Emergence of Quantum Phases in Novel Materials

VIII Edition ICMM School

Fermi Liquid Theory

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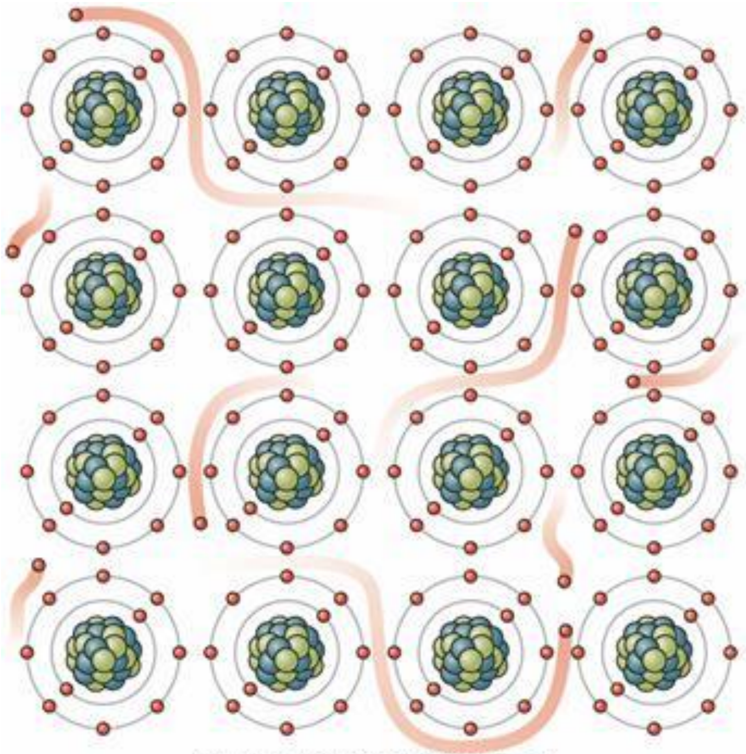
Outline

- ❑ The Fermi gas
- ❑ Why does band theory work? Concepts: Adiabaticity and quasiparticles
  - Definition of adiabaticity and quasiparticles
  - Electronic distribution function
  - Quasiparticle decay
  - Quasiparticle weight and spectral function
- ❑ Energy as a functional of the number of quasiparticles. Measurable quantities.
  - Renormalized mass: Specific heat and mass in ARPES
  - Interaction parameters : Spin susceptibility, resistivity
- ❑ Fermi liquid behavior and instabilities of the Fermi liquid

Some references

- Introduction to many body physics. Piers Coleman. Cambridge University Press.
- Theory of Quantum Liquids. Phillipe Nozieres, David Pines. Advanced Books Classics.
- A guide to Feynmann Diagrams in the Many Body problem. E.D. Mattuck. Dover Books on Physics
- Metal-insulator transitions. M. Imada, A. Fujimori, Y. Tokura. Rev. Mod. Phys. 70, 1039 (1998)

Why is band theory so successful?



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Interactions:

Electrons interact between them (and with the lattice)

Interaction not small vs kinetic energy

We cannot solve the interacting problem exactly

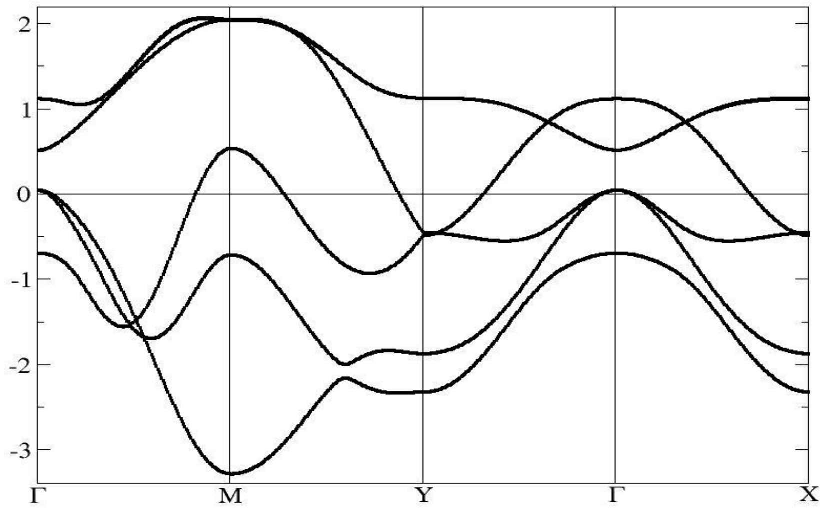
Approximations

Band theory: The basis of our understanding of solids

Band theory:

Basis of our understanding of solids

- Successful description
- Metals and insulators
- Dependence on temperature of measurable quantities (Cv, c, ..)



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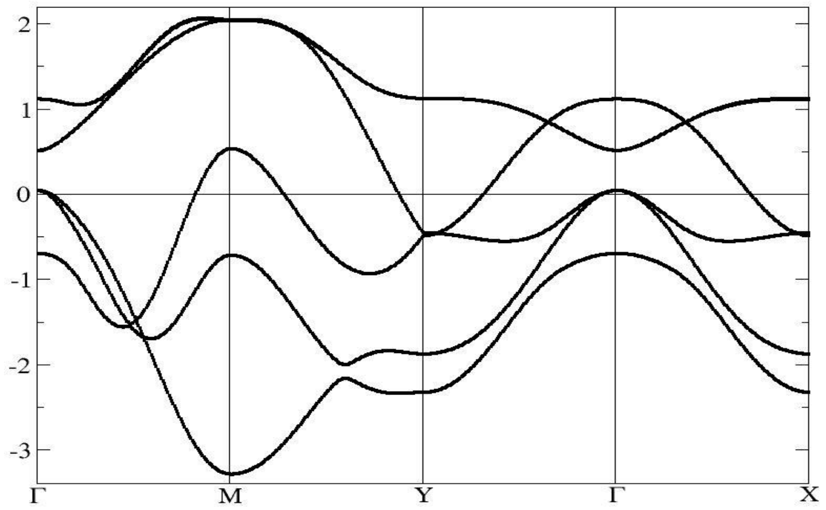

Interactions:

Electrons interact between them (and with the lattice)

Interaction not small vs kinetic energy

Interactions not included in the band picture beyond simple mean field

⬇

"Independent" electron model

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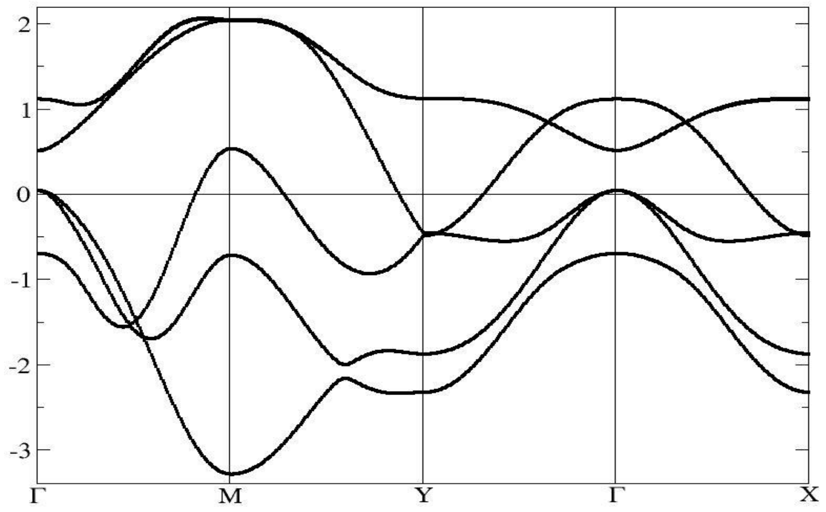

Interactions:

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Interaction not small vs kinetic energy

Interactions not included in the band picture beyond simple mean field

Why does band theory work?

Fermi liquid theory

Does it always work?

NO (Mott physics, Luttinger liquids …)

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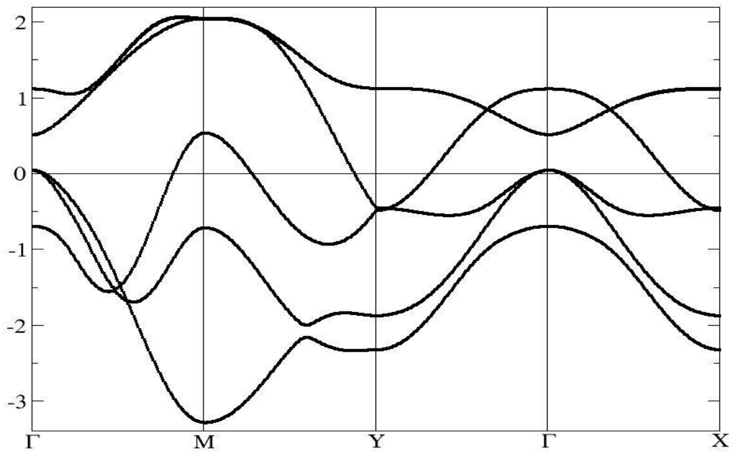
What do we mean with “independent” electron model?

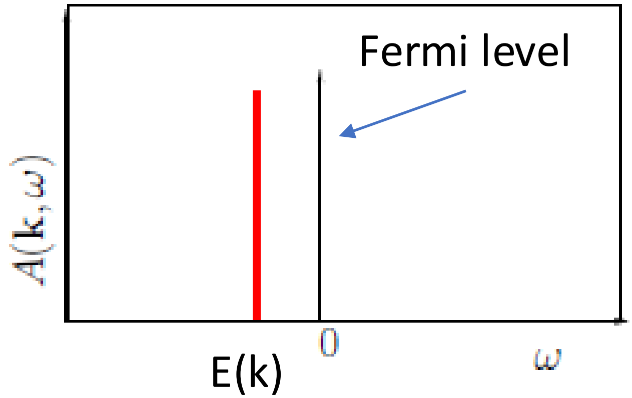
- Well defined eigenstates with momentum k E(k)
- E(k) filled following Fermi-Dirac. Fermi surface.
- If we add or remove an electron with momentum k to the ground state. The new electron occupies an eigenstate E(k).
- Energy levels are not modified, just filled. Rigid Band shift



The Spectral function

Distribution of excitations created when
an electron (a real electron) with momentum k
is added or removed from the system

- Non interacting system



Eigenstate, well defined energy
for a given momentum.

Infinite lifetime

In an independent electron system, if we add or remove

an electron with momentum k to the ground state

the new electron occupies an Eigenstate E(k)

In an independent electron system

the spectral function A(k, w) is a delta function and "determines the bands"

$$A(k,w)=\delta(\omega=E(k))$$

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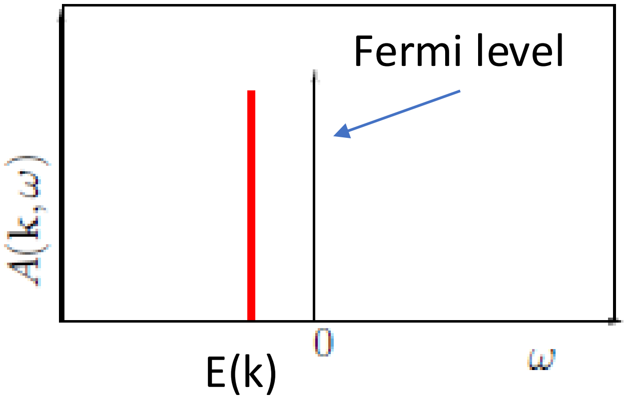

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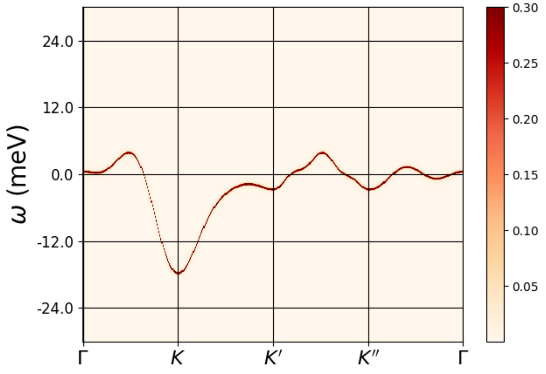


In an independent electron system

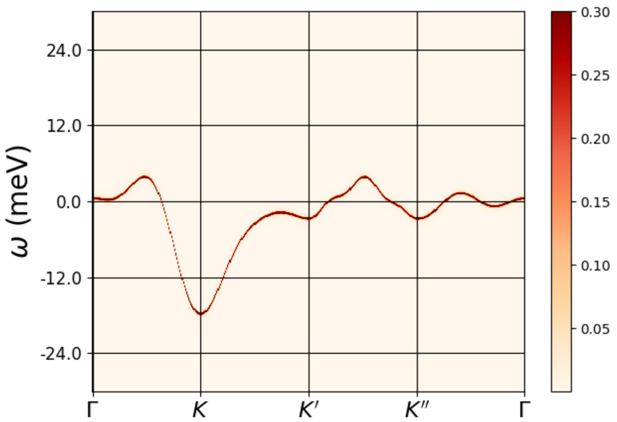
the spectral function A(k, w) is a <u>delta function</u> and "determines the bands"

$$A(k,w)=\delta(\omega=E(k))$$

What does it mean that the "independent" electron model works?

Independent electron model (Band theory):

- Well defined "single particle states" E(k)
- Rigid band shift.



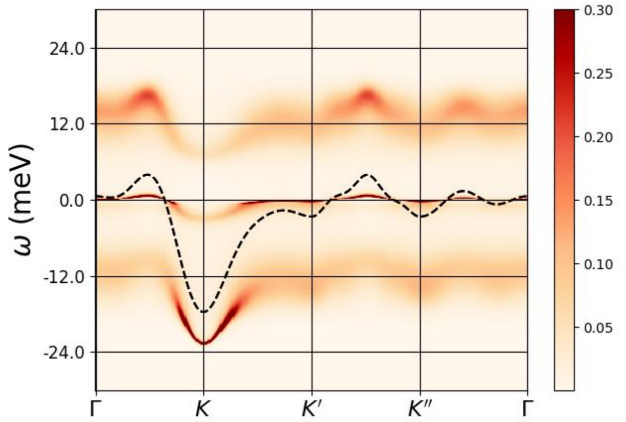
Interacting system:

- If we add electron with momentum k to ground state of a Fermi sea we expect the energy levels E(k) to be modified due the interactions, not just filled

Non rigid band shift expected

Electronic state E(k) expected to decay in excitations of the system

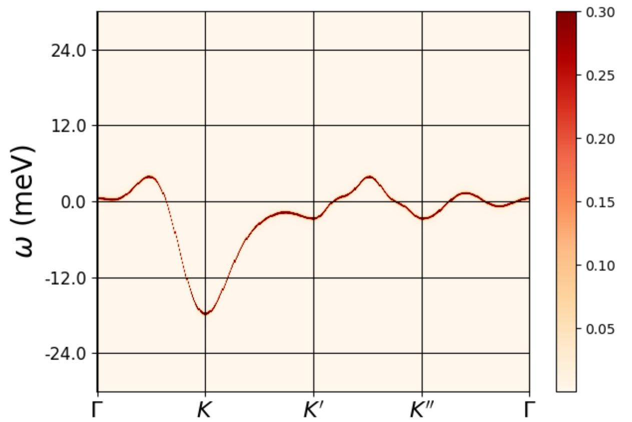
Description in terms of single particle states "does not seem possible".



What does it mean that the "independent" electron model works?

Independent electron model (Band theory):

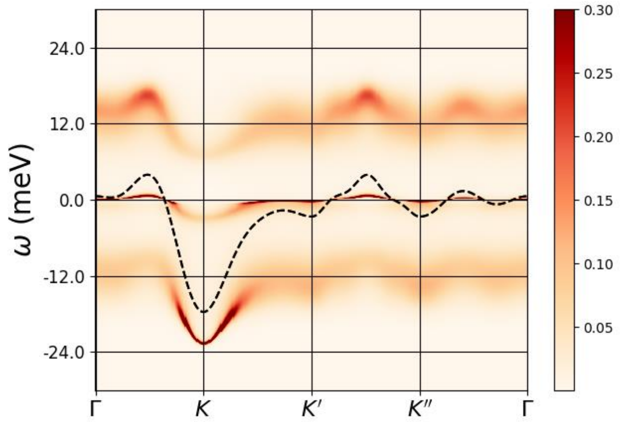
- Rigid band shift. Single particle states E(k)



Interacting system:

- Description in terms of single particle states does not seem possible.
- Electronic state expected to decay in excitations of the system.

Fermi liquid theory



- Pauli principle restricts the phase space for decay of excitations
- Description based on elementary excitations not on ground state

Fermi liquid theory: general idea

- It justifies the success of band theory
- Focus not in ground state but in low energy excitations. Introduces the concept of quasiparticles. Valid only at low energies and temperatures.
- Theory written in terms of parameters. It goes beyond particular models
- Phenomenological theory, but it can be justified with perturbation theory
- Perturbative theory, but not restricted to weak interactions.
- Proposed for 3-He: isotropic, no charge, short range interactions, in the continuum limit but it can be generalized to describe electrons in a metal
- Sometimes it fails. Strongly correlated electron systems. Non-Fermi liquid behavior

Adiabaticity and quasiparticles

Assume N interacting fermions

$$H = H_0 + V$$

H Interacting electronic Hamiltonian

(we do not know how to solve it)

Ground state $|\Psi^*_g>$

H_0 : Non interacting electronic Hamiltonian

(we know how to solve it)

Ground state $|\Psi_g>$

Interactions

See Coleman's book

Adiabaticity

Assume N interacting fermions

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H Interacting electronic Hamiltonian

(we do not know how to solve it)

Ground state $|\Psi^*_g>$

H_0 : Non interacting electronic Hamiltonian

(we know how to solve it)

Ground state $|\Psi_g>$

Interactions

Assume adiabaticity

$|\Psi^*_g>$ and $\Psi_g>$ adiabatically connected $\Rightarrow$ Effect of interactions V is perturbative (No phase transition)

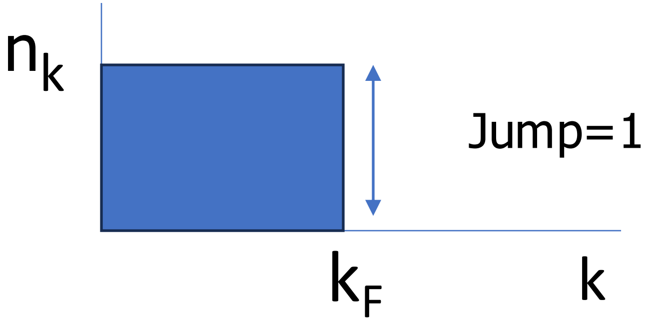
See Coleman's book

Introducing quasiparticles

- Assuming adiabaticity (perturbative effect of interactions) we will see that the system has elementary excitations with momentum k, spin ½ and charge **e** called quasiparticles $a^{+}_{k\sigma}$

(Assume there is no spin-orbit interaction)

- The quasiparticles $a^{+}_{k\sigma}$ follow a Fermi-Dirac distribution at T=0 (step). Fermi surface



- The quasiparticle energies $\varepsilon^*(K)$ are well defined at small T and small energy (with respect to Fermi level)

$\Gamma << \varepsilon^*(K)$ in 3D and 2D (But not in 1D)

Away from T=0 and/or from the Fermi Surface $\tau^{-1}=\Gamma$ is finite (excitation decay)

See Coleman's book

Introducing quasiparticles

- Assuming adiabaticity (perturbative effect of interactions) we will see that the system has elementary excitations with momentum k, spin ½ and charge **e** called quasiparticles $a^{+}_{k\sigma}$

The quasiparticles follow a Fermi-Dirac distribution and have well defined energy at small T and small energy

Distinguish:

Real electron $c^{+}_{k\sigma}$

Quasiparticle $a^{+}_{k\sigma}$

$$c^{+}_{k\sigma} \neq a^{+}_{k\sigma}$$

Not equal, but one to one correspondence when Fermi liquid works

- In an experiment (STM, ARPES) we add or remove a real electron

We probe the spectral function of the real electrons

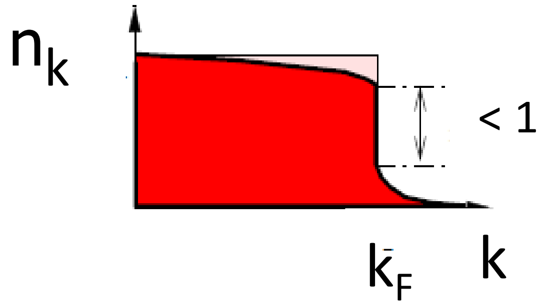
real electron

$c^{+}_{k\sigma}\,|\Psi^*_g>$

STM

$c_{k\sigma}\,|\Psi^*_g>$

ARPES



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From adiabaticity to quasiparticles

Assume N interacting fermions

$$H=H_0 + V$$

H Interacting electronic Hamiltonian

(we do not know how to solve it)

Ground state $|\Psi^*_g>$

H_0 : Non interacting electronic Hamiltonian

(we know how to solve it)

Ground state $|\Psi_g>$

Interactions

Assume adiabaticity

$|\Psi^*_g>$ and $\Psi_g>$ adiabatically connected ⇒ Effect of interactions V is perturbative (No phase transition)

See Coleman's book

From adiabaticity to quasiparticles

$|\Psi^*_g>$ ground state of N-particles of interacting system $H=H_0+V$

$|\Psi_g>$ ground state of N-particles of non-interacting system H_0

Imagine that we switch on the interactions very slowly in time starting from H_0 at $t=-\infty$

$t=-\infty$ $\quad$ t $\quad$ $t=0$

H_0 $\quad$ $H=H_0+V$

$|\Psi_g>$ $\quad$ $|\Psi^*_g>$

$H(t)=H_0+\lambda(t)V$

with $\lambda(t)=e^{-\delta|t|}$

δ arbitrarily small

Interaction completely switched on at t=0

See Coleman's book

From adiabaticity to quasiparticles

$|\Psi^*_g>$ ground state of N-particles of interacting system $\quad H=H_0+V$

$|\Psi_g>$ ground state of N-particles of non-interacting system $\quad H_0$

Imagine that we switch on the interactions very slowly in time starting from H_0 at $t=-\infty$

$t=-\infty \qquad t \qquad t=0$

$H_0 \qquad\qquad H=H_0+V$

$|\Psi_g> \qquad\qquad |\Psi^*_g>$

$H(t)=H_0+\lambda(t)V$

with $\lambda(t)=e^{-\delta|t|}$ $\quad$ δ arbitrarily small

Interaction completely switched on at t=0

Follow the evolution of the ground state from $|\Psi_g>$ to $|\Psi^*_g>$

$$|\Psi_g^*(t)>= V^{ev}\,|\Psi_g>$$

Adiabaticity: $|\Psi^*_g>$ and $|\Psi_g>$ are perturbatively connected

See Coleman's book

From adiabaticity to quasiparticles

$t=-\infty$ $\quad$ t $\quad$ $t=0$

N particles

H_0 $\qquad$ $H=H_0 + V$

$|\Psi_g>$ $\qquad$ $|\Psi^*_g>= V^{ev}\ |\Psi_g>$

We add an electron

(N+1particles (fermions))

$$|k\sigma>=C^+_{k\sigma}\ |\Psi_g>$$

Eigenstate
(Clean system)

The new particle added to an excited state with momentum **k** charge e and spin σ.

Ground state
non-interacting
system with N particles

See Coleman's book

From adiabaticity to quasiparticles

$t=-\infty$ $\quad$ t $\quad$ $t=0$

N particles

H_0 $\quad\longrightarrow\quad$ $H=H_0 + V$

$|\Psi_g>$ $\quad$ $|\Psi^*_g>= V^{ev}\ |\Psi_g>$

We add **an electron**
(N+1particles (fermions))

Switch on interactions and let the system evolve

$$|k\sigma>=C^+_{k\sigma}\ |\Psi_g> \qquad\longrightarrow\qquad |k\sigma*>=V^{ev}|k\sigma>= V^{ev}\, C^+_{k\sigma}\ |\Psi_g>$$

Eigenstate

The new particle added to an excited state with momentum **k** charge e and spin σ.

Ground state **non-interacting** system with N particles

State which results from the evolution of $|k\sigma>$

See Coleman's book

From adiabaticity to quasiparticles

$t=-\infty$ $\quad$ t $\quad$ $t=0$ $\quad$ $V^{ev} c^{\dagger}_{k\sigma} V^{ev\dagger}$

N particles

H_0 $\quad$ $H=H_0 + V$

$|\Psi_g>$ $\quad$ $|\Psi^*_g>= V^{ev} \; |\Psi_g>$

We add **an electron**

(N+1particles (fermions))

Switch on interactions and let the system evolve

$$|k\sigma>=C^{+}_{k\sigma} \; |\Psi_g>$$

The new particle added to an excited state with momentum **k** charge e and spin σ.

$$|k\sigma*> = V^{ev}|k\sigma>= V^{ev} \, C^{+}_{k\sigma} \; |\Psi_g>$$

$$|k\sigma*> = V^{ev} \, C^{+}_{k\sigma} \, (V^{ev})^{-1} \; |\Psi^*_g>$$

$$|k\sigma*> \neq C^{+}_{k\sigma} \; |\Psi*_g>$$

See Coleman's book

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From adiabaticity to quasiparticles

$t=-\infty$ $\quad$ t $\quad$ $t=0$ $\quad$ $V^{ev} c^{\dagger}_{k\sigma} V^{ev\dagger}$

N particles

H_0 $\qquad$ $H=H_0+V$

$|\Psi_g\rangle$ $\qquad$ $|\Psi^*_g\rangle = V^{ev}\,|\Psi_g\rangle$

We add **an electron**
(N+1particles (fermions))

Switch on interactions and let the system evolve

$|k\sigma\rangle = C^{+}_{k\sigma}\,|\Psi_g\rangle$

$|k\sigma*\rangle = V^{ev}|k\sigma\rangle = V^{ev}\,C^{+}_{k\sigma}\,|\Psi_g\rangle$

$|k\sigma*\rangle = V^{ev}\,C^{+}_{k\sigma}\,(V^{ev})^{-1}\,|\Psi^*_g\rangle$

Same spin, momentum and charge
(Adiabaticity)

$|k\sigma*\rangle \neq C^{+}_{k\sigma}\,|\Psi*_g\rangle$

Interactions conserve charge, momentum and spin
(Assume there is no spin-orbit interaction)

See Coleman's book

From adiabaticity to quasiparticles

$t=-\infty$ $\qquad$ t $\qquad$ $t=0$ $\qquad$ $V^{ev} c^{\dagger}_{k\sigma} V^{ev\dagger}$

N particles

H_0 $\qquad$ $H=H_0+V$

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Same spin, momentum and charge

(Adiabaticity)

$$|k\sigma*\rangle \neq C^{+}_{k\sigma}\,|\Psi*_g\rangle$$

Create an excitation with charge e,

momentum k and spin σ

$$|k\sigma*\rangle = a^{+}_{k\sigma}\,|\Psi*_g\rangle$$

See Coleman's book

From adiabaticity to quasiparticles

We add an electron $C^{+}_{k\sigma}$

with momentum k and spin σ

$$|k\sigma*\rangle = a^{+}_{k\sigma}\,|\Psi*_{g}\rangle$$

Create an excitation with momentum k and spin σ in the interacting state

Quasiparticle

Quasiparticle: Elementary excitation of the interacting system with momentum k, spin 1/2, and charge

$$|k\sigma*\rangle \neq C^{+}_{k\sigma}\,|\Psi*_{g}\rangle$$

Quasiparticles and real electrons are not the same but they have a one to one connection

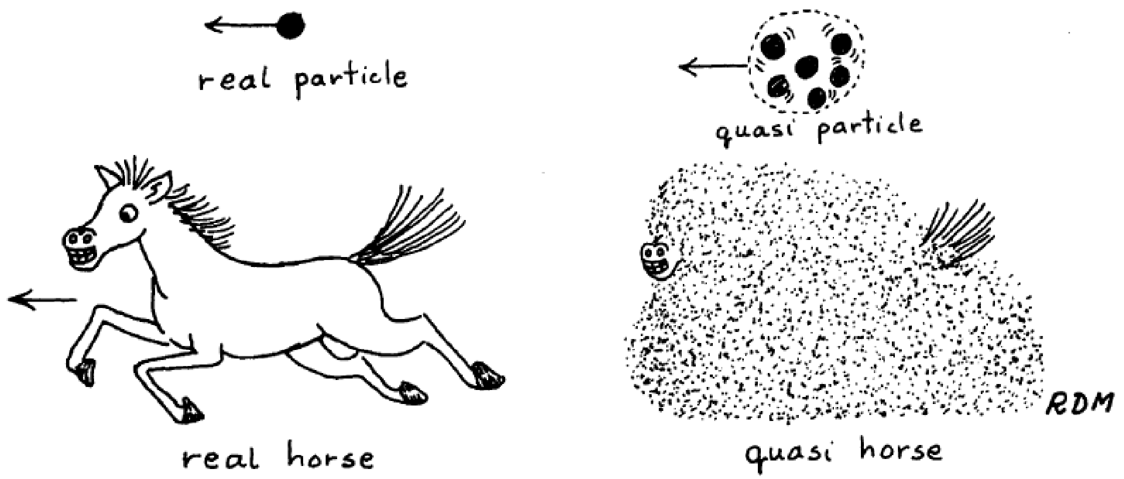
See Coleman's book

From adiabaticity to quasiparticles

We add an electron $C^{+}_{k\sigma}$
with momentum k and spin σ

$$|k\sigma*\rangle = a^{+}_{k\sigma}\,|\Psi*_{g}\rangle$$

Create an excitation with momentum k and spin σ in the interacting state → Quasiparticle



Remember adiabaticity

Mattuck's book

Quasiparticle: Elementary excitation of the interacting system with momentum k, spin 1/2, and charge

$$|k\sigma*\rangle \neq C^{+}_{k\sigma}\,|\Psi*_{g}\rangle$$

Quasiparticles and real electrons are not the same but they have a one to one connection

See Coleman's book

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Quasiparticles: Fermi Surface

Existence and definition of quasiparticles

$$|k\sigma *\rangle = a^{+}_{k\sigma}\,|\Psi *_g\rangle$$

Quasiparticles and real electrons
are not the same

$c^{\dagger}_{k\sigma}|\Psi *_g\rangle \longrightarrow$ $a^{\dagger}_{k\sigma}$ Quasiparticle

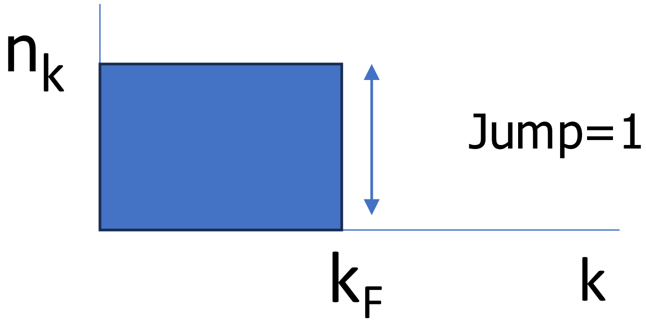
$c_{k\sigma}|\Psi *_g\rangle \longrightarrow$ $a_{k\sigma}$ Quasihole

Elementary excitation
of the interacting system
with momentum k,
spin 1/2, and charge

Fermionic excitations

Fermi-Dirac distribution

Fermi surface



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Quasiparticles: Fermi Surface and decay

Non-interacting system

$|k\sigma\rangle = C^{+}_{k\sigma}\,|\Psi_g\rangle$ Eigenstate (Infinite lifetime)

Interacting system

$|k\sigma*\rangle = a^{+}_{k\sigma}\,|\Psi*_g\rangle$ **Not an Eigenstate** $\longrightarrow$ Finite lifetime $\tau = \Gamma^{-1}$ (width level)

Stability of the quasiparticle requires $\Gamma \ll \varepsilon$ Decay rate of the quasiparticle Much smaller than its energy

See Coleman's book

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Quasiparticle decay

A quasiparticle with $k>k_F$ cannot decay into an occupied state below the FS with $k<k_F$

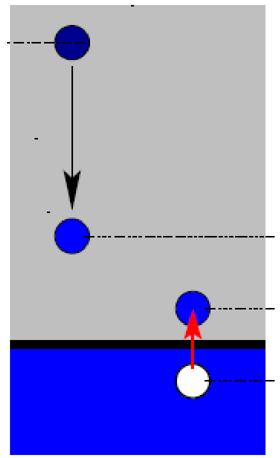
Decay of quasiparticles conserves momentum, charge & spin

$a^\dagger_{k\sigma}$

2 particles and 1 hole $\longrightarrow$ $a^\dagger_{k1\sigma}a^\dagger_{k2\sigma}a_{k3\sigma}$

3 particles and 2 holes $\longrightarrow$ $a^\dagger_{k1\sigma}a^\dagger_{k2\sigma}a^\dagger_{k4}a_{k3\sigma}a_{k3\sigma}$

…



Decay of one quasiparticle
3-body process

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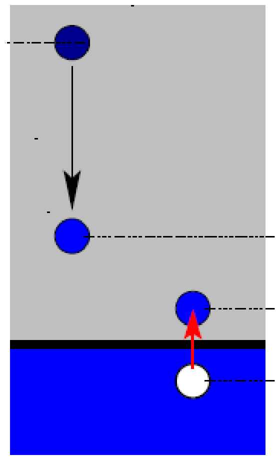

Decay of one quasiparticle
3-body process

$$\tau^{-1}=\frac{\varepsilon^{*2}+\pi^2T^2}{E^*_F}$$

$\varepsilon^*=E^*-E^*_F$

Energy from Fermi Surface

E^*_F Fermi energy of the quasiparticles

Estimate in 3D based
on phase space considerations
(Pauli principle) & 3 body decay

See Coleman's book

Quasiparticle decay

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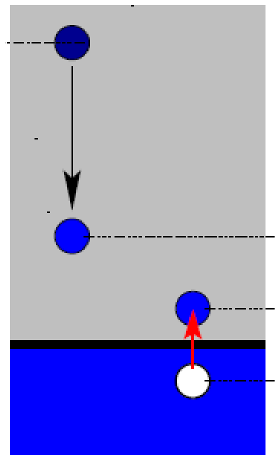

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$\varepsilon^*=E^*-E^*_F$

Energy from Fermi Surface

Estimate in 3D based
on phase space considerations
(Pauli principle) & 3 body decay

Decay of one quasiparticle
3-body process

E^*_F Energy which control the range of temperature and energy at which the quasiparticle is well defined It can be very small!

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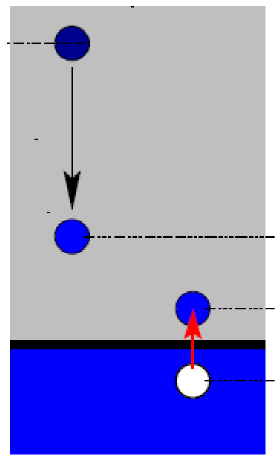
Quasiparticle decay



Decay of one quasiparticle
3-body process

$\varepsilon = E - E^*_F$

Energy from
Fermi Surface

$$\frac{\Gamma}{\varepsilon} \sim \varepsilon$$ Estimate in 3D

$$\frac{\Gamma}{\varepsilon} \sim \varepsilon \,\mathrm{Ln}\, E^*_F/\varepsilon$$ Estimate in 2D

$$\frac{\Gamma}{\varepsilon} \sim \mathrm{const}$$ Estimate in 1D

Quasiparticles well defined at low energies & temperatura with infinite lifetime at the Fermi surface at zero temperature in 3D and 2D but not 1D.

Fermi liquid theory fails in 1D

Giuliani and Quinn
PRB 26, 4421 (1982)

Quasiparticle decay

3 dimensions

©Michael Paraskevas

2 dimensions

©Sebastian Dubiel

1 dimension

©Carlos Valenzuela

Interactions are never perturbative
in 1 dimensión

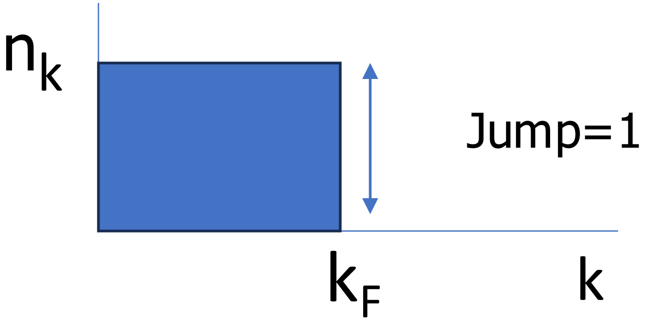
The quasiparticles are not well defined
even at very low temperature and energy

Adiabaticity and quasiparticles. Summary

- Provided that the interacting and non-interacting ground states are adiabatically connected (Perturbative effect of interactions. No phase transition):

  - The interacting system has elementary excitations with momentum k, spin ½ and charge **e** called quasiparticles $a^{+}_{k\sigma}$

  - The quasiparticles $\mathbf{a}^{+}_{ks}$ follow Fermi Dirac statistics and we can define a Fermi surface



  - The quasiparticles $\mathbf{a}^{+}_{ks}$ are not eigenstates of the interacting system. They are expected to decay and have a finite lifetime. But this decay is restricted by Pauli's principle. Quasiparticles are well defined in 2D and 3D at low temperatures and energies.

$$\tau^{-1} = \frac{\varepsilon^2 + \pi^2 T^2}{E_F}$$

in 3D

Interactions are not perturbative in 1D
Non-Fermi liquid behavior

For simplicity we drop the *
to refer to the energies

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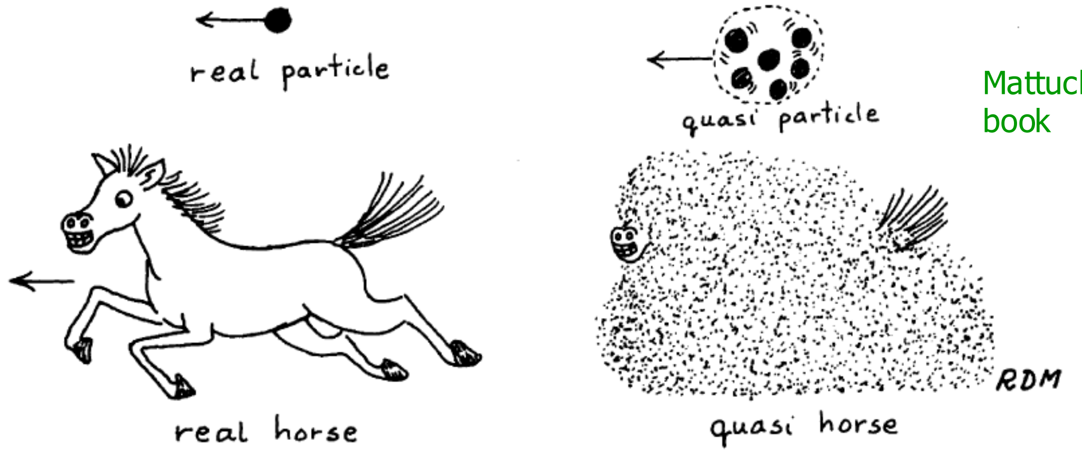
Connecting quasiparticles and real electrons

Adding an electron (hole) and creating a quasiparticle is not the same thing, but there is a one to one correspondence between them.

$$c^{+}_{k\sigma} \neq a^{+}_{k\sigma}$$

Real electron

Quasiparticle



Mattuck's book

In an experiment:

$$c^{+}_{k\sigma}\,|\Psi^{*}_{g}\rangle \qquad c_{k\sigma}\,|\Psi^{*}_{g}\rangle$$

real electron

STM

ARPES

Experimentally it is posible to measure the electron spectral function $A(k,\omega)$

Write $c^{+}_{k\sigma}$ in terms of $a^{+}_{k\sigma}$ → Quasiparticle weight

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Connecting quasiparticles and real electrons

- Write the electron operator $c^{\dagger}{}_{k\sigma}$ in terms of the elementary excitations of the interacting system, the quasiparticles $a^{\dagger}{}_{k\sigma}$

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} \underbrace{A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)}_{} a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

Generic expression which conserves charge, momentum & spin

If different from zero the electron decays into particle-hole quasiparticle excitations

higher order decay processes

Decay of the electron into electron-hole quasiparticles excitations
(not to be confused with the decay of the quasiparticle)

See Coleman's book

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Connecting quasiparticles and real electrons

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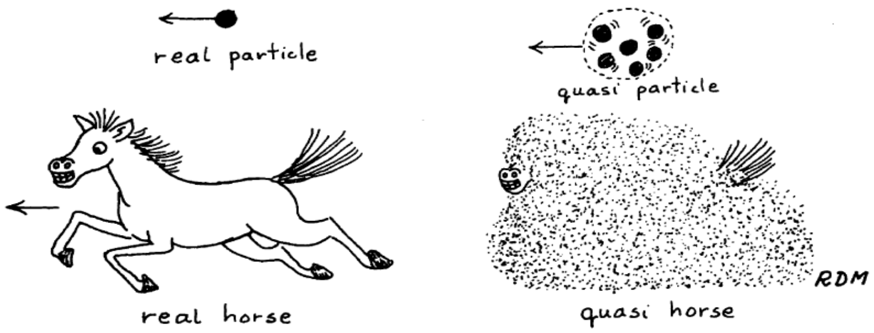
A single particle description of $C^{\dagger}_{k\sigma}|\Psi^*_g>$ will "make sense" if Z_k is finite

Z_k Quasiparticle weight → Measurable



The one-to-one connection can be defined only if Z_k is finite

Mattuck's book

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Connecting quasiparticles and real electrons

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A single particle description of $C^{\dagger}_{k\sigma}|\Psi*_g>$ will "make sense" if Z_k is finite

Z_k Quasiparticle weight

$$Z_k = |<\Psi*_g|a_{k\sigma}c^{\dagger}_{k\sigma}|\Psi*_g>|^2 > 0$$

Overlap between the state reached adding an electron to the interacting system and the state which results from adding an elementary excitation to the interacting system

See Coleman's book

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Connecting quasiparticles and real electrons

❑ Write the electron operator $c^{\dagger}_{k\sigma}$ in terms of the elementary excitations of the interacting system, the quasiparticles $a^{\dagger}_{k\sigma}$

$$c^{\dagger}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}} a^{\dagger}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma) a^{\dagger}_{\mathbf{k}_4\sigma_4} a^{\dagger}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

$$Z_k = |\langle \Psi *_g | a_{k\sigma} c^{\dagger}_{k\sigma} | \Psi *_g \rangle|^2$$

Quasiparticle weight

$$0 \leq Z_k \leq 1$$

Z_k Measures the strength of correlations & the validity of Fermi liquid description

- $Z_k=1$ non interacting system
- $Z_k>0$ ensures one to one correspondence between electron and quasiparticle.
- $Z_k=0$ Fermi liquid theory not applicable

See Coleman's book

Electronic distribution. Jump at Fermi Surface

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

$$n_k = <\Psi^*{}_g | C^{\dagger}{}_{k\sigma} C_{k\sigma} | \Psi^*{}_g> = Z_k <\Psi^*{}_g | a^{\dagger}{}_{k\sigma} a_{k\sigma} | \Psi^*{}_g> + \text{continuum}$$

Distribution of real electrons

Electronic excitations which are in the quasiparticle state

Continuum or incoherent part:
Electronic excitations which are not in the quasiparticle state

Do not confuse this continuum with the continuum limit vs lattice

See Coleman's book

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Electronic distribution. Jump at Fermi Surface

$$c^{\dagger}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathrm{k}}}\,a^{\dagger}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\,a^{\dagger}_{\mathbf{k}_4\sigma_4} a^{\dagger}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \dots$$

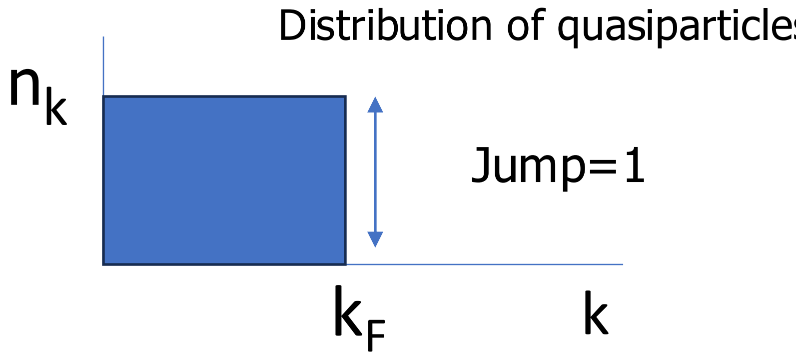
$$n_k = \langle \Psi^*_g | C^{\dagger}_{k\sigma} C_{k\sigma} | \Psi^*_g \rangle = Z_k \langle \Psi^*_g | a^{\dagger}_{k\sigma} a_{k\sigma} | \Psi^*_g \rangle + \text{continuum}$$

Distribution of real electrons

The quasiparticles follow Fermi Dirac statistics

Continuum or incoherent part: Electronic excitations which are not in the quasiparticle state

At T=0 the distribution of quasiparticles has jump of height 1 at the Fermi surface $\Theta(-\varepsilon_k)$



See Coleman's book

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Electronic distribution. Jump at Fermi Surface

$$c^\dagger{}_{\mathbf{k}\sigma} = \sqrt{Z_\mathbf{k}}\, a^\dagger{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^\dagger{}_{\mathbf{k}_4\sigma_4} a^\dagger{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

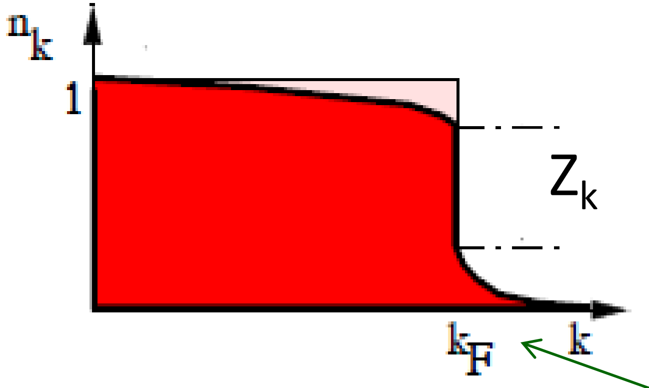
$$n_k = <\Psi^*_g | C^\dagger_{k\sigma} C_{k\sigma} | \Psi^*_g> = Z_k <\Psi^*_g | a^\dagger_{k\sigma} a_{k\sigma} | \Psi^*_g> + \text{continuum}$$

$$Z_k\ \Theta(-\varepsilon_k)$$

Jump of Height 1
at Fermi surface

Multiplied by Z_k

At T=0 the distribution of electrons has a jump of height Z_k (not 1) at the Fermi level



Leakage of electrons away from the Fermi surface (stronger correlations larger leakage)

See Coleman's book

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The Spectral Function $A(k,\omega)$

ARPES

STM (sum of all K)

In an experiment:

$c_{k\sigma} \,|\Psi^*_g>$

$c^+_{k\sigma} \,|\Psi^*_g>$

The spectral function $A(k,\omega)$: Distribution of excitations created when an electron (a real electron) with momentum k is added or removed from the system

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The Spectral function $A(k,\omega)$

In an experiment:

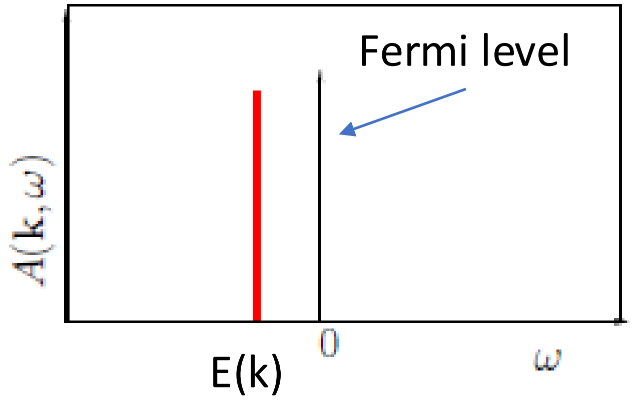
ARPES

$c_{k\sigma} \, |\Psi^*{}_g>$

STM (sum of all K)

$c^+{}_{k\sigma} \, |\Psi^*{}_g>$

- ❑ Non interacting system



Eigenstate, well defined energy
for a given momentum.

Infinite lifetime

The spectral function $A(k,\omega)$: Distribution of excitations created when an electron (a real electron) with momentum k is added or removed from the system

In an independent electron system

the spectral function A(k, w) is a delta function and "measures the bands"

E(k)

$$A(k,\omega)=\delta(\omega=E(k))$$

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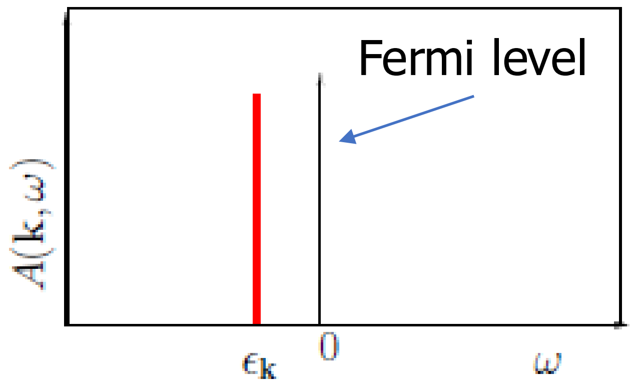
The Interacting Spectral Function A(k,ω)

- Non interacting

Distribution of excitations created when an electron (a real electron) with momentum k is added or removed from the system

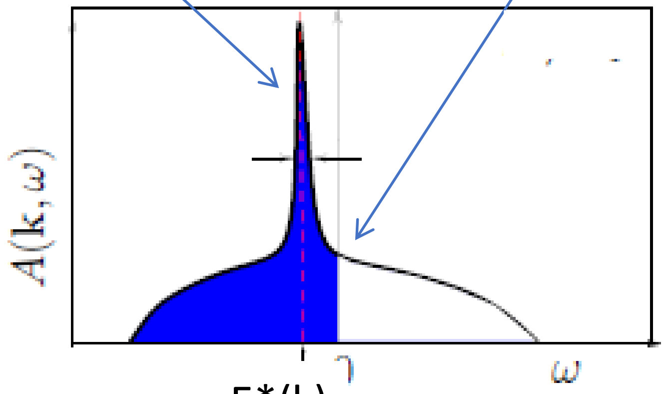


- Interacting system (Fermi liquid)

Quasiparticle peak:
Part of the electronic excitation which is in the quasiparticle state
Coherent part

Incoherent part:
Amount of the Electronic excitation which is not in the quasiparticle state

(also called continuum)



E*(k)

Not the same energy
as in the non-interacting limit

Figs. Coleman's book

See Coleman's book

The Interacting Spectral Function A(k,ω)

$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

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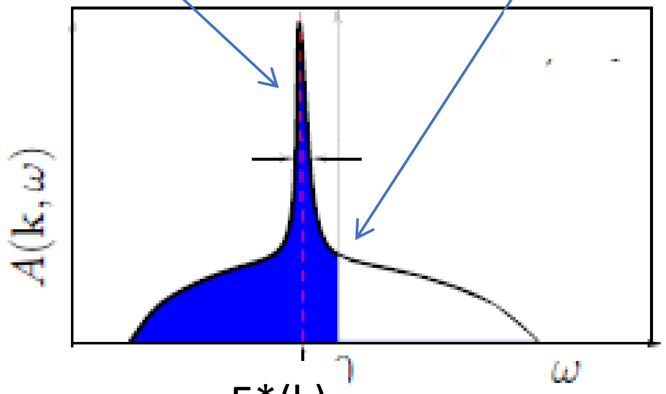

E*(k)

Not the same energy
as in the non-interacting limit

Figs. Coleman's book

See Coleman's book

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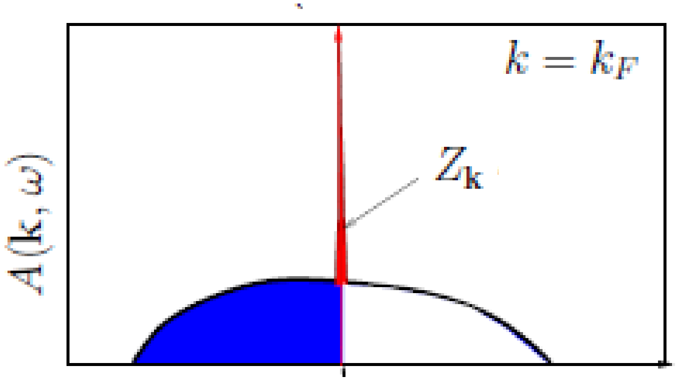
The Interacting Spectral Function $A(k,\omega)$

Distribution of excitations created when an electron (a real electron) with momentum k is added or removed from the system

At the Fermi Surface the quasiparticle peak is infinitely narrow (at T=0)

Fermi surface

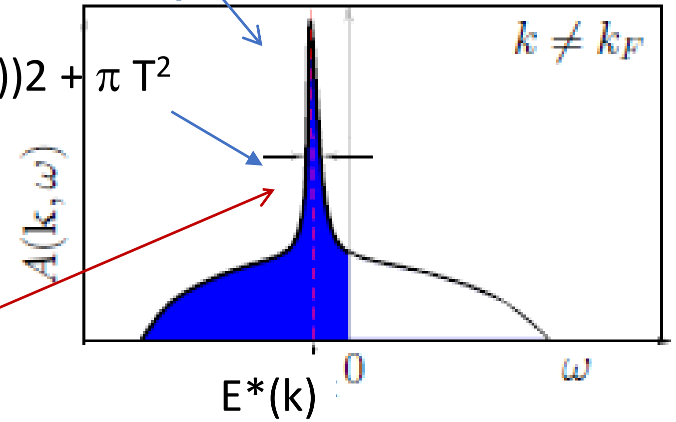


E^*_F

Quasiparticle peak: Part of the electronic excitation which is in the quasiparticle state
Coherent part

Away from Fermi surface

$\Gamma \propto (E^*(k))2 + \pi T^2$



$E^*(k)$

Peak width:
Decay of
the quasiparticle

Figs. Coleman's book

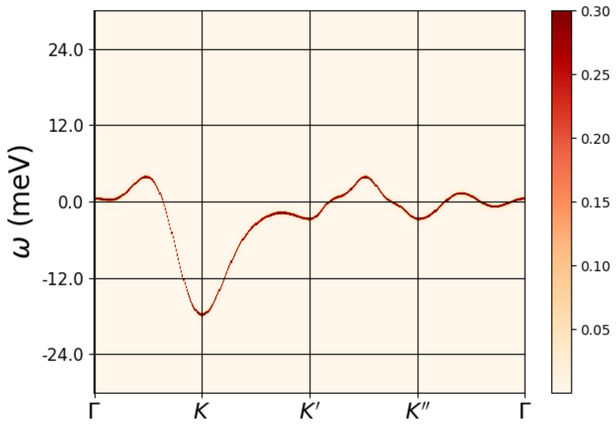
See Coleman's book

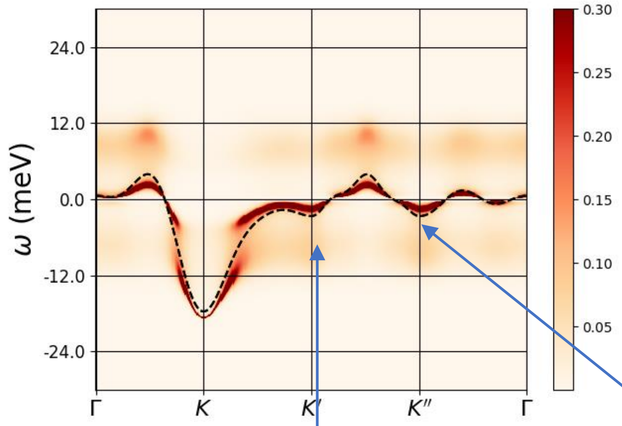
The Interacting Spectral Function A(k,ω)

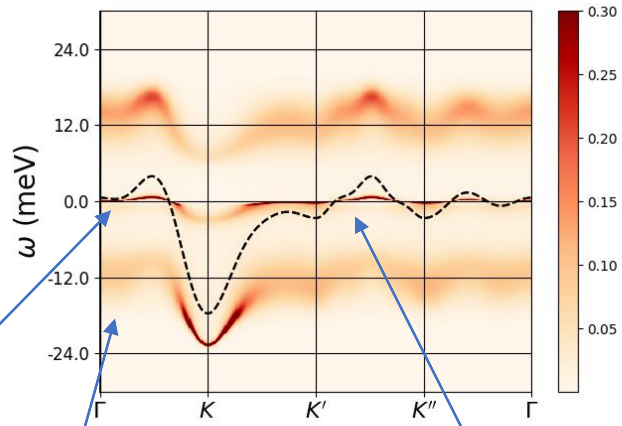
Finite temperature

Larger interaction

Non interacting







Quasiparticle
(peak with finite width)

Energy of the quasiparticle peak
different from non-interacting E(K)
(much narrower band)

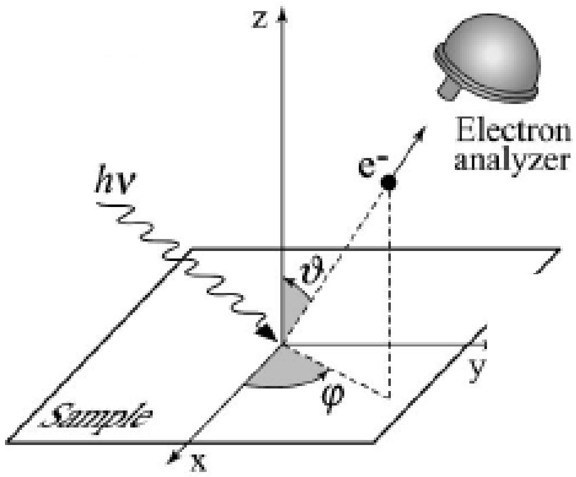
Continuum or incoherent
part of the spectrum
(non-quasiparticle like)

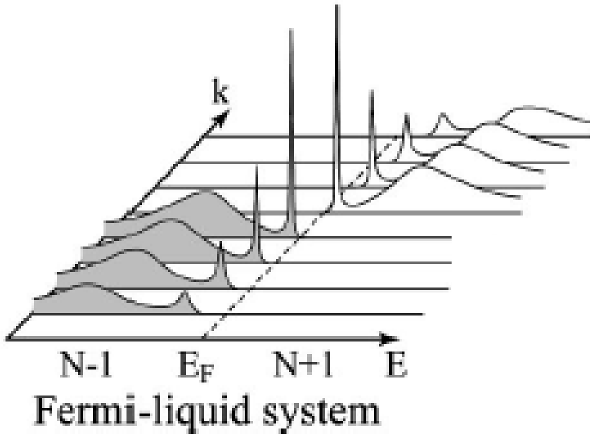
Calculated spectrum for a
2-orbital Hubbard model
at half-filling

Calderón, Camjayi & EB,
PRB 106, L081123 (2022)

The Spectral Function A(k,ω) and ARPES

(angle resolved photoemission)

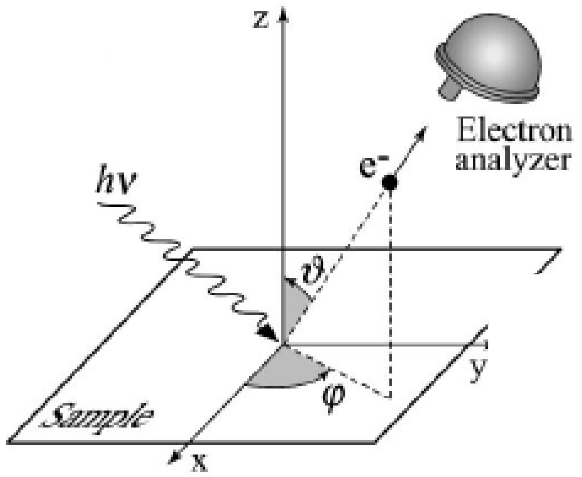


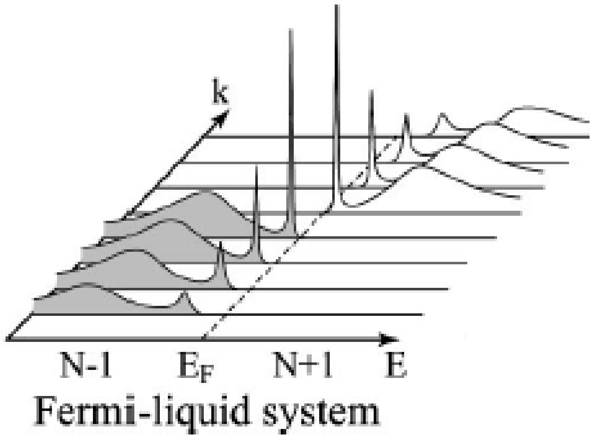


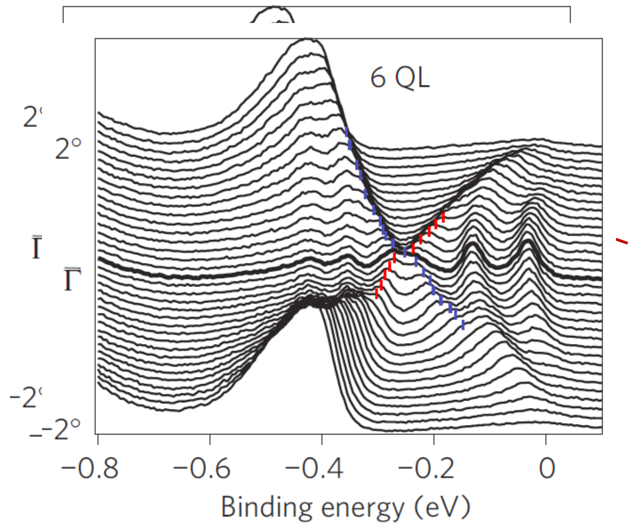
Figs: Damascelli et al, RMP 2003

The Spectral Function $A(k,\omega)$ and ARPES

(angle resolved photoemission)







Bi_2Se_3

Weakly correlated system

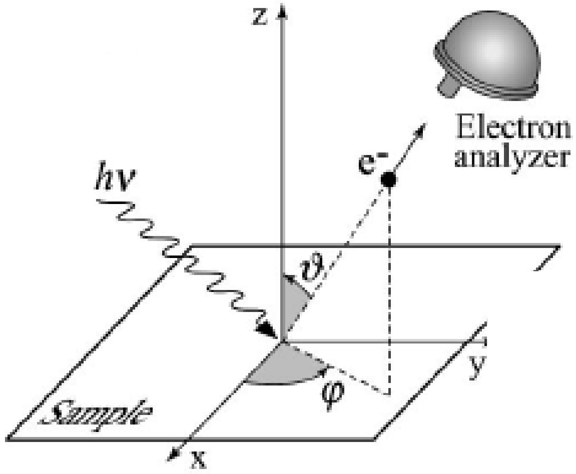
Well defined
Quasiparticle
peaks

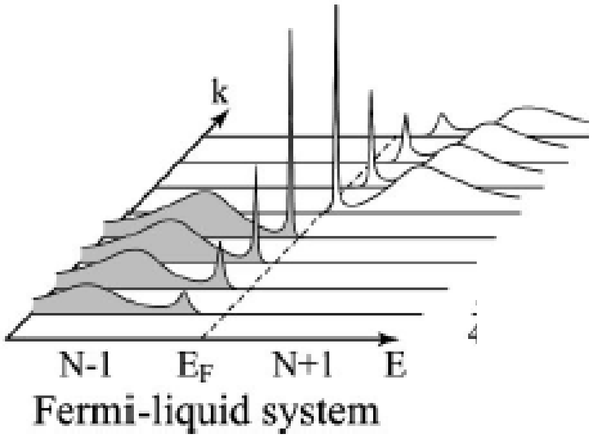
Possibility to map the bands

Figs: Damascelli et al, RMP 2003
Zhang et al, Nat Phys. 2010

The Spectral Function A(k,ω) and ARPES

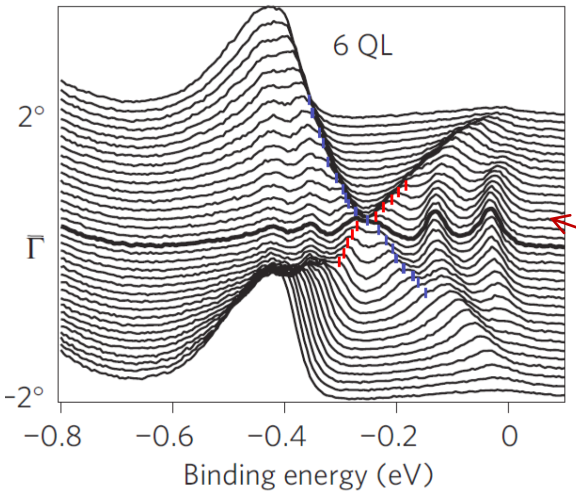
(angle resolved photoemission)





Very strongly correlated system

Cuprate

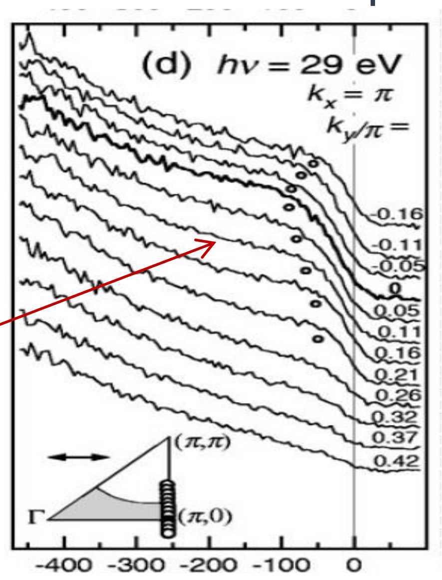


Bi_2Se_3

Weakly correlated system

Well defined Quasiparticle peaks

Badly defined quasiparticle peak



Possibility to map the bands

Figs: Damascelli et al, RMP 2003
Zhang et al, Nat Phys. 2010

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The Spectral Function A(k,ω) and ARPES

Weakly correlated system

Well defined bands

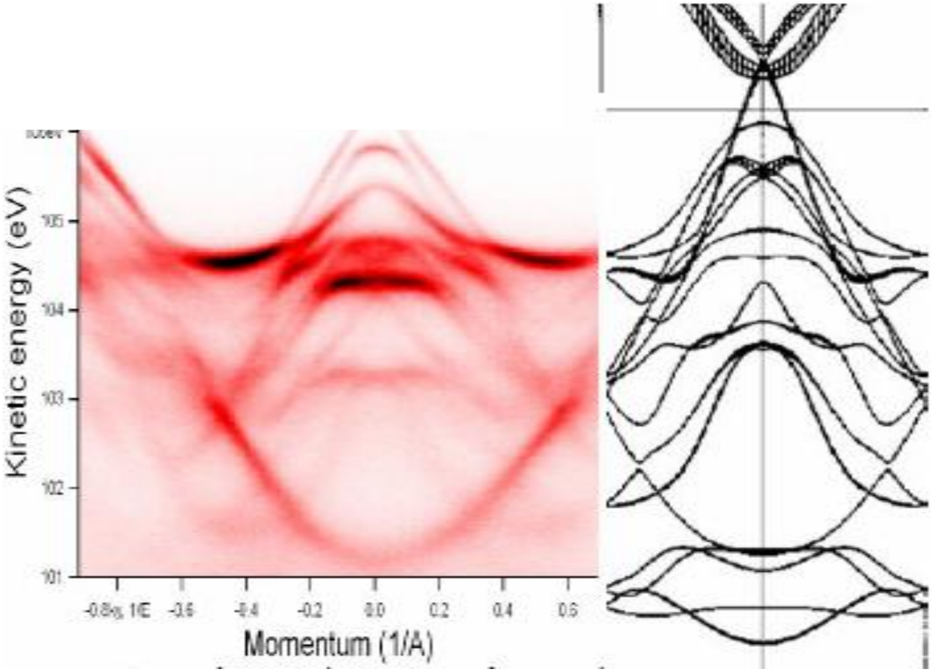
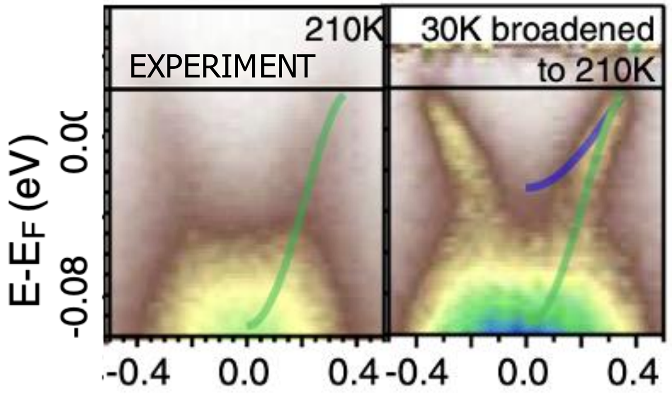


Fig. : Evtushinksy lectures

Strongly correlated system

Blurred spectrum and anomalous temperature behavior

Iron chalcogenide



PRL 110, 067003 (2013)

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Summary: Connecting quasiparticles and electrons. The spectral function

Electron → $c^{+}_{k\sigma} \neq a^{+}_{k\sigma}$ ← Quasiparticle: Elementary excitation

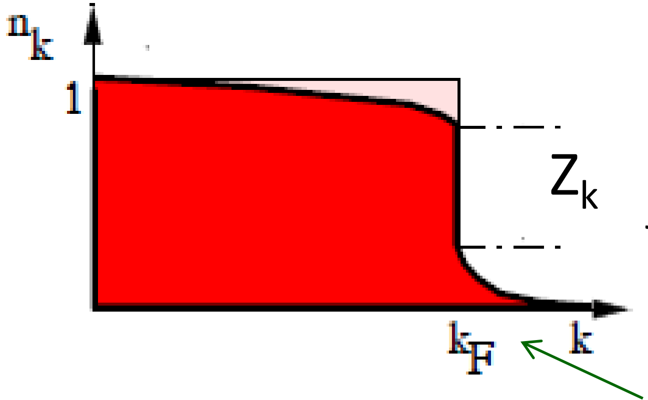
$$c^{\dagger}{}_{\mathbf{k}\sigma} = \sqrt{Z_{\mathbf{k}}}\, a^{\dagger}{}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}_4+\mathbf{k}_3=\mathbf{k}_2+\mathbf{k}} A(\mathbf{k}_4\sigma_4, \mathbf{k}_3\sigma_3; \mathbf{k}_2\sigma_2, \mathbf{k}\sigma)\, a^{\dagger}{}_{\mathbf{k}_4\sigma_4} a^{\dagger}{}_{\mathbf{k}_3\sigma_3} a_{\mathbf{k}_2\sigma_2} + \ldots$$

Quasiparticle weight (overlap)

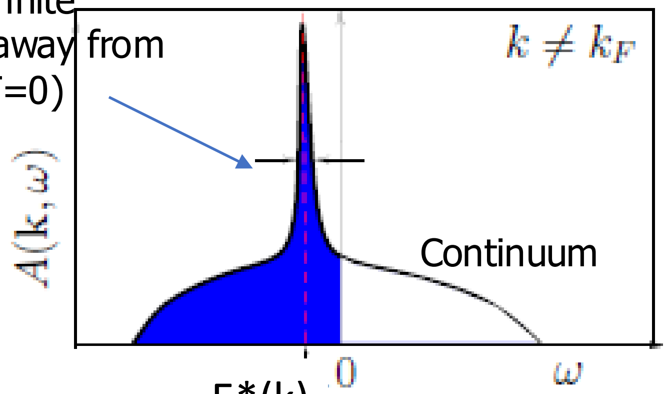
Decay of the electron (continuum)

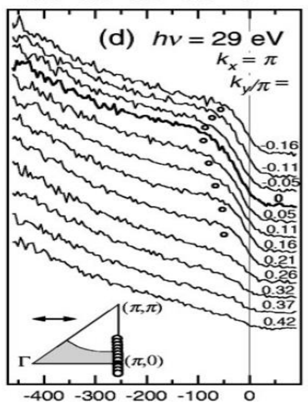
Coleman's book
Damascelli, RMP 2003
Zhang et al, N Phys. 2010

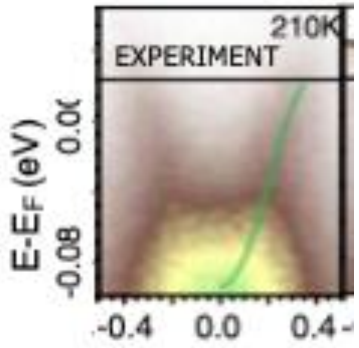
$0 < Z \leq 1$ measures correlations



Spectral function







Fermi Liquid Theory

- Assume adiabaticity: the interacting ground state is perturbatively connected

- Quasiparticles in a metal: fermionic excitations with spin ½ and charge e. Well defined Fermi surface

- Quasiparticle decay rate in 3D: $$\tau^{-1} = \frac{(\varepsilon^*)^2 + \pi^2 T^2}{E^*_F}$$

- Quasiparticles: elementary excitations of the interacting system have a one to one correspondence with elementary excitations of the non-interacting one. Implies quasiparticle weight Z_k finite.

- Description of the system in terms not of the ground state of the system but in terms of the low energy excitations$\rightarrow$ deviation from equilibrium δn_p.

- Energy functional $F[\delta n_p]$ in terms of parameters which can be measured experimentally (specific heat, mass, susceptibilities …)

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Description of the energy in terms of excitations (quasiparticles)

- Consider non-interacting ground state

$$H = H_0 - \mu N = \sum_{\sigma} (E_{\mathbf{k}} - \mu) c^{\dagger}{}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} \qquad\qquad |\psi_g\rangle = \prod_{|\mathbf{k}|<k_F,\ \sigma} c^{\dagger}{}_{\mathbf{k}\sigma} |0\rangle$$

- Define the transformation

$$\mathrm{d}^{\dagger}{}_{\mathbf{k}\sigma} = \begin{cases} c^{\dagger}{}_{\mathbf{k}\sigma} & (k > k_F) \qquad \text{particle} \\ \mathrm{sgn}(\sigma) c_{-\mathbf{k}-\sigma} & (k > k_F) \qquad \text{hole} \end{cases}$$

- Rewrite the hamiltonian

Ground state

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} |(E_{\mathbf{k}} - \mu)| \, \mathrm{d}^{\dagger}{}_{\mathbf{k}\sigma} \mathrm{d}_{\mathbf{k}\sigma} + F_g$$

Excitations

Deviation from
equilibrium occupation

Fermionic excitations
Charge e
Spin σ

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Description in terms of excitations (quasiparticles)

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} (E_{\mathbf{k}} - \mu)\, d^\dagger_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + F_g$$

Excitations (the occupation of the states in equilibrium is included in F_g)

$\delta n_{k\sigma} = d^\dagger_{k\sigma} d_{k\sigma}$ → Density of excitations

$$\boxed{H_0 - \mu N = F_g + \Sigma_{k\sigma} \varepsilon_{k\sigma} \delta n_{k\sigma} = F[\delta n_{k\sigma}]}$$

Energy of the excitation measured with respect to the ground state

Energy written as a functional of the density of excitations

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Fermi liquid theory: Description in terms of excitations (quasiparticles)

$$H_0 - \mu N = \sum_{\mathbf{k}\sigma} |(E_{\mathbf{k}} - \mu)|\, d^{\dagger}_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + F_g$$

Excitations (the occupation of the states in equilibrium is included in F_g)

$\delta n_{k\sigma} = d^{\dagger}_{k\sigma} d_{k\sigma}$ → Density of excitations

$$\boxed{H_0 - \mu N = F_g + \Sigma_{k\sigma} \varepsilon_{k\sigma} \delta n_{k\sigma} = F[\delta n_{k\sigma}]}$$

Energy of the excitation measured with respect to the ground state

Energy written as a functional of the density of excitations

❑ Landau's idea: Energy of interacting system is a functional of the density of excitations, the quasiparticles, $F[\delta n_{k\sigma}]$ and do an expansion around equilibrium assuming small $\delta n_{k\sigma}$

$\delta n_{k\sigma} = a^{\dagger}_{k\sigma} a_{k\sigma}$

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Fermi liquid theory: Description in terms of excitations (quasiparticles)

- Landau's idea: Energy of interacting system is a functional of the density of quasiparticles $F[\delta n_{k\sigma}]$

Expansion around equilibrium. Small $\delta n_{k\sigma}$

$$\delta n_{k\sigma} = a^{\dagger}_{k\sigma} a_{k\sigma}$$

$$\boxed{F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma} \varepsilon^*_{k\sigma} \delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'} \delta n_{k\sigma} \delta n_{k'\sigma'} + O(\delta n^3)}$$

First order in the expansion

second order in the expansion

$$\varepsilon^*_{k\sigma} = \left. \frac{\partial F}{\partial(\delta n_{k\sigma})} \right|_{\delta n_{k\sigma}=0}$$

$$f_{k\sigma k'\sigma'} = \left. \frac{\partial^2 F}{\partial(\delta n_{k\sigma})\partial(\delta n_{k\sigma})} \right|_{\delta n_{k\sigma}=0}$$

Fermi liquid theory: Description in terms of excitations (quasiparticles)

- Landau's idea: Energy of interacting system is a functional of the density of quasiparticles $F[\delta n_{k\sigma}]$

Expansion around equilibrium. Small $\delta n_{k\sigma}$

$$\delta n_{k\sigma} = a^{\dagger}_{k\sigma} a_{k\sigma}$$

$$\boxed{F\,[\delta n_{k\sigma}] = F_g + \sum_{k\sigma} \varepsilon^*_{k\sigma} \delta n_{k\sigma} + \sum_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'} \delta n_{k\sigma} \delta n_{k'\sigma'} + O(\delta n^3)}$$

Second order: Residual interactions between the quasiparticles (responsable for quasiparticle decay and instabilities)

First order: Energy of a quasiparticle in the absence of other quasiparticles

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Comparison with non-interacting case

Renormalized energy of the quasiparticle $\varepsilon^*_{k\sigma}$

No second order term in the non-interacting case (no interactions)

$$\boxed{H_0-\mu N = F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon_{k\sigma}\delta n_{k\sigma}}$$

Fermi liquid theory: Description in terms of excitations (quasiparticles)

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$$\Downarrow$$

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\left(\varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k'\sigma'}\right)\delta n_{k\sigma}$$

Fermi liquid theory: Description in terms of excitations (quasiparticles)

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$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\,(\varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k'\sigma'})\,\delta n_{k\sigma} = F_g + \Sigma_{k\sigma}\varepsilon'_{k\sigma}\delta n_{k\sigma}$$

$$\varepsilon'_{k\sigma} = \varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k'\sigma'} ©$$

Non-rigid band shift

The energy of a quasiparticle is modified by the presence of other excitations

$$\delta n_{k\sigma}=n_{k\sigma}(T,\mu)-n_{k\sigma}(0,\mu)$$

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Fermi liquid theory: Parameters of the model. The renormalized mass

- To first order:

Energy of a quasiparticle in the absence of other quasiparticles

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma} \varepsilon^*_{k\sigma} \delta n_{k\sigma}$$

Fermi liquid theory: Parameters of the model. The renormalized mass

- To first order:

Energy of a quasiparticle in the absence of other quasiparticles

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma}$$

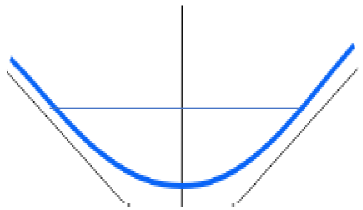
In a non-interacting state (continuum)

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon_{k\sigma}\delta n_{k\sigma}$$

$$\varepsilon_{k\sigma} = k^2/2m - \mu = (k^2 - k_F^2)/2m$$

close to k_F we can linearize the spectrum
(first order expansion in $k-k_F$)

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$



k_F

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Fermi liquid theory: Parameters of the model. The renormalized mass

- To first order:

Energy of a quasiparticle in the absence of other quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma}$$

By analogy, in the interacting system, we linearize the spectrum close to k_F and **define m***

In a non-interacting state (continuum)

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon_{k\sigma}\delta n_{k\sigma}$$

$$\varepsilon_{k\sigma} = k^2/2m - \mu = (k^2 - k_F^2)/2m$$

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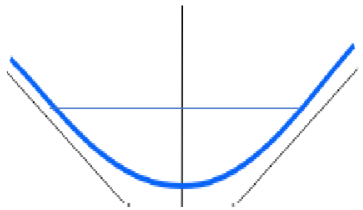
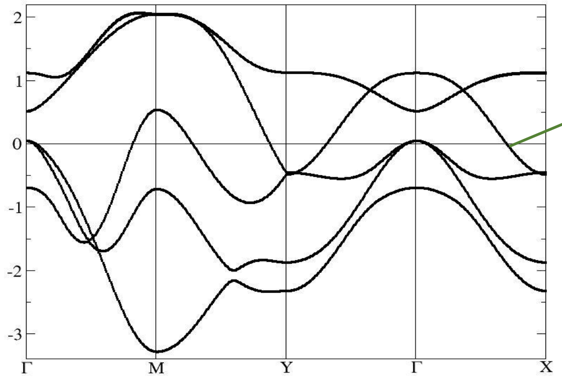


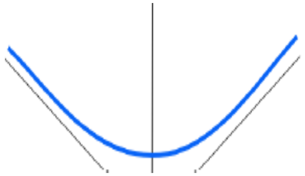
$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

Renormalized mass or quasiparticle mass

Fermi liquid theory: Parameters of the model. The renormalized mass

- Non-interacting system:





Band mass different to the free electron mass
Modification due to ionic potential

$$m^{-1} = |\partial^2\varepsilon/\partial k^2|$$

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$

- Interacting system:

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

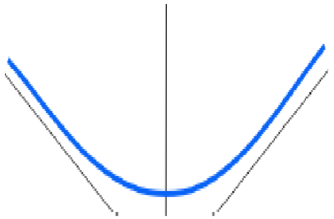
Quasiparticle mass different from the band mass m due to the electronic interactions

(also v^*_F in Dirac materials)

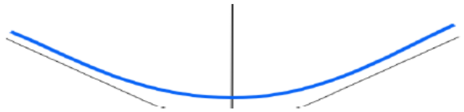
Fermi liquid theory: Parameters of the model. The renormalized mass

Mass/bandwidth renormalization

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$

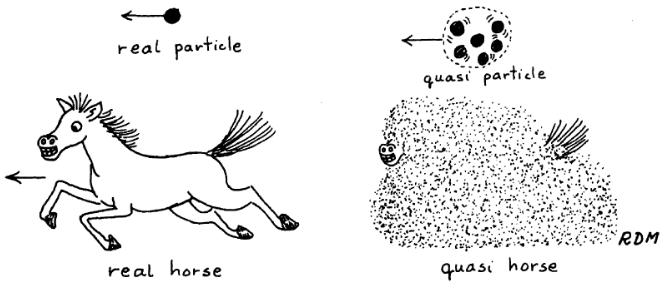


$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$



m*>m always

The backflow of the surrounding fluid enhances the mass



In simple models $m^* = \frac{m}{Z}$ ← Quasiparticle weight

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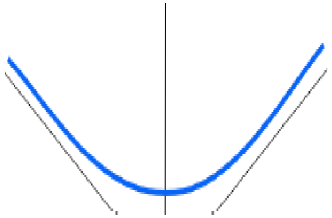
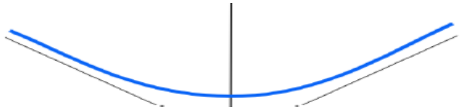
Fermi liquid theory: Parameters of the model. The renormalized mass

Mass/bandwidth renormalization

$$\varepsilon_{k\sigma} = \frac{k_F}{m}(k-k_F)$$



$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$



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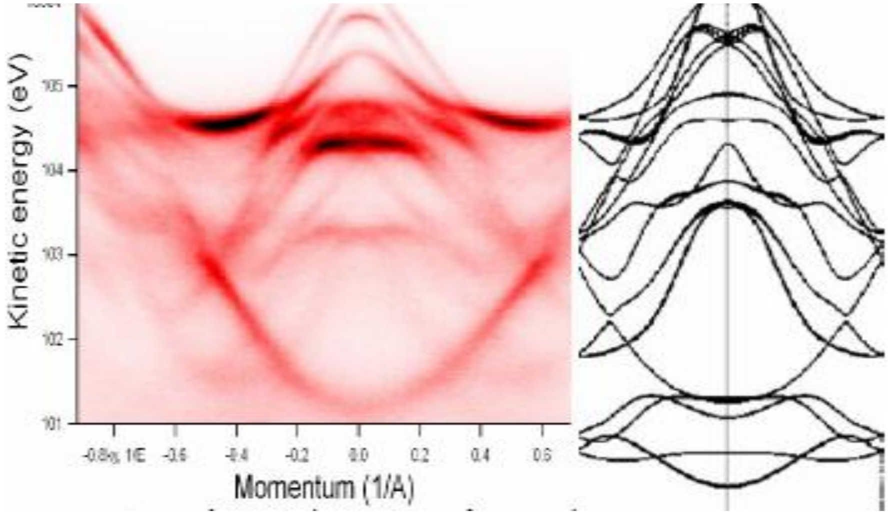

How large is m*/m?

Experimental

LDA or similar

Fig. : Evtushinksy lectures

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Fermi liquid theory: Measurable quantities

Assumed
$K_B T << \mu$

Non-interacting Fermi gas:

From Fermi-Dirac statistics $$n(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/K_B T}+1}$$

- Specific heat $$C_v = \left.\frac{\partial F}{\partial T}\right|_{\mu}$$

$C_v=\gamma\ T$ Linear in temperature $\gamma=(\pi/3)K^2{}_B N(E_F) \propto m$

- Spin susceptibility $$\chi_s = \frac{\partial M}{\partial H}$$

$\chi_s = \mu_B{}^2 N(E_F) \propto m$ independent of T

Fermi liquid theory: Measurable quantities

Interacting Fermi liquid:

$$n_{k\sigma} = \frac{1}{e^{-(\varepsilon_{k\sigma}'-\mu)/K_BT}+1}$$

Fermi-Dirac distribution for "interacting" quasiparticle energies

Quasiparticle weight Z_k not present because this is the distribution for the quasiparticle energies

$$\varepsilon'_{k\sigma} = \varepsilon^*_{k\sigma} + \Sigma_{k'\sigma'} f_{k\sigma k'\sigma'} \delta n_{k'\sigma'}©$$

- Specific heat $C_v = \left.\frac{\partial F}{\partial T}\right|_{\mu}$
- Spin susceptibility $\chi_s = \frac{\partial M}{\partial H}$

See Pines & Nozieres' and Coleman's books for details on the derivation

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Fermi liquid theory: Parameters of the model. The renormalized mass

Specific heat $C_v = \left.\dfrac{\partial F}{\partial T}\right|_{\mu}$

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

F(T) $\quad \propto T^2 \quad \propto T^4$

$C^*_v = \gamma^* \, T \qquad \gamma^* = (\pi/3) K^2_B N^*(E^*_F) \propto m^*$

Same temperature dependence as in Fermi gas but with a renormalized proportionality constant γ^*

Fermi liquid theory: Parameters of the model. The renormalized mass

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Same temperature dependence as in Fermi gas but with a renormalized proportionality constant γ^*

Check experimental dependence and measure g
Can be also compared with
Model calculations

Interacting $\qquad$ Non-Interacting

$$C^*_v = \frac{m^*}{m} C_v$$

Comparison with
LDA like calculations

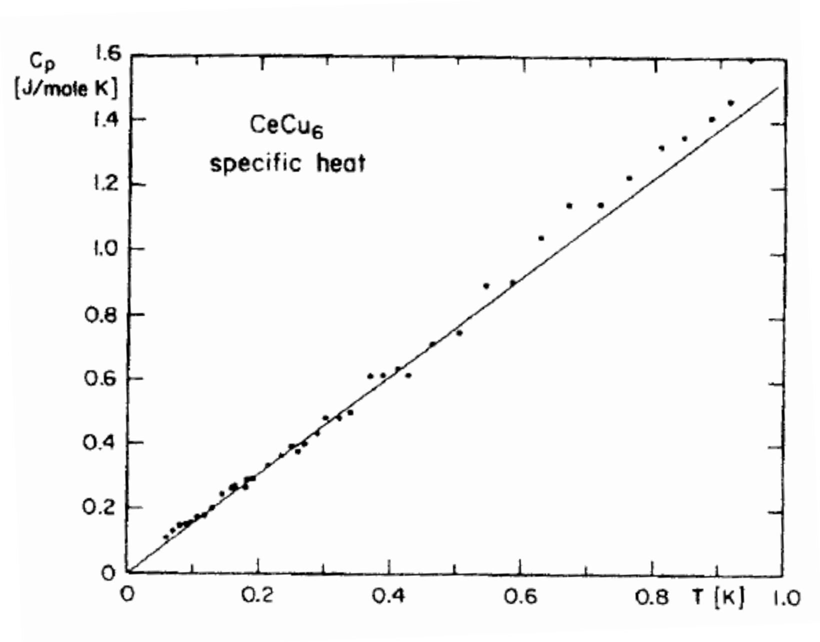
Fermi liquid theory: Parameters of the model. The renormalized mass

Specific heat

$C_v=\gamma^* T$ $\qquad \gamma^*=(\pi/3)K^2_B N^*(E^*_F) \propto m^*$

$\gamma^*_{Au,Ag}= 0.67$ mJ mol^{-1} K^{-1} (vs 0.63 mJ mol^{-1} K^{-1} in absence of interaction)

$\gamma^*_{CeCu6}= 1,5$ J mol^{-1} K^{-1} → Heavy fermion



Fermi liquid theory: Parameters of the model. The renormalized mass

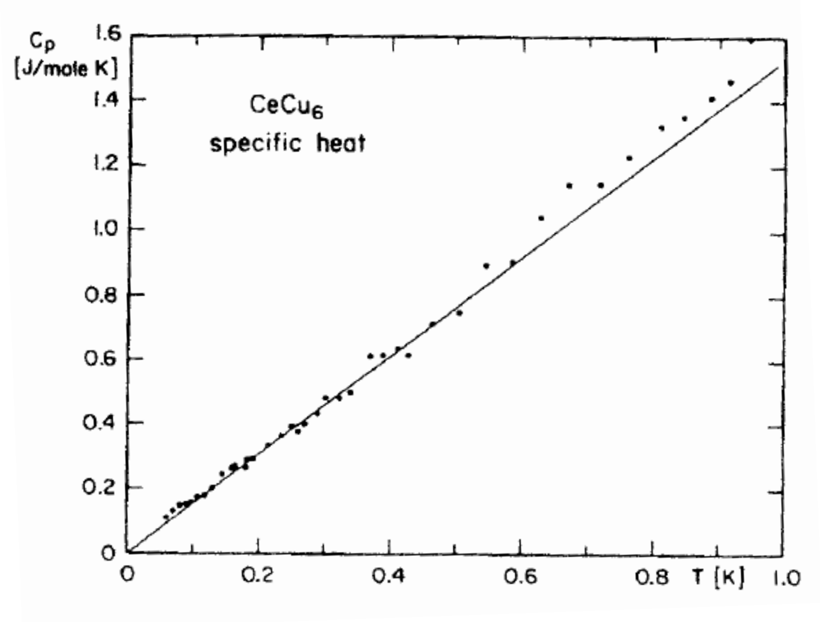
Specific heat

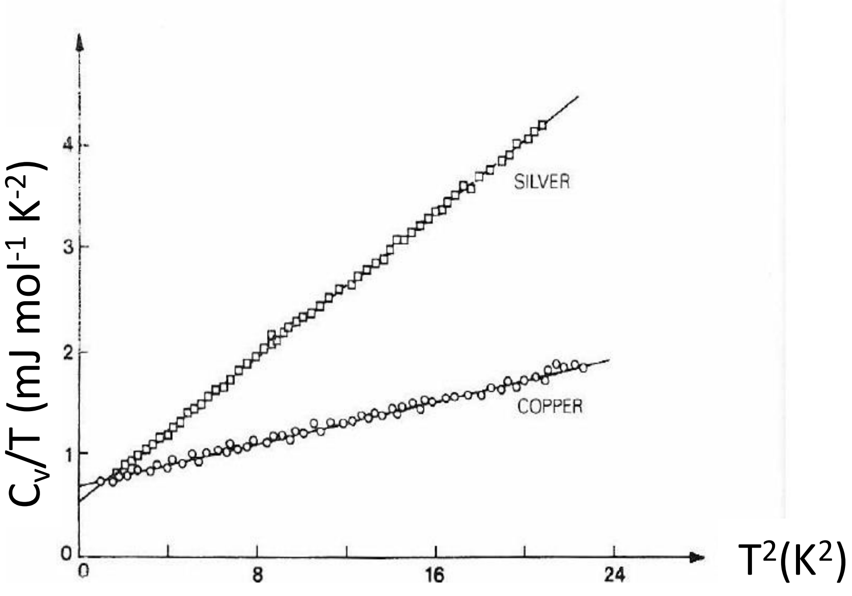
$C_v = \gamma^* T$ $\qquad \gamma^* = (\pi/3) K^2_B N^*(E^*_F) \propto m^*$

(note phonon contribution can dominate Cv $\propto T^3$)

$\gamma^*_{Au,Ag} = 0.67$ mJ mol^{-1} K^{-1} (vs 0.63 mJ mol^{-1} K^{-1} in absence of interaction)

$\gamma^*_{CeCu6} = 1,5$ J mol^{-1} K^{-1} $\rightarrow$ Heavy fermion





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Fermi liquid theory: Parameters of the model. Comparing to experiment

Specific heat

Iron superconductors

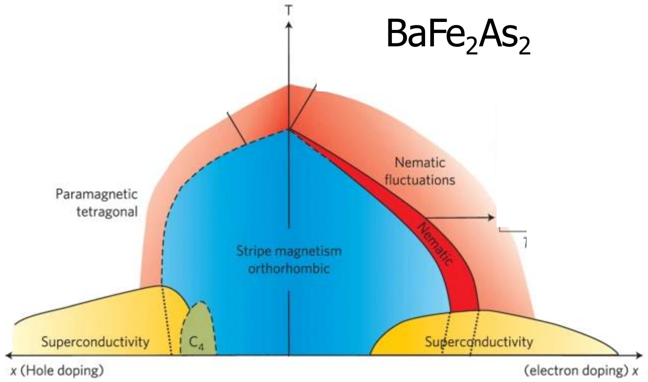
Non-interacting electrons

$C_v=\gamma\, T \qquad \gamma=(\pi/3)K^2_B N(E_F) \propto m$

Fermi liquid

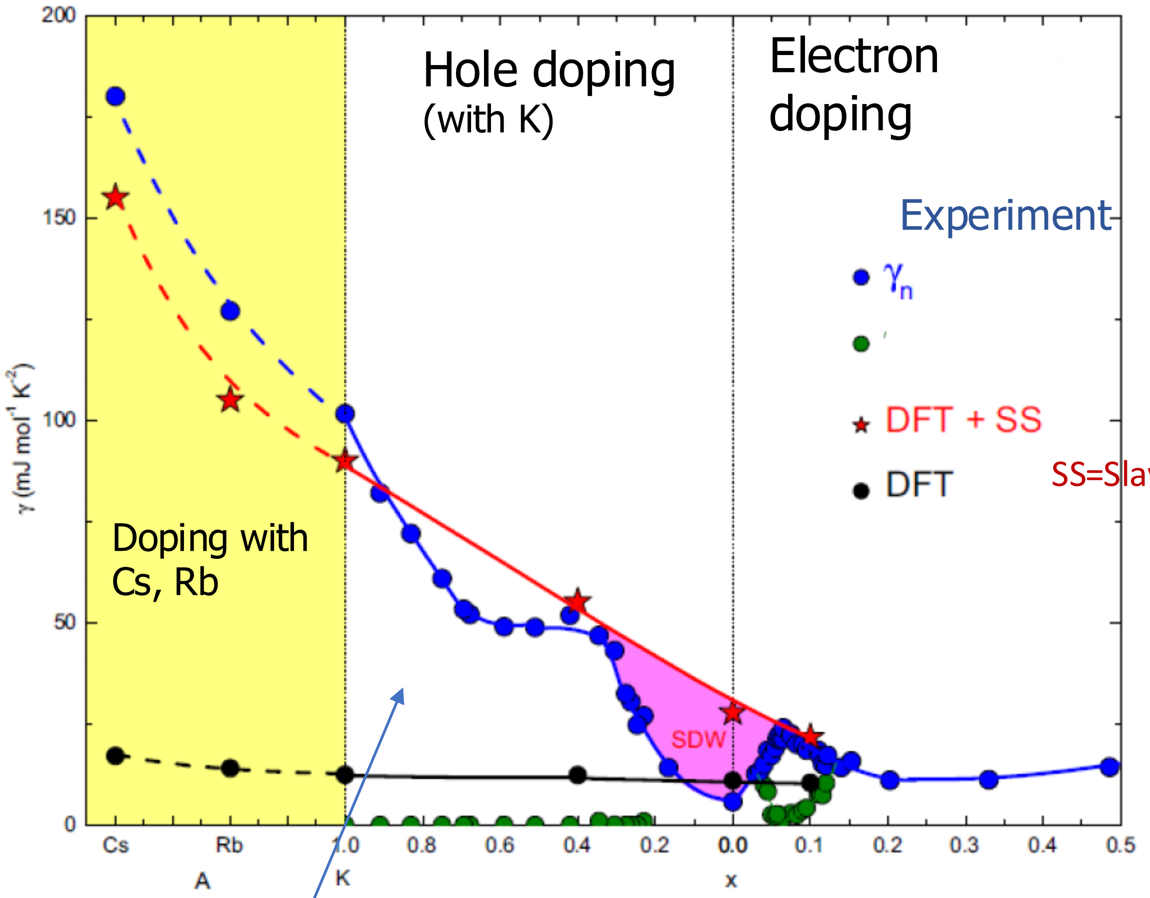
$C_v=\gamma^*\, T \qquad \gamma \propto m^*$

Temperature



Hole doping

Electron doping



SS=Slave Spin

The mass is highly enhanced with hole doping

de Medici et al

Doping

Fermi liquid theory: Interactions between quasiparticles

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

Focus on the Fermi surface $|k|=|k'|=K_F$

Remember assumption of isotropic system

Time-reversal invariance (no magnetic field) $f_{k\sigma,k'\sigma'} = f_{-k-\sigma,-k'-\sigma'}$

Fermi surface invariant under reflection $k \rightarrow -k$ $f_{k\sigma,k'\sigma'} = f_{k-\sigma,k'-\sigma'}$

If spin is conserved in general the dependence on spin enters only via their relative orientation

$$f_{k\sigma,k'\sigma'} = f_{kk'}^{s} + f_{kk'}^{a}\,\sigma.\sigma'$$

Spin symmetric

Spin antisymmetric (exchange)

Pines & Nozieres and Coleman's books

Fermi liquid theory: Interactions between quasiparticles

$$F[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

$f_{kk'}{}^{s}$ $\quad$ $f_{kk'}{}^{a}$

Focus on the Fermi surface $|k|=|k'|=K_F$. Isotropic system

$f_{kk'}{}^{s,a}$: dependence only on the angle ξ between k and k' → Expansion in Legendre Polynomials

$$f_{kk'}{}^{s,a} = \Sigma_{l=0}{}^{\infty} \mathbf{f_l{}^{s,a}} P_l(\cos\xi)$$

Dimensionless parameters $N^*(E^*_F)\, f_l{}^{s,a} = \mathbf{F_l{}^{s,a}}$

Density of quasiparticle states at the Fermi level In the interacting system

Interaction parameters
$F_0{}^s, F_0{}^a, F_1{}^s, F_1{}^a \ldots$
Can be extracted from experiment and model calculations

Fermi liquid theory: Interactions and parameters. Summary

Expansion of the free energy in terms of the quasiparticle density

$$F\,[\delta n_{k\sigma}] = F_g + \Sigma_{k\sigma}\varepsilon^*_{k\sigma}\delta n_{k\sigma} + \Sigma_{k\sigma k'\sigma'} f_{k\sigma k'\sigma'}\delta n_{k\sigma}\delta n_{k'\sigma'} + O(\delta n^3)$$

Energy of a quasiparticle
in the absence of other quasiparticles

Residual interactions
between the quasiparticles

Linearized dispersion around K_F

$$\varepsilon^*_{k\sigma} = \frac{k_F}{m^*}(k-k_F)$$

Renormalized mass or
quasiparticle mass

$$f_{kk'}^{s,a} = \Sigma_{l=0}^{\infty} f_l^{s,a} P_l(\cos\xi)$$

$$N^*(E^*_F)\, f_l^{s,a} = F_l^{s,a}$$

Small number of parameters l=0,1 …

Fermi liquid theory: Measurable quantities. Spin susceptibility

- Spin susceptibility

$$\chi_s = \frac{\partial M}{\partial H}$$

$$\chi^*_s = \frac{\mu_B^2 N^*(E_F)}{1 + F_0^a} = \frac{(m^*/m)}{1 + F_0^a} \chi_s$$

independent of T
as in Fermi gas

interacting

Susceptibility
Spin dependent
Isotropic response

Non-interacting

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Fermi liquid theory: Measurable quantities. Spin susceptibility

- Spin susceptibility

$$\chi_s = \frac{\partial M}{\partial H}$$

$$\chi^*_s = \frac{\mu_B^2 N^*(E_F)}{1+F_0^a} = \frac{(m^*/m)}{1+F_0^a}\chi_s$$

interacting

Susceptibility
Spin dependent
Isotropic response

Non-interacting

independent of T
as in Fermi gas

Wilson ratio or Stoner enhancement factor

$$W = \frac{(\gamma^*/\chi^*_s)}{(\gamma/\chi_s)} = \frac{1}{1+F_0^a}$$

experiment

Constant

LDA

$C_v = \gamma^* T \qquad \gamma^* \propto m^*$

Deviation from unity,
signature of electronic correlations

A way to obtain F_0^a

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Fermi liquid theory: Measurable quantities. Resistivity

Band theory: classification into metals and insulators

Metal ρ
increases with T

Not changed by interactions in a Fermi liquid

Insulator ρ
decreases with T

Fermi liquid theory: Measurable quantities. Resistivity

Band theory: classification into metals and insulators

Metal ρ
increases with T

Not changed by interactions in a Fermi liquid

Insulator ρ
decreases with T

Drude conductivity $\sigma = ne^2\tau/m^*$ $\qquad \tau^{-1} = \dfrac{\pi^2 T^2}{E^*_F} \propto m^* T^2$

Fermi liquid theory: Measurable quantities. Resistivity

Band theory: classification into metals and insulators

Metal ρ increases with T

Not changed by interactions in a Fermi liquid

Insulator ρ decreases with T

Drude conductivity $\sigma = ne^2\tau/m^*$ $\qquad \tau^{-1} = \dfrac{\pi^2 T^2}{E^*_F} \propto m^* T^2$

$$\boxed{\rho = \rho_0 + AT^2 \qquad A \propto (m^*)^2}$$

High resistivity due to interactions

disorder

Quadratic dependence on T

Constant strongly dependent on interactions

Many materials show a resistivity which shows a T^2 behavior

Phonon resistivity $\propto T^5$ can dominate in simple metals

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Fermi liquid theory: Measurable quantities.

$C^*_v = \gamma^* T \qquad \gamma^* \propto m^*$

$\rho = \rho_0 + AT^2 \qquad A \propto (m^*)^2$

$$\frac{A}{(\gamma^*)^2} \sim \text{constant}$$

Kadowaki –Woods ratio

Fermi liquid theory: Measurable quantities.

$$C^*_v=\gamma^* T \qquad \gamma^* \propto m^*$$

$$\rho=\rho_0 + AT^2 \qquad A \propto (m^*)^2$$

$$\frac{A}{(\gamma^*)^2} \sim \text{constant}$$

Kadowaki –Woods ratio

A and γ corrected by degeneracy

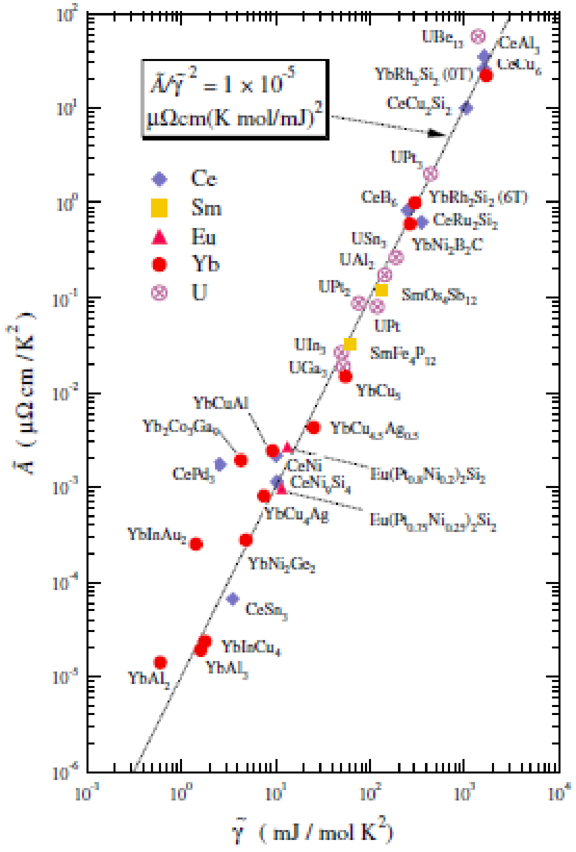


Fig: Tsuji et al, PRL 94, 057201 (2005)

Fermi liquid theory: Measurable quantities. Resistivity

$$\rho = \rho_0 + AT^2 \qquad A \propto (m^*)^2$$

High-Tc superconducting cuprates

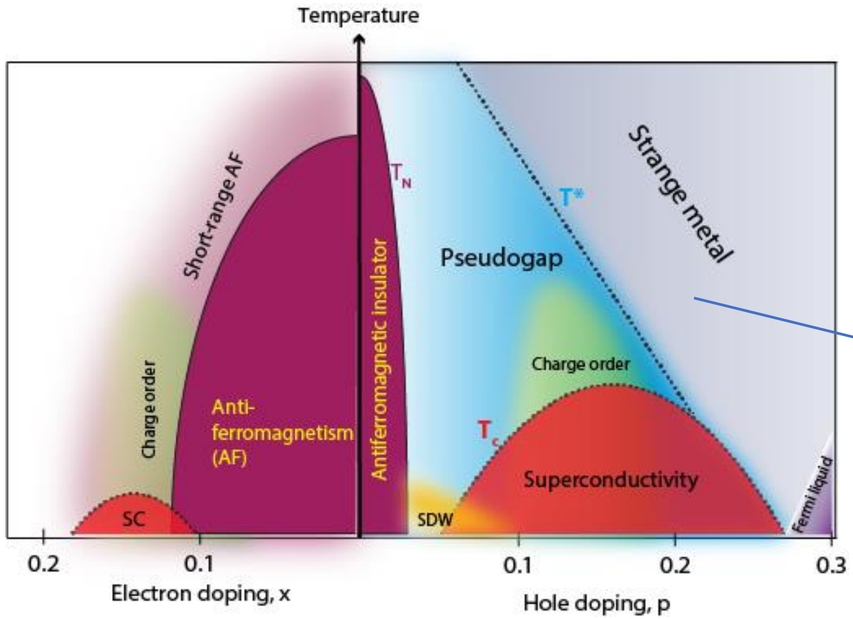


Fig: cme.physics.ucdavis.edu

Strange metal

$$\rho \sim T$$

Observed in many correlated systems. Not the behavior expected from quasiparticles

Key whether this behavior reaches T → 0

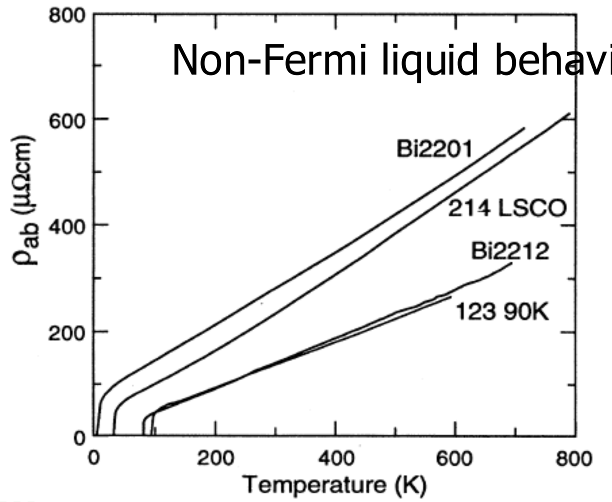


Fig: Dagotto, RMP 66, 763 (1993)

In cuprates the resistivity does not seem to saturate

Linear in T resistivity also observed in iron superconductors, heavy fermions, Twisted Bilayer Graphene, Twisted dichalcogenides …

Fermi liquid theory: Measurable quantities.

Mass renormalization

Looking at the quasiparticle current

$$m^* = m\,(1 + F_1^s) = \frac{m}{1 - N(E_F)f^1_s}$$

Spin independent
But directional response

Coleman's book

Fermi liquid behavior

- Bands observed in photoemission

Reduction in bandwidth as compared to LDA estimate of m*/m

- Specific heat linear in temperature.

Enhancement of g estimate of m*/m

Careful in materials with multiple Fermi pockets

$$C^*_v = \gamma^* T$$

- Temperature independent spin susceptibility

Enhancement: mass renormalization + Stoner enhancement

- Resistivity quadratic in temperature

Interactions enhance resistivity as $(m^*)^2$

$$\rho = \rho_0 + AT^2$$

Correlated electrons: frequently non-Fermi liquid behavior

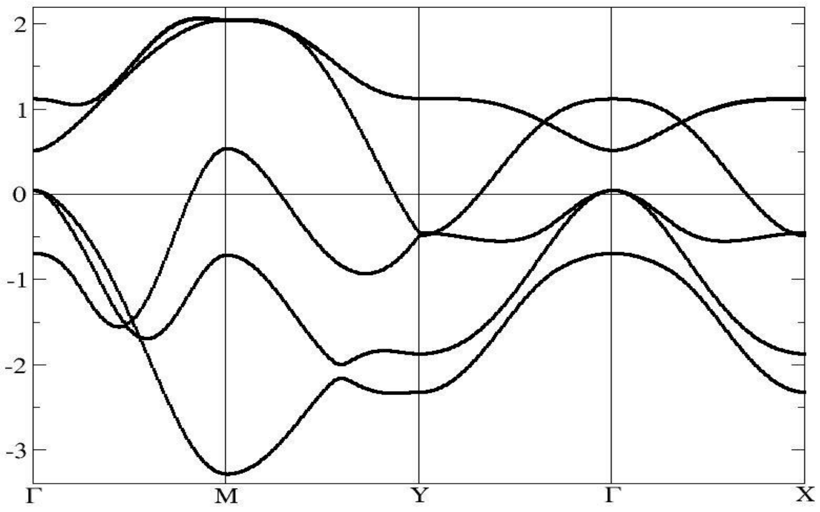
The notion of
Quantum Materials

Remember: Fermi liquid behavior does not mean simple band theory

Band theory:

Basis of our understanding of solids

- Successful description
- Metals and insulators
- Dependence on temperature of measurable quantities (Cv, c, ..)



m*/m can be large

Enhancement of susceptibility and specific heat

$$\rho=\rho_0 + AT^2$$

Due to decay of quasiparticles

Renormalized parameters tell us about the strength of interactions

Larger deviations of ”standard” behavior at large energies and temperatures