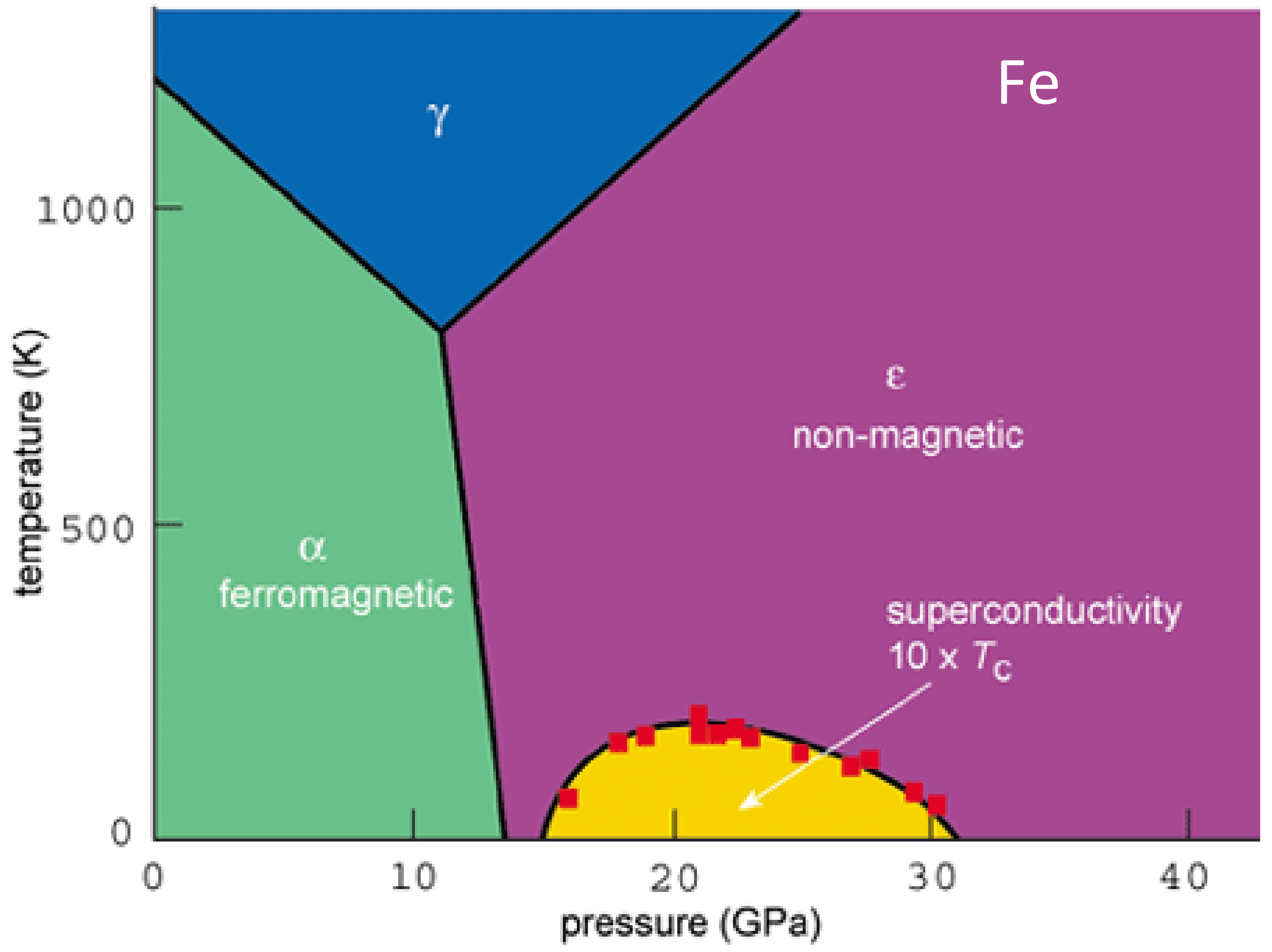
Nat. Phys. 4, 186

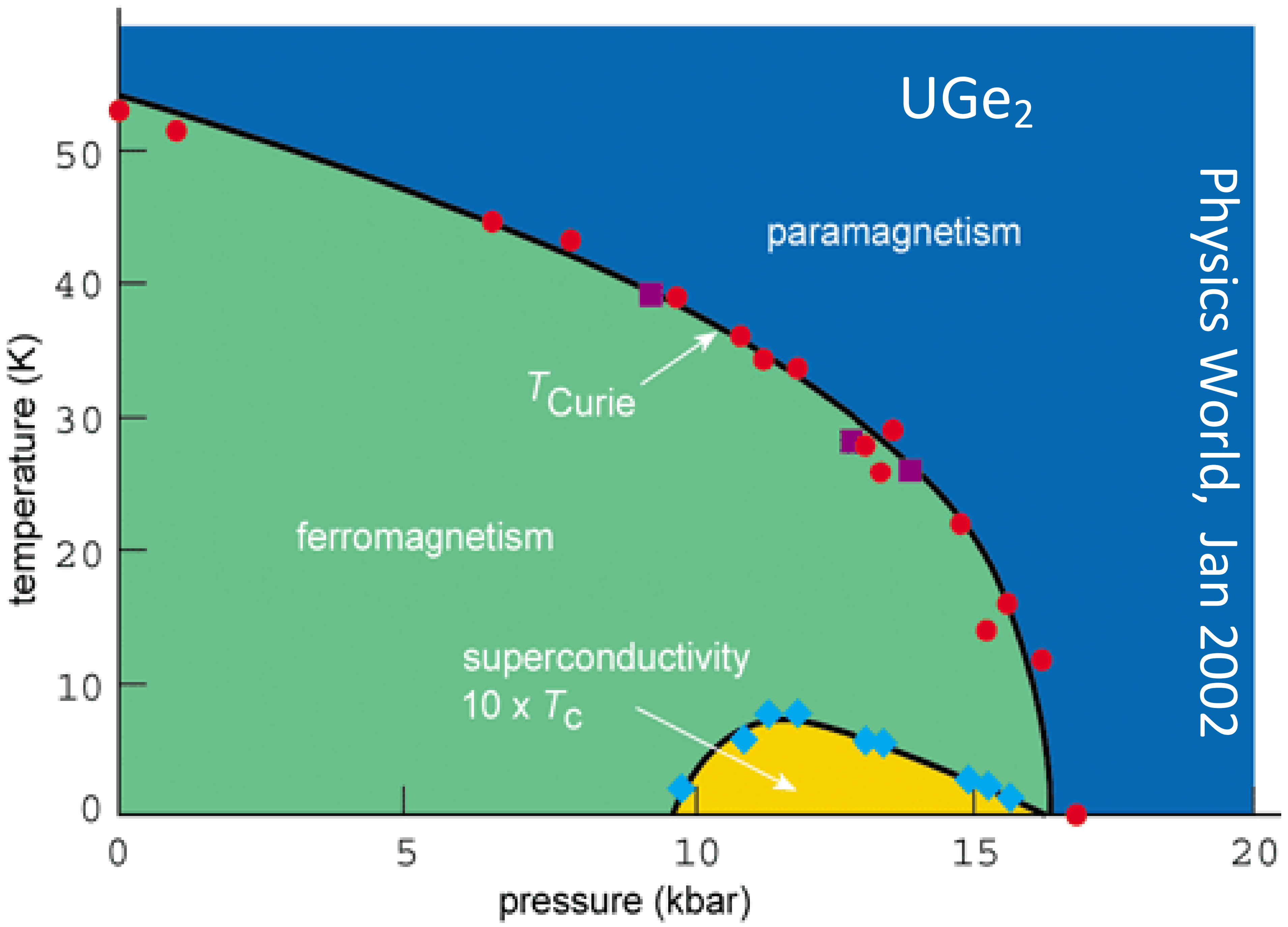


PRL 43, 1892 (1979)

M.J. Calderón calderon@icmm.csic.es

FERROMAGNETIC SUPERCONDUCTORS





Proximity of quantum critical point can lead to coexistence

PRL 94, 097003 (2005)

Possible triplet pairing

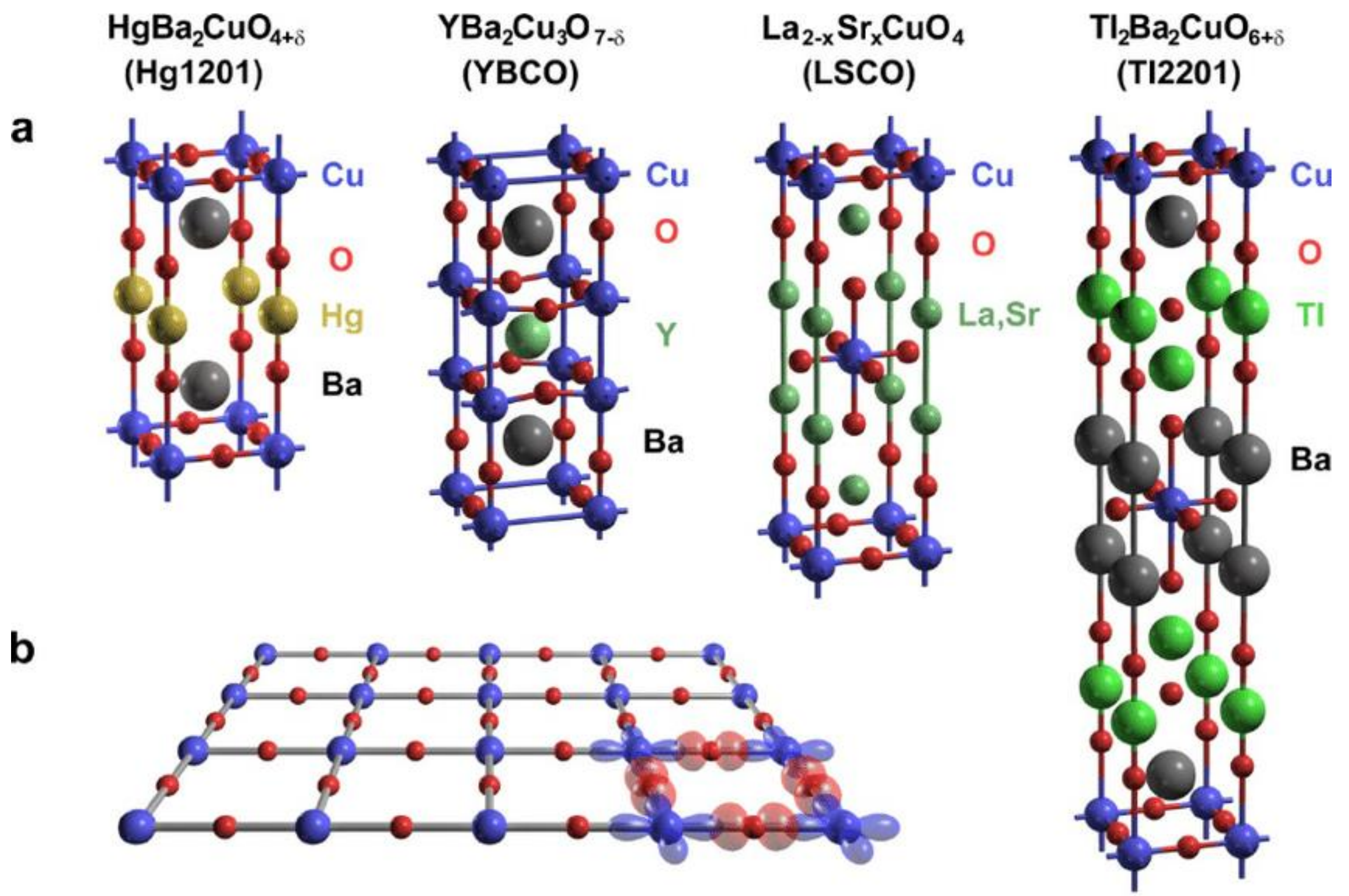
UPt_3, UTe_2 Science 365, 684 (2019)

URhGe Science 309, 1343 (2005)

UCoGe Nature Comm 8, 14480 (2017)

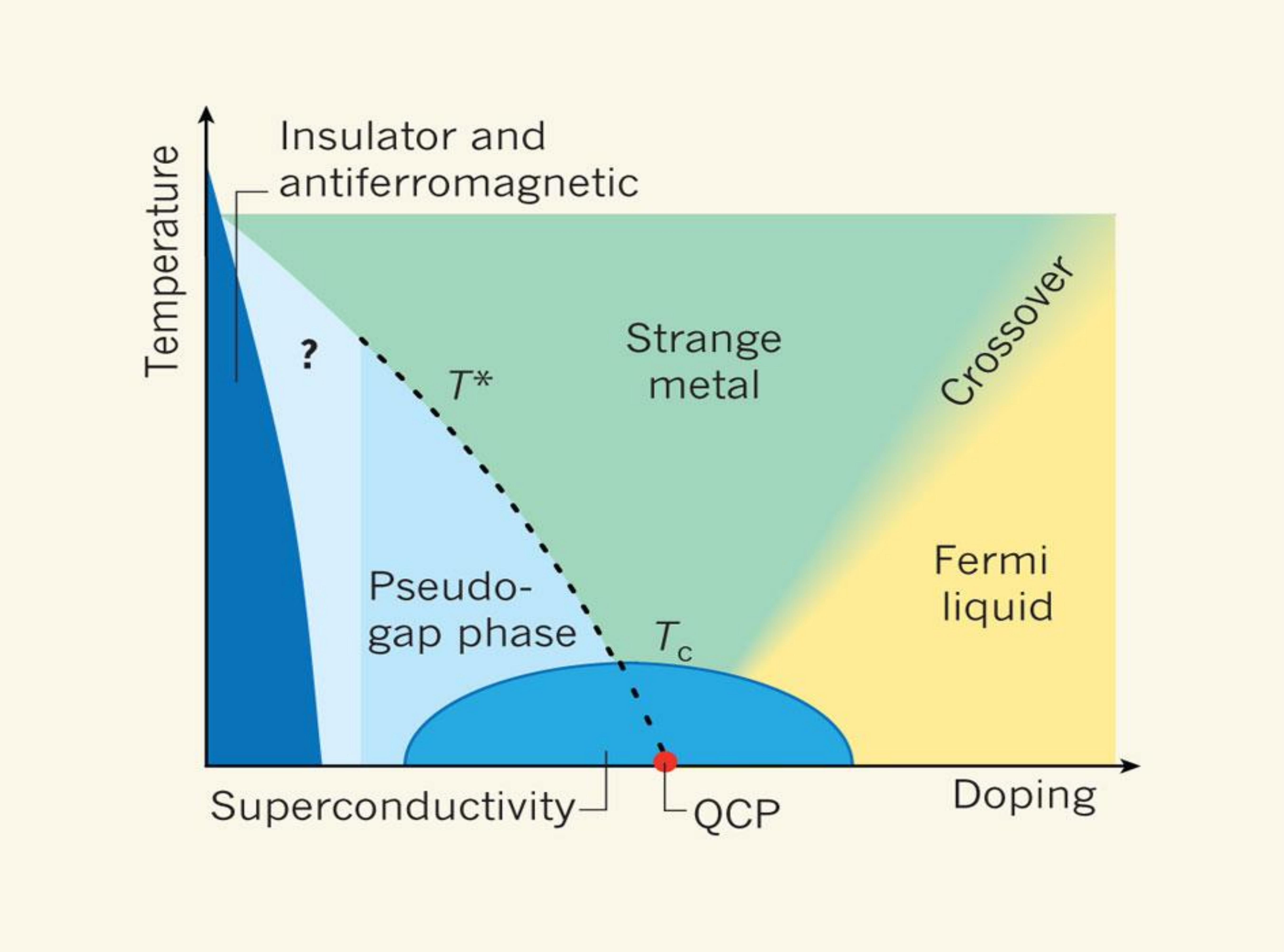
CUPRATES

- Layers of CuO_2. Different related structures.
- But note(!): SPC requires coherence in 3dim.
- Highest T_c ~ 134K (at ambient pressure).
- T_c increases with number of CuO_2 planes in the unit cell (up to n=3).
- Pairs are singlets.
- d-wave pairing was proposed in the cuprates early on. Scalapino, Phys. Rep. 250, 329 (1995)
- Undoped cuprates are correlated insulators and AF (π,π).



10.1073/pnas.1301989110

CUPRATES: COMPLEX “NORMAL” STATE

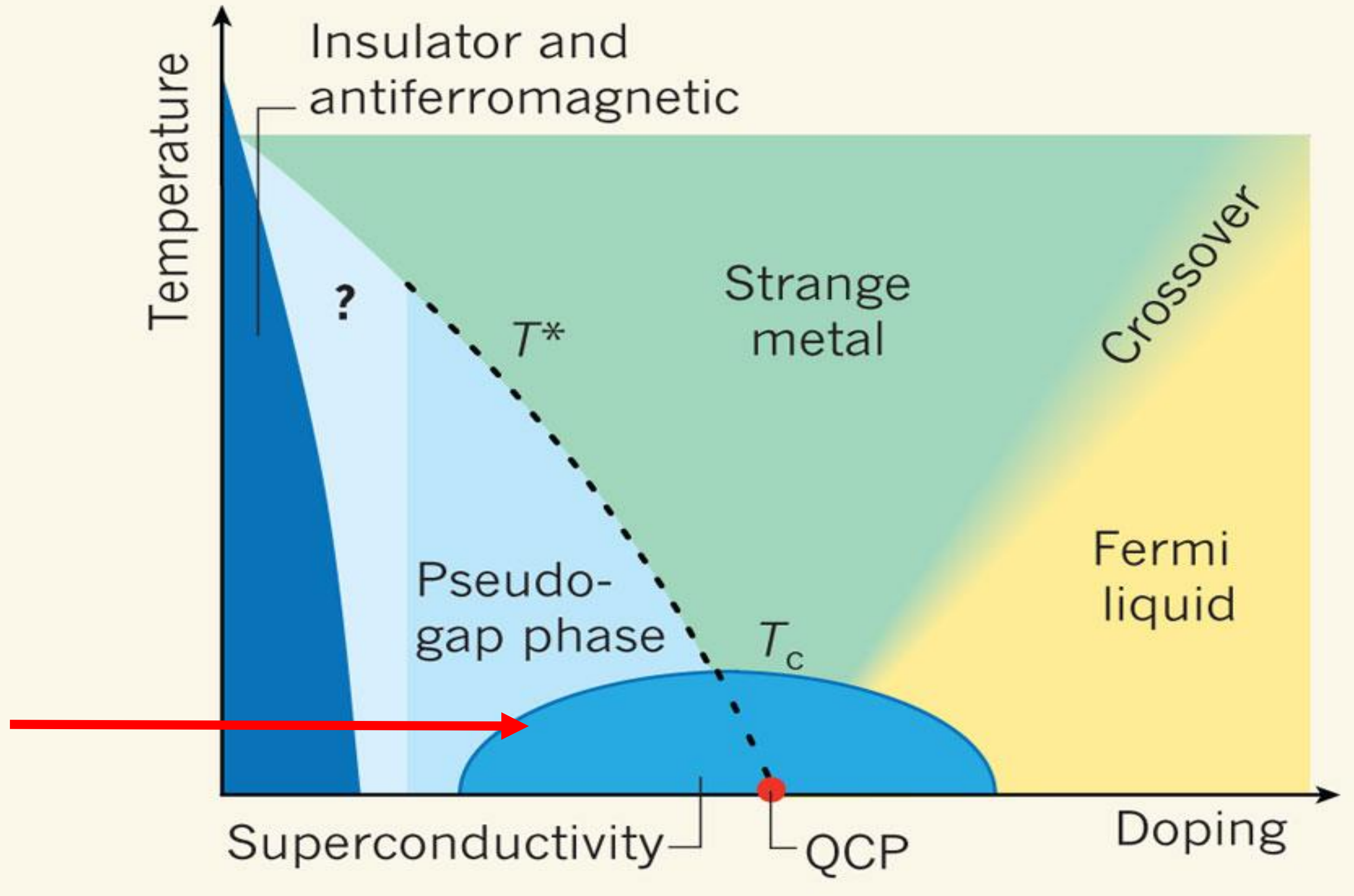


Nature 468, 184–185

PSEUDOGAP

Spin quenching sets up at T*.

underdoped



Nature 468, 184–185

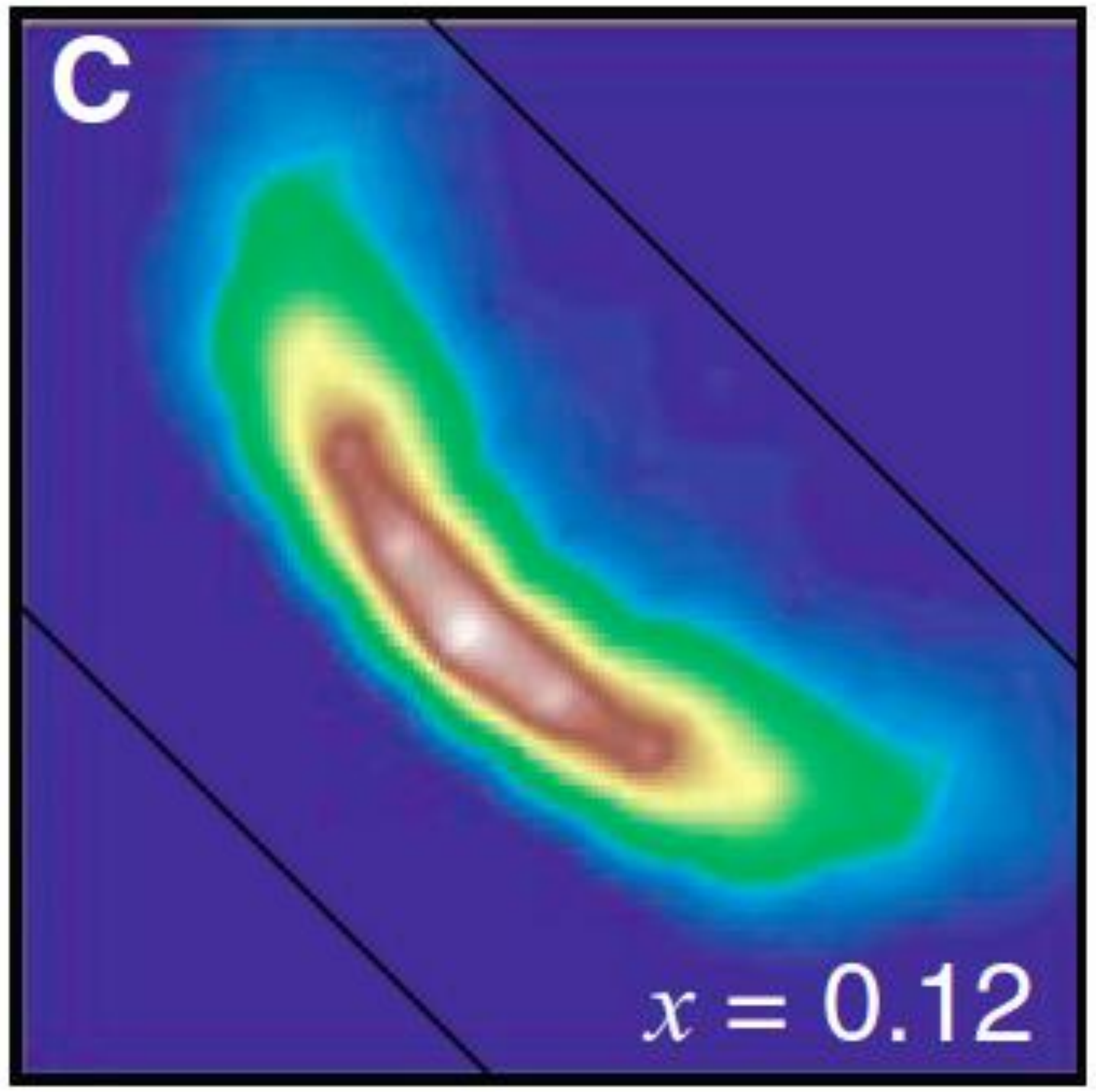
Origin?: spin-singlet formation (Anderson), pairing with short range order (preformed pairs), antiferromagnetic fluctuations, charge density wave

Is it due to fluctuations or is it a new phase (with a related broken symmetry)? Transition or crossover?

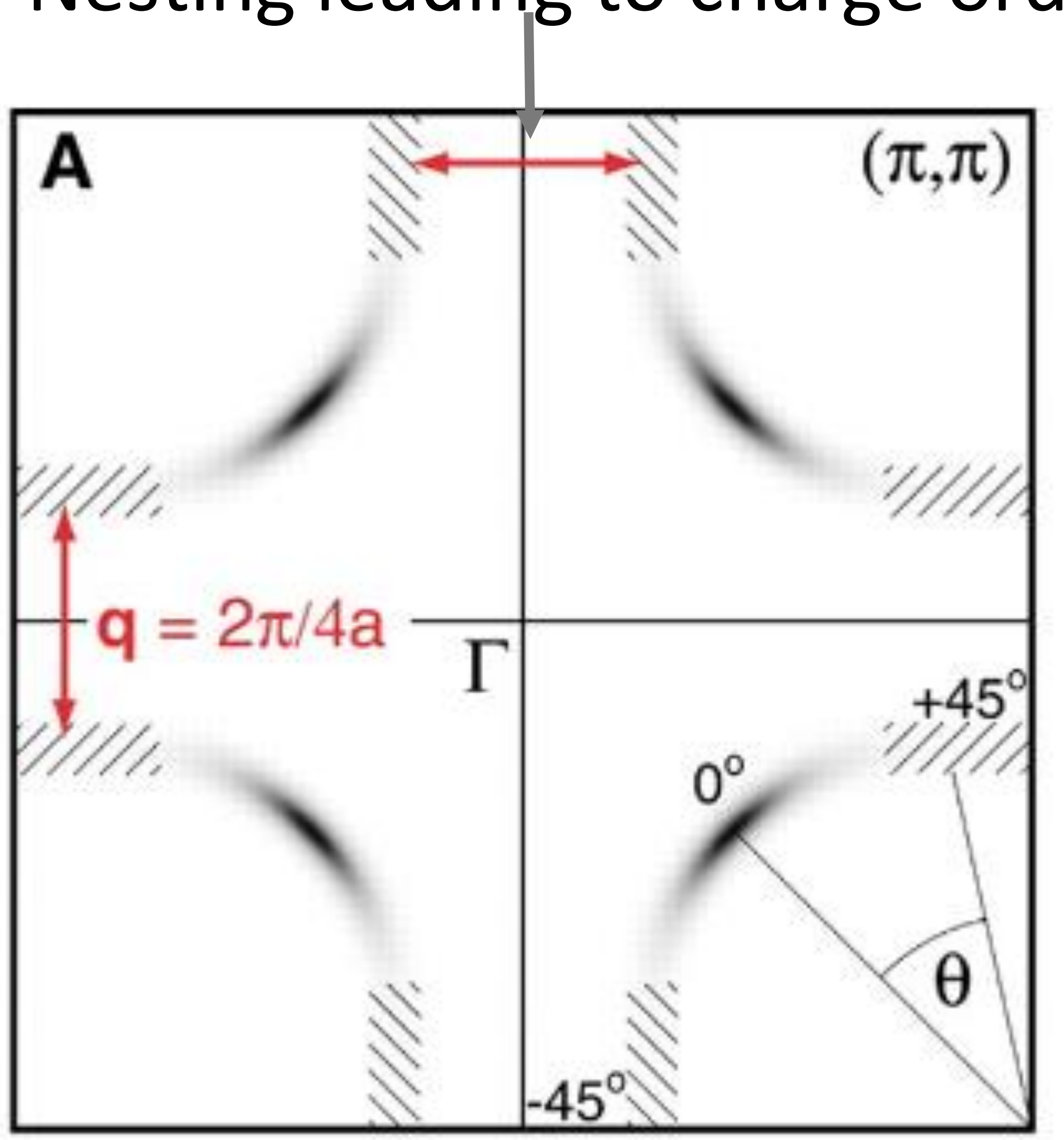
PSEUDOGAP

In other words:
Is it a precursor or a competing phase?

ARPES



Nesting leading to charge order

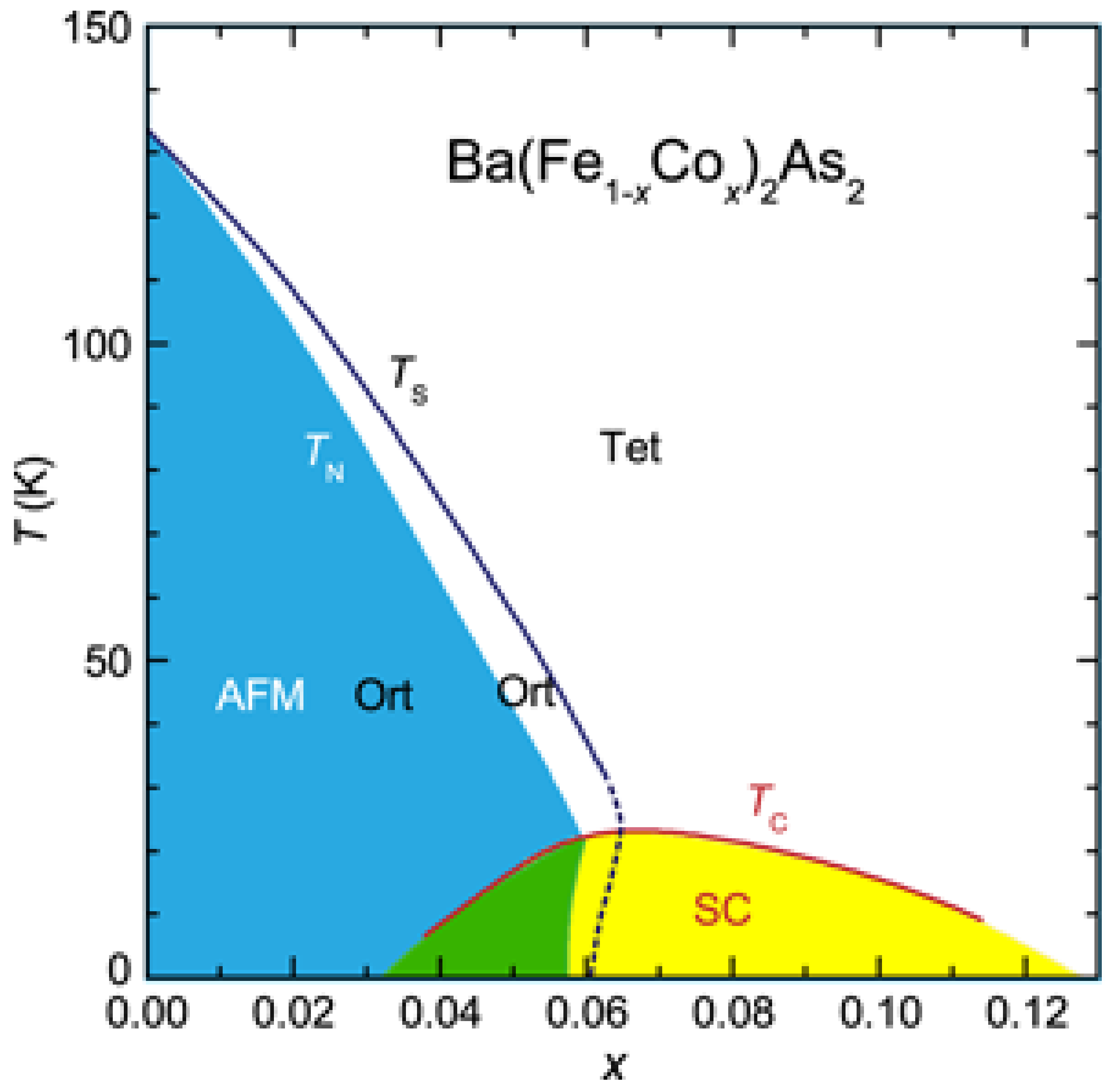


Science 307, 901

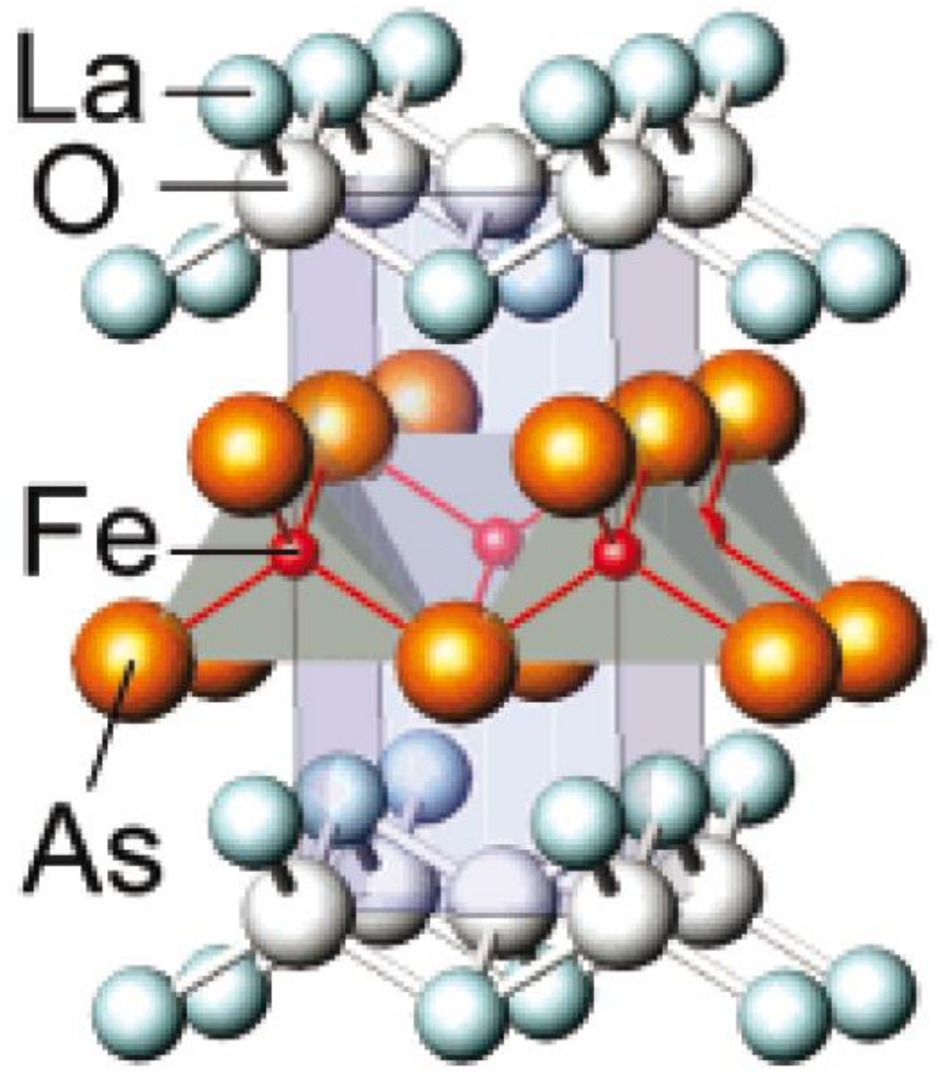
Norman et al, Adv. Phys. 54, 715 (2005)
http://www.msd.anl.gov/files/msd/cuprates-columbia.pdf

FE BASED SUPERCONDUCTORS

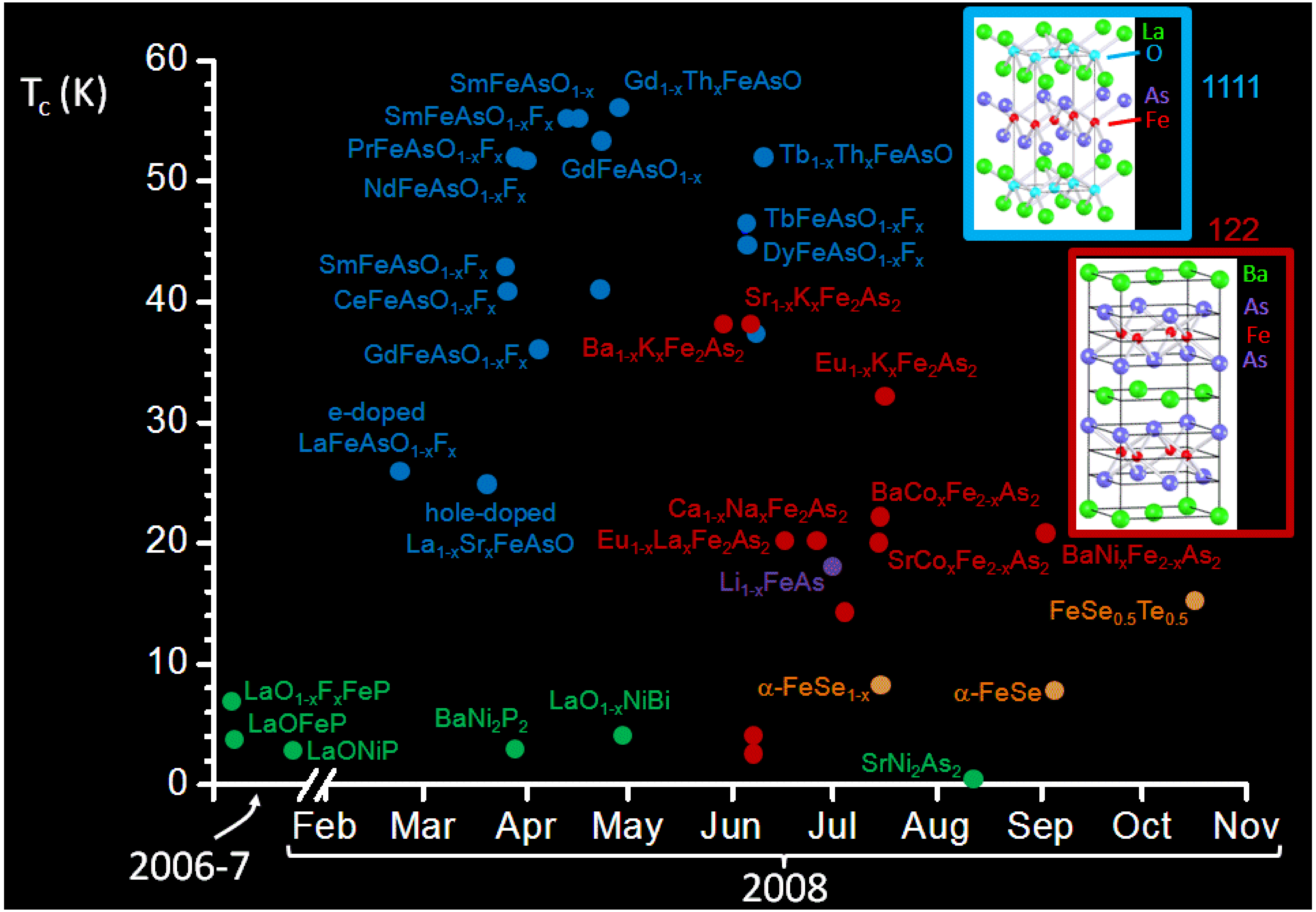
Many different families discovered, all sharing a Fe plane



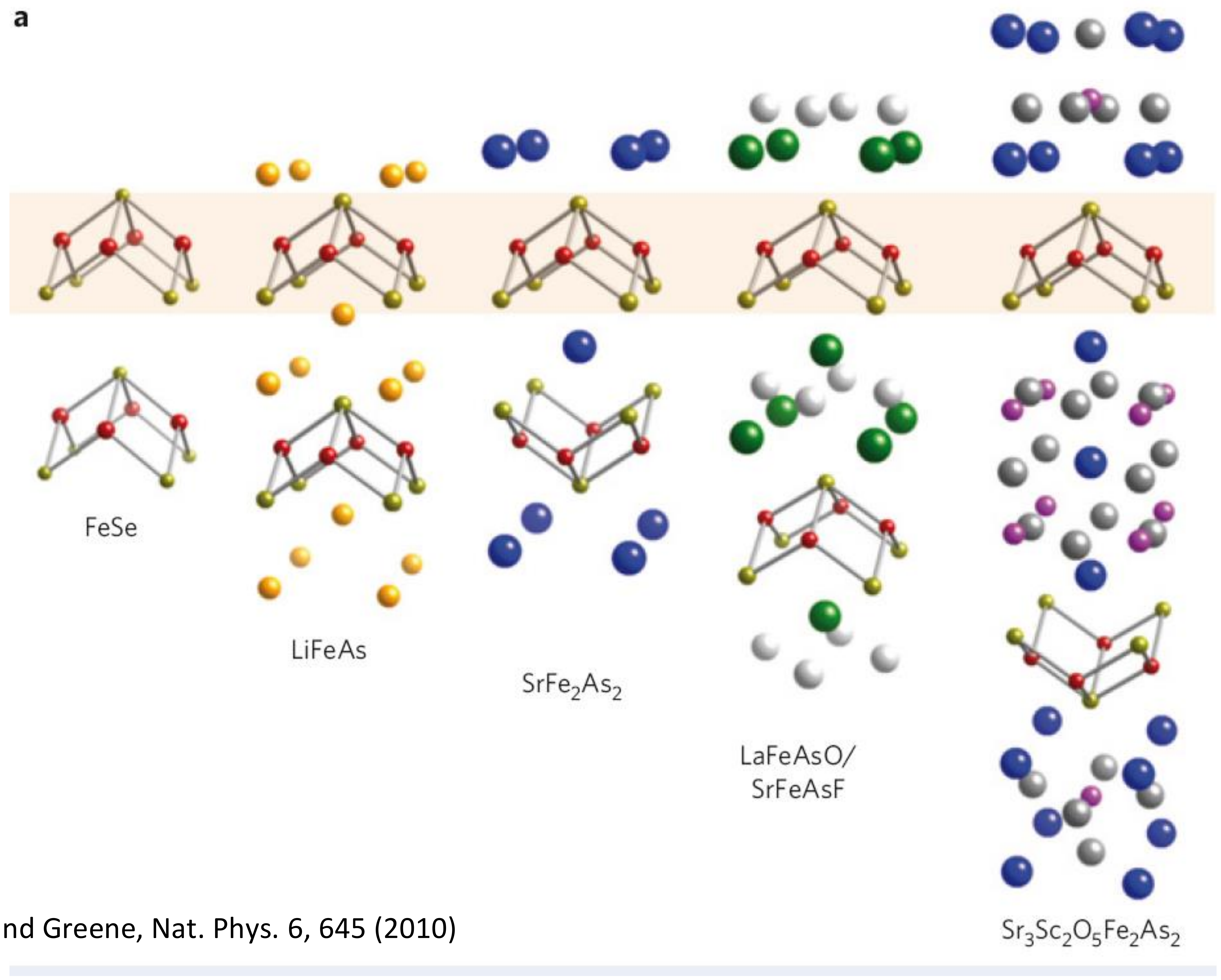
Nandi et al, PRL 104, 057006 (2010)



An important breakthrough in 2008 because it could help understand cuprates


T_c (K)
60
50
40
30
20
10
0
$SmFeAsO_{1-x}$
$Gd_{1-x}Th_xFeAsO$
$SmFeAsO_{1-x}F_x$
$PrFeAsO_{1-x}F_x$
$GdFeAsO_{1-x}$
$Tb_{1-x}Th_xFeAsO$
$NdFeAsO_{1-x}F_x$
$TbFeAsO_{1-x}F_x$
$DyFeAsO_{1-x}F_x$
$SmFeAsO_{1-x}F_x$
$CeFeAsO_{1-x}F_x$
$Sr_{1-x}K_xFe_2As_2$
$GdFeAsO_{1-x}F_x$
$Ba_{1-x}K_xFe_2As_2$
$Eu_{1-x}K_xFe_2As_2$
e-doped
$LaFeAsO_{1-x}F_x$
hole-doped
$La_{1-x}Sr_xFeAsO$
$BaCo_xFe_{2-x}As_2$
$Ca_{1-x}Na_xFe_2As_2$
$Eu_{1-x}La_xFe_2As_2$
$SrCo_xFe_{2-x}As_2$
$BaNi_xFe_{2-x}As_2$
$Li_{1-x}FeAs$
$FeSe_{0.5}Te_{0.5}$
$LaO_{1-x}F_xFeP$
$LaOFeP$
$LaONiP$
$BaNi_2P_2$
$LaO_{1-x}NiBi$
α-$FeSe_{1-x}$
α-FeSe
$SrNi_2As_2$
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
2006-7
2008
La
O
As
Fe
1111
122
Ba
As
Fe
As

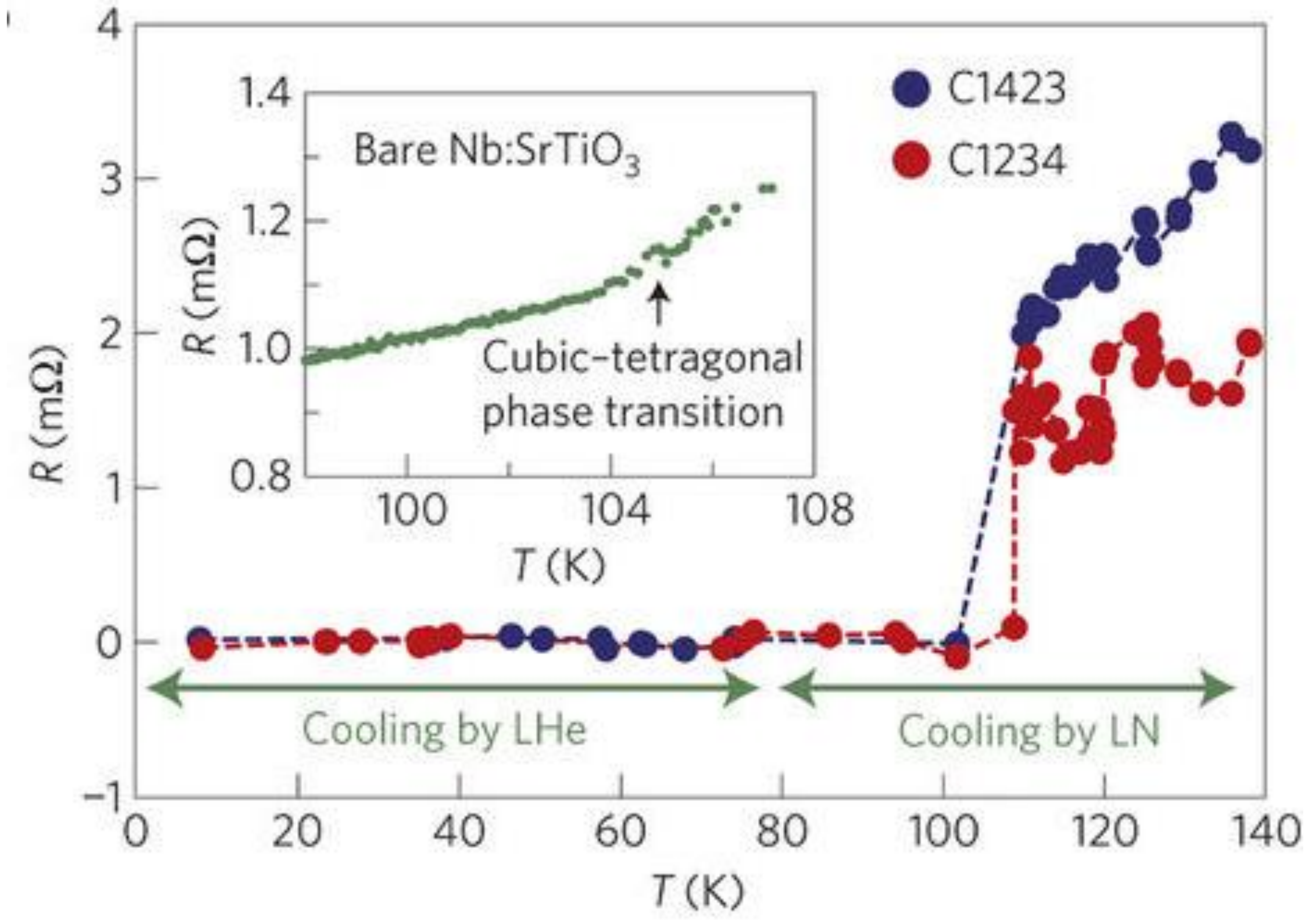
Fe-As or Fe-Se planes



Paglione and Greene, Nat. Phys. 6, 645 (2010)

HIGHEST T_C IN Fe SUPERCONDUCTORS

Single layer FeSe on doped $SrTiO_3$



Nature Materials 14, 285–289 (2015)

Fe BASED SUPERCONDUCTORS

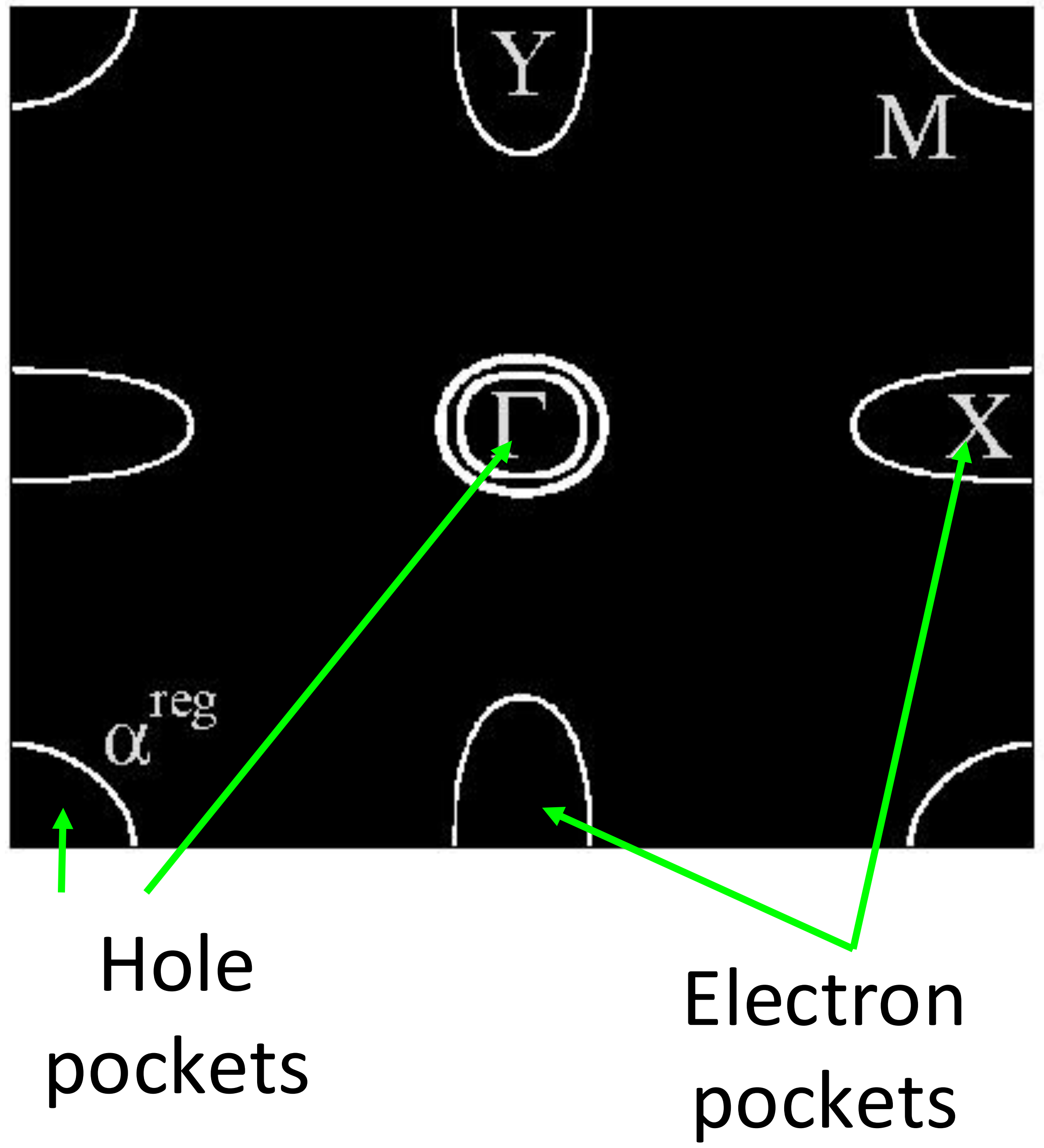
Differences with cuprates:

- The AF state is metallic (not Mott insulator): Hund metal (correlations).
- Multiorbital system (more than 1 gap possible)
- More 3dim-like (less anisotropy in c-direction)

Proposed mechanisms for superconductivity

- Spin fluctuations (π,0)
- Orbital fluctuations
- Possible role of nematicity

Extended BZ (1 Fe unit cell)



Gap symmetry

Family	Full gap	Highly anisotropic	Strong nodal
1111	$PrFeAsO_{1-y}$[52K] [293] SmFeAs(O,F)[55K] [295]	LaFeAs(O,F)[26K] [214] NdFeAs(O,F) [214]	LaFePO[6K] [203, 204, 294]
122	$(Ba,K)Fe_2As_2$[40K] [146, 236, 296, 242] $Ba(Fe,Co)_2As_2$ [OP,23K] [238, 208]	$Ba(Fe,Co)_2As_2$ [OD] [238, 241]* $Ba(Fe,Ni)_2As_2$ [297]* $Ba(Fe,Co)_2As_2$ [UD] [241]*	KFe_2As_2 [4K] [211, 309] $BaFe_2(As,P)_2$[OP,31K] [205, 149] $(Ba,K)Fe_2As_2$ [UD] [242]
111	LiFeAs [18K] [298, 258]		LiFeP [6K] [299]
11		Fe(Se,Te) [27K] [231, 246]	

arXiv:1106.3712

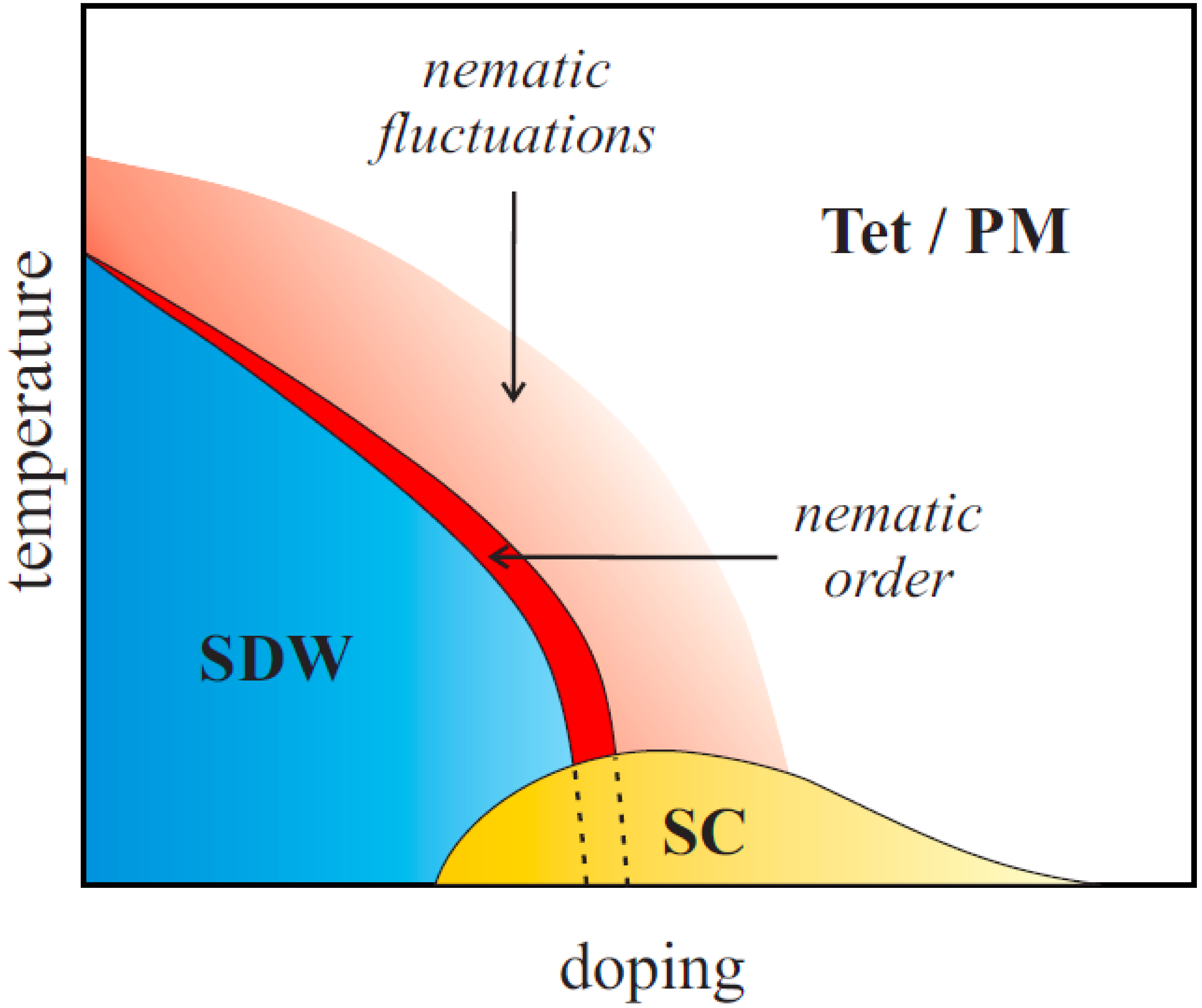
Fe BASED SUPERCONDUCTORS

Role of nematicity?

Breaks the tetragonal symmetry.

Related to magnetic fluctuations.

Nematic order also postulated for the pseudogap in cuprates.



http://www.ifsc.usp.br/coloquio/2013/Fernandes.pdf

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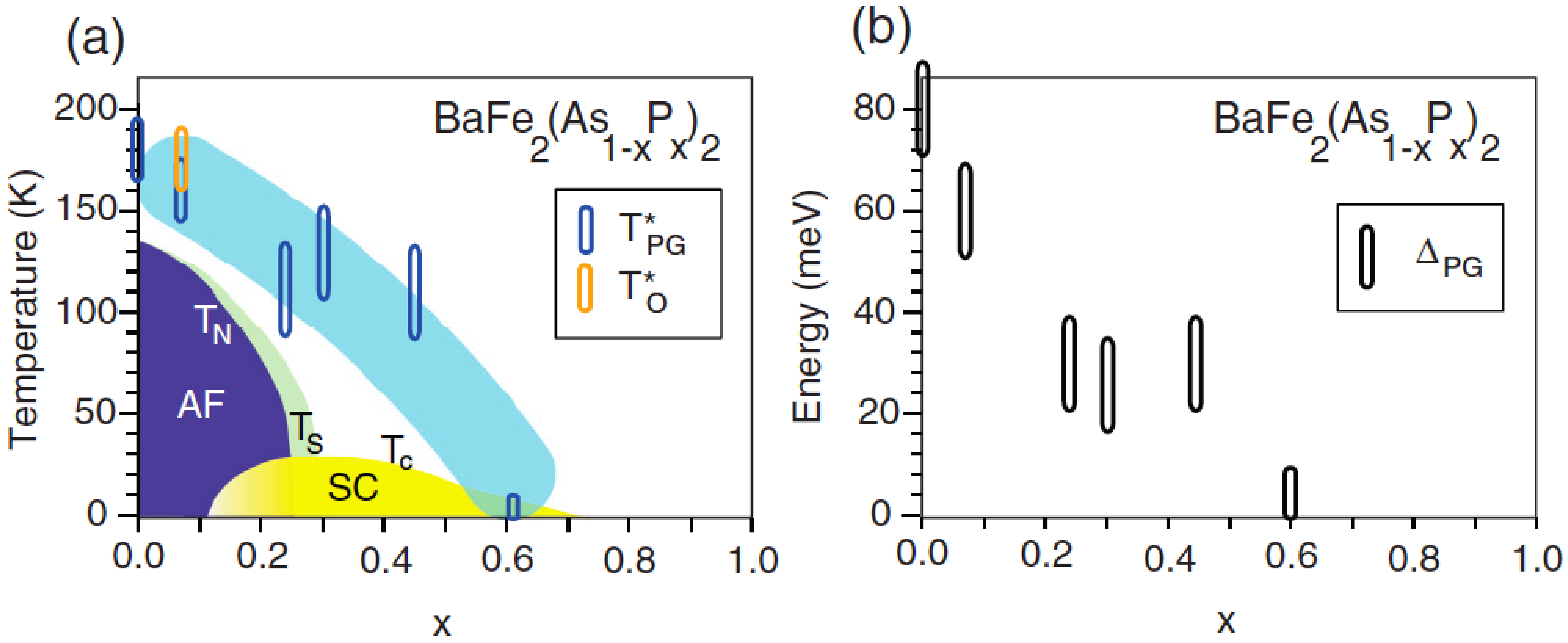
Fe BASED SUPERCONDUCTORS

Pseudogap

Gap opening due to spin fluctuations: Nat. Comm. **2**, 392 (2011)



Accompanied by orbital ordering. PRB 89, 045101 (2014)

MECHANISMS

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SPIN-FLUCTUATIONS

Spin fluctuations play the role of phonons (à la BCS).

The "normal" state can be described as a nearly AF Fermi liquid (unconventional Fermi liquid close to an AF instability).

Note: calculating the effective interaction is not trivial because vertex corrections can be important (Migdal's theorem doesn't apply)

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SPIN-FLUCTUATIONS

If the magnetic susceptibility has a peak at q (remember nesting) the interaction is also peaked at q and positive.

$$\Gamma_s(\boldsymbol{k},\boldsymbol{k}') = \frac{3}{2}U^2\frac{\chi_0(\boldsymbol{q})}{1-U\chi_0(\boldsymbol{q})}$$

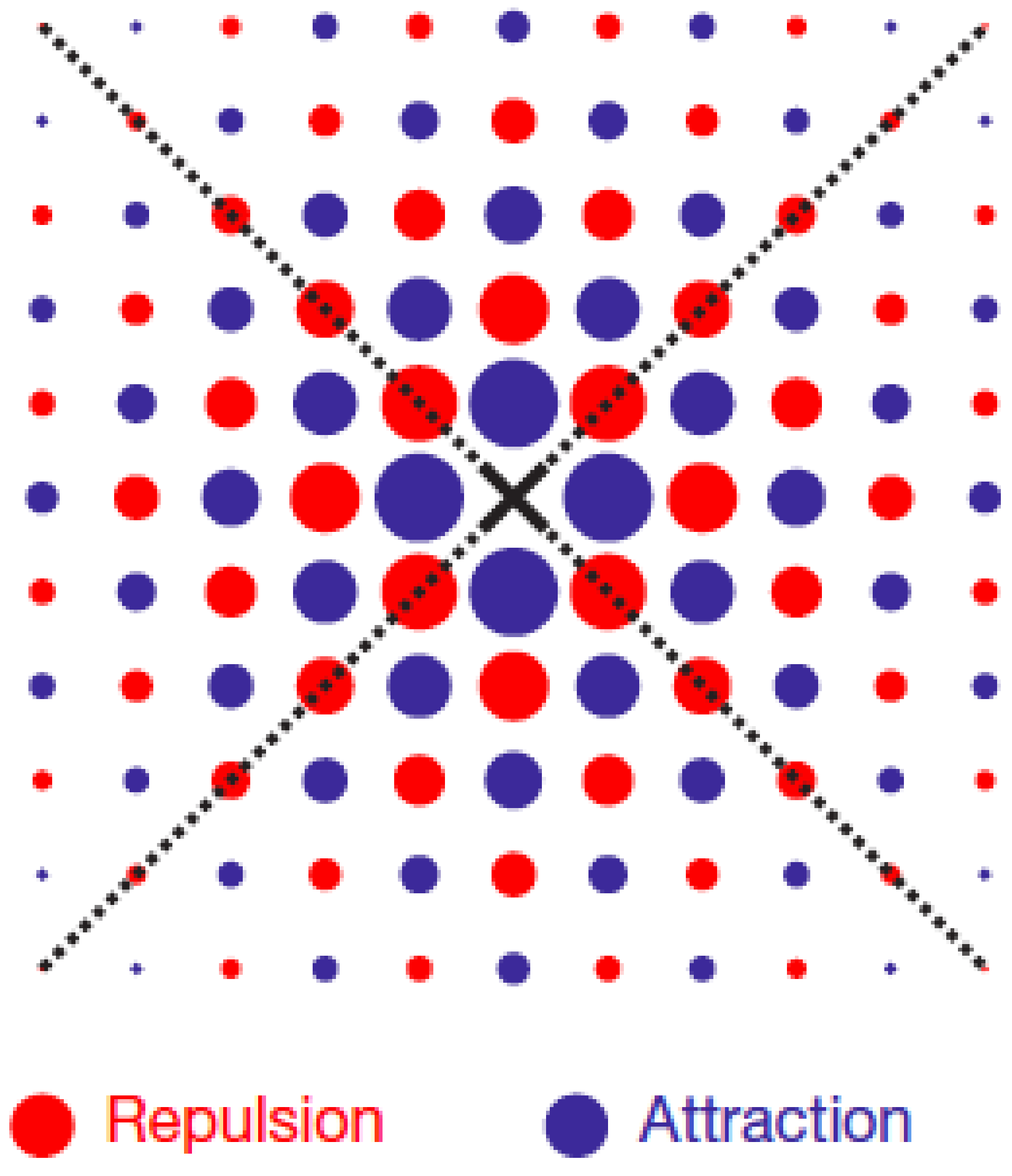
If you look at the interaction in real space

It changes sign with position!!

Attractive interaction by <u>space avoidance</u>

Scalapino, Phys. Rep. 1995
Hirshfield et al. 1106.3712

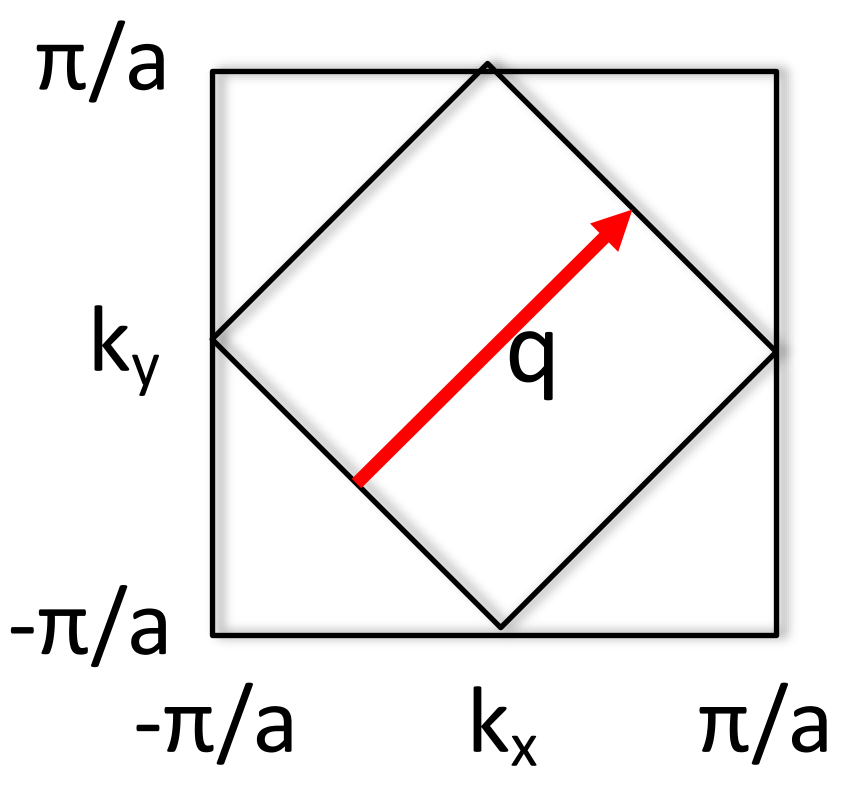
q=(π,π)



Nature 450, 1177 (2007)

SPIN-FLUCTUATIONS

Square lattice at half-filling:



At q=(π,π) the spin susceptibility is maximal (nesting)

$$V_{k,k'} = V_{k-k'} = V_q \propto \chi_q > 0 \qquad q = (\pi,\pi)$$

To fullfil the gap equation, you need an anisotropic gap such that

$$\Delta_k = -\frac{1}{\Omega}\sum_{kk'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \qquad \mathrm{sign}(\Delta_{\mathbf{k}}) = -\mathrm{sign}(V_{\mathbf{k},\mathbf{k}'})\mathrm{sign}(\Delta_{\mathbf{k}'})$$

d-wave

$$\boxed{\begin{aligned}\Delta_k &= \frac{\Delta_0}{2}(\cos k_x - \cos k_y)\\ \Delta_{k+(\pi,\pi)} &= \frac{\Delta_0}{2}(-\cos k_x + \cos k_y) = -\Delta_k\end{aligned}}$$

Scalapino, Phys. Rep. 1995
Hirshfield et al. 1106.3712

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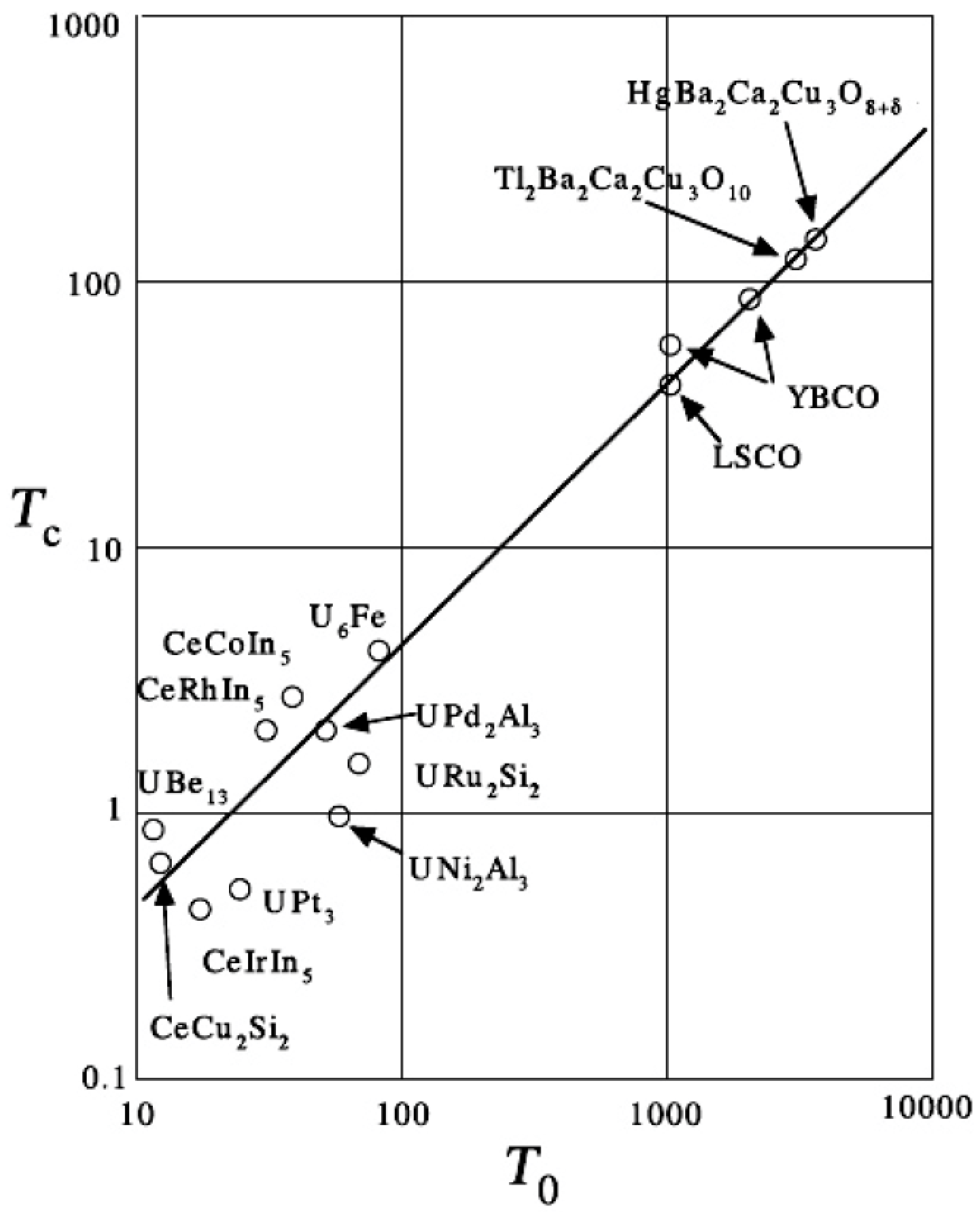
SPIN-FLUCTUATIONS

Indications that support spin-fluctuations:
Correlation between Tc and a typical energy scale of spin fluctuations.



In the Scalapino approach, the t-J model is the basis for understanding.

Moriya-Ueda, Rep. Prog. Phys. 66, 1299 (2003)

Scalapino, Rev. Mod. Phys. 84, 1383 (2012)

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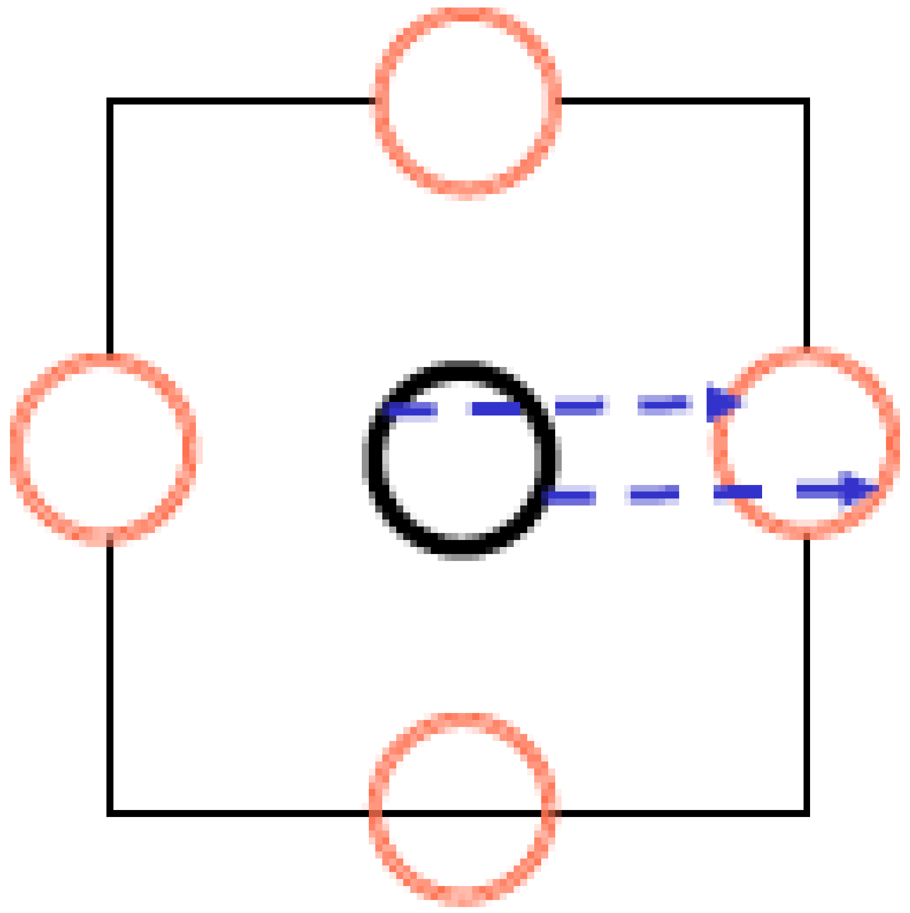
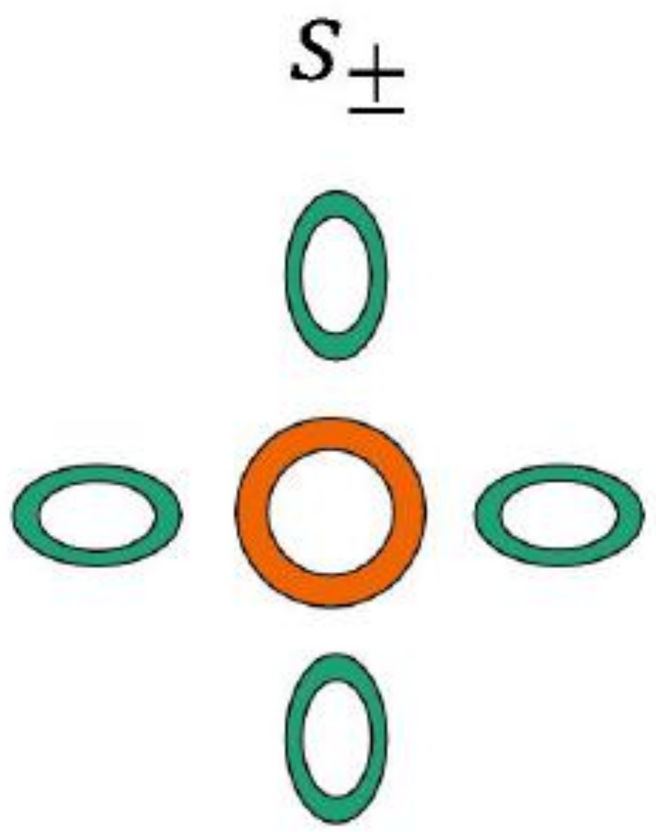
SPIN-FLUCTUATIONS

For a murtiorbital system (as Fe-superconductors)

For instance, spin fluctuations related to nesting between electron and hole pockets: q=(π,0)





arXiv:0901.4790

This would lead to an $s_{\pm}$

(as the order parameter averages to zero on the Fermi surface, you also get "Coulomb avoidance")

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ELECTRON-PHONON

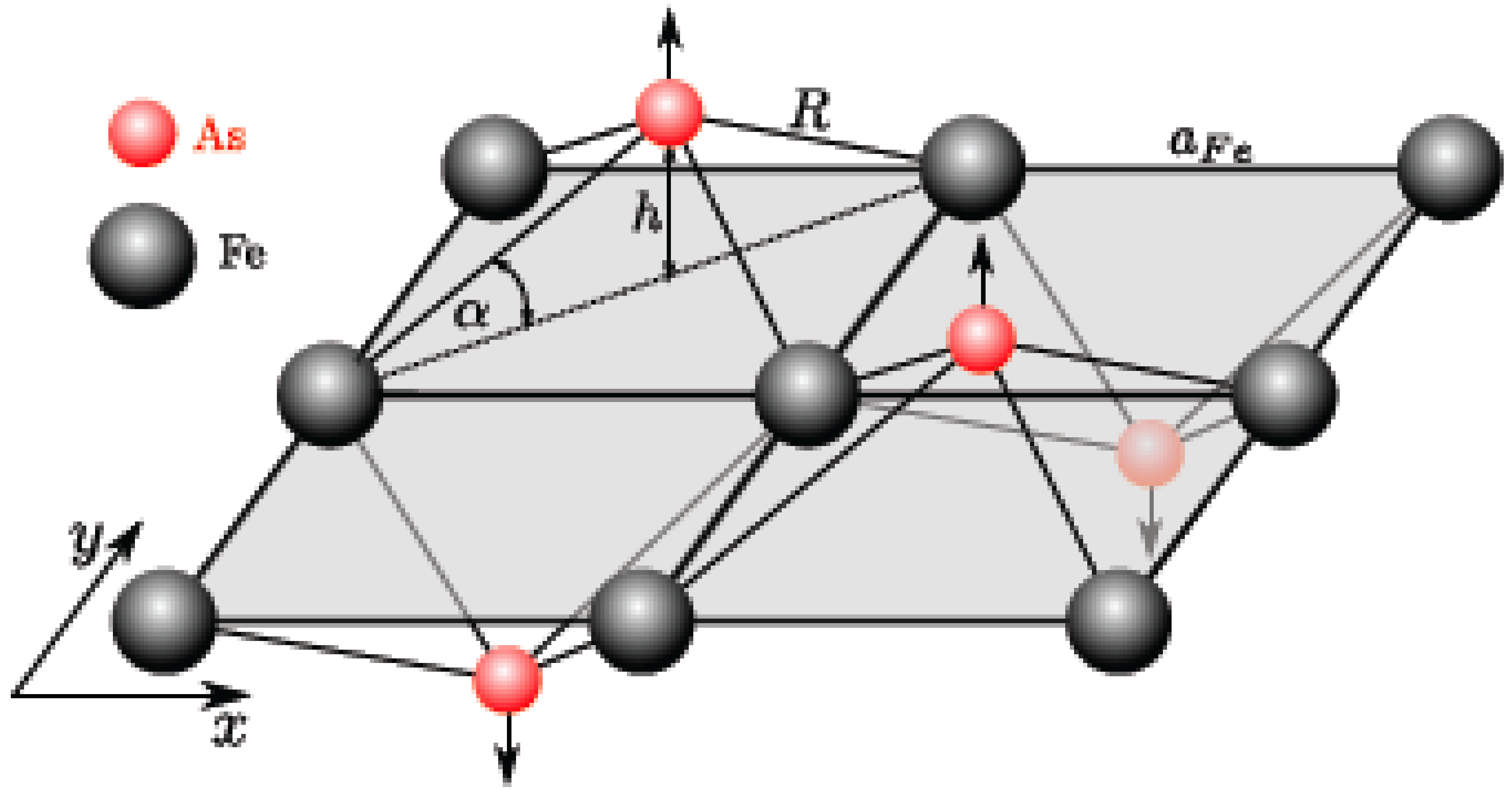
Not discarded!

For cuprates: Phys. Rev. Lett. 105, 257001

Maybe coupled to other degrees of freedom?

PRB 88, 165106

For Fe-superconductors



Spin-phonon?

Adv. in Cond. Matt. Phys. 2010, 164916 (2010)

Orbital fluctuations induced by electron-phonon.

PRL 104, 157001

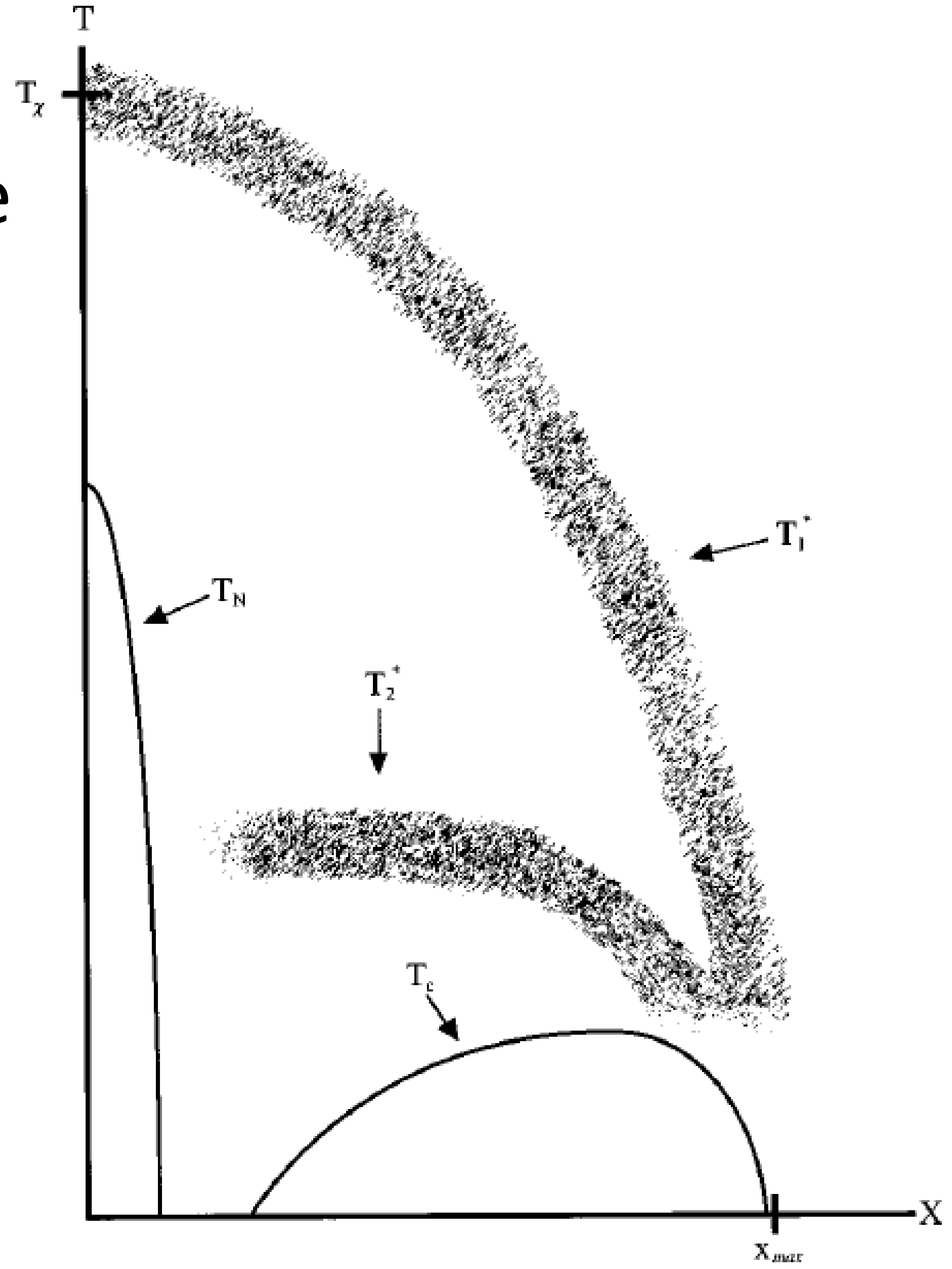
Interplay of many degrees of freedom may be relevant.

KINETIC ENERGY DRIVEN MECHANISM

Different energy scales involved for underdoped cuprates.

$$T_c < T^*_{pair} < T^*_{stripe}$$

Carlson et al, Chapter 21



PRB 56, 6120

At T^*_{stripe}: stripe formation. Stripes are rivers of charge where holes can move (gain kinetic energy in 1dim). In between, AF regions where the carriers are localized.

At T^*_{pair}: local pairing (spin-gap) within the 1DEGs (stripes). Pairs can tunnel to neighboring 1DEG (which in principle has another k_F). This way the system gains kinetic energy in a perpendicular direction as well. There is finite Δ but not phase coherence.

At T_c: phase coherence sets in (Josephson coupling between stripes) → superconductivity.

T*: Phase transition or crossover (no exp. evidence of phase transition)

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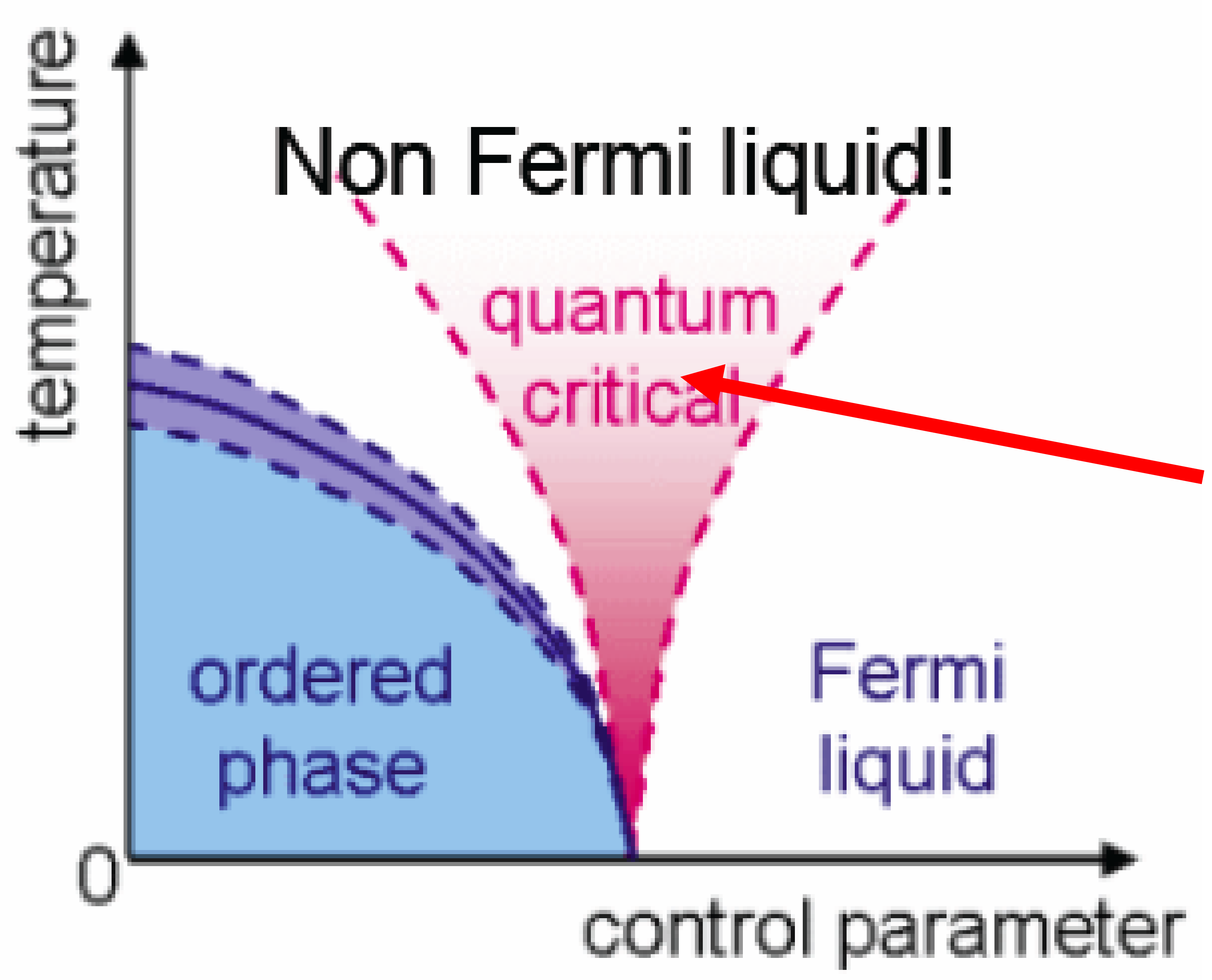
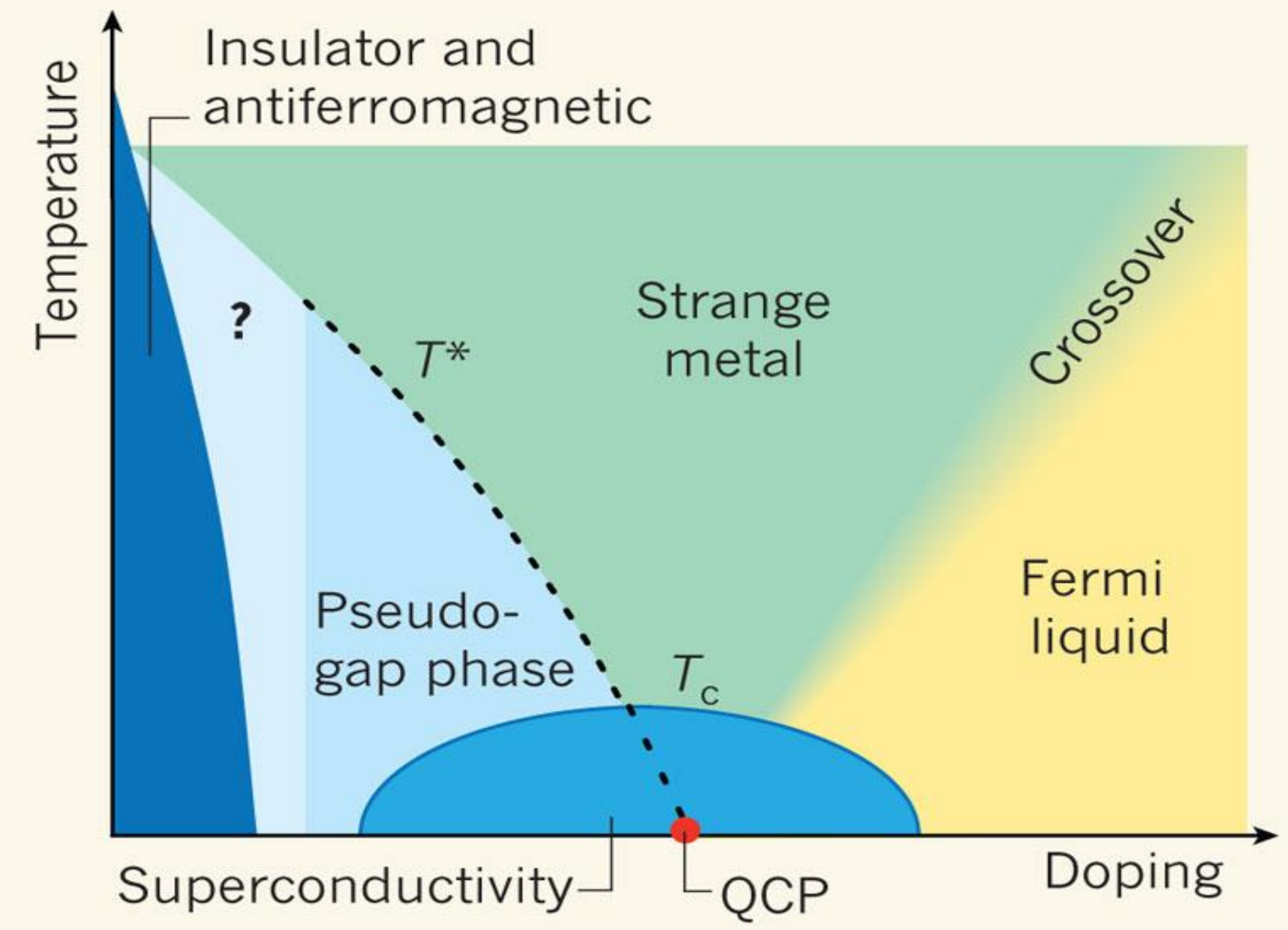
QUANTUM PHASE TRANSITIONS



Region in which thermal and quantum fluctuations are equally important

Transition at T=0 with consequences for T>0.

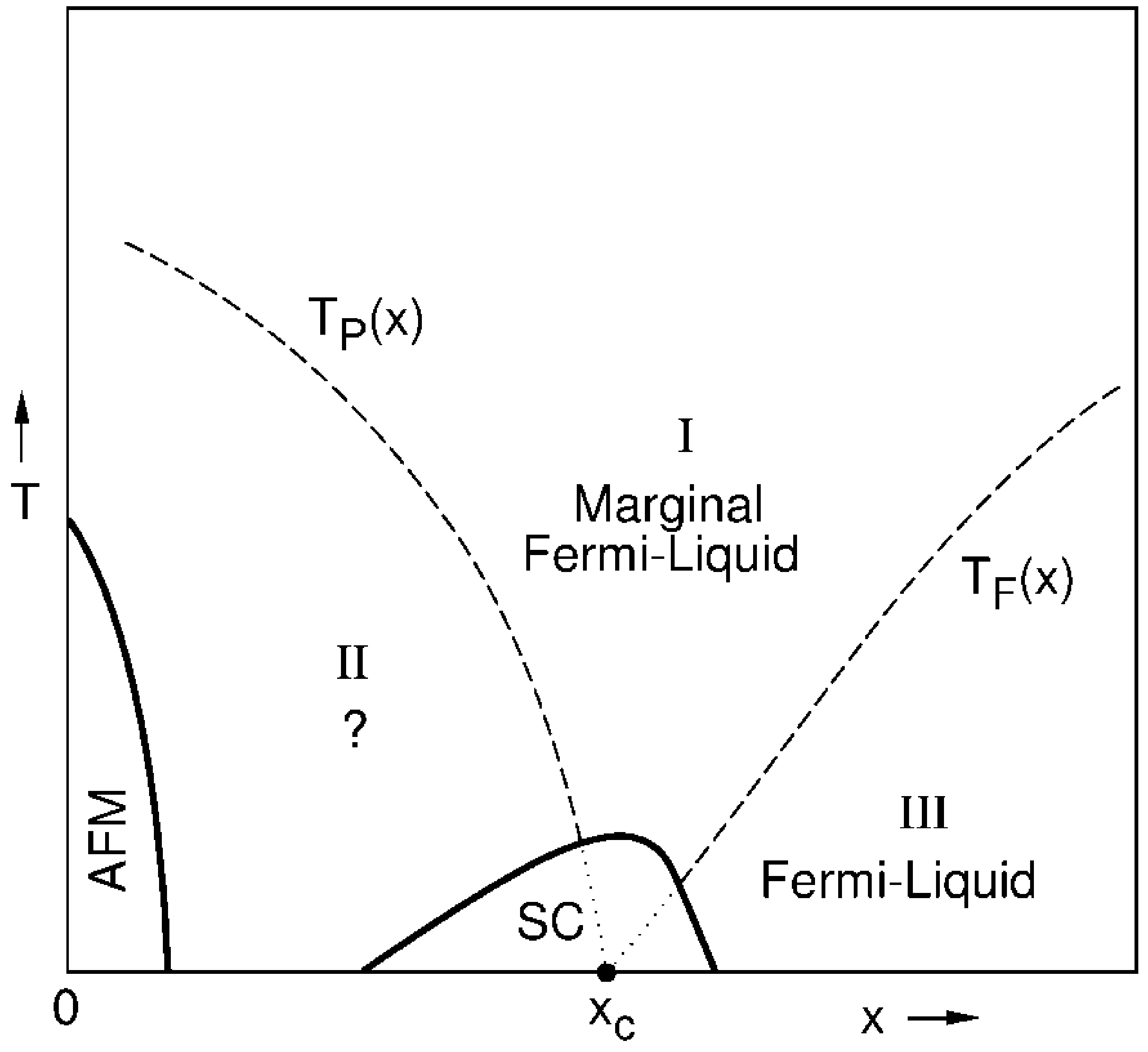
Divergence of time coherence.



Nature 468, 184–185

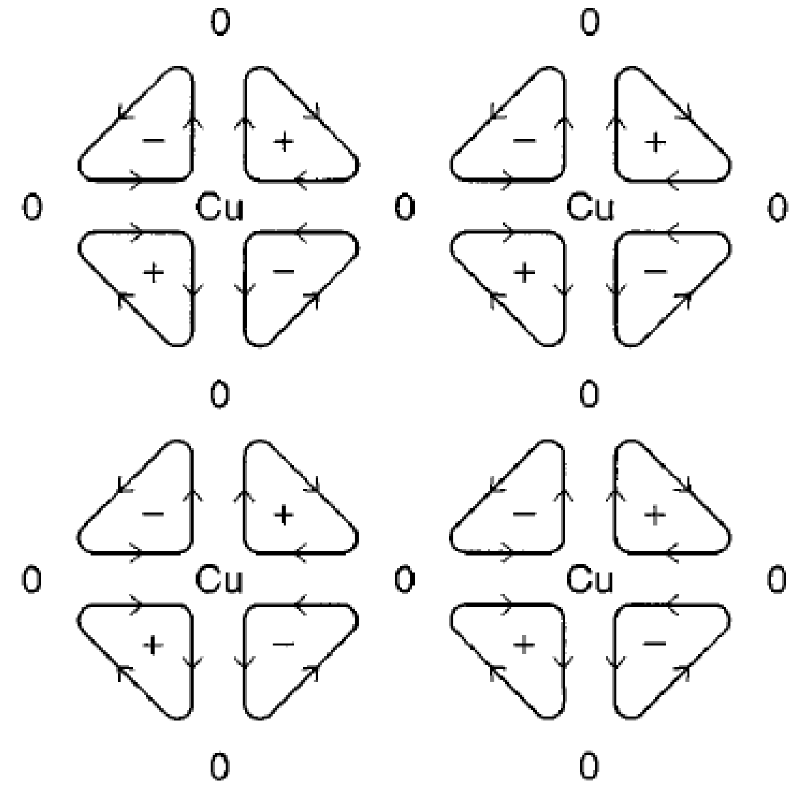
HIDDEN ORDER

Idea: the pseudogap phase is in fact a gapped phase (pseudo in experiment due to imperfections or dynamic effects)



PRL 83, 3538

Circulating currents, PRB 55, 14554



Charge order PRL 87, 056401

d-density wave PRB 63, 094503

Nematic phase arXiv:1404.0362

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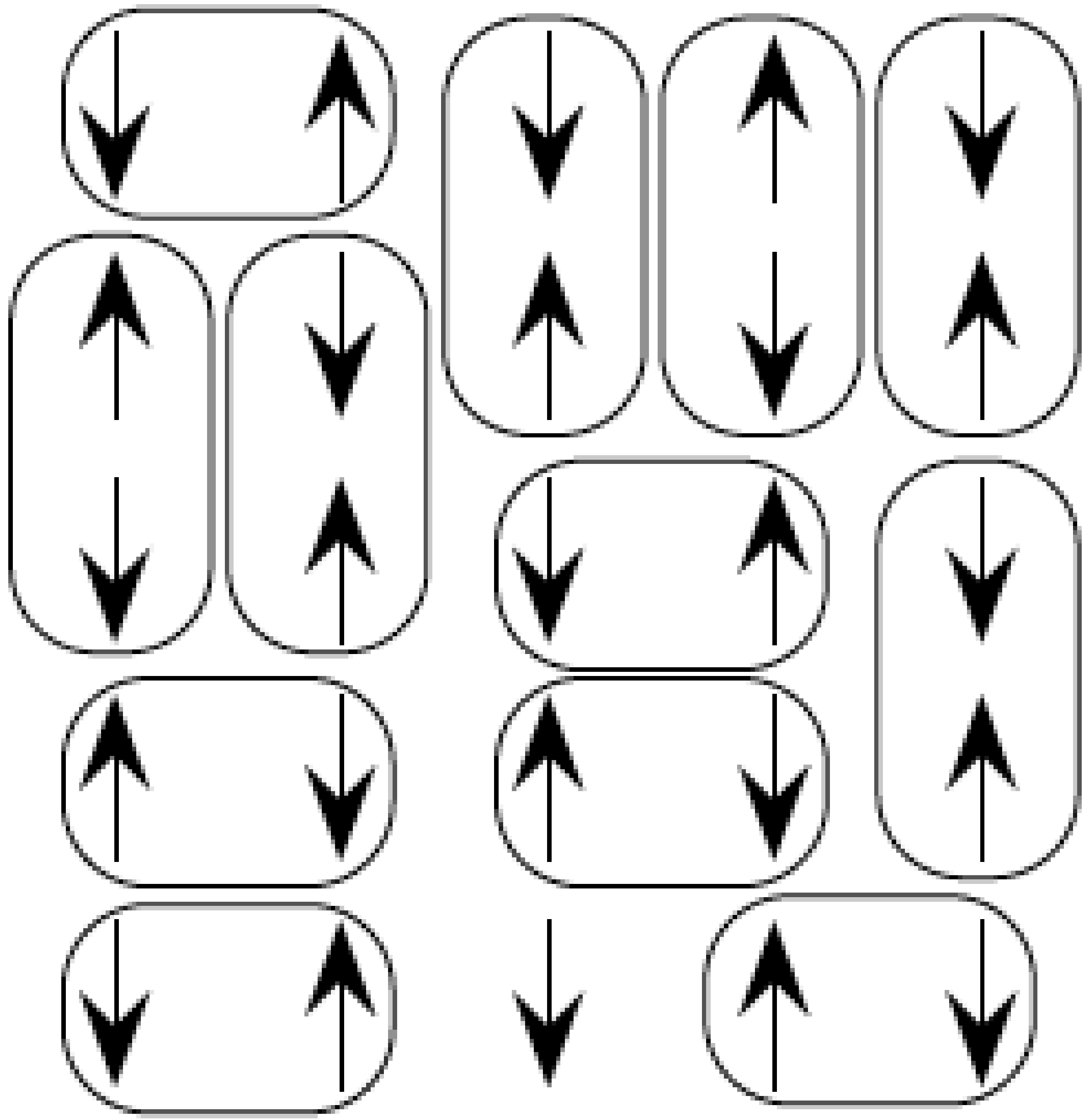
RESONATING VALENCE BONDS (Anderson)

Mottness from the start.

Pairing scale very large (related to T*)

Quantum fluctuations destroy long range order leading to a spin liquid of singlets.



http://www.msd.anl.gov/files/msd/cuprates-columbia.pdf

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POST HIGH T_C PRINCIPLES TO FIND SPC

MAZIN, NATURE 464, 183 (2010)

- Layered structures
- Carrier density should not be too high (compared to conventional metals)
- Transition metals of the fourth period (3d) are good
- Magnetism is essential
- Proper Fermi surface geometry is essential (in relation to spin excitations, nesting)

Corollary: work with solid state chemists (you need complex chemical compounds)

.... Really?

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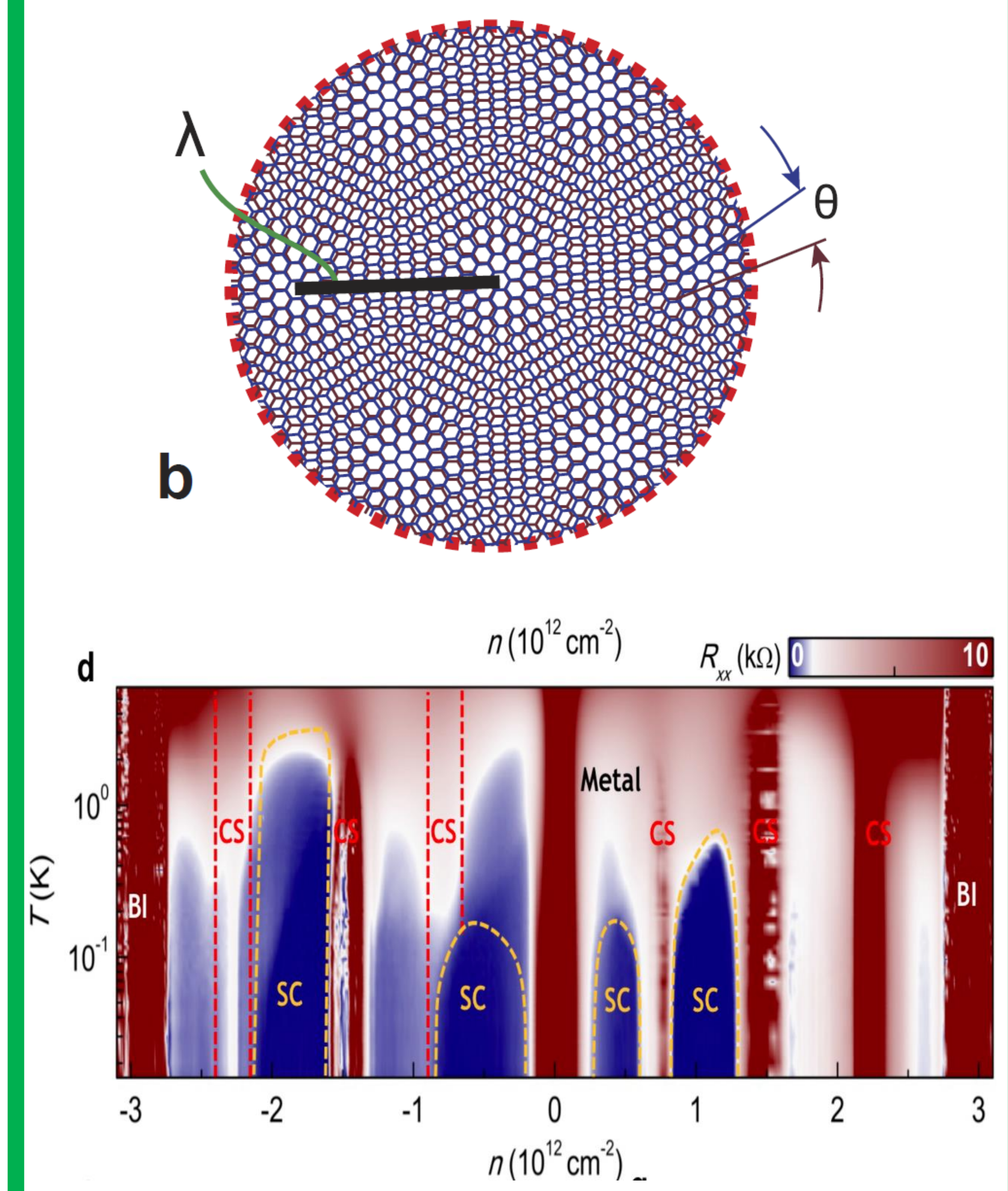
SUPERCONDUCTIVITY IN MULTILAYERED GRAPHENE (2018-)

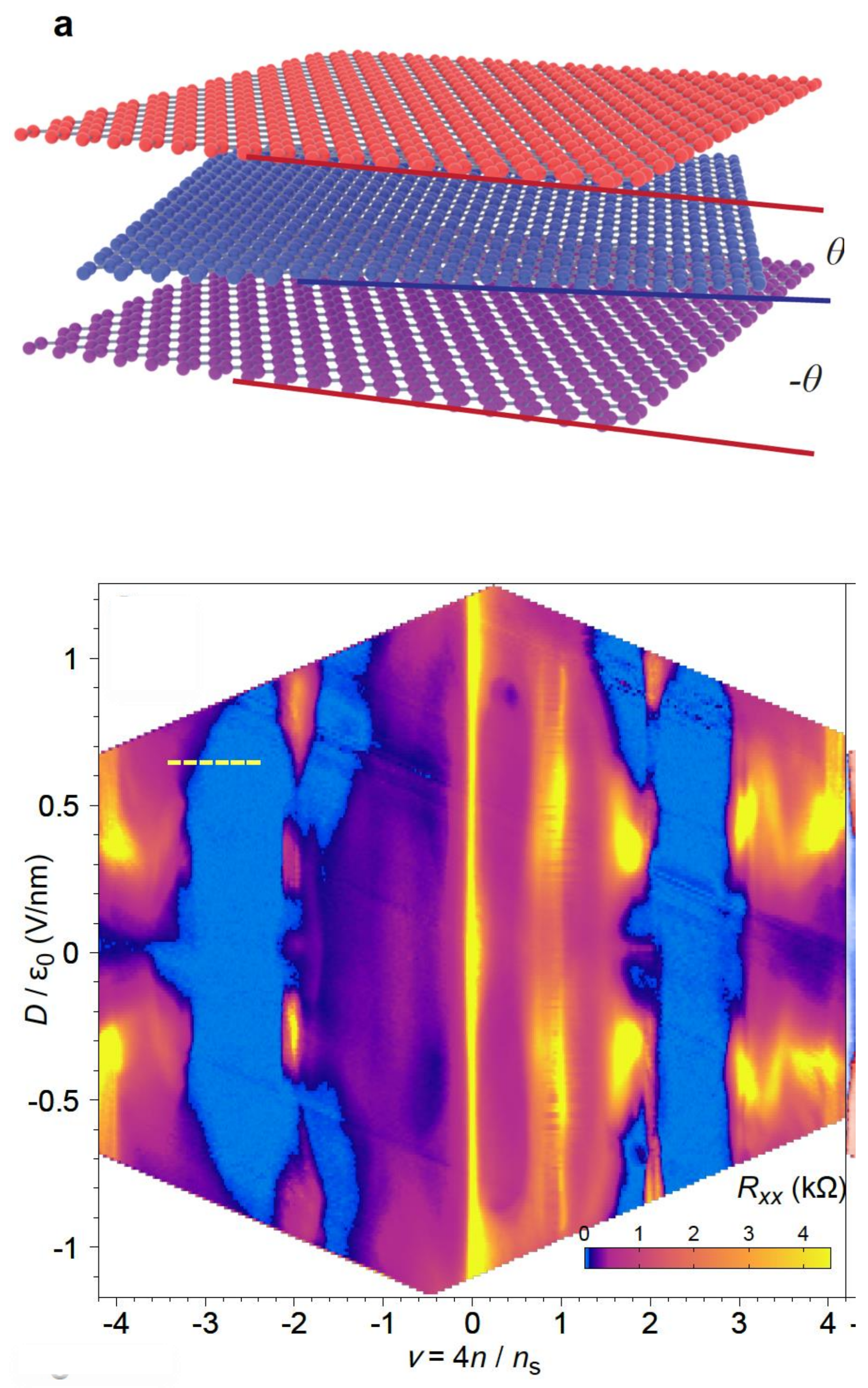
Moiré systems:

Twisted bilayer graphene



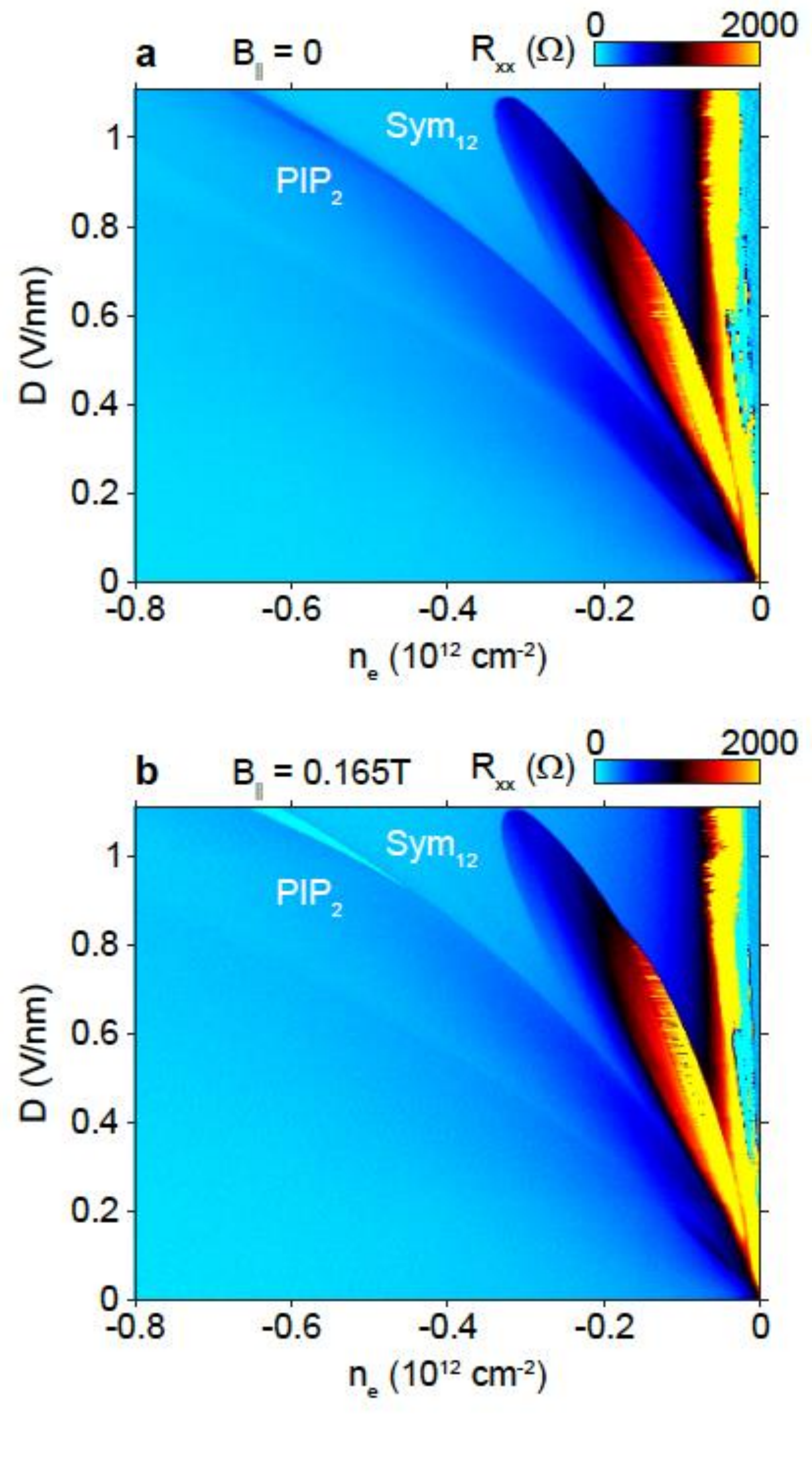
Cao et al, Nature 556, 80 (2018); Nature 556, 43 (2018)
Lu, et al Nature 574, 653 (2019)

Mirror-symmetric twisted trilayer, quatrilayer and pentalayer graphene



Park et al, Nature 590, 249 (2021)
Zhang et al, Science 377, 1538 (2022)
Park et al Nat. Mater. 21, 877 (2022)

Non-moiré



Bernal bilayer: Zhou et al Science 375, 774 (2022)

ABC trilayer: Zhou et al, Nature 598, 429; 598, 434 (2021)

Glue: interactions; quantum geometry
https://arxiv.org/pdf/2308.08248
Chiral and topological superconductivity

OUTLINE

- **Superconductivity**
  - Properties (zero resistivity, Meissner effect)
  - Understanding (pairing, BCS, Ginzburg-Landau)
  - Electron-phonon interaction (conventional superconductivity)
- **Unconventional superconductivity (unsolved)**
  - What are the new issues.
  - What are the proposals.

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