GRAPHENE

The running of the constants

To first approximation, the dispersion relation around the Fermi energy of single-layer graphene is linear, making its charge carriers behave like massless relativistic subatomic particles. More careful inspection of its low-energy band structure suggests the picture is more complex, extending the analogy even further.

Maria A. H. Vozmediano

The best times in Physics are those when physicists of different expertise meet around a problem of common interest. And this is now happening in the case of graphene. From the early days of the isolation of single sheets of graphene, the relativistic nature of its charge carriers was clear[1]. These carriers, known as Dirac fermions, are described by equations similar to those that describe the quantum electrodynamic (QED) interactions of relativistic charged particles. A meticulous study performed by Elias and co-workers[2] of the electronic structure of graphene shows that at very low energies reaching a few meV of graphene's Dirac point, where its cone-like valence and conduction bands touch, the shape of the conduction and valence bands diverge from a simple linear relation. The result implies that the analogy between graphene and high-energy physics is deeper than first expected. In particular, it implies that the electromagnetic coupling of graphene does renormalize, as occurs in quantum field theory.

The magnitude of the coupling constants that characterize the strengths of the fundamental forces of nature vary profoundly. Between gravity, the weakest force, and the strong force, the strongest, the difference is a mind-bending 40 orders of magnitude. Between these extremes, electromagnetism governs essentially all the interactions between atoms and molecules and determines the behaviour of everyday condensed matter systems like graphene. Despite the name, coupling constants are not in fact constant, but can change depending on the energy scale of the experiments in which they are measured. For instance, the fine structure constant of QED (α_{QED}) is approximately 1/137 in experiments performed at energies of the order of electron mass (about 0.5 MeV), but is found to increase to approximately 1/128 in experiments carried out at the much higher energy of around 90 GeV in the Large Electron–Positron collider (LEP) at CERN. This so-called running of coupling constants is a consequence of quantum field theory and is related to renormalization — the solution to the 'divergence problem' and the infinities that arise when computing physical properties in perturbation theory (see Fig. 1).

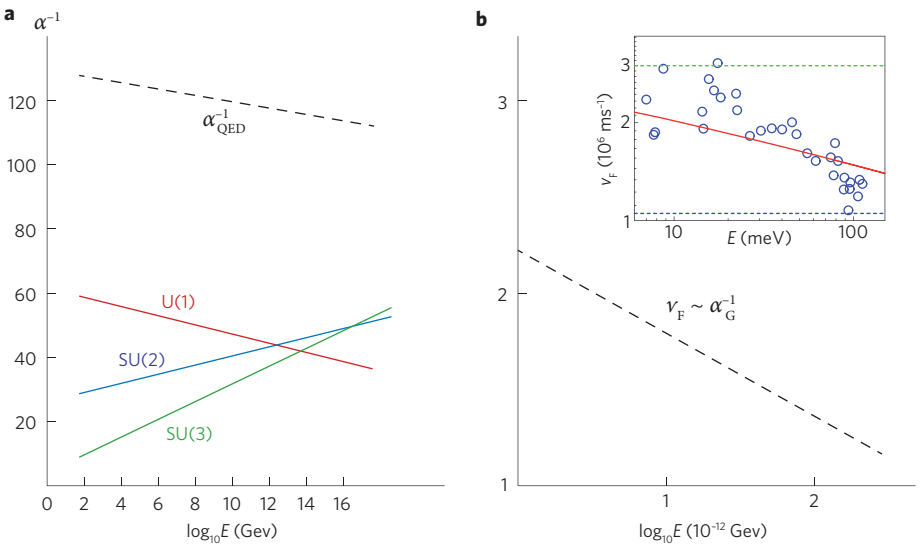


Figure 1 | Initial formulations of quantum field theory predicted infinite values of basic physical quantities when calculated with perturbative techniques. This 'divergence problem' is solved by renormalization of the coupling constants. **a**, Typical picture of the running of the inverse of the coupling constants (α^{-1}) as a function of energy scale (E) in the standard model SU(3)×SU(2)×U(1) of elementary particles, and QED (α_{QED}). **b**, The running of the Fermi velocity in graphene is directly proportional to the inverse coupling constant. The horizontal axis represents the log of the energy in meV. The inset shows a plot of the actual measurement with error bars represented by the radius of the circles. The steeper slope of graphene with respect to the QED case is a result of the larger value of the coupling constant in graphene.

One of the most notable things discovered about graphene when it was first isolated was the linear shape of its conduction and valence bands[1]. This implies that its electrons move as if they are free of mass. But what does all this really mean? QED describes free electrons moving through a vacuum and interacting with other charged particles by the exchange of photons with a strength given by α_{QED}. The Dirac fermions of graphene are not real electrons in the strictest sense of the word, but collective degrees of freedom that just happen to have the same charge and spin as electrons. They are an effective description produced from the free remainders of the carbon orbitals in the graphene's honeycomb lattice. It is a surprising and happy coincidence that they do behave as free electrons in most respects.

The experiment carried out by Elias *et al.* push the analogy to its limits. The coupling constant in QED is defined as a function of the fundamental electron charge, e, and the speed of light, c, by $\alpha_{QED}=e^2/4\pi c$. In graphene, c is replaced by the Fermi velocity, v_F, which is of the order of $c/300$. This increases the effective fine structure constant in graphene by the same amount, so that $\alpha_G \approx 300\alpha_{QED}$. The running of this constant in QED is due to the renormalization of the electron charge and c remains constant. The increase in α_{QED} at higher energies has been demonstrated in accelerators in the energy range from 1 MeV to 100 GeV. In contrast, in the case of graphene, the electric

charge stays constant and the upward renormalization of α_G is due to a decreasing Fermi velocity at increasing energies. In both QED and graphene, the renormalization of the coupling between two different energies E_1 and E_2 is given by the relation

$$\alpha(E_2)=\frac{\alpha(E_1)}{1-A\alpha(E_1)\ln(E_2/E_1)}$$

where A is a constant that depends on the number of fermion species that contribute to the renormalization at energy E_2.

The idea that such a renormalization would occur in graphene was suggested almost a decade before it had actually been successfully isolated[3]. The reason it has taken so long since graphene's initial isolation to confirm it experimentally is that it only becomes evident within 1 eV of the Dirac point and a clear demonstration of the validity of any logarithmic relation naturally requires a dataset that spans several orders of magnitude. In this sense, the experiments performed by Elias *et al.* represent a real tour-de-force, probing graphene's electronic structure down to fractions of meV of the Dirac point, and confirming the logarithmic behaviour all the way down to this point. Beyond establishing the QED-like behaviour of graphene further than any physicist might have reasonably expected, the result improves our understanding of the often controversial nature of electron–electron interactions in neutral graphene.

Maria A. H. Vozmediano is at the Instituto de Ciencia de Materiales de Madrid (CSIC), Sor Juana Inés de la Cruz 3, Madrid 28049, Spain.
e-mail: vozmediano@icmm.csic.es

References
1. Novoselov, K. S. *et al. Nature* **438,** 197–200 (2005).
2. Elias, D. C. *et al. Nature Phys.* **7,** 701–704 (2011).
3. González, J., Guinea, F. & Vozmediano, M. A. H. *Nucl. Phys. B* **424,** 595–618 (1994).

Published online: 24 July 2011

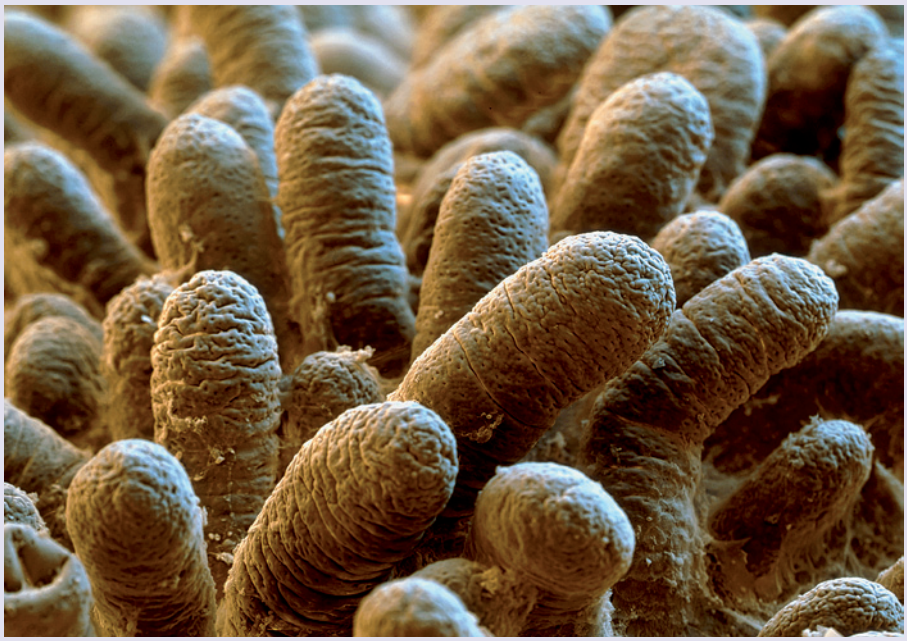
BIOPHYSICS

On mechanics and morphology

When D'Arcy Thompson penned his 1917 book *On Growth and Form* he boldly declared that the morphologist — devoted to understanding the structure of organisms — is *ipso facto* a student of physical science. His meaning was clear: the growth of complex structures mediating specific biological function is underpinned by an intrinsic mechanics, an appreciation of which is crucial to a broader understanding of both form and function.

Thierry Savin and colleagues refer to Thompson's tome in their investigation, published in *Nature*, of the elaborate looped morphology that arises in the vertebrate gut (*Nature* **476,** 57–62; 2011). Using experiment, simulation, and an innovative physical mock-up comprising rubber tubing stitched to latex, they have examined the forces arising from relative growth between the gut tube and a neighbouring sheet of tissue known as the dorsal mesentery. The study reveals a mechanism for the formation of loops based on differential strain between the two tissues.

This is a timely nod to Thompson's century-old ideas, given the recent surge of physicists and mathematicians into the biological sciences, problem-solving artillery engaged. In another paper, published in *Physical Review Letters*, Edouard Hannezo, Jacques Prost and Jean-François Joanny adopt a similarly mechanical approach to understanding the complex structures seen lining the small intestine (pictured), invoking an analogy with the buckling of metallic plates under compression (*Phys. Rev. Lett.* **107,** 078104; 2011). They have developed a model that implicates cellular division and death as sources of internal stress, which in turn influences morphology and induces mechanical feedback on organ and tissue development.



© SPL

One of the most interesting aspects of Thompson's treatise is an emphasis on the degree to which structures in different tissues and organisms can be related to one another by means of mathematical transformation. Both of the new papers offer striking evidence to this effect. For Savin *et al.*, scaling arguments for the size, number and radius of loops account for qualitative and quantitative variation across different species, including chick, quail, finch and mouse. In a similar spirit, Hannezo and colleagues report that by tuning their model for the morphology of the small intestine, the markedly different structures populating the colon can also be reproduced.

The upshot of this and related work is that macroscopic mechanics drives morphology during the formation of tissues and organisms — bringing the formalism of physics to bear on long-standing problems in developmental biology.

ABIGAIL KLOPPER