

Electronic correlations in Hund metals

E. Bascones

Instituto de Ciencia de Materiales de Madrid



Laura Fanfarillo
ICMM-CSIC



Outline

□ Introduction:

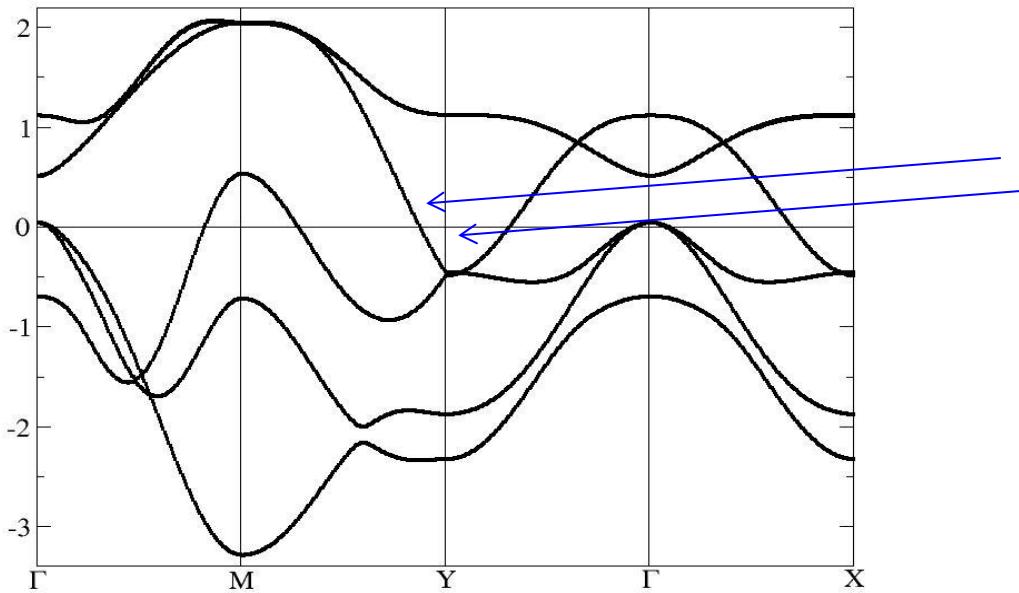
- * Mott physics and Quasiparticle weight: basic notions
- * Iron superconductors as Hund metals
- * The role of Hund's coupling in correlations. Hund vs Mott

□ Correlations in Multi-orbital systems

- * Quasiparticle weight vs interactions, filling and number of orbitals
- * Charge and spin fluctuations
- * Simple understanding of correlations in Hund metals

□ Summary

Metals and Insulators in band theory. Independent electrons



Metallicity
in clean systems
Bands crossing
the Fermi level
(finite DOS)

Fig: Calderón et al, PRB, 80, 094531 (2009)

Insulating behaviour
in clean systems
Bands below
Fermi level filled

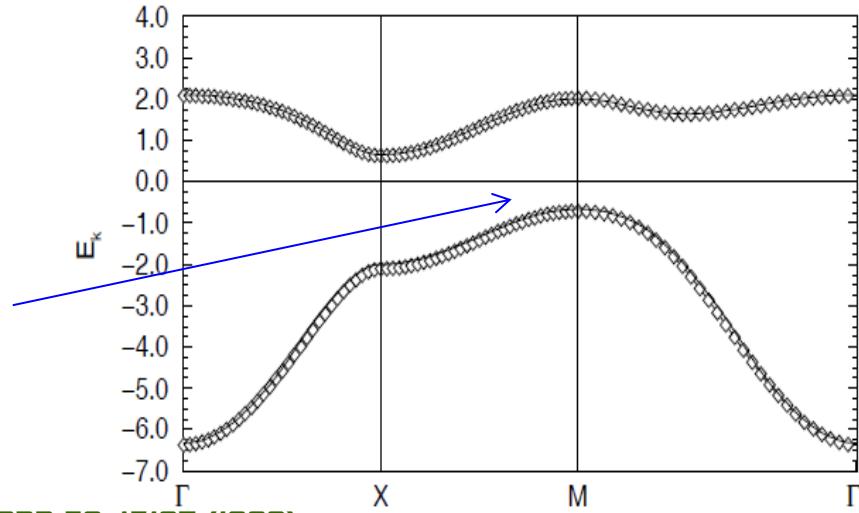


Fig: Hess & Serene, PRB 59, 15167 (1999)

Metals and Insulators in band theory. Independent electrons

Spin degeneracy:
Each band can hold 2 electrons per unit cell

Odd number
of electrons
per unit cell

→ Metallic

Even number
of electrons
per unit cell

→ Insulating

→ Metallic (in case
of band overlap)

Failure of independent electron picture. Mott insulators

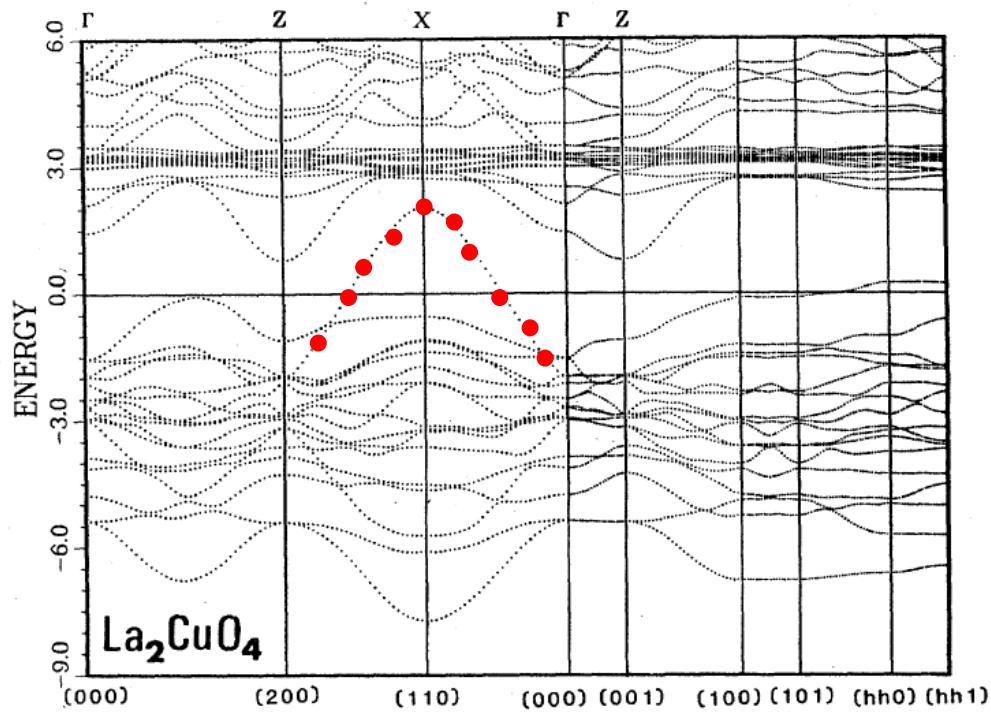


Fig: Pickett, RMP 61, 433 (1989)

Electron counting La_2CuO_4 :

$$\begin{aligned} & 2 \text{ La } (57 \times 2) + \text{Cu } (29) + 4 \text{ O } (4 \times 8) \\ & = 175 \text{ electrons} \end{aligned}$$

Metallic behavior expected

Failure of independent electron picture. Mott insulators

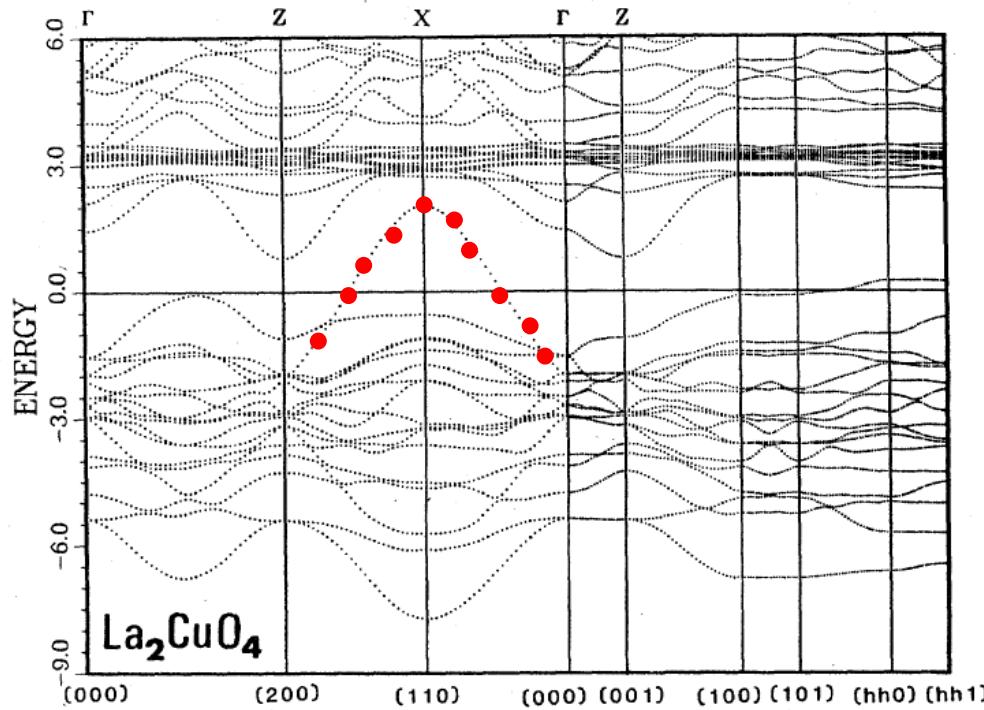


Fig: Pickett, RMP 61, 433 (1989)

Electron counting La_2CuO_4 :
2 La (57x2)+Cu (29) + 4 O (4x8)
=175 electrons

**Metallic behavior expected
but
Insulating behavior is found**

Mott insulator:
Insulating behavior due to electron-electron interactions
Breakdown of independent electron picture

Mott insulators. The single-orbital Hubbard model at half-filling

$$H = \sum_{i,j,\sigma} t c_i^\dagger c_j, \sigma + h.c. + U \sum_j n_{j,\uparrow} n_{j,\downarrow}$$

Kinetic energy *Intra-orbital repulsion*

Atomic lattice with a single orbital per site and average occupancy 1 ([half filling](#))

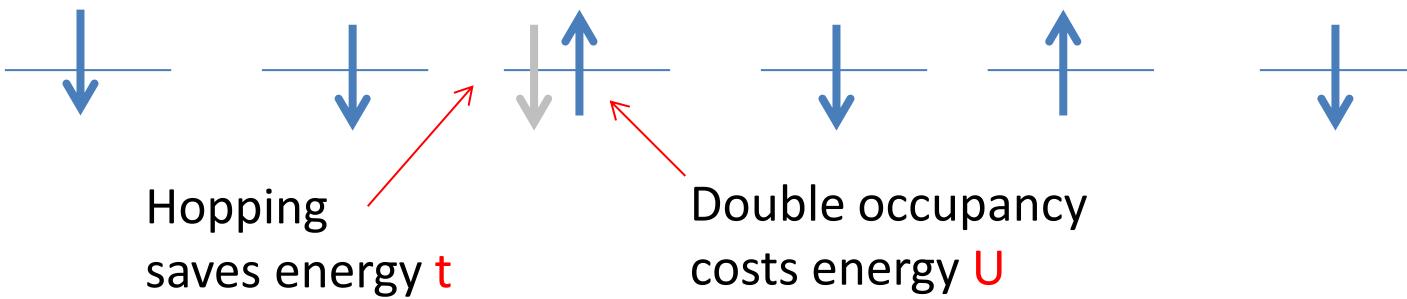


Mott insulators. The Hubbard model at half-filling

$$H = \sum_{i,j,\sigma} t c_i^\dagger c_j, \sigma + h.c. + U \sum_j n_{j,\uparrow} n_{j,\downarrow}$$

Kinetic energy *Intra-orbital repulsion*

Atomic lattice with a single orbital per site and average occupancy 1 ([half filling](#))



For $U \gg t$ electrons localize: Mott insulator
Mott transition at U_c

But away from half-filling the system is always metallic → Correlated metal

The quasiparticle weight in the single-orbital Hubbard model

Z: a way to quantify the correlations

$$0 \leq Z \leq 1$$

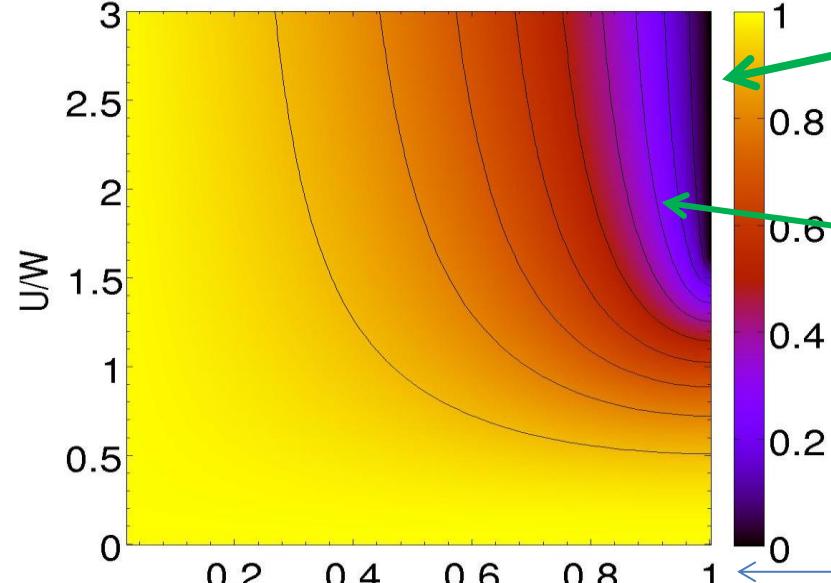
Z=1 Single-particle picture

Z=0 There are no quasiparticles
Breakdown of single-particle picture

Simple description: Heavy electron

$$Z^{-1} \propto m^*/m$$

Z colour plot for the Hubbard model



Z vanishes at the Mott transition

Correlated metal as U is increased
or half-filled is approached
(only close to the Mott transition)

W bandwidth
of non interacting system

Half-filling

n: number of electrons per atom

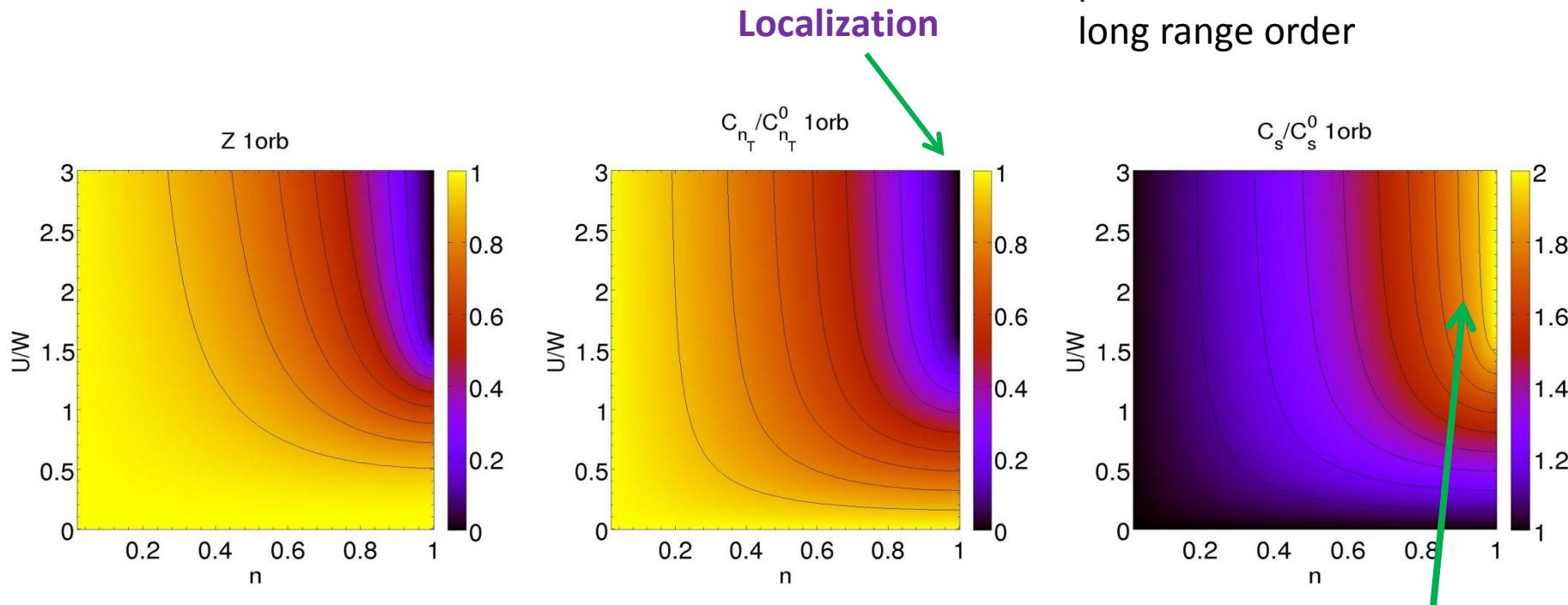
Charge and spin fluctuations in the single-orbital Hubbard model

$$n = \langle n \rangle + \delta n$$

$$C_T = \langle n^2 \rangle - \langle n \rangle^2 = \langle (\delta n)^2 \rangle$$

$$C_S = \langle S^2 \rangle - \langle S \rangle^2 = \langle S^2 \rangle$$

C_S larger when atoms are spin polarized even if there is no long range order



Mott insulator:

Suppression of charge fluctuations \rightarrow Localized spins

$$n = \langle n \rangle$$

Enhancement of spin fluctuations

High-T_c superconductors cuprates are doped Mott insulators

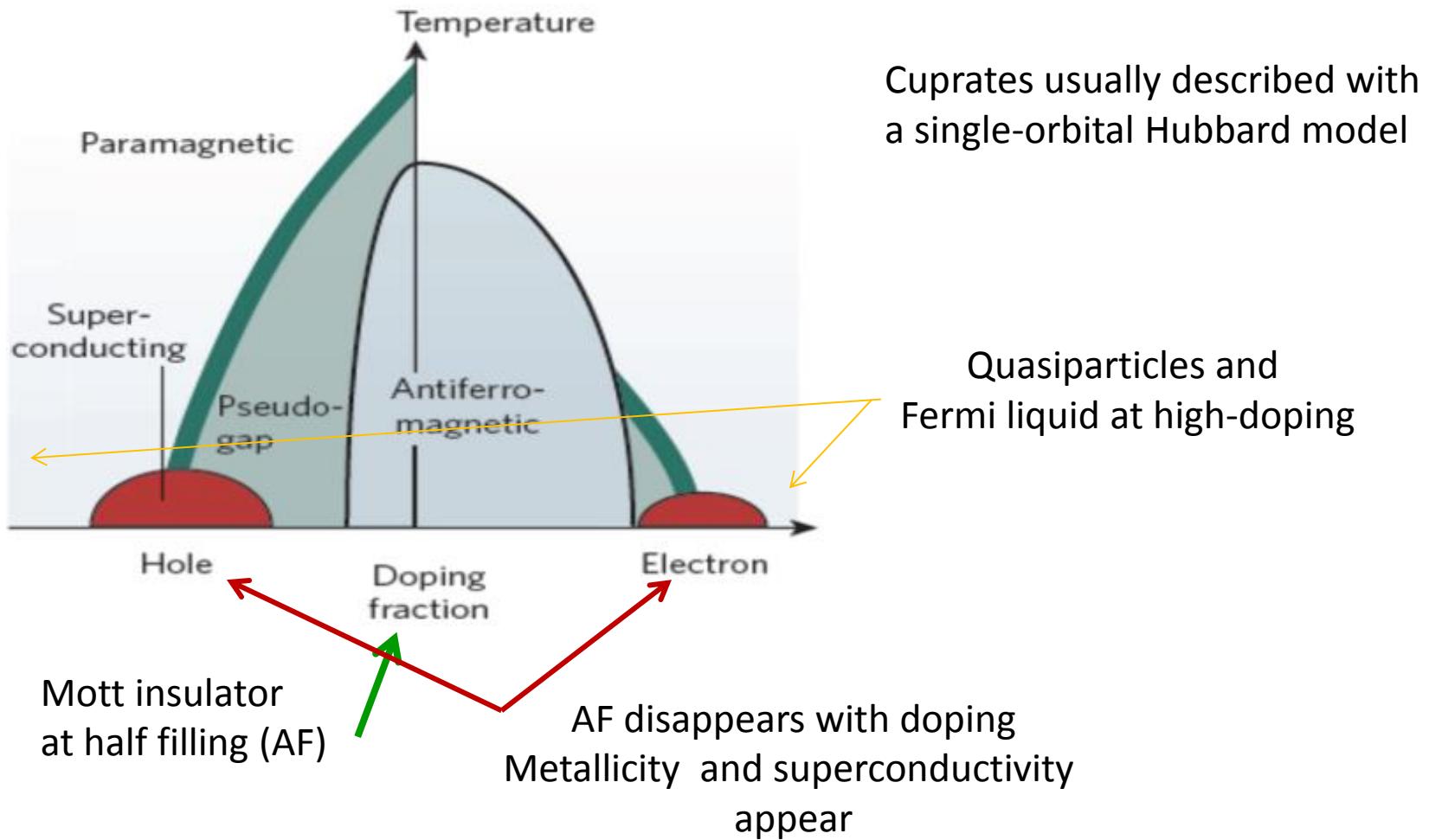
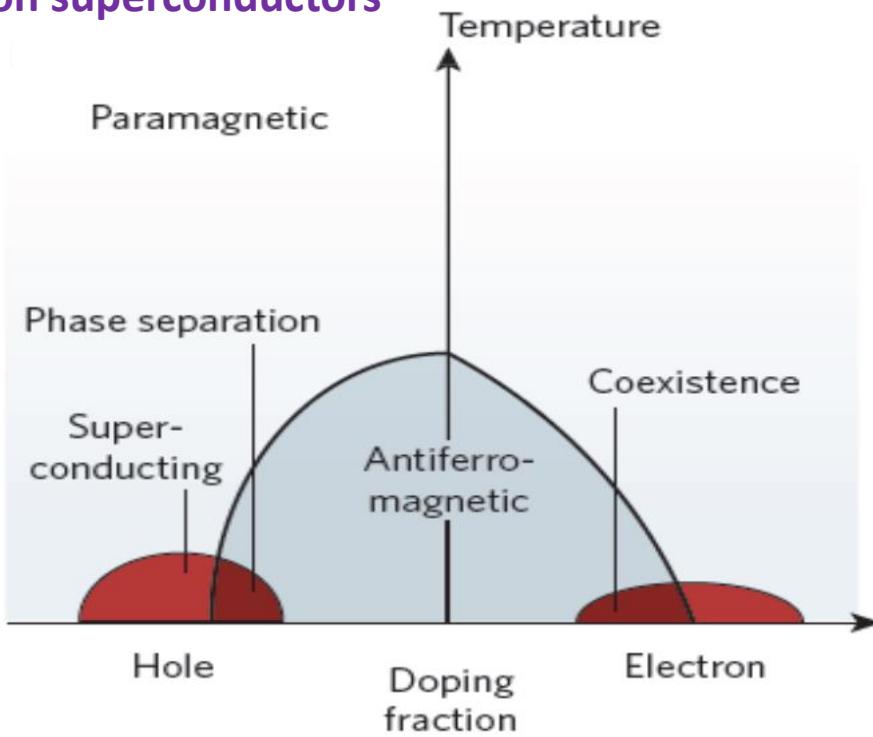


Fig: Nature 464,183 (2010)

Iron superconductors: metallic when “undoped”

Iron superconductors



“Undoped” iron superconductors
are antiferromagnetic but metallic

Cuprates

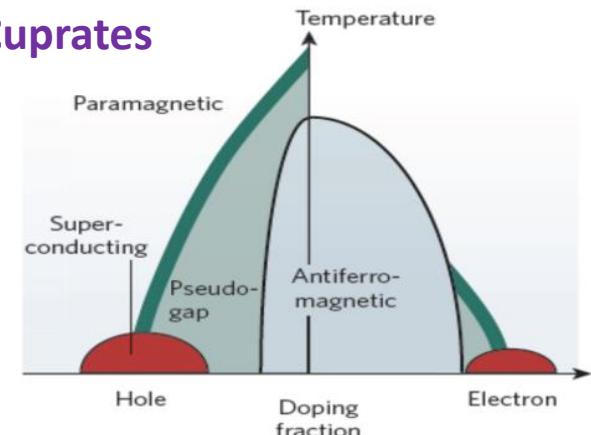
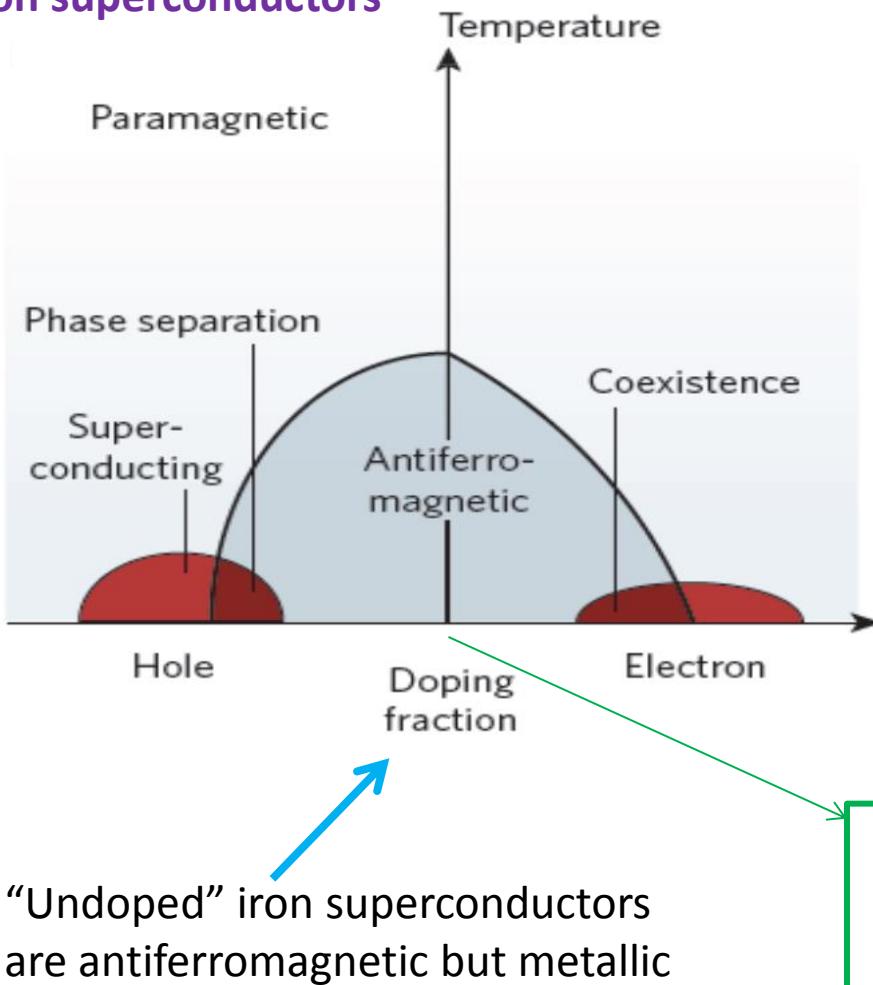


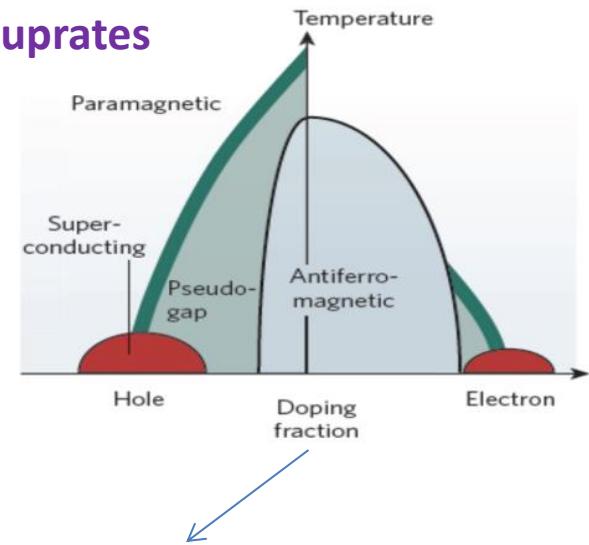
Fig: Nature 464,183 (2010)

Iron superconductors: metallic when “undoped”

Iron superconductors



Cuprates



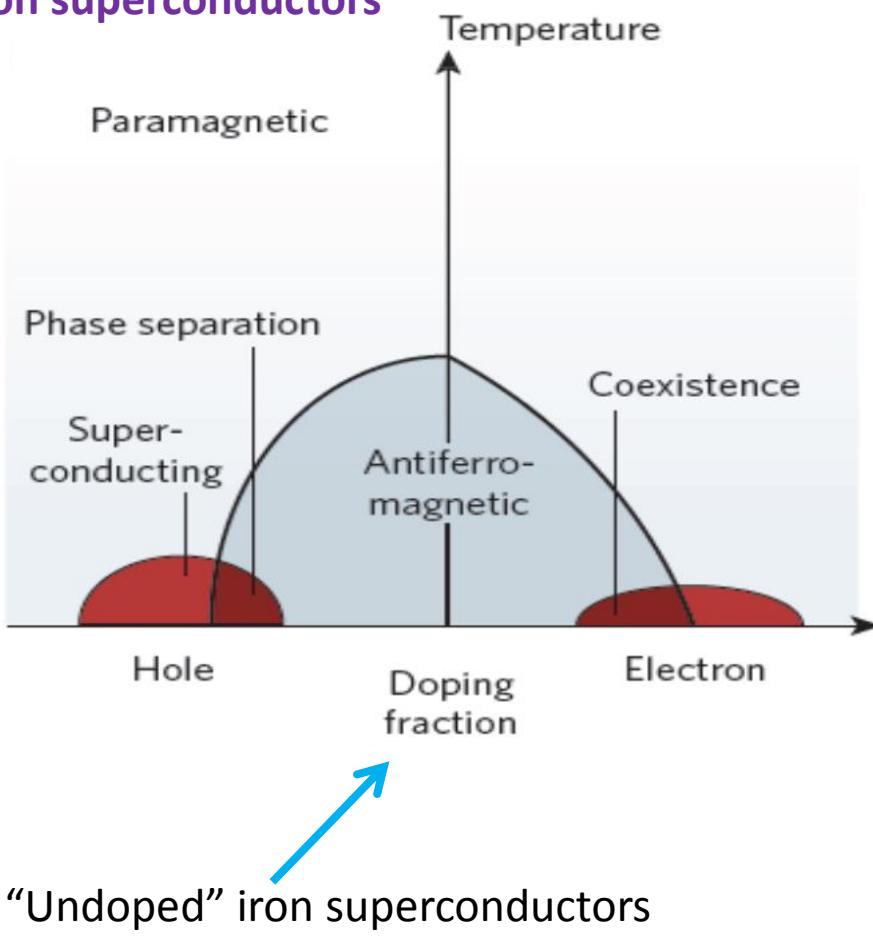
Single orbital system:
1 electron in 1 orbital
(half-filling)

**Multi-orbital system:
6 electrons in 5 orbitals**

Fig: Nature 464,183 (2010)

Iron superconductors: metallic when “undoped”

Iron superconductors



Multi-orbital system:
6 electrons in 5 orbitals

- Correlated metal $m^*/m \sim 3$ ($Z \sim 0.33$)
- Far from a Mott transition $U/W < 1$ (U interorbital repulsion, W bandwidth)
- Hund’s coupling plays a key role in the correlations



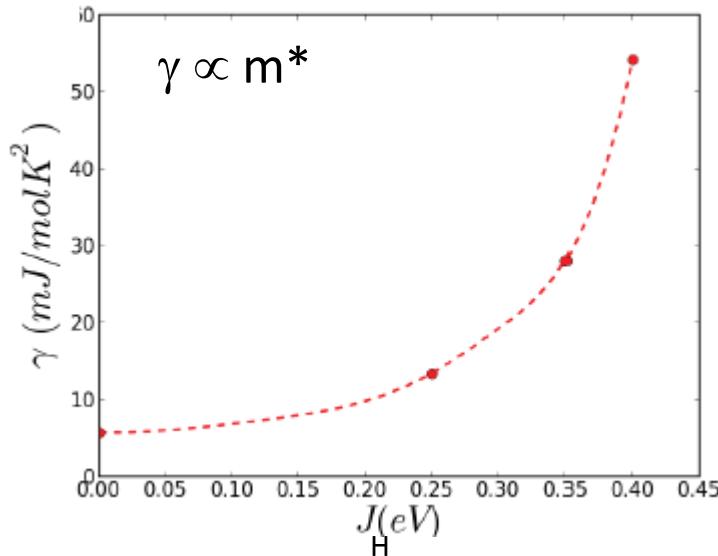
Iron superconductors as Hund metals

Fig: Nature 464,183 (2010)

Correlations in Hund metals and iron superconductors: Hund vs Mott

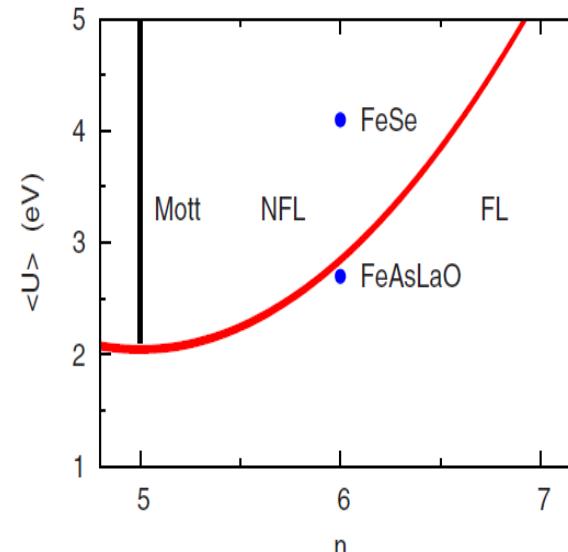
Iron superconductors as Hund metals

- Correlations driven by Hund J_H weakly dependent on U ,
- Not in proximity to a Mott insulator
- Properties essentially different to a doped Mott insulator



Haule & Kotliar NJP 11,025021 (2009)

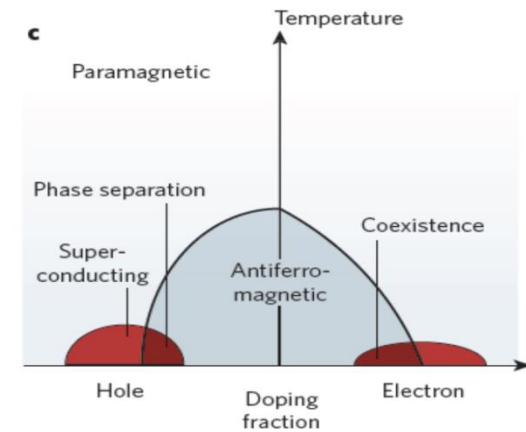
Iron superconductors as doped Mott insulators



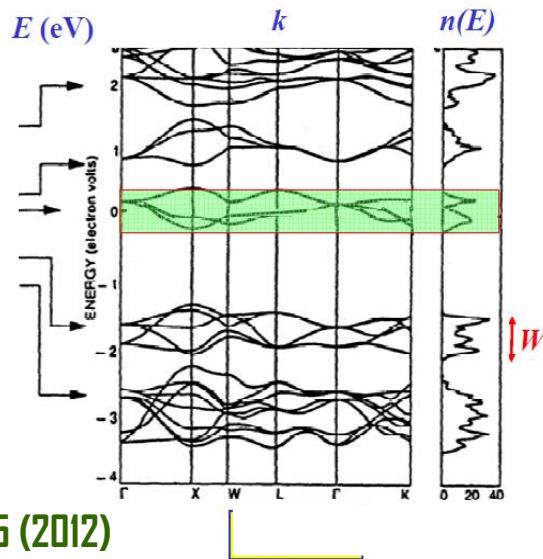
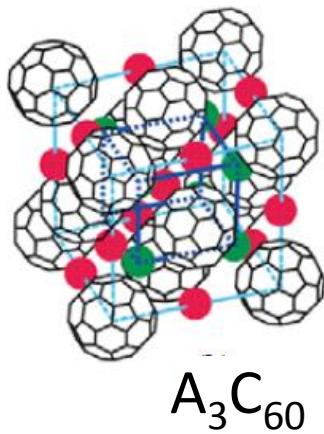
$$J_H/U=0.25$$

Ishida & Liebsch,
PRB 81, 054513 (2010)
Liebsch & Ishida,
PRB 82, 155106 (2010)

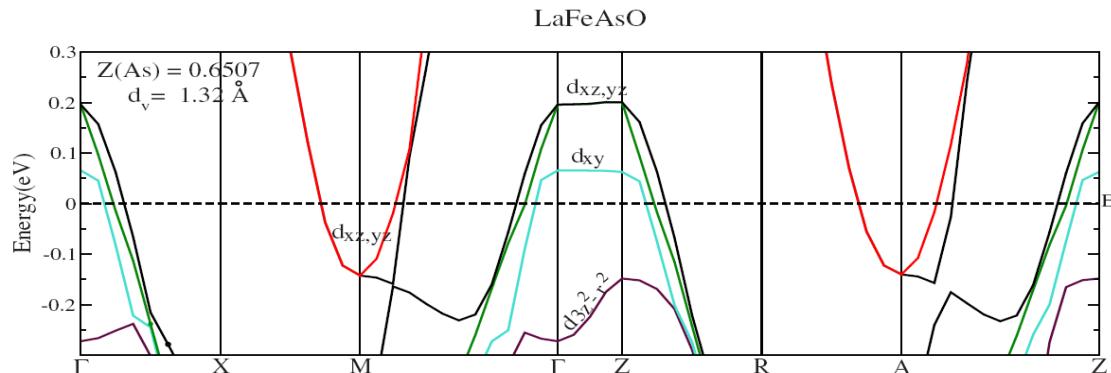
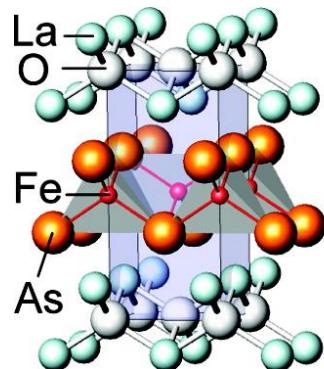
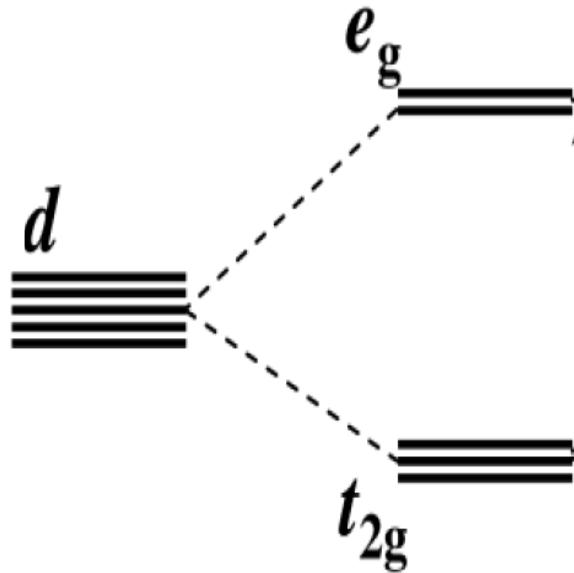
Correlations controlled by the **proximity** in **doping** to a Mott insulating state



Multi-orbital systems



Oxides with cubic symmetry



Kamihara et al, JACS, 130, 3296 (2008).

Vildosola et al, PRB 78, 064518 (2008)

Hubbard-Kanamori Hamiltonian for multi-orbital systems

$$\begin{aligned}
 H = & \sum_{i,j,\gamma,\beta,\sigma} t_{i,j}^{\gamma,\beta} c_{i,\gamma,\sigma}^\dagger c_{j,\beta,\sigma} + h.c. + U \sum_{j,\gamma} n_{j,\gamma,\uparrow} n_{j,\gamma,\downarrow} \\
 & \quad \text{Tight-binding (hopping)} \quad \text{Intra-orbital repulsion} \\
 + & \left(U' - \frac{J_H}{2} \right) \sum_{j,\gamma>\beta,\sigma,\tilde{\sigma}} n_{j,\gamma,\sigma} n_{j,\beta,\tilde{\sigma}} - 2J_H \sum_{j,\gamma>\beta} \vec{S}_{j,\gamma} \vec{S}_{j,\beta} \\
 & \quad \text{Inter-orbital repulsion} \quad \text{Hund's coupling} \\
 + & J' \sum_{j,\gamma \neq \beta} c_{j,\gamma,\uparrow}^\dagger c_{j,\gamma,\downarrow}^\dagger c_{j,\beta,\downarrow} c_{j,\beta,\uparrow} + \sum_{j,\gamma,\sigma} \epsilon_\gamma n_{j,\gamma,\sigma} \\
 & \quad \text{Pair hopping} \quad \text{Crystal-field}
 \end{aligned}$$

In the presence of rotational symmetry $U' = U - 2J_H$ $J' = J_H$

Two interaction parameters: U , J_H

Hubbard-Kanamori Hamiltonian: Interaction terms

Intra-orbital
repulsion

Inter-orbital
Repulsion
(different spin)

Inter-orbital
Repulsion
(same spin)

$$U \sum_m \hat{n}_{m\uparrow} \hat{n}_{m\downarrow} + U' \sum_{m \neq m'} \hat{n}_{m\uparrow} \hat{n}_{m'\downarrow} + (U' - J_H) \sum_{m < m', \sigma} \hat{n}_{m\sigma} \hat{n}_{m'\sigma} +$$

$$-J_H \sum_{m \neq m'} d_{m\uparrow}^+ d_{m\downarrow} d_{m'\downarrow}^+ d_{m'\uparrow} + J \sum_{m \neq m'} d_{m\uparrow}^+ d_{m\downarrow}^+ d_{m'\downarrow} d_{m'\uparrow}$$

Spin flip

Pair hopping

$$U' = U - 2J_H$$

Hubbard-Kanamori Hamiltonian: Interaction terms

Intra-orbital
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Inter-orbital
Repulsion
(different spin)

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Spin flip

Pair hopping

$$U' = U - 2J_H$$

Hubbard-Kanamori Hamiltonian: Interaction terms

Intra-orbital
repulsion

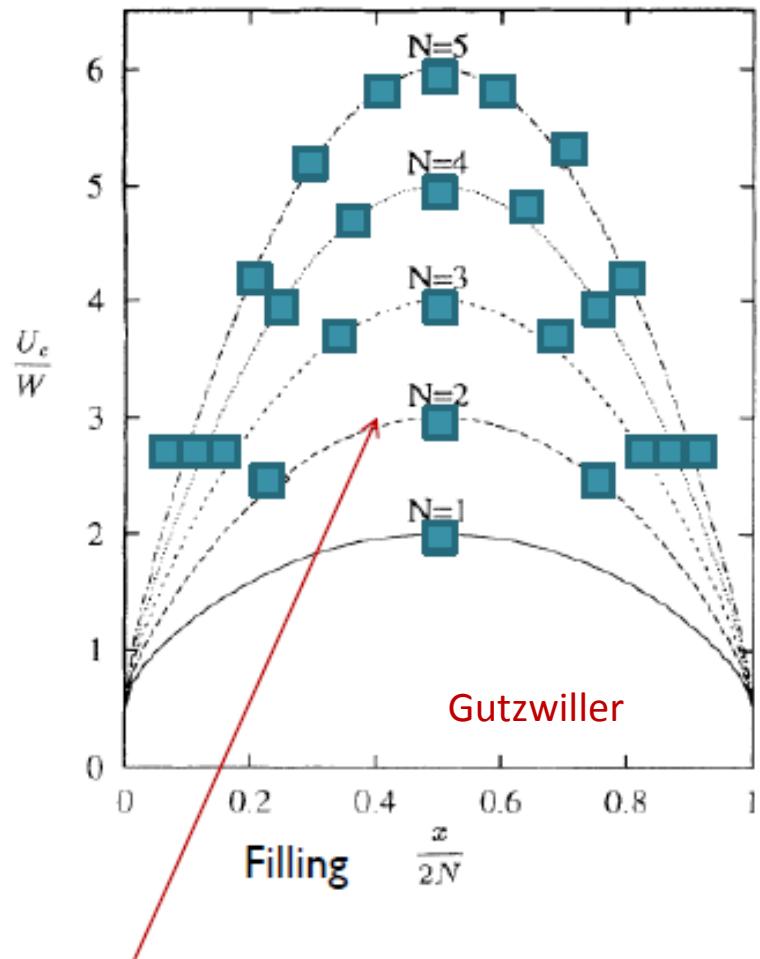
Inter-orbital
Repulsion
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$$U \sum_m \hat{n}_{m\uparrow} \hat{n}_{m\downarrow} + U' \sum_{m \neq m'} \hat{n}_{m\uparrow} \hat{n}_{m'\downarrow} + (U' - J_H) \sum_{m < m', \sigma} \hat{n}_{m\sigma} \hat{n}_{m'\sigma} +$$

$$U' = U - 2J_H$$

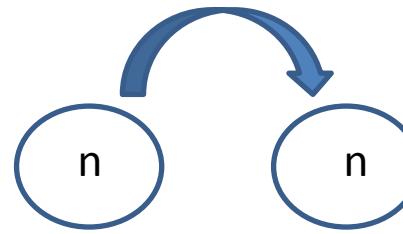
Mott transition in multi-orbital systems at Zero Hund



Zero Hund. U_c larger at half filling

- N-orbital system with n electrons/atom:
- It can hold up to $2N$ electrons
 - Half-filling $n=N$ (1 electron/orbital)
 - Mott transition not only at half-filling but for all integer n

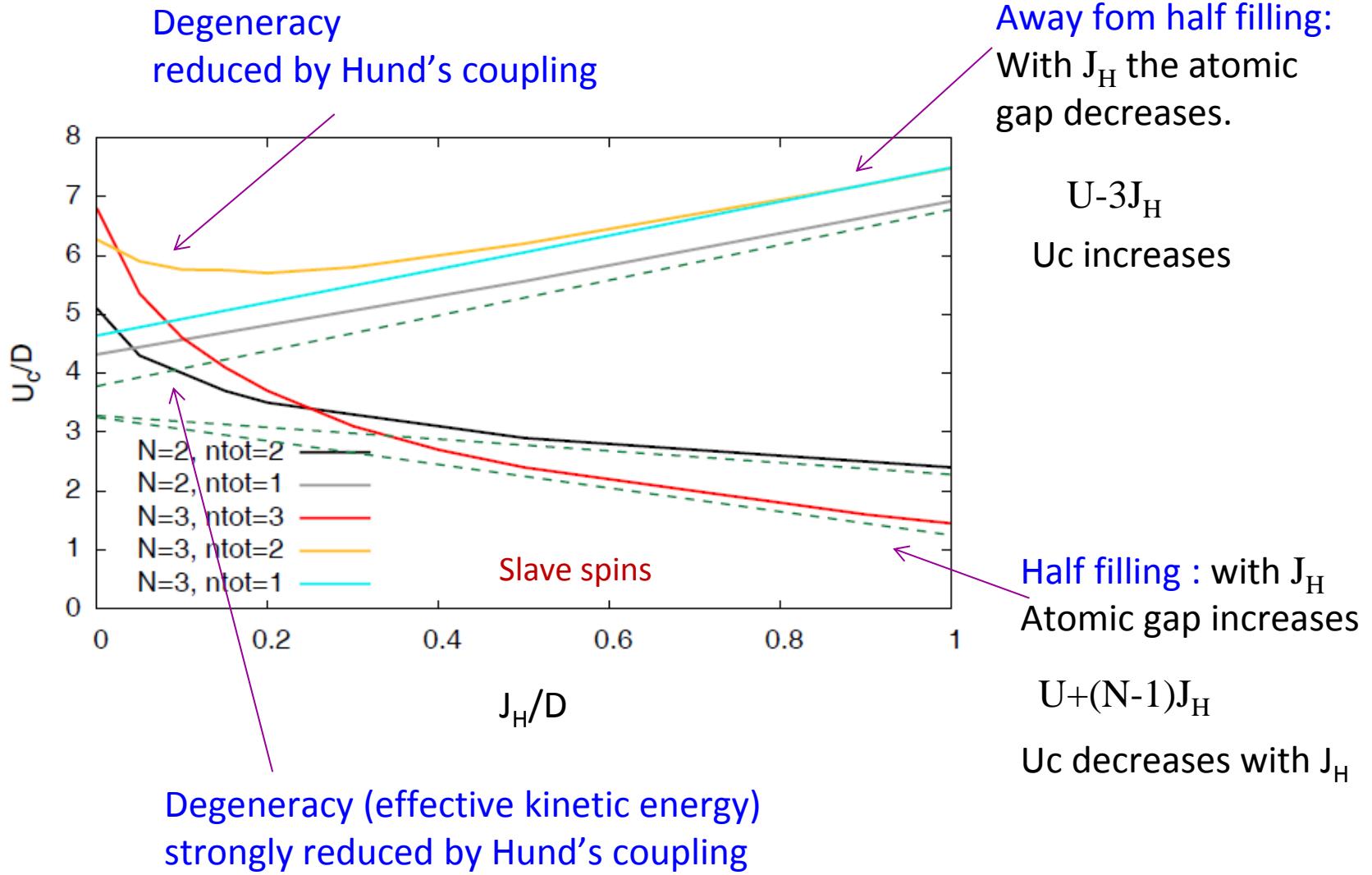
Lu PRB 49,
5687 (1994)



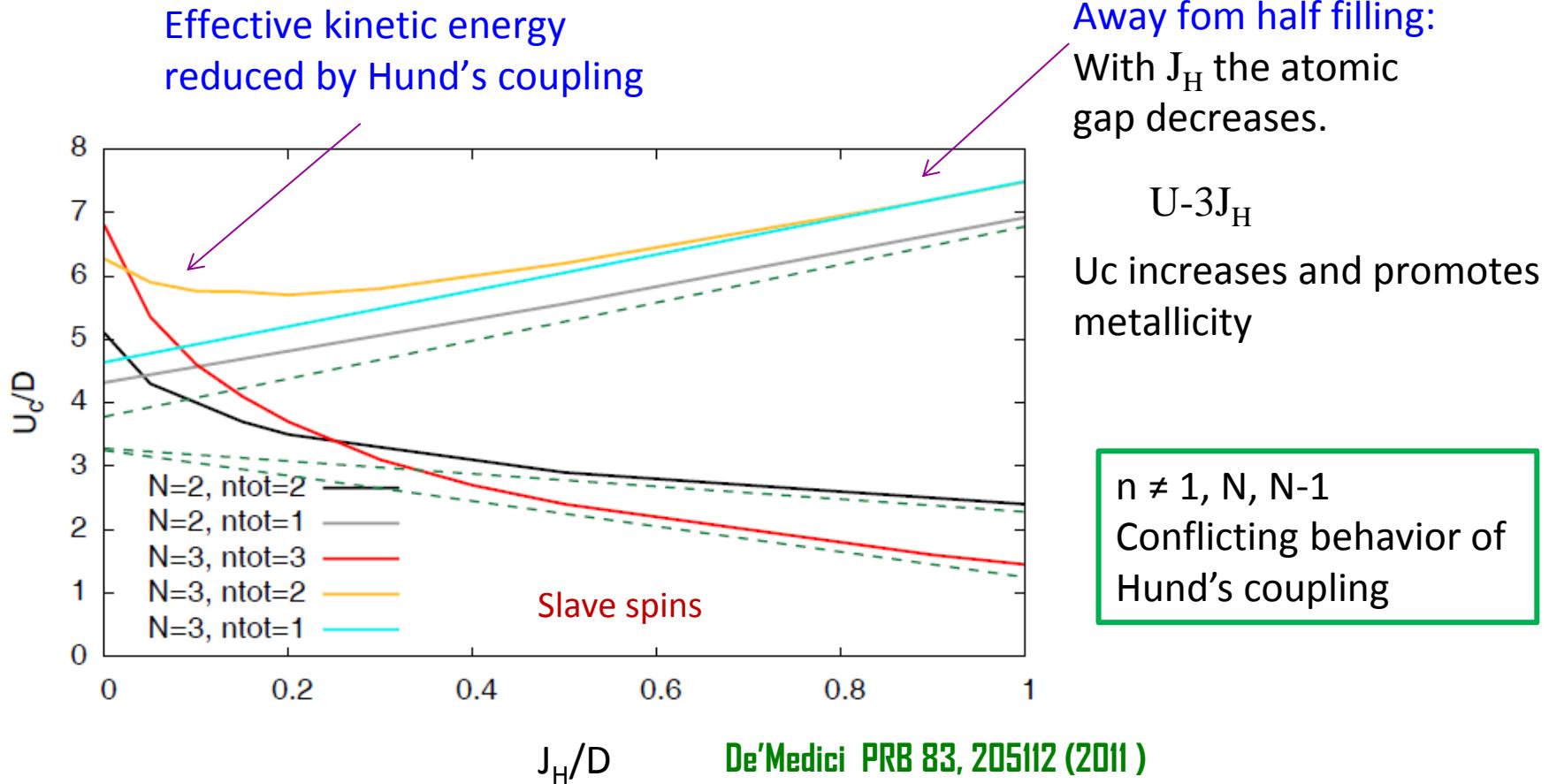
Atomic gap: $E(n+1)+E(n-1)-2E(n)=U$
vs
Kinetic Energy

Effective kinetic energy larger due to orbital degeneracy

Mott transition in multi-orbital systems at Finite Hund



Mott transition in multi-orbital systems at Finite Hund



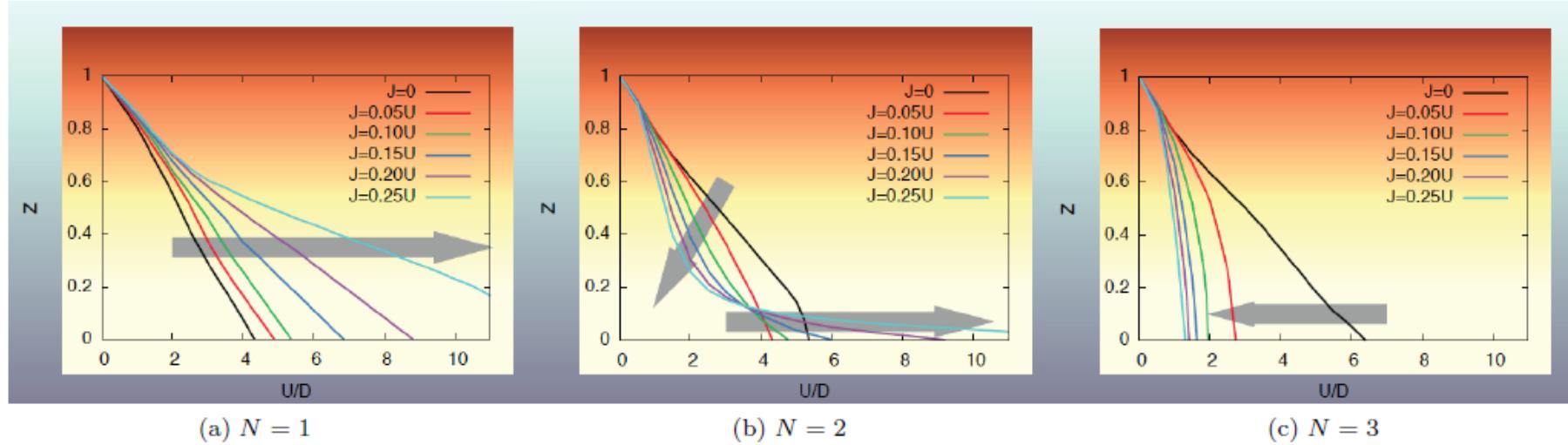
Examples:

- * iron superconductors 6 electrons in 5 orbitals
- * ruthenates 4 electrons in 3 orbitals
- * SrCrO_3 2 electrons in 3 orbitals

Hund's coupling and bad metallicity

3 orbital system

DMFT



n=1 electron or 1 hole

J_H promotes metallicity

U_c increases

Quasiparticle weight increases

n=half-filling

J_H promotes insulating behavior

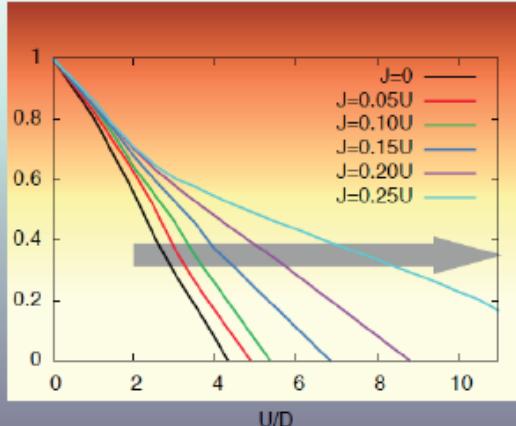
U_c increases

Z decreases

De'Medici et al PRL 107, 255701 (2011)

Hund's coupling and bad metallicity

3 orbital system



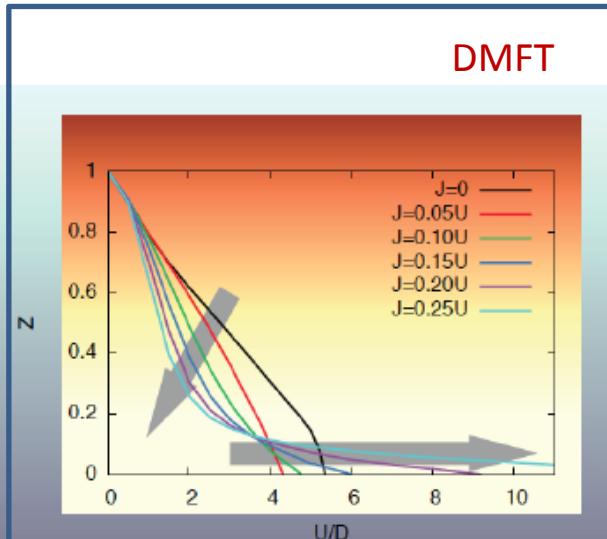
(a) $N = 1$

n=1 electron or 1 hole

J_H promotes metallicity

U_c increases

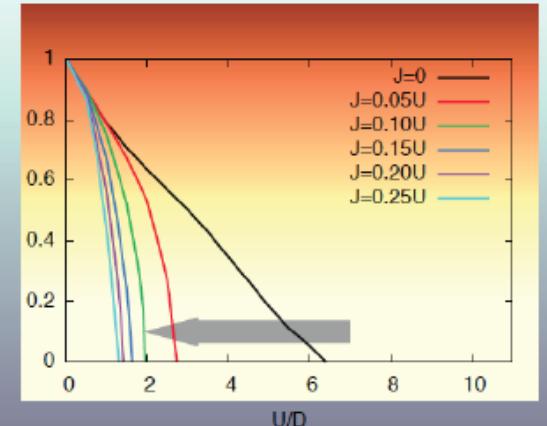
Q_p weight Z increases



(b) $N = 2$

**$n \neq 1$ electron ,1 hole,
half-filling**

J_H promotes bad metallic
Behavior (correlated metal)
Large U_c but small Z



(c) $N = 3$

$n=$ half-filling

J_H promotes insulating
behavior

U_c increases

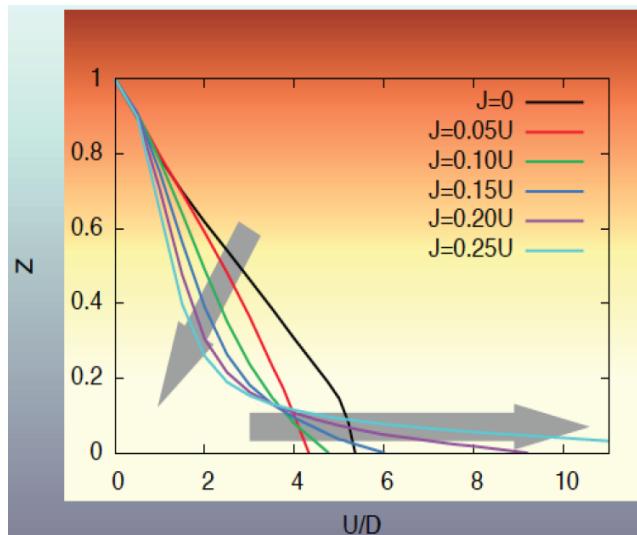
Z decreases

De'Medici et al PRL 107, 255701 (2011)

Hund metals and spin freezing

3 orbital system

DMFT



$n \neq 1$ electron ,1 hole,
half-filling

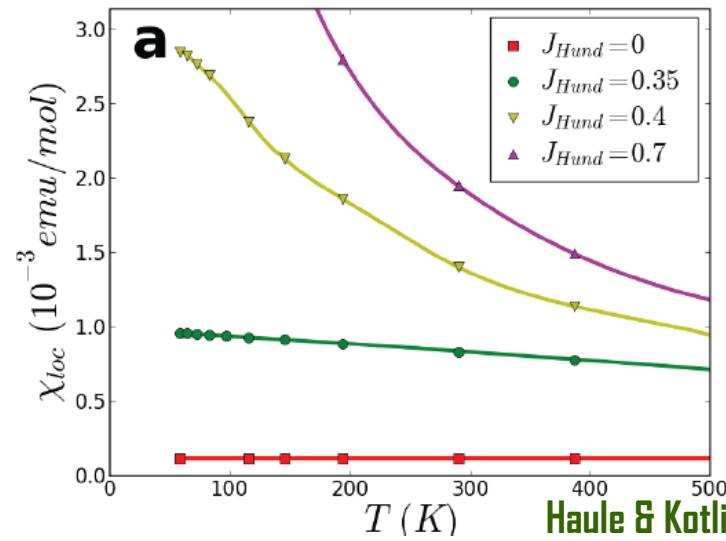
J_H promotes bad metallic
Behavior (correlated metal)

Large U_c but small Z

De'Medici et al PRL 107, 255701 (2011)

Iron superconductors

DMFT

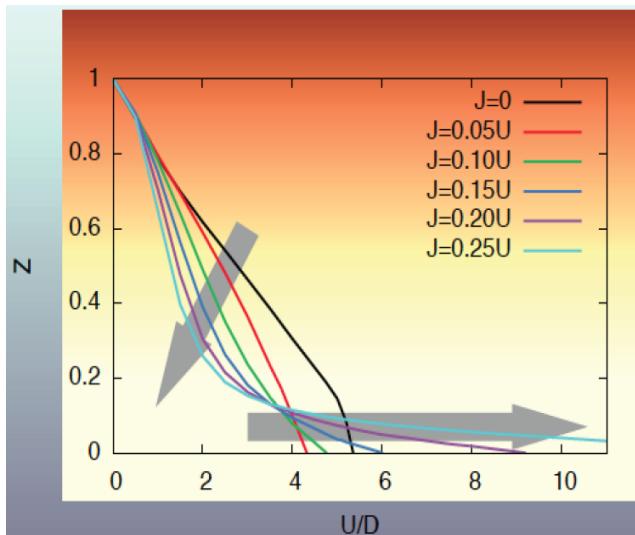


Haule & Kotliar
NJP 11,025021 (2009)

Hund metals and spin freezing

3 orbital system

DMFT



$n \neq 1$ electron ,1 hole,
half-filling

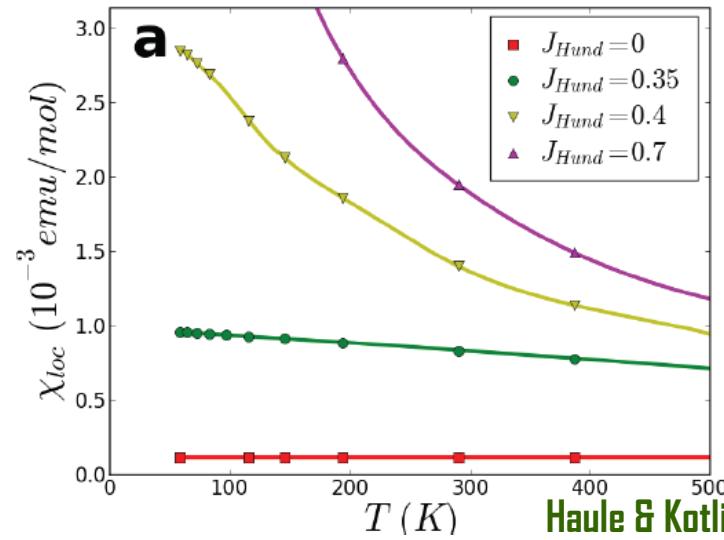
J_H promotes bad metallic
Behavior (correlated metal)

Large U_c but small Z

De'Medici et al PRL 107, 255701 (2011)

Iron superconductors

DMFT



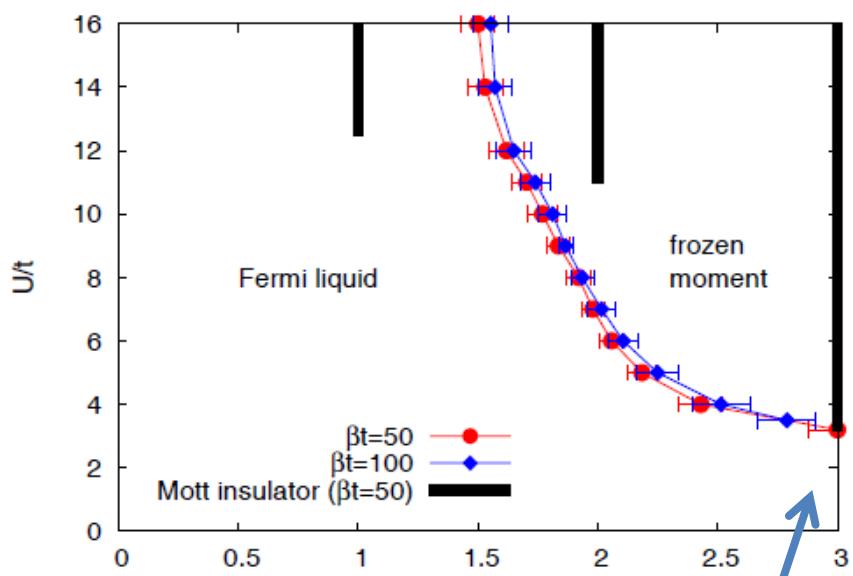
Haule & Kotliar
NJP 11,025021 (2009)

Hund metals

- Correlations driven by Hund J_H weakly dependent on U ,
- Not in proximity to a Mott insulator
- Properties essentially different to a doped Mott insulator (wide Hubbard bands)

Doping dependence in Hund metals

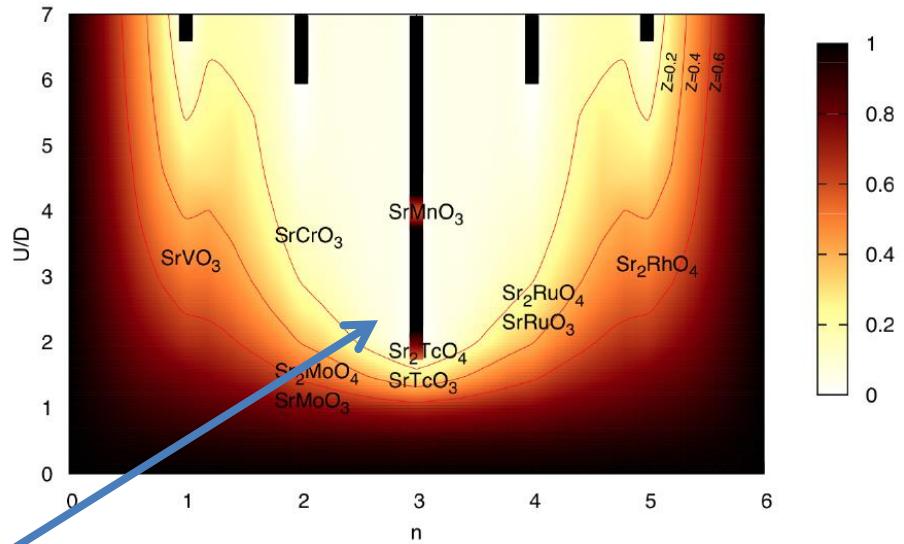
DMFT 3 orbitals $J_H/U=1/6=0.167$



Werner et al PRL 101, 166405 (2008)

Fermi liquid behavior with very small Z

DMFT N=3 orbitals and $J/U=0.15$
Z colour plot



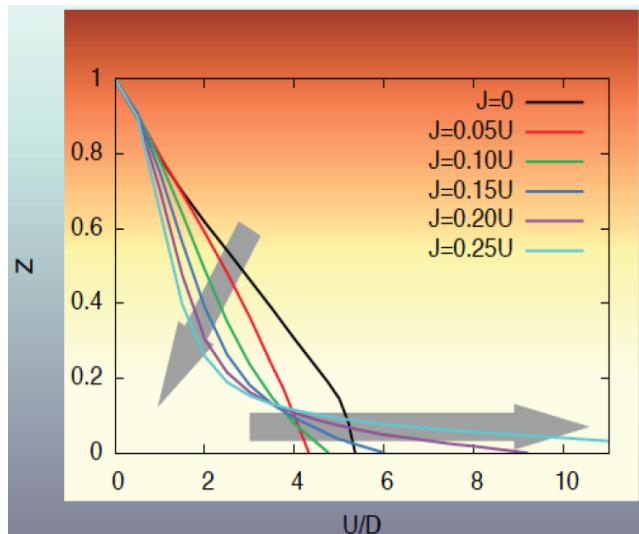
De'Medici et al PRL 107, 255701 (2011),
Georges et al, Ann. Rev. Cond. Matt. Phys. 4, 137 (2013)

The Mott transition at half-filling seems
to play a special role in setting the value of Z

Correlations in Hund metals: Hund vs Mott

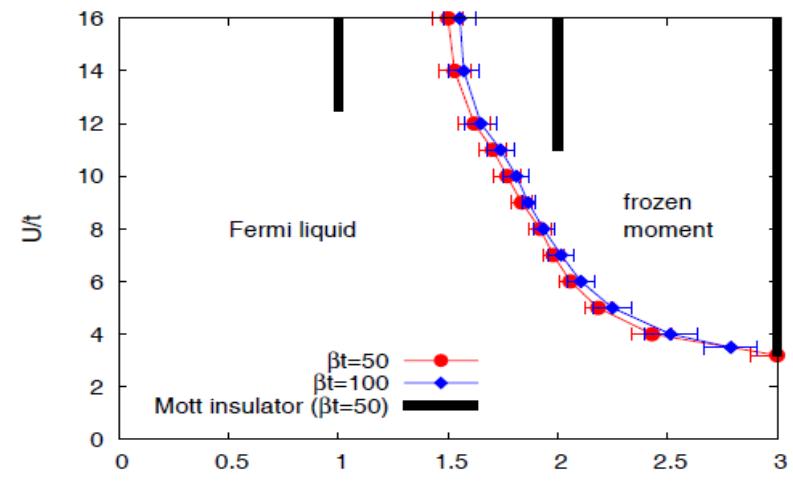
Hundness as opposed to Mottness

- Correlations driven by Hund J_H weakly dependent on U ,
- Not in proximity to a Mott insulator
- Properties essentially different to a doped Mott insulator



Haule & Kotliar NJP 11,025021 (2009)
Georges et al, Ann. Rev. Cond. Matt. Phys. 4,137 (2013)

Hund metals as doped Mott insulators



Correlations controlled by the **proximity in doping** to a Mott insulating state

Werner et al PRL 101, 166405 (2008)
Ishida & Liebsch, PRB 81, 054513 (2010)

Summary

- Single-orbital Hubbard model:
 - * Mott transition at half-filling $U \gg t$ ($U/W \sim 1.5$)
 - * Away from half-filling or for $U < U_c$: correlated metal. Suppression of Quasiparticle weight Z and Charge fluctuations & enhancement of spin fluctuations
- Multi-orbital Hubbard-Kanamori Hamiltonian:
 - * Mott transition at all integer filling. U_c depends on J_H and filling n
 - * J_H promotes metallic behavior for $n=1$ electron or 1 hole. Z and U_c increase.
 - * J_H promotes insulating behavior at half-filling. Z and U_c decrease.
 - * J_H promotes bad metallic behavior for $n \neq 1$ electron, 1 hole, half-filling. **Hund metal** U_c non-monotonic, but increases at large J_H
 Z small. Coherent-incoherent crossover with temperature. Spin freezing
Dependence on doping. **Hund vs Mott ?**

The model: Degenerate orbitals with density-density interactions

- Multi-orbital systems with **N** orbitals ($N=2-5$) and n electrons (half-filling $n=N$)
- **Equivalent orbitals:** No crystal field splitting or hybridization between orbitals
 - Hopping to 1st nearest neighbors equal for all the orbitals
 - Non-interacting bandwidth W (2D but generic results)

Intra-orbital
repulsion

Inter-orbital
Repulsion
(different spin)

Inter-orbital
Repulsion
(same spin)

$$U \sum_m \hat{n}_{m\uparrow} \hat{n}_{m\downarrow} + U' \sum_{m \neq m'} \hat{n}_{m\uparrow} \hat{n}_{m'\downarrow} + (U' - J_H) \sum_{m < m', \sigma} \hat{n}_{m\sigma} \hat{n}_{m'\sigma}$$

$$U' = U - 2J_H \longrightarrow U, J_H \text{ two interaction parameters } \max J_H/U = 1/3$$

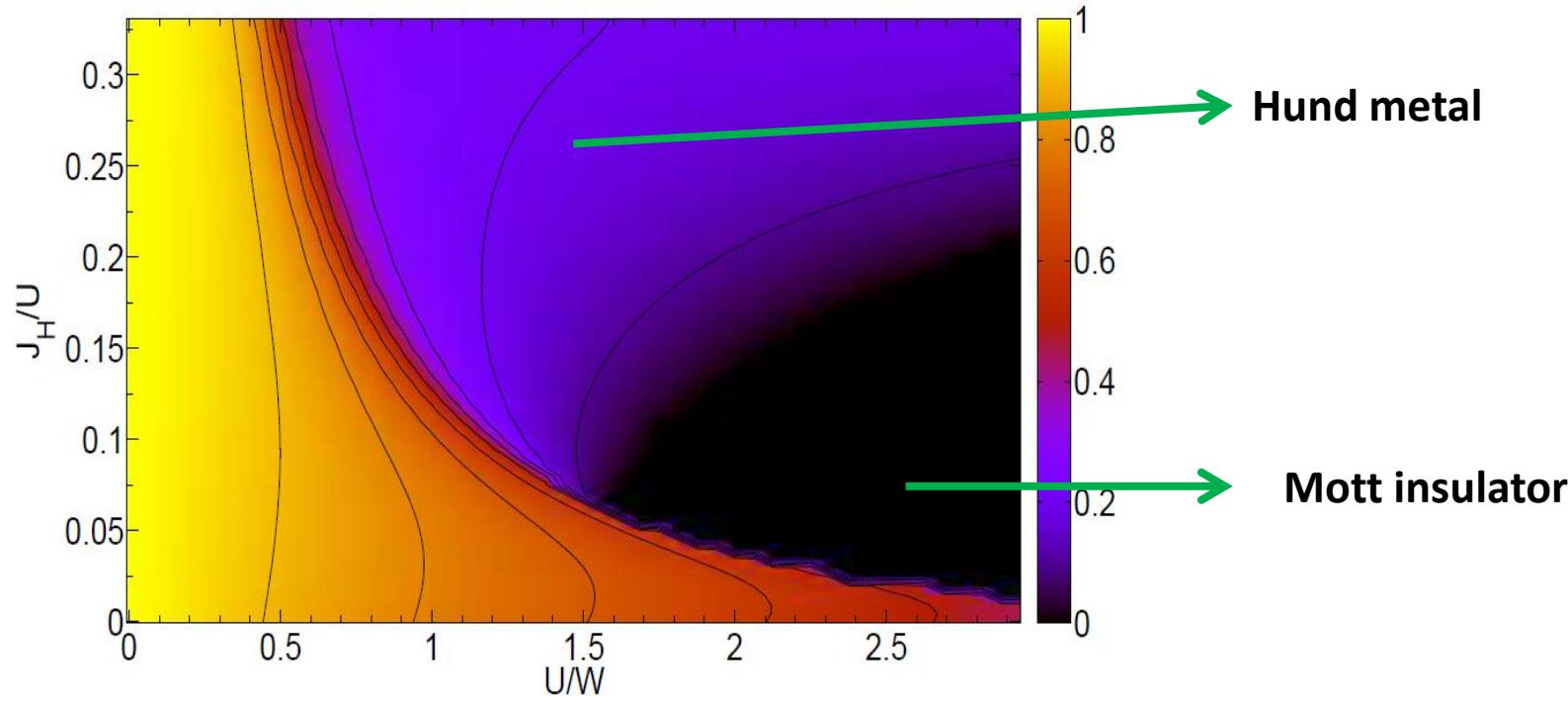
- Hamiltonian solved with Slave Spin Technique: Z, Charge & Spin fluctuations

The quasiparticle weight as a function of interactions

6 electrons in 5 orbitals, as in “undoped” iron superconductors (or 4 electrons in 5 orbitals)

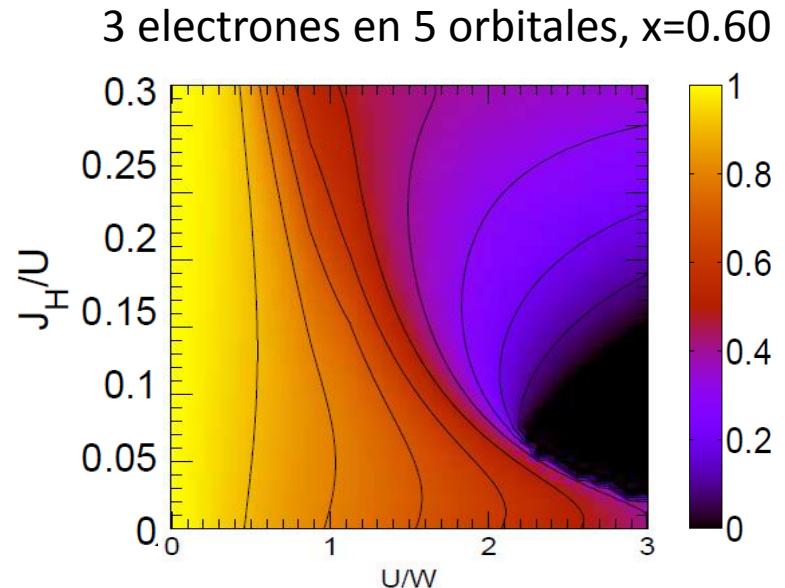
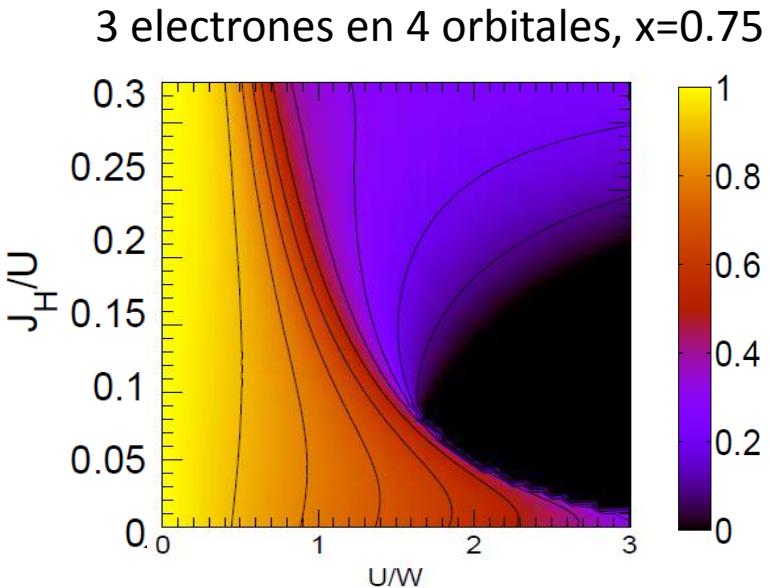
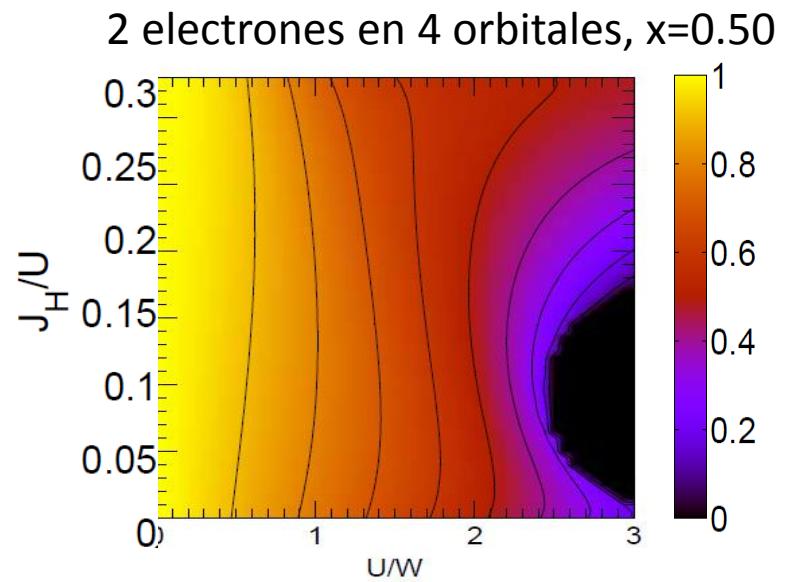
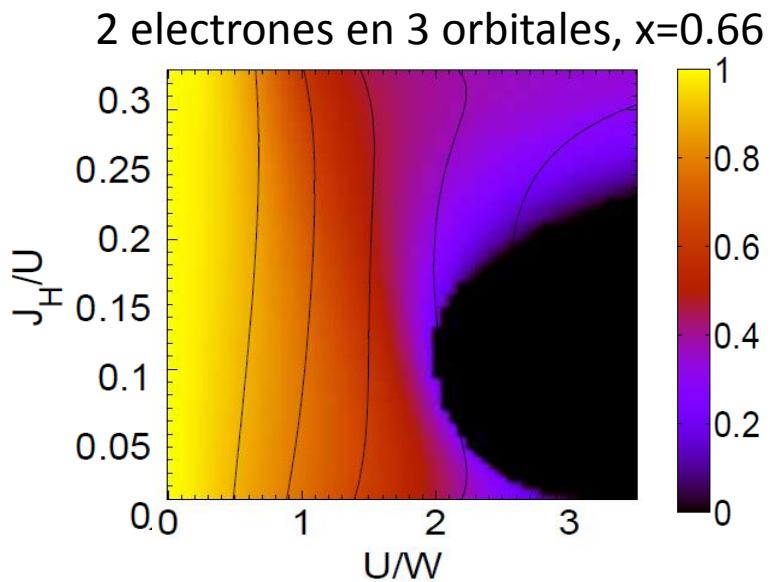
**Particle-hole symmetry
with respect to half-filling**

Quasiparticle weight, colour plot



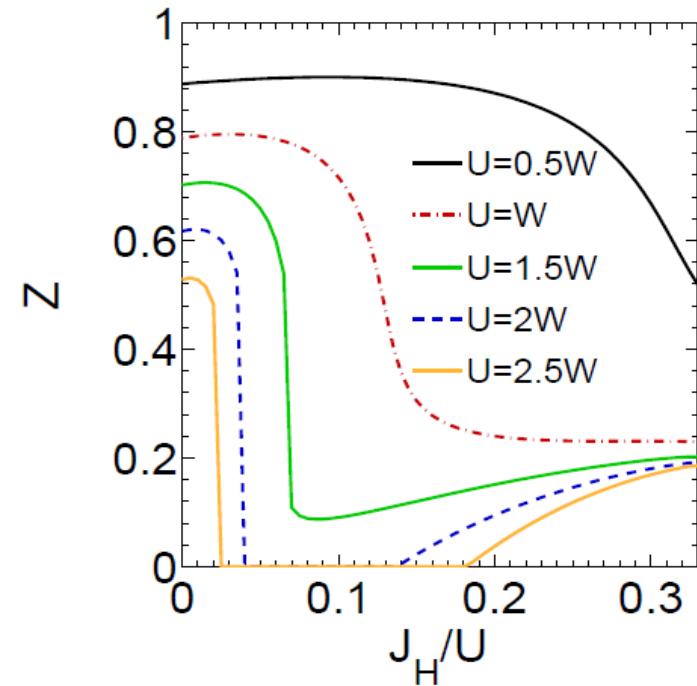
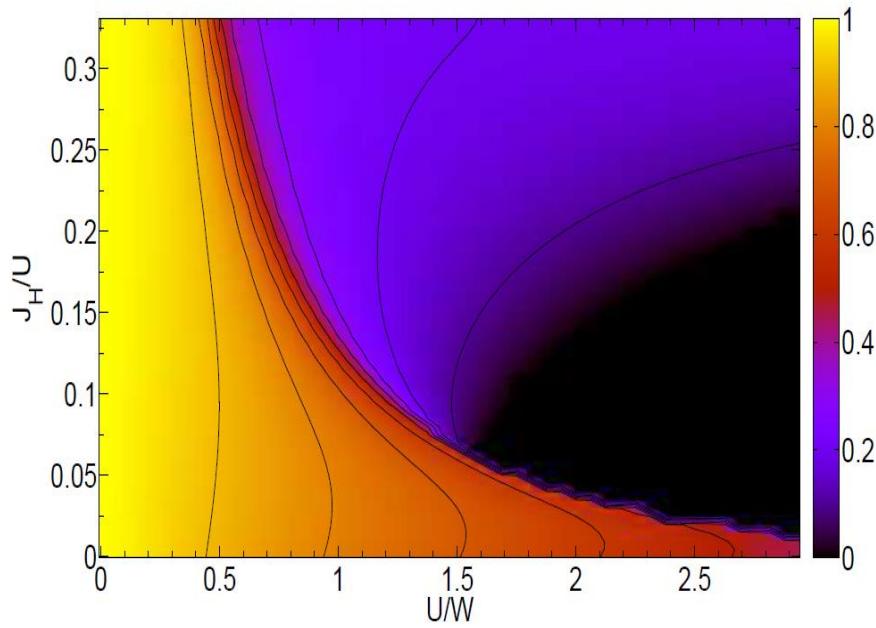
L. Fanfarillo, EB arXiv:1501.04607

The quasiparticle weight as a function of interactions



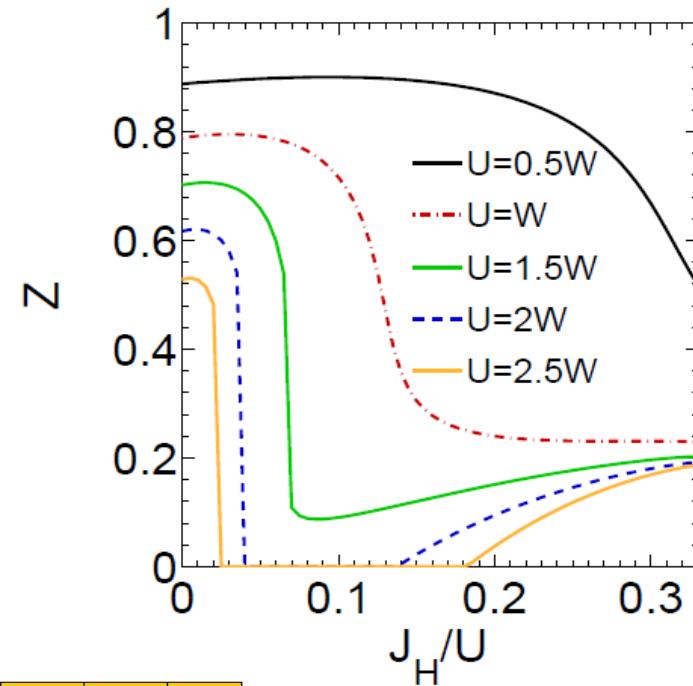
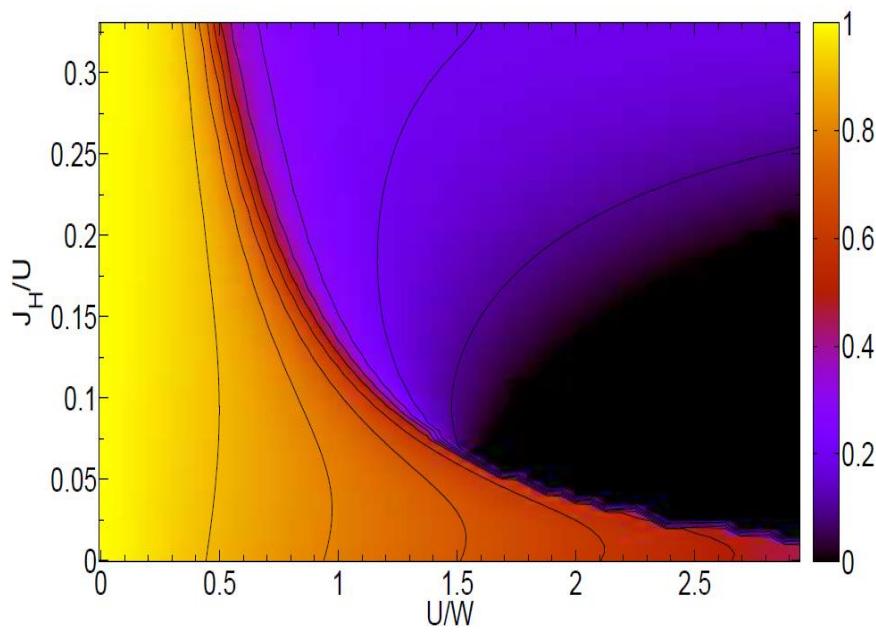
The quasiparticle weight as a function of interactions

6 electrons in 5 orbitals,

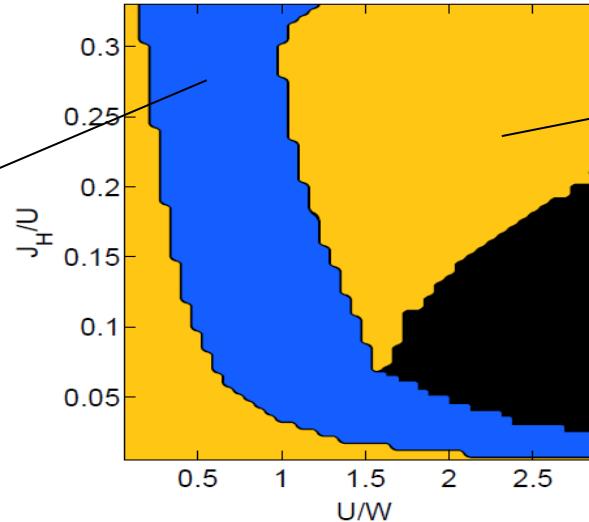


The quasiparticle weight as a function of interactions

6 electrons in 5 orbitals,

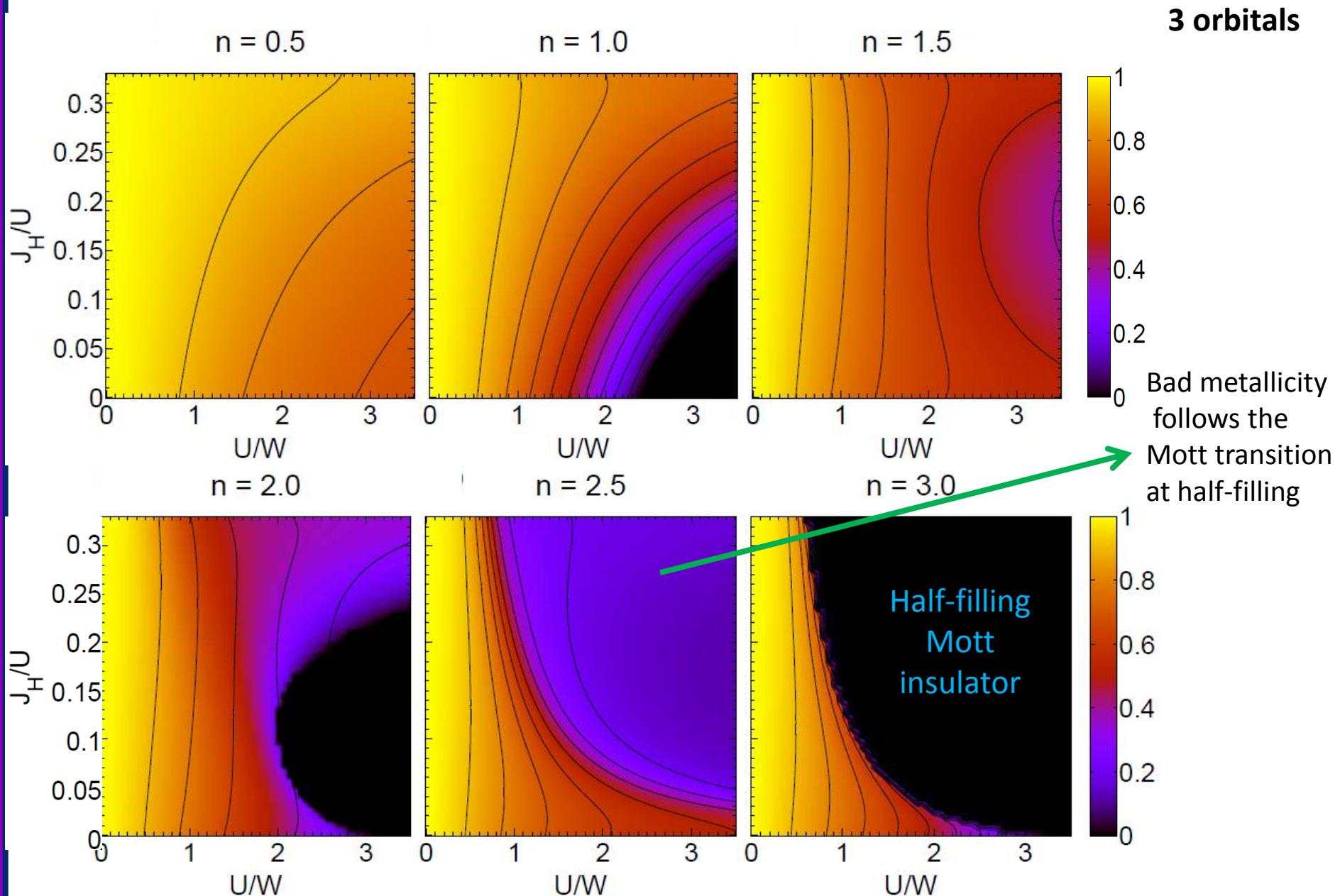


Z decreases
with J_H



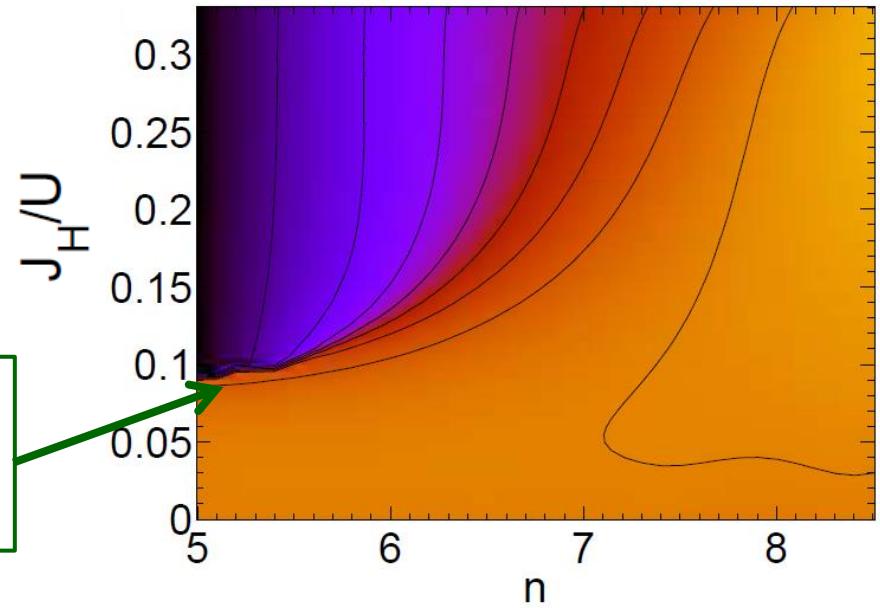
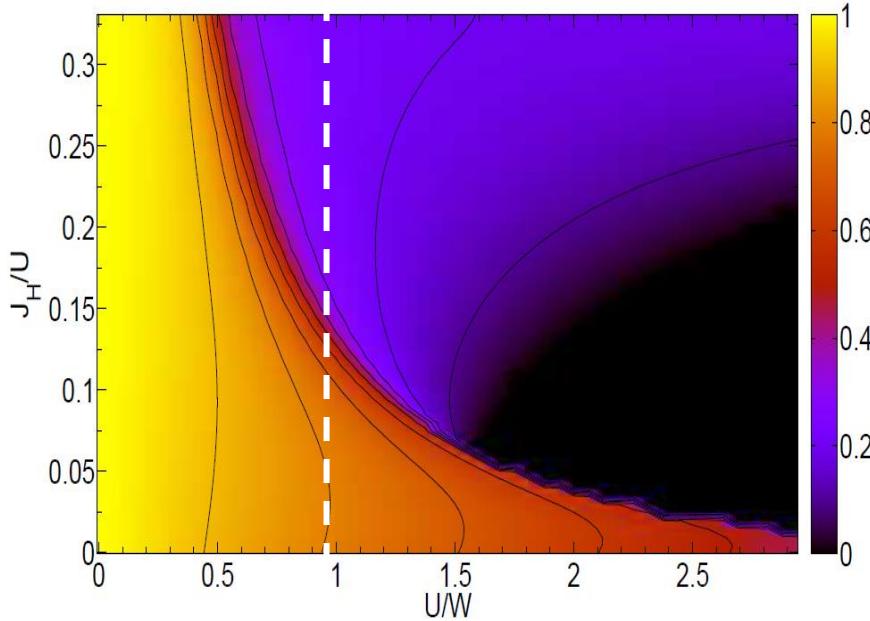
Z increases with J_H

The quasiparticle weight as a function of doping



The quasiparticle weight as a function of doping

6 electrons in 5 orbitals,



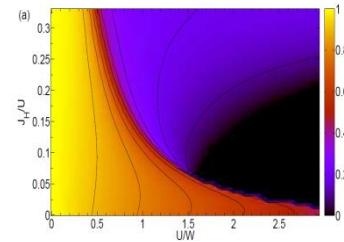
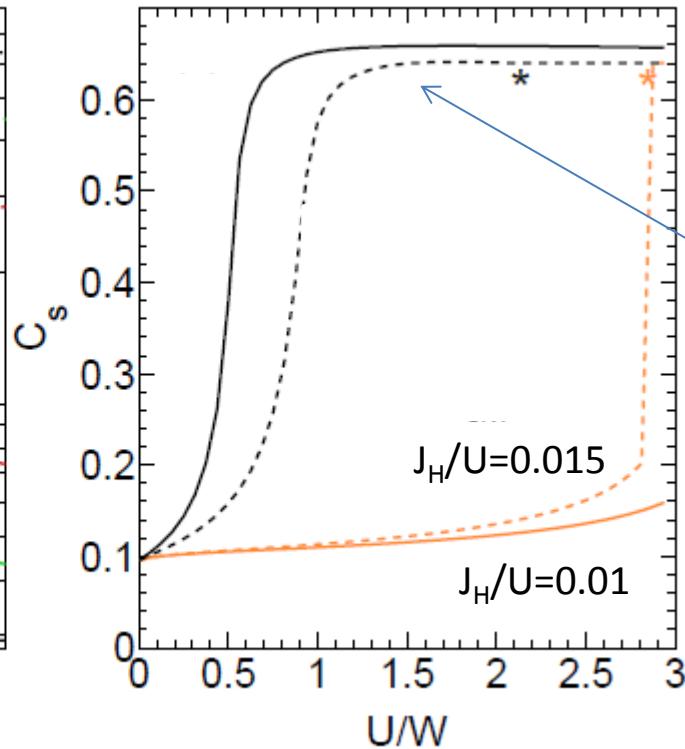
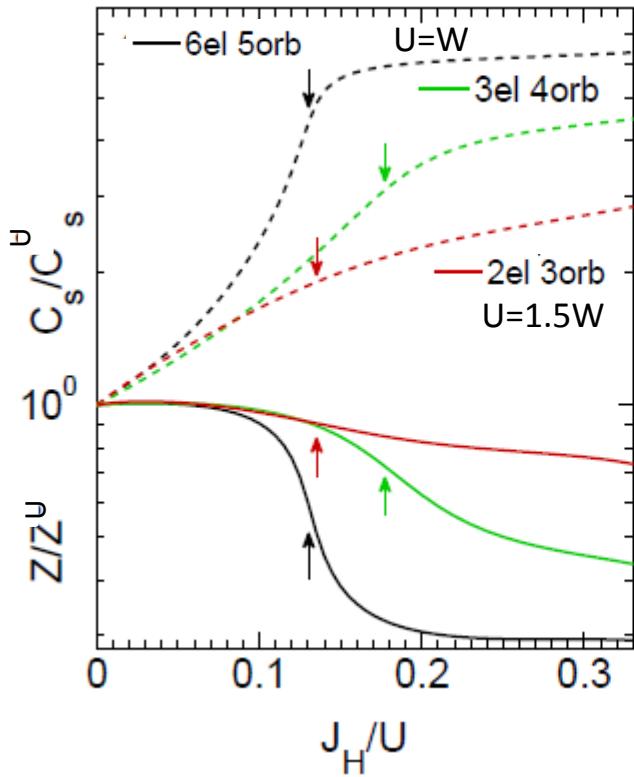
The suppression of the quasiparticle weight Z in the Hund metal is clearly connected with the half-filling Mott transition

L. Fanfarillo, EB arXiv:1501.04607

Spin fluctuations

- Single-orbital: Suppression of Z concomitant with the enhancement of spin fluctuations

Multi-orbital: Suppression of Z concomitant
with a very strong enhancement of spin fluctuations



Spin fluctuations
almost saturated
in the Hund metal

Charge fluctuations

- Single-orbital: Suppression of Z concomitant with the suppression of charge fluctuations

Mott physics, small Z  Localization

Charge fluctuations

- Single-orbital: Suppression of Z concomitant with the suppression of charge fluctuations

Mott physics, small Z



Localization

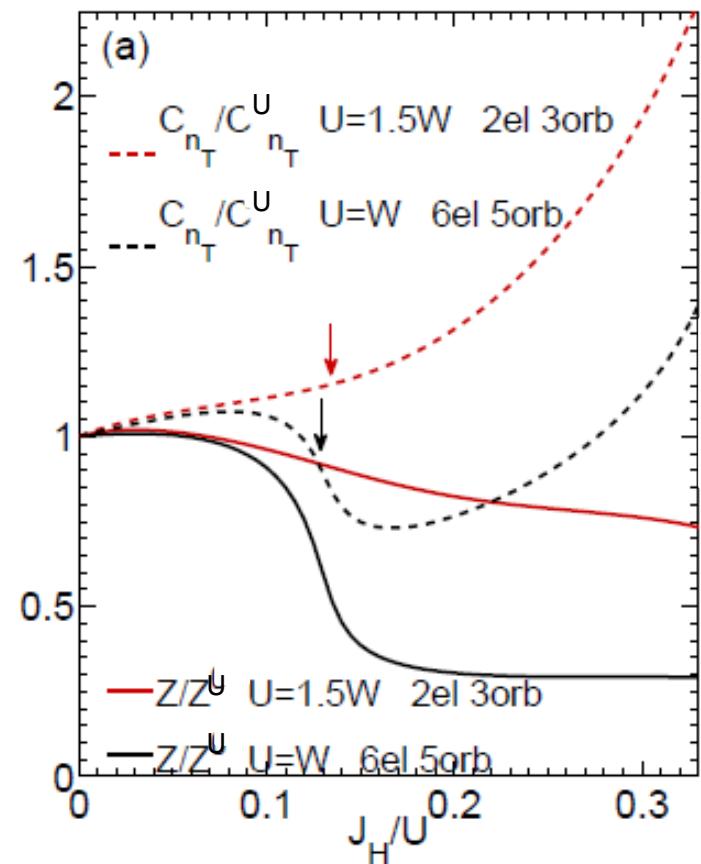
- Multi-orbital: Suppression of Z due to Hund's coupling is not always accompanied by the suppression of charge fluctuations

Hund correlations

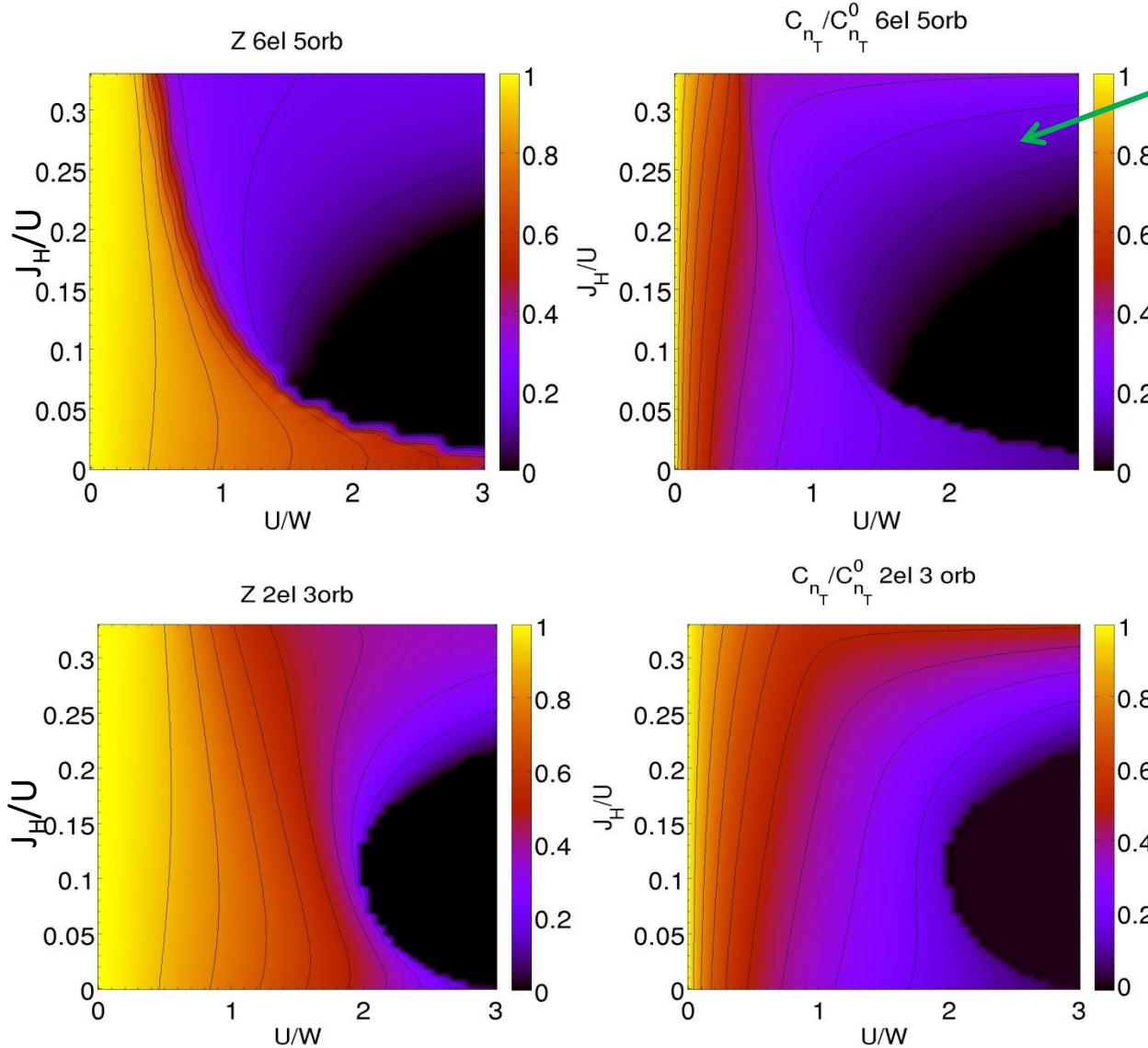


Not always localization,
sometimes increases itinerancy

Hund correlations not equal to Mott correlations



Charge fluctuations



Itinerancy

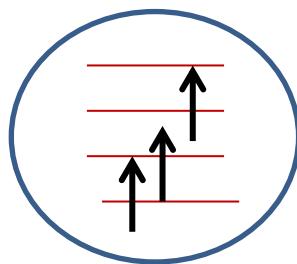
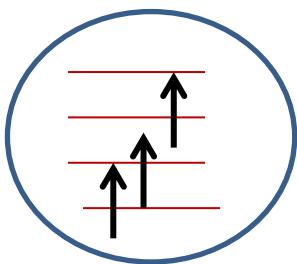
Weaker suppression of charge fluctuations for a given Z , as compared to a single-orbital system or small Hund's coupling

Different dependence of charge fluctuations and quasiparticle weight Z on interactions

Understanding the correlations in Hund metals

N orbitals

n electrons

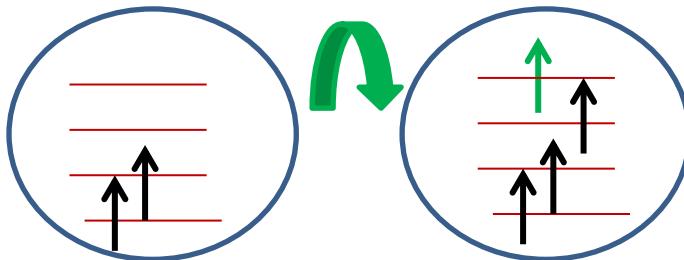


2 spin polarized atoms

Atomic gap: $E(n+1)+E(n-1)-2E(n)$
vs
Kinetic Energy

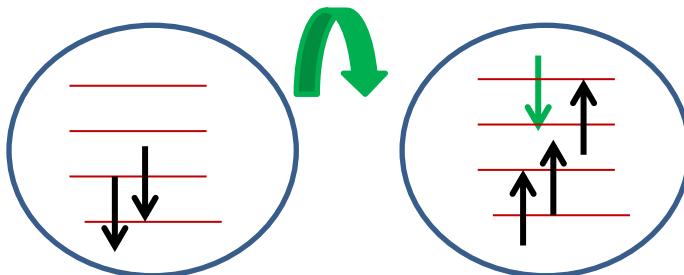
Understanding the correlations in Hund metals

Hopping processes



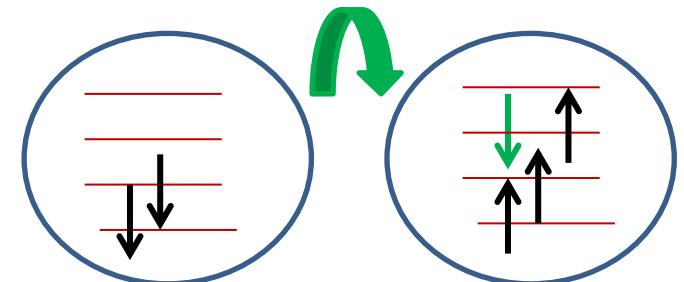
Spin parallel hopping to an empty orbital

$$E^{\uparrow\uparrow} = U - 3J_H$$



Spin anti-parallel hopping to an empty orbital

$$E^{inter\uparrow\downarrow} = U + (n - 3)J_H$$

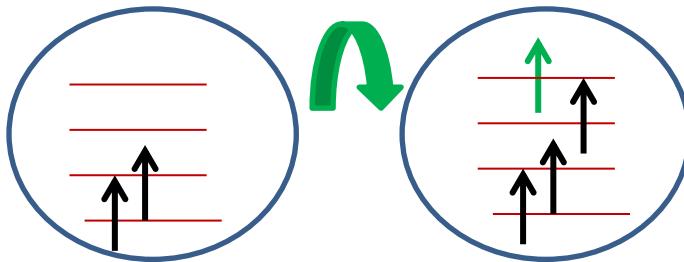


Spin anti-parallel hopping to an occupied orbital

$$E^{intra\uparrow\downarrow} = U + (n - 1)J_H$$

Understanding the correlations in Hund metals

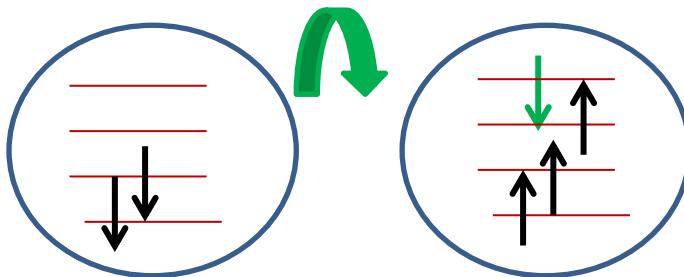
Hopping processes



Spin parallel hopping to an empty orbital

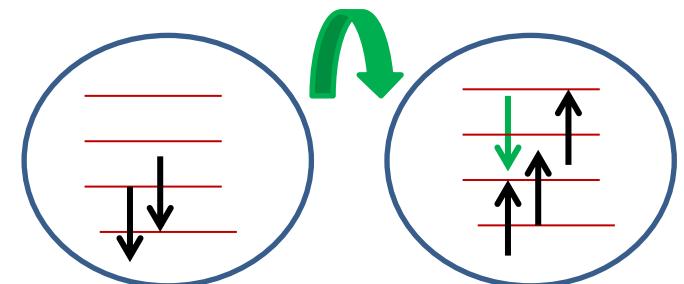
$$E^{\uparrow\uparrow} = U - 3J_H$$

Responsible for the Mott transition away from half-filling



Spin anti-parallel hopping to an empty orbital

$$E^{\text{inter}\uparrow\downarrow} = U + (n - 3)J_H$$

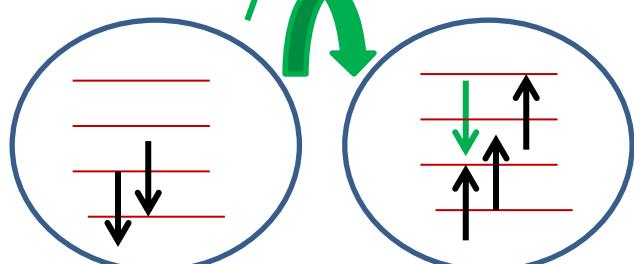
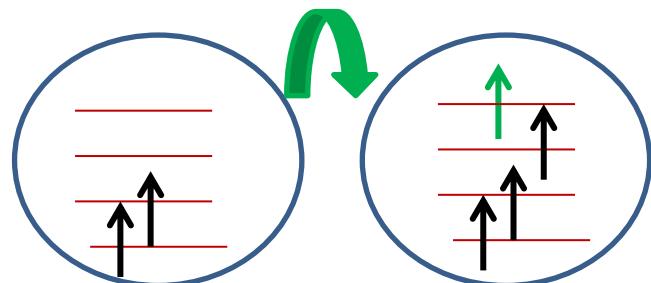
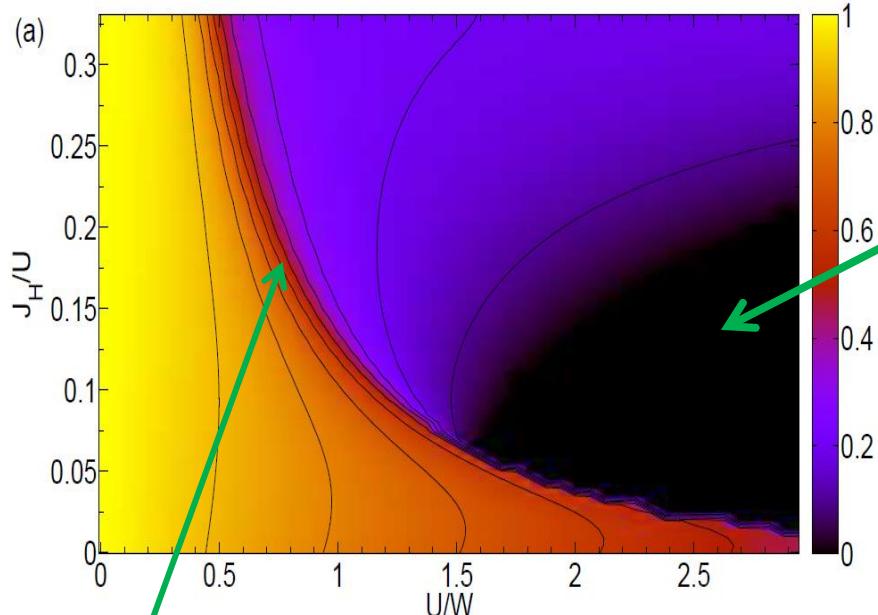


Spin anti-parallel hopping to an occupied orbital

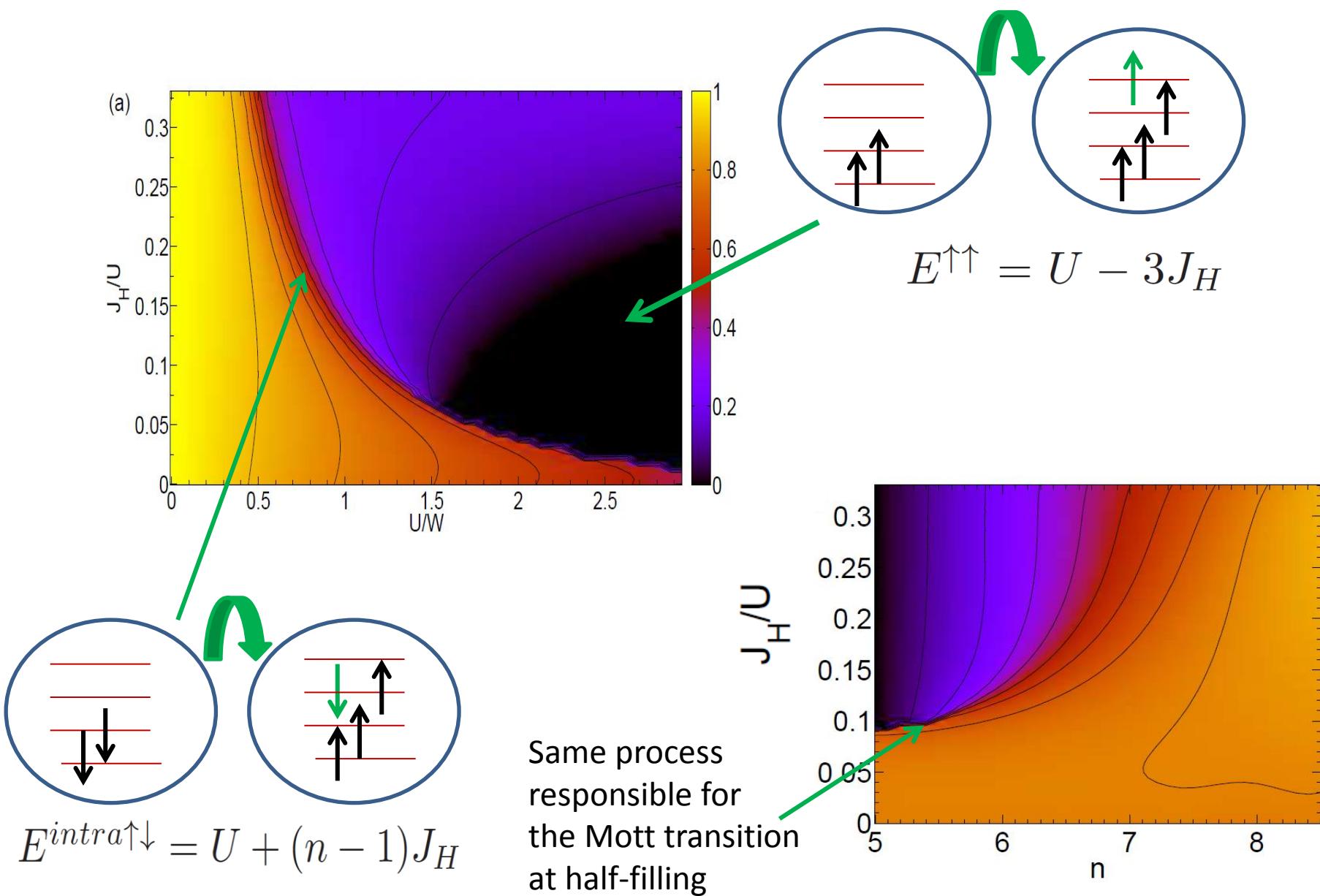
$$E^{\text{intra}\uparrow\downarrow} = U + (n - 1)J_H$$

Responsible for the Mott transition at half-filling

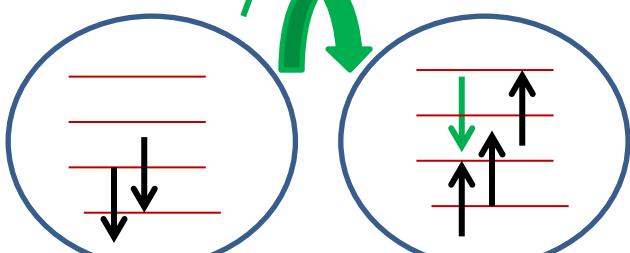
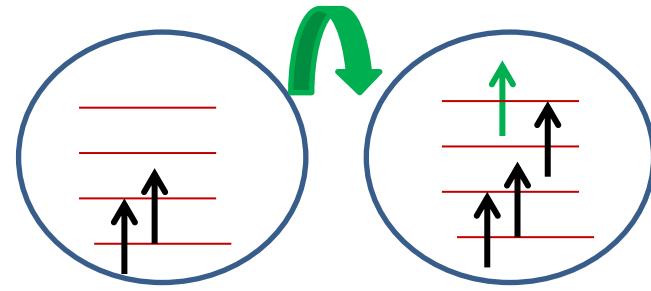
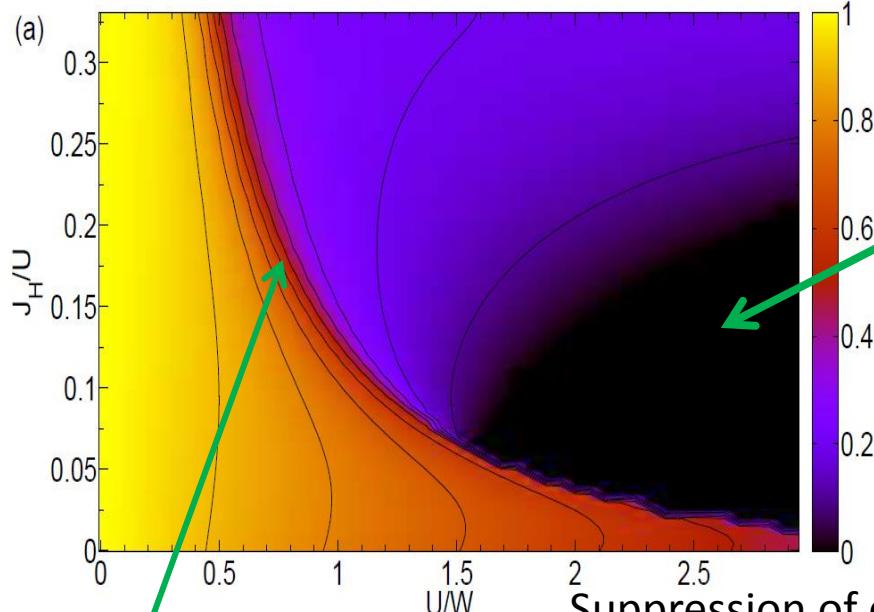
Understanding the correlations in Hund metals



Understanding the correlations in Hund metals

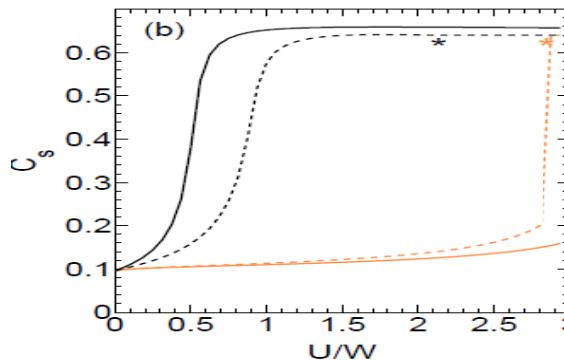


Understanding the correlations in Hund metals

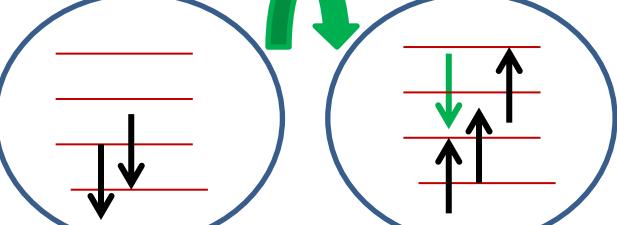
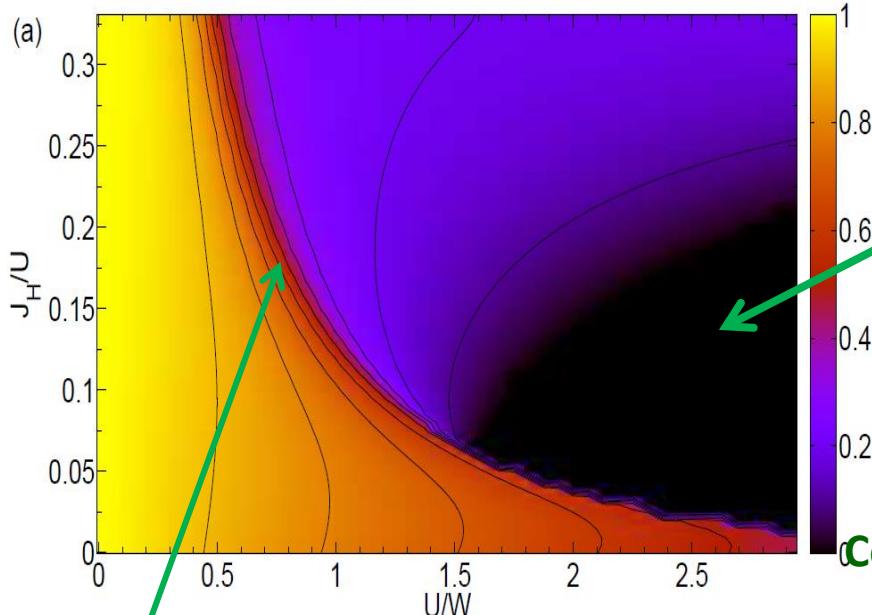


Suppression of configurations with double occupancy and for $n > 2$ hopping with antiparallel spins to empty orbitals

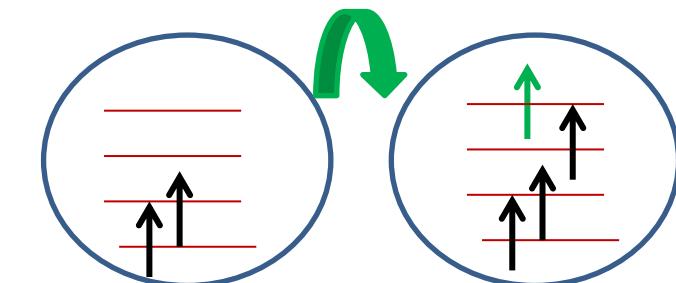
→ **Enhancement of spin fluctuations**



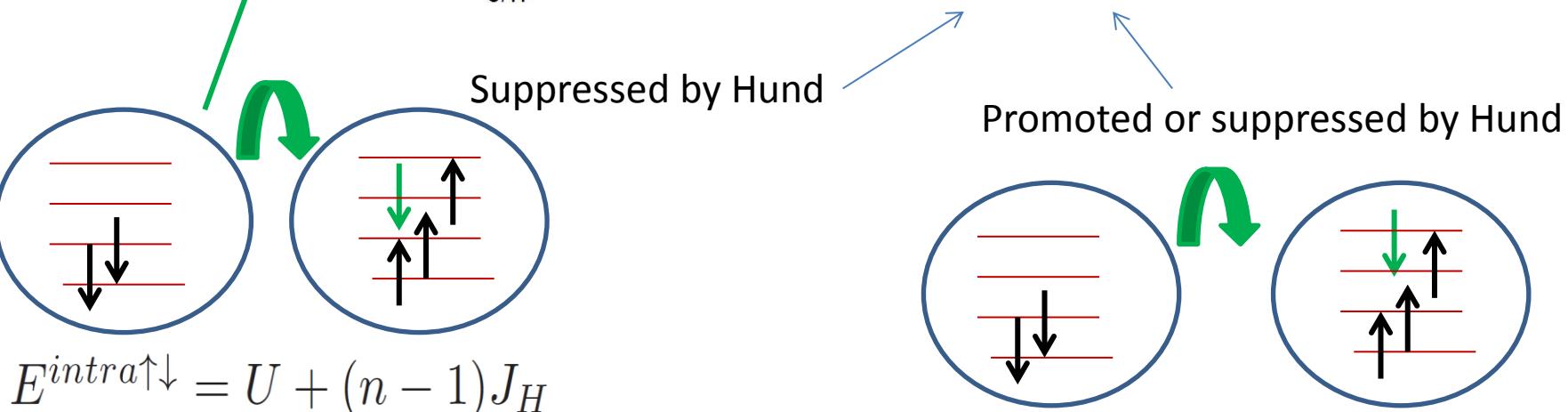
Understanding the correlations in Hund metals



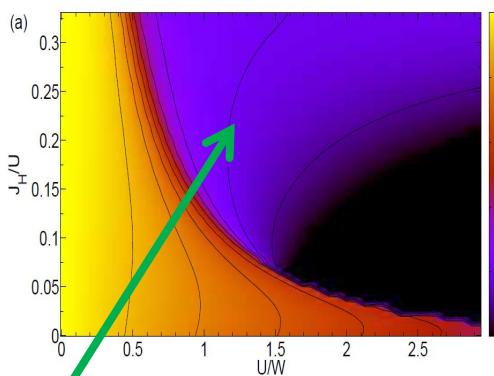
$$E^{intra\uparrow\downarrow} = U + (n - 1)J_H$$



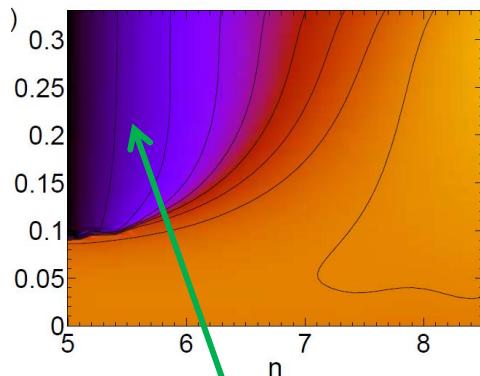
Promoted by Hund



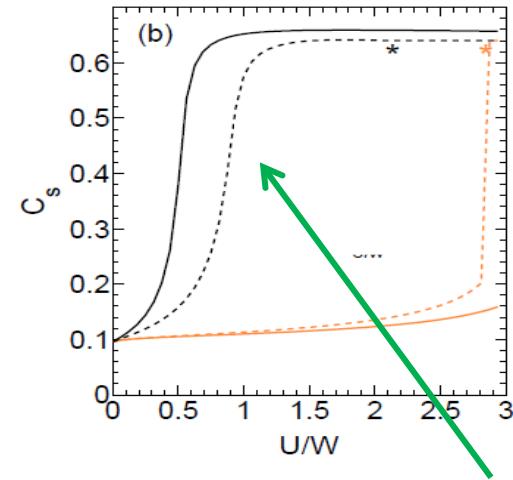
Summary: Strong influence of Hund's coupling on electronic correlations



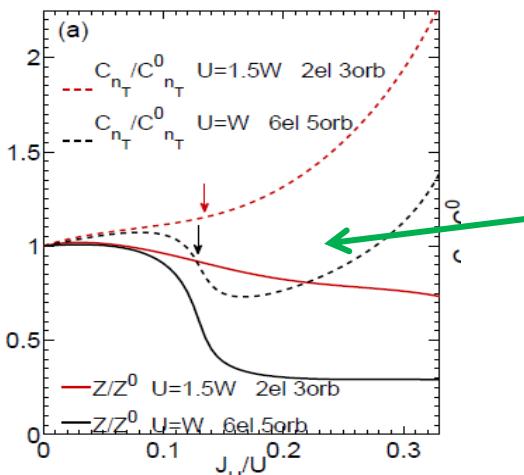
Hund metal with small Q_p weight Z : strong correlations



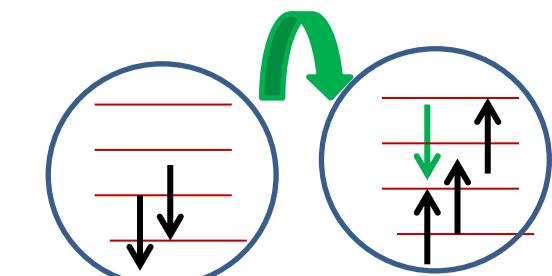
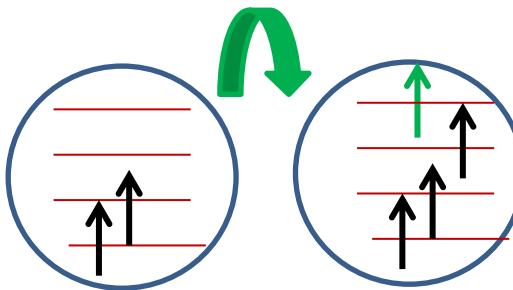
Suppression of Q_p weight connected to half-filling Mott insulator



Enhancement of spin fluctuations (atoms polarized)



Charge fluctuations do not follow the behavior expected in Mott systems



Behavior understood in terms of which hopping processes are promoted or suppressed by Hund's coupling

Revisit oxides and other materials

Correlated metals. The quasiparticle weight (Fermi liquid description)

□ Write the electron $c_{k\sigma}^\dagger$ in terms of the excitations of the interacting system $a_{k\sigma}^\dagger$

Same charge, momentum & spin

$$c_{k\sigma}^\dagger = \sqrt{Z_k} a_{k\sigma}^\dagger + \sum_{k_4+k_3=k_2+k} A(k_4\sigma_4, k_3\sigma_3; k_2\sigma_2, k\sigma) a_{k_4\sigma_4}^\dagger a_{k_3\sigma_3}^\dagger a_{k_2\sigma_2} + \dots$$

A single particle description
of $C_{k\sigma}^\dagger |\Psi_g^* \rangle$
makes sense if Z_k is finite

higher order
decay processes

$$Z_k = | \langle \Psi_g^* | a_{k\sigma} c_{k\sigma}^\dagger | \Psi_g^* \rangle |^2 > 0$$

Quasiparticle weight

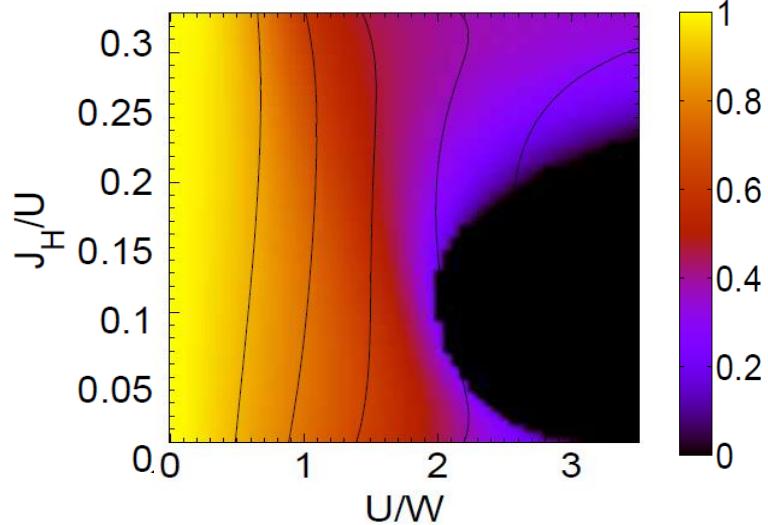
A way to quantify the correlations

Overlap between the
elementary excitations
of the interacting and
non-interacting system

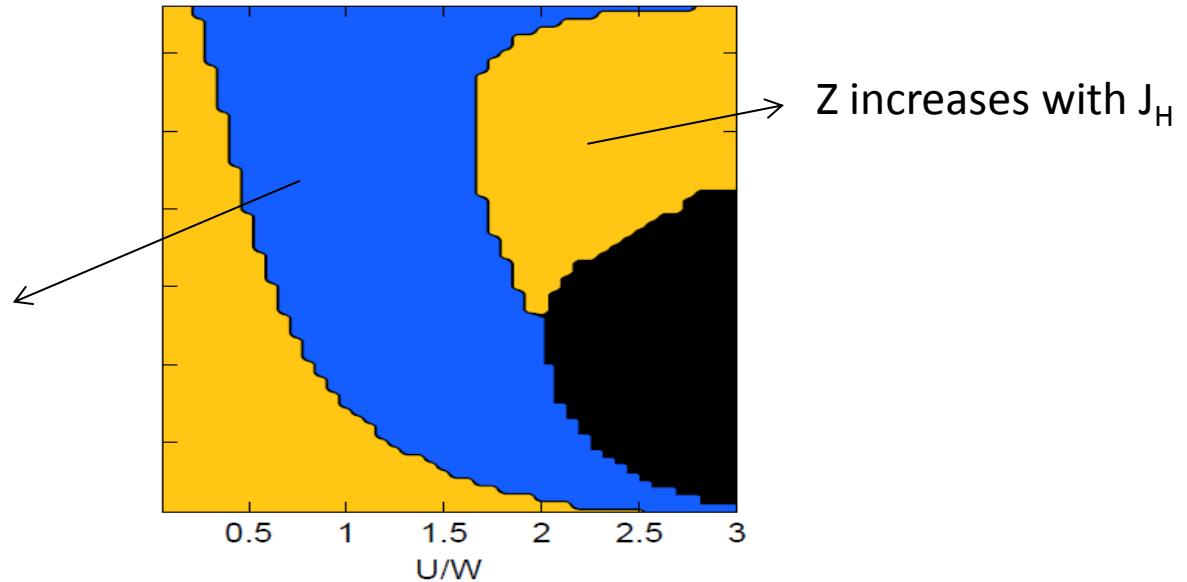
See Coleman's book

The quasiparticle weight as a function of interactions

2 electrones en 3 orbitales, $x=0.66$

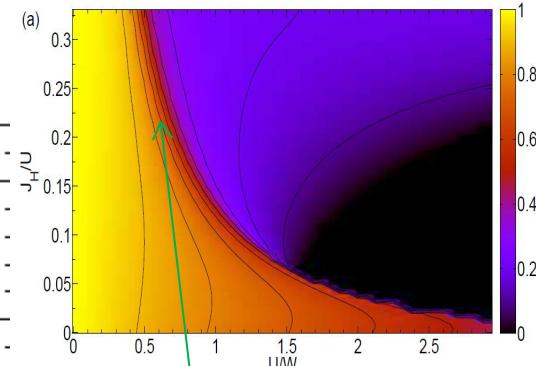
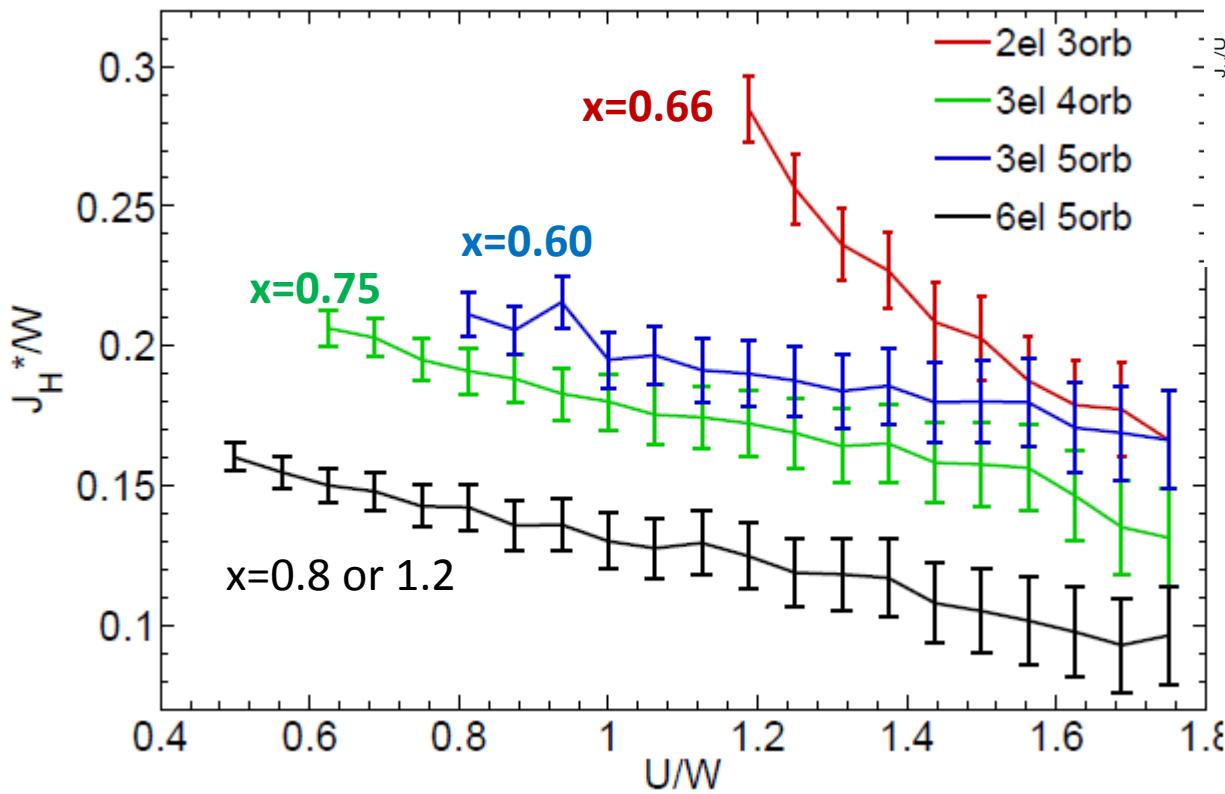


Z decreases
with J_H

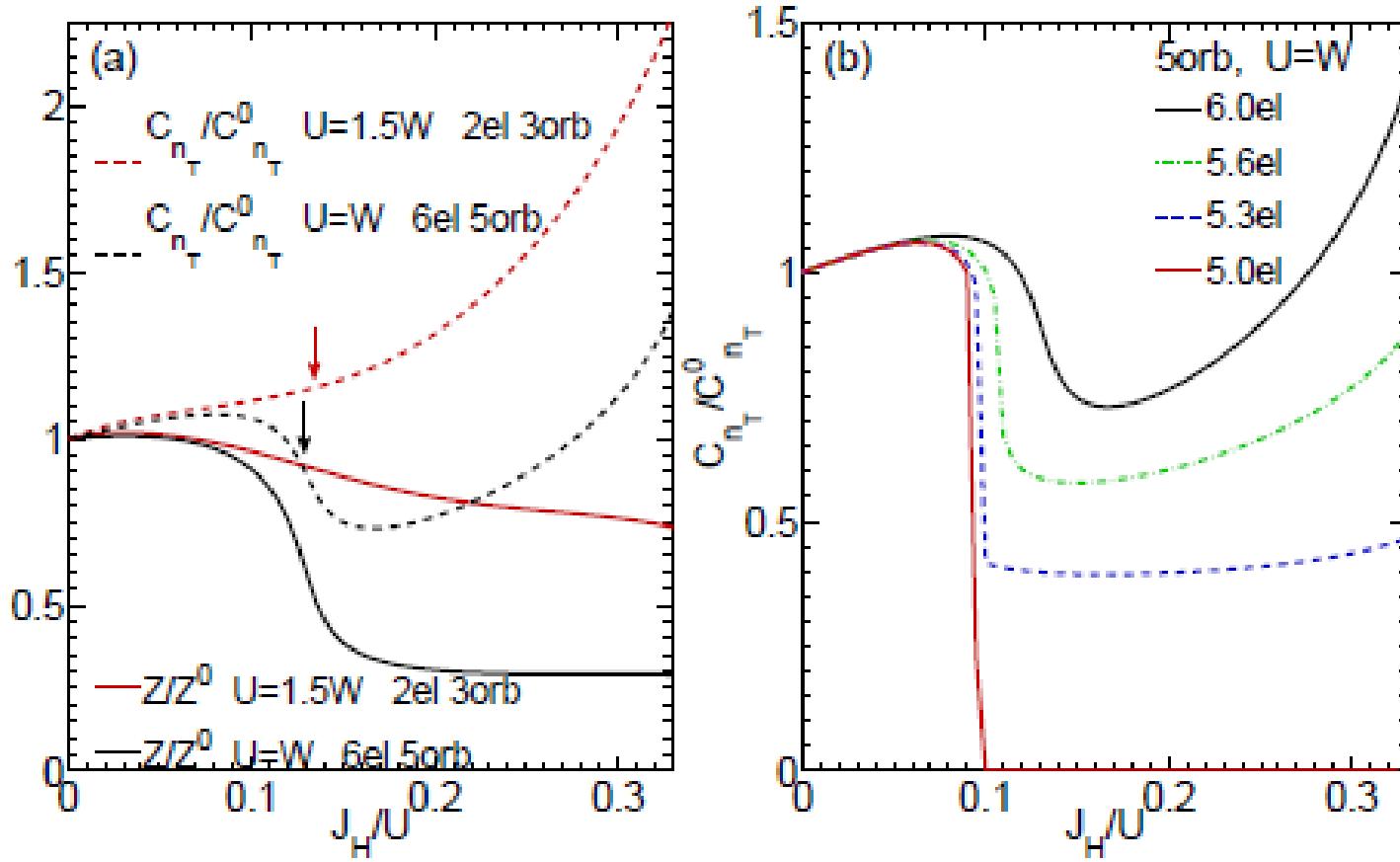


L. Fanfarillo, EB arXiv:1501.04607

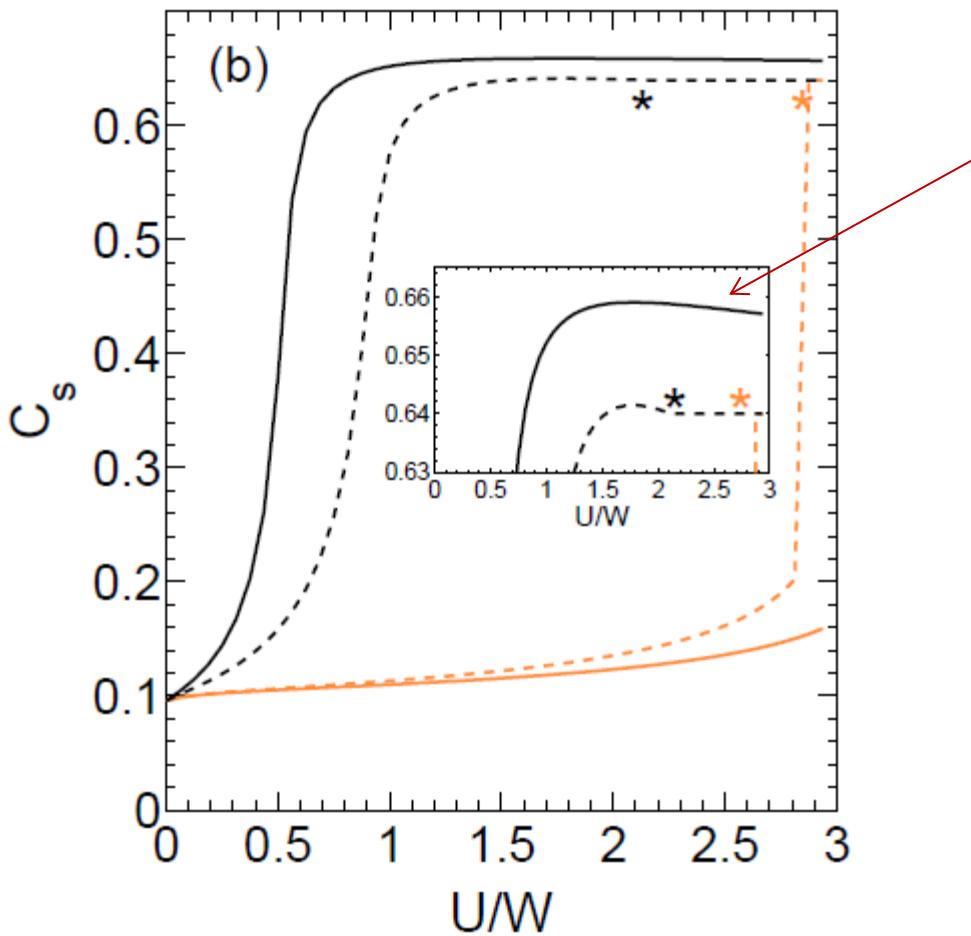
The quasiparticle weight as a function of doping



Charge correlations in Hund metals

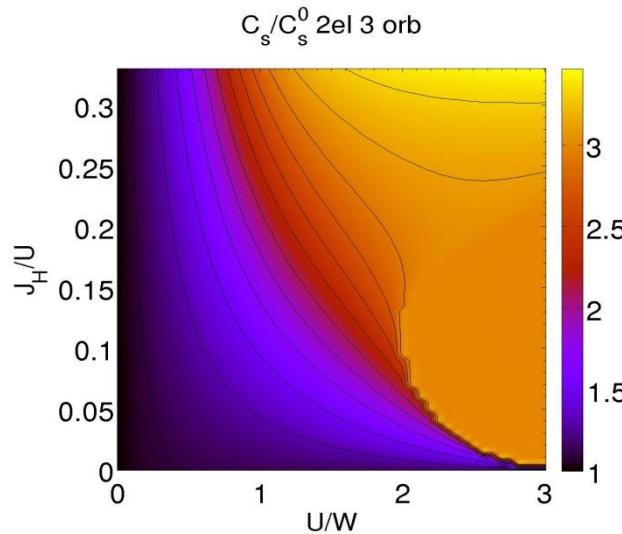
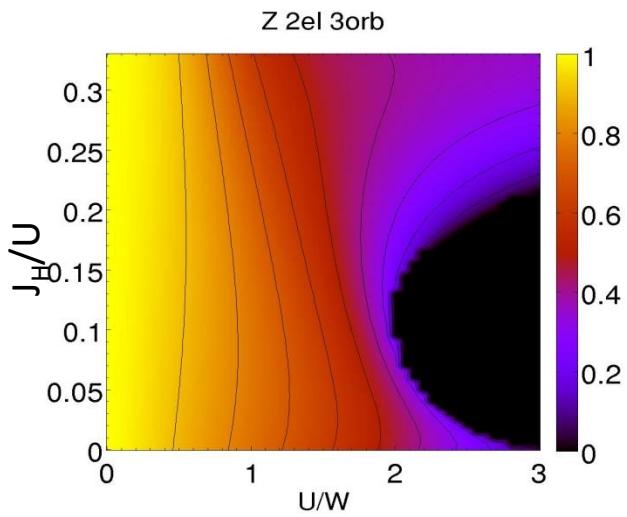
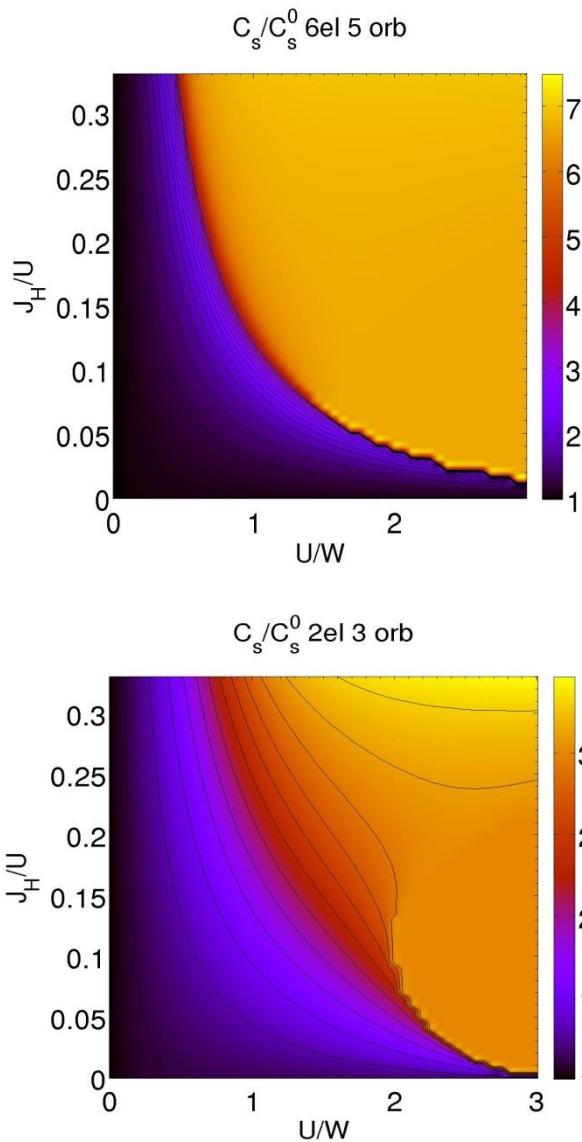
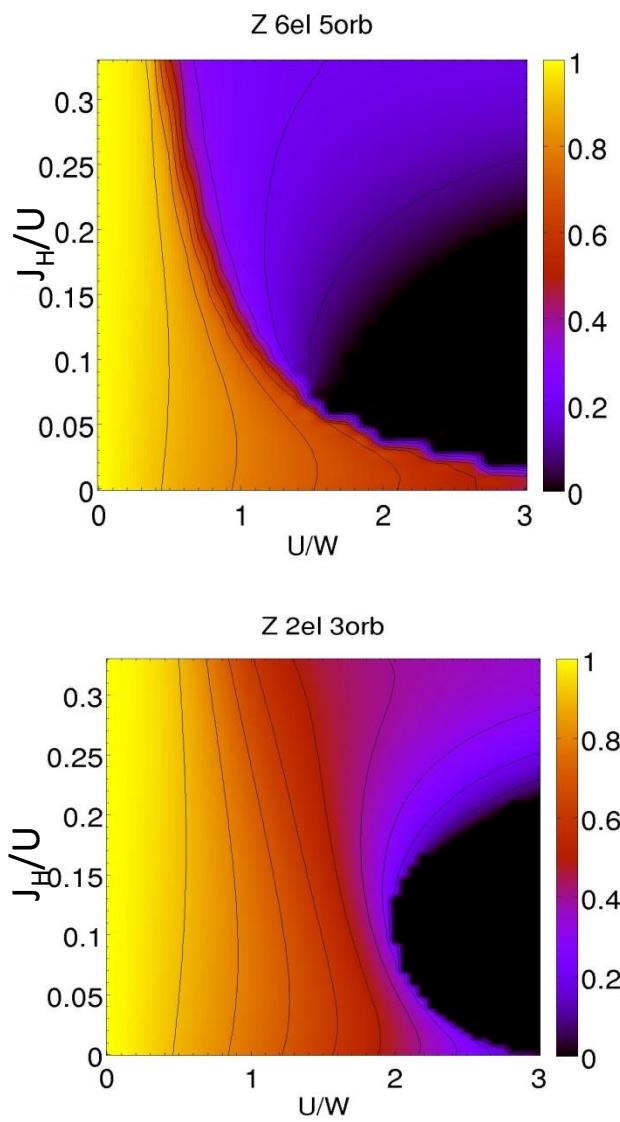


Spin fluctuations in Hund metals



The spin fluctuations are larger in the metal.
U enhances the tendency to localization.

Spin fluctuations in Hund metals



Atomic moments are formed in the

Z₂- Slave spin technique

$$H = U \sum_m \hat{n}_{m\uparrow} \hat{n}_{m\downarrow} + U' \sum_{m \neq m'} \hat{n}_{m\uparrow} \hat{n}_{m'\downarrow} + (U' - J) \sum_{m < m', \sigma} \hat{n}_{m\sigma} \hat{n}_{m'\sigma}$$

$$n_{m\sigma} = d_{m\sigma}^\dagger d_{m\sigma}$$

Physical states

$d_{i\sigma}$	Auxiliary fermion $f_{i\sigma}$ $ n_{i\sigma}^d = 1\rangle \Leftrightarrow n_{i\sigma}^f = 1, S_{i\sigma}^z = +1/2\rangle,$
	Pseudospin variable (slave-spin) $S_{i\sigma}$ $ n_{i\sigma}^d = 0\rangle \Leftrightarrow n_{i\sigma}^f = 0, S_{i\sigma}^z = -1/2\rangle.$

Unphysical states

Constraint

$$\begin{aligned} & |n_{i\sigma}^f = 0, S_{i\sigma}^z = +1/2\rangle \\ & |n_{i\sigma}^f = 1, S_{i\sigma}^z = -1/2\rangle \end{aligned} \longrightarrow f_{i\sigma}^\dagger f_{i\sigma} = S_{i\sigma}^z + \frac{1}{2}$$

de Medici et al, PRB 72, 205124 (2005)
Hassan & de Medici, PRB 81, 035106 (2010)

Z₂- Slave spin technique

$$d_{i\sigma} = f_{i\sigma} O_{i\sigma}, \quad d_{i\sigma}^\dagger = f_{i\sigma}^\dagger O_{i\sigma}^\dagger \quad \text{For non-diagonal operators}$$

$$O_{i\sigma} = \begin{pmatrix} 0 & c_{i\sigma} \\ 1 & 0 \end{pmatrix} \quad \rightarrow \quad c = \frac{1}{\sqrt{n(1-n)}} - 1.$$

$$H_0 = - \sum_m t_m \sum_{<ij>, \sigma} O_{im\sigma}^\dagger O_{jm\sigma} (f_{im\sigma}^\dagger f_{jm\sigma} + h.c) + \sum_{i,m\sigma} (\epsilon_m - \mu) f_{im\sigma}^\dagger f_{im\sigma}$$

$$\frac{U}{2} \sum_i \left(\sum_{m,\sigma} S_{im\sigma}^z \right)^2 + \frac{U'}{2} \sum_i \left(\sum_{m,\sigma} S_{im\sigma}^z \right)^2 + J \sum_{i,m} \left(\sum_{\sigma} S_{im\sigma}^z \right)^2 - \frac{J}{2} \sum_{i,\sigma} \left(\sum_m S_{im\sigma}^z \right)^2$$

$$H_{int}[\{\vec{S}_{im\sigma}\}]$$

de Medici et al, PRB 72, 205124 (2005)
 Hassan & de Medici, PRB 81, 035106 (2010)

Z_2 - Slave spin technique

- Constraint treated on average with static and site independent Lagrange multiplier λ_m . Spin variables and auxiliary fermions are decoupled

$$H_{eff}^f = - \sum_m t_m^{eff} \sum_{\langle ij \rangle, \sigma} (f_{im\sigma}^\dagger f_{jm\sigma} + h.c.)$$

$$+ \sum_{i,m\sigma} (\epsilon_m - \mu - \lambda_m) f_{im\sigma}^\dagger f_{im\sigma}$$

$$H_{eff}^S = - \sum_m J_m^{eff} \sum_{\langle ij \rangle, \sigma} O_{im\sigma}^\dagger O_{jm\sigma}$$

$$+ \sum_{i,m\sigma} \lambda_m (S_{im\sigma}^z + \frac{1}{2}) + H_{int}[\{\vec{S}_{im\sigma}\}]$$

$$t_m^{eff} = t_m \langle O_{im\sigma}^\dagger O_{jm\sigma} \rangle$$

$$J_m^{eff} = t_m \langle f_{im\sigma}^\dagger f_{jm\sigma} + f_{jm\sigma}^\dagger f_{im\sigma} \rangle$$

de Medici et al, PRB 72, 205124 (2005)
Hassan & de Medici, PRB 81, 035106 (2010)

Z_2 - Slave spin technique

- Spin hamiltonian treated at a single site mean field level

$$H_{eff}^f = \sum_{\mathbf{k}, m\sigma} (Z_m \epsilon_{\mathbf{k}m} + \epsilon_m - \mu - \lambda_m) f_{\mathbf{k}m\sigma}^\dagger f_{\mathbf{k}m\sigma}$$

$$H_s = \sum_{m\sigma} h_m O_{m\sigma}^\dagger + \sum_{m\sigma} \lambda_m (S_{m\sigma}^z + \frac{1}{2}) + H_{int}[\vec{S}_{m\sigma}]$$

$$h_m = \langle O_{im\sigma} \rangle \frac{1}{N} \sum_{\mathbf{k}} \epsilon_{\mathbf{k}m} \langle f_{\mathbf{k}m\sigma}^\dagger f_{\mathbf{k}m\sigma} \rangle$$

$$Z_m = \langle O_{im\sigma}^\dagger \rangle^2$$

- Solve self-consistently both coupled equations to calculate λ_m , h_m , Z_m

de Medici et al, PRB 72, 205124 (2005)
Hassan & de Medici, PRB 81, 035106 (2010)